Chapter 4. Benefits of Atomic Clock Augmentation

4.1 Background

Most GPS receivers use inexpensive quartz oscillators as a time reference. The receiver has a clock bias from GPS time but this bias is removed by treating it as an unknown when solving for position. In effect, the receiver clock is continually calibrated to GPS time. If a highly stable reference were used, however, the receiver time could be based on this clock without solving for a bias. "Clock coasting" (with no clock model), as it is referred to, requires an atomic clock with superior long term stability. The analysis in this chapter shows the potential for clock coasting to improve vertical positioning accuracy.

4.2 Atomic Clock Benefits

Currently, GPS receivers need four measurements to solve for three-dimensional position and time. The receiver clocks are not synchronized with GPS time which necessitates the fourth measurement. Using an atomic clock synchronized to GPS time in the receiver would eliminate the need for one of the measurements. Clock coasting has been shown to provide a navigation solution during periods which otherwise might be declared GPS outages [Sturza, 1983], [Knable & Kalafus, 1984]. Misra (May 1994) has proposed a clock model in order to make the receiver clock available as a measurement continuously. Although five or more satellites are visible at any time with the current 24-satellite constellation, availability can be compromised by satellite failures. Therefore, the negative impact of satellite failures could be reduced by atomic clock
augmentation. Redundant oscillators could be used in the receiver to lower the probability of a receiver clock failure. Thus, availability of GPS positioning would be increased.

van Graas (1994) has noted that adding an atomic clock improves availability more than adding three GPS satellites or a geostationary satellite. Also, a perfect clock helps more than including the altimeter as a measurement. This is a direct consequence of removing the clock bias terms from Eq. 2.11. The new equations decrease the correlation between user states, which has tremendous benefit in the vertical direction as will be shown later in this chapter. Reducing VDOP is crucial to increasing the availability of landing with GPS in low visibility conditions. If VDOP is greater than some nominal number, the landing system may be declared unavailable. Thus, using an atomic clock increases the availability of DGPS autolanding by reducing VDOP.

GPS currently does not provide sufficient integrity information for stand-alone receivers. While satellites may be marked unhealthy in the GPS navigation message, a timely alarm in the event of a satellite failure is desirable. Therefore, integrity must be monitored by receiver software, although for DGPS applications the ground station can provide sufficient integrity monitoring. A typical stand-alone integrity monitor relies on redundant measurements in order to check for inconsistencies [Kline, 1991]. Thus, having an atomic clock available as a measurement increases the availability of the integrity monitoring function. Further information on clock-aided integrity monitoring can be found in [Misra, November 1994], [McBurney & Brown, 1988], and [Lee, 1993].
4.3 Atomic Clock Stability

The stability of a clock is often presented in terms of Allan variance. From [Allan et. al., 1974]:

\[
\sigma^2_A(\tau) = \left\{ \frac{[\phi(t+2\tau) - 2\phi(t+\tau) + \phi(t)]^2}{2\tau\omega_0} \right\}
\]  

(4.1)

where:  
\(\sigma^2_A(\tau)\) is the Allan variance  
\(\tau\) is the averaging time (s)  
\(\phi(t)\) is the clock signal phase at time \(t\) (rad)  
\(\omega_0\) is the natural frequency of the source (rad/s)  
\(< >\) signifies an infinite time average

The Allan variance is approximated using a series of samples because an infinite time average is not practical. Using the definition:

\[
\bar{y}_k = \frac{\phi(t_k + \tau) - \phi(t_k)}{2\omega_0\tau}
\]

(4.2)

we write the Allan variance based on \(N\) samples:

\[
\sigma^2_A(\tau) = \frac{1}{2(N-1)} \sum_{k=1}^{N-1} (\bar{y}_{k+1} - \bar{y}_k)^2
\]

(4.3)

Clock stability is usually expressed in terms of the deviation, \(\sigma_A(\tau)\).
The most common oscillators available are quartz, rubidium cell, cesium beam, and hydrogen maser. Table 4.1 shows the typical stability $\sigma_4(\tau)$ that can be expected from each oscillator [Knable & Kalafus, 1984]. Quartz oscillators show an Allan deviation of $10^{-12}$ over short periods of about one second to one minute. This is comparable to a cesium beam and better than a rubidium cell over short periods. In the longer term, for instance a day or a month, quartz oscillators perform much worse than atomic standards. The stability of a hydrogen maser is about an order of magnitude better than the cesium beam for periods of up to one day.

For GPS receivers, a rubidium standard is attractive due to the combination of stability and portability. However, development of a miniaturized cesium cell has been underway at Westinghouse. Also, hydrogen masers are becoming less bulky and may be practical for airborne use at some point in the future.
<table>
<thead>
<tr>
<th></th>
<th>$\tau = 1$ sec</th>
<th>$\tau = 1$ day</th>
<th>$\tau = 1$ month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>$10^{-12}$</td>
<td>$10^9$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Rubidium</td>
<td>$10^{-11}$</td>
<td>$10^{-12}/10^{-13}$</td>
<td>$10^{-11}/10^{-12}$</td>
</tr>
<tr>
<td>Cesium Beam</td>
<td>$10^{10}/10^{-11}$</td>
<td>$10^{-13}/10^{-14}$</td>
<td>$10^{-13}/10^{-14}$</td>
</tr>
<tr>
<td>Hydrogen Maser</td>
<td>$10^{-13}$</td>
<td>$10^{-14}/10^{-15}$</td>
<td>$10^{-13}$</td>
</tr>
</tbody>
</table>
4.4 Reducing VDOP with a Perfect Clock

The Achilles’ heel of GPS is the vertical accuracy or lack thereof. As mentioned in Chapter 3, SPS users can expect vertical position error to be about 50% worse than horizontal position error. A similar ratio exists in DGPS, even though the magnitude of the position errors is decreased. An improvement in vertical accuracy would have a positive impact, particularly in applications involving final approach and landing of aircraft.

A GPS solution can be obtained using four or more measurements as shown in Appendix A. If a highly stable oscillator is incorporated, then the receiver clock can be integrated over some interval to yield what can be considered a constant offset from GPS time. Misra et. al., 1994, have proposed clock estimation using a polynomial fit after observing the clock for some time, perhaps half an hour. The receiver could coast on the atomic clock, using it as a measurement, to yield a clock-aided GPS solution. Thus, the geometry matrix becomes:

\[
H = \begin{bmatrix}
\hat{X} - X_1 & \hat{Y} - Y_1 & \hat{Z} - Z_1 \\
\hat{R}_1 & \hat{R}_1 & \hat{R}_1 \\
\hat{X} - X_2 & \hat{Y} - Y_2 & \hat{Z} - Z_2 \\
\hat{R}_2 & \hat{R}_2 & \hat{R}_2 \\
\hat{X} - X_3 & \hat{Y} - Y_3 & \hat{Z} - Z_3 \\
\hat{R}_3 & \hat{R}_3 & \hat{R}_3 
\end{bmatrix}
\] (4.4)

where three measurements are now required to solve for X, Y, and Z position. Letting

\[ G = (H^TH)^{-1} \] as in Eq. 3.9, the covariance matrix for the position errors is now:
\[
\begin{bmatrix}
\sigma_x^2 & \text{cov}(X,Y) & \text{cov}(X,Z) \\
\text{cov}(Y,X) & \sigma_y^2 & \text{cov}(Y,Z) \\
\text{cov}(Z,X) & \text{cov}(Z,Y) & \sigma_z^2
\end{bmatrix} = \begin{bmatrix}
G_{xx} & G_{xy} & G_{xz} \\
G_{yx} & G_{yy} & G_{yz} \\
G_{zx} & G_{zy} & G_{zz}
\end{bmatrix} \sigma_r^2
\] (4.5)

where: \( \sigma_r^2 \) is the variance of the range measurement errors
\( \sigma_x^2, \sigma_y^2, \sigma_z^2 \) are variances of the position errors

Thus, we calculate the horizontal dilution of precision (HDOP) as:

\[
\text{HDOP} = \frac{\sqrt{\sigma_x^2 + \sigma_y^2}}{\sigma_r} = \sqrt{G_{xx} + G_{yy}}
\] (4.6)

and the vertical dilution of precision (VDOP) is given by:

\[
\text{VDOP} = \frac{\sigma_z}{\sigma_r} = \sqrt{G_{zz}}
\] (4.7)

These values are smaller with clock aiding because the covariance matrix now has no dependence on the clock bias state. In particular, the VDOP term is reduced because of the strong correlation between the vertical axis and the clock state. Misra (May 1994) explains this phenomenon by the fact that a clock bias is very similar to moving the antenna along the vertical axis in terms of impact on the pseudorange measurements. A clock bias adds or subtracts the same amount from each pseudorange, while moving the antenna vertically changes each
pseudorange in the same direction although not equally. Hence, there is a strong correlation between the z and t states.

It is important to note that this reduction in VDOP represents only the potential improvement that can be achieved through clock aiding. If the clock is not synchronized with GPS time, another error term is included that adds to the vertical error of the clock-aided solution. In other words, a clock error that affects each measured pseudorange would be translated into a position error via the geometry matrix. Thus, the vertical error is:

\[
\text{vert. error} = VDOP_{\text{clock}} \sigma_r + (\text{geometry term}) \cdot (\text{clock bias})
\]

where: \( VDOP_{\text{clock}} \) is VDOP assuming perfect clock

(\text{geometry term}) relates the clock bias to a vertical position error

Here, (\text{geometry term}) is not the same as VDOP or \( VDOP_{\text{clock}} \), but is a multiplier based on the navigation solution and translates the clock bias into a vertical position error. This shows the importance of synchronization in clock-aided navigation.

To illustrate the VDOP improvement with a perfect clock, a GPS simulation was conducted in which HDOP and VDOP were calculated for Roanoke, VA over a one day period. Two scenarios were considered, one with a perfect receiver clock and one without. The horizontal dilution of precision (HDOP) and the vertical dilution of precision (VDOP) were calculated using all satellites in view. Three simulations were run using satellite elevation mask
angles of 5°, 10°, and 15°. HDOP and VDOP were calculated every 30 s over a simulation run time of one day. The GPS 24 Optimized constellation was used [Martinez, 1993].

The number of satellites in view for the 5° mask angle case is plotted in Fig. 4.1. Plots of HDOP and VDOP are shown in Figs. 4.2-4.3 with and without clock aiding. The number of satellites in view varies between 5 and 11 with an average of 7-8 satellites visible during the simulation. While there is some improvement in the horizontal accuracy (Fig. 4.2), the most striking benefit is the VDOP improvement (Fig. 4.3). The maximum VDOP is 3.35 without the clock. With the clock the worst VDOP is 0.69. The HDOP is somewhat improved with a maximum HDOP of 2.23 without the perfect clock; using a perfect clock the worst HDOP is 1.38.

Similar comparisons can be made for the cases of 10° elevation mask (Figs. 4.4-4.6) and 15° elevation mask (Figs. 4.7-4.9). Such elevation mask angles can be experienced if an aircraft is banking, but this could be thought of as a simulation of satellite failures as well. Also, a ground station in a DGPS system may eliminate a satellite due to high multipath. van Graas (1995) noted that low elevation satellites exhibit the worst multipath because the satellite has a tendency to linger in approximately the same spot in the sky. Thus, a low elevation satellite can produce slow-changing multipath errors unlike higher elevation satellites, whose direction cosines relative to the ground station change more rapidly.
### Table 4.2 Worst Case DOPs for Roanoke Simulation

<table>
<thead>
<tr>
<th>Elevation Mask</th>
<th>Without Clock</th>
<th>With Perfect Clock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max HDOP</td>
<td>Max VDOP</td>
</tr>
<tr>
<td>5°</td>
<td>2.23</td>
<td>3.35</td>
</tr>
<tr>
<td>10°</td>
<td>158.09</td>
<td>542.89</td>
</tr>
<tr>
<td>15°</td>
<td>158.09</td>
<td>542.89</td>
</tr>
</tbody>
</table>

### Table 4.3 Average DOPs for Roanoke Simulation

<table>
<thead>
<tr>
<th>Elevation Mask</th>
<th>Without Clock</th>
<th>With Perfect Clock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. HDOP</td>
<td>Avg. VDOP</td>
</tr>
<tr>
<td>5°</td>
<td>1.06</td>
<td>1.57</td>
</tr>
<tr>
<td>10°</td>
<td>1.39</td>
<td>2.63</td>
</tr>
<tr>
<td>15°</td>
<td>1.64</td>
<td>3.34</td>
</tr>
</tbody>
</table>
Tables 4.2-4.3 show worst case and average DOP values for all the scenarios considered. The important result is that the VDOP is 0.77 or better for all cases when a perfect clock is used. The large maximum values of HDOP (158.1) and VDOP (542.9) shown in Table 4.2 correspond to cases where only four satellites are in view as seen in Figs. 4.4 and 4.7. Here we have a case where the geometry "blows up" resulting an large DOPs, although not all four-satellite geometries are this poor. However, it is important to note the trend that geometry gets worse as satellites are removed. Thus, poor geometry could be experienced if one or more satellites fail. This also brings up the issue of constellation management. Currently, GPS is operational with 24 satellites in six orbital planes. The U.S. Department of Defense will maintain a 24-satellite constellation by launching new satellites when necessary to replace faulty satellites [Federal Radionavigation Plan, 1994]. The Federal Radionavigation Plan states that the availability of the standard positioning service is 99.16%, which means that there will be times when poor geometry is experienced. Unexpected satellite failures would also cause degraded coverage as well. The scenarios presented in this chapter are intended to show the effect of having fewer satellites available, particularly in terms of increased HDOP and VDOP.
Figure 4.1     Number of Satellites Visible at Roanoke During 24 Hour GPS Simulation (5° Elevation Mask)
Figure 4.2 HDOP at Roanoke During 24 Hour GPS Simulation (5° Elevation Mask)
Figure 4.3 VDOP at Roanoke During 24 Hour GPS Simulation (5° Elevation Mask)
Figure 4.4  Number of Visible Satellites at Roanoke During 24 Hour GPS Simulation (10° Elevation Mask)
Figure 4.5 HDOP at Roanoke During 24 Hour GPS Simulation (10° Elevation Mask)
Figure 4.6  VDOP at Roanoke During 24 Hour GPS Simulation (10° Elevation Mask)
Figure 4.7  Number of Visible Satellites at Roanoke During 24 Hour GPS Simulation (15° Elevation Mask)
Figure 4.8 HDOP at Roanoke During 24 Hour GPS Simulation (15° Elevation Mask)
Figure 4.9 VDOP at Roanoke During 24 Hour GPS Simulation (15° Elevation Mask)
4.5 Conclusions From Roanoke Simulation

One of the intriguing elements of this simulation is the fact that the average VDOP remained approximately constant for each case with a perfect clock. This shows that investigating atomic clock augmentation of GPS receivers is a worthwhile task. It is important to note, however, that these results only indicate the potential that can be achieved with a perfect clock. The vertical accuracy improvement would be less if the receiver clock were not synchronized with GPS time. Chapters 5, 6, and 7 examine the sources of error that can cause the receiver clock to drift.