THE EXPERIMENTAL TESTING OF AN ACTIVE MAGNETIC BEARING/ROTOR SYSTEM UNDERGOING BASE EXCITATION

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(ABSTRACT)

Active Magnetic Bearings (AMB) are a relatively recent innovation in bearing technology. Unlike conventional bearings, which rely on mechanical forces originating from fluid films or physical contact to support bearing loads, AMB systems utilize magnetic fields to levitate and support a shaft in an air-gap within the bearing stator. This design has many benefits over conventional bearings. The potential capabilities that AMB systems offer are allowing this new technology to be considered for use in state-of-the-art applications. For example, AMB systems are being considered for use in jet engines, submarine propulsion systems, energy storage flywheels, hybrid electric vehicles and a multitude of high performance space applications. Many of the benefits that AMB systems have over conventional bearings makes them ideal for use in these types of vehicular applications. However, these applications present a greater challenge to the AMB system designer because the AMB-rotor system may be subjected to external vibrations originating from the vehicle’s motion and operation. Therefore these AMB systems must be designed to handle the aggregate vibration of both the internal rotor dynamic vibrations and the external vibrations that these applications will produce.

This paper will focus on the effects of direct base excitation to an AMB/rotor system because base excitation is highly possible to occur in vehicular applications. This type of excitation has been known to de-stabilize AMB/rotor systems therefore this aspect of AMB system operation needs to be examined. The goal of this research was to design, build and test a test rig that has the ability to excite an AMB system with large amplitude base excitation. Results obtained from this test rig will be compared to predictions obtained from linear models commonly used for AMB analysis and determine the limits of these models.
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CHAPTER 1
Project Introduction

1.1 Introduction

Active Magnetic Bearings (AMB) are a relatively recent innovation in bearing technology. Unlike conventional bearings, which rely on mechanical forces originating from fluid films or physical contact to support bearing loads, AMB systems utilize magnetic fields to levitate and support a shaft in an air-gap within the bearing stator. This design has many benefits over conventional bearings. The non-contacting support of the shaft eliminates losses in the bearings due to friction, increases the life of the bearing due to the lack of wear, and reduces power consumption through the elimination of elaborate oil supply systems. In addition, the AMB systems are actively controlled allowing for the characteristics of the bearing to be manipulated by changing the control system parameters. This gives these bearings a level of versatility not readily available with conventional bearings.

The potential capabilities that AMB systems offer are allowing this new technology to be considered for use in state-of-the-art applications. For example, AMB systems are being considered for use in jet engines, submarine propulsion systems, energy storage flywheels, hybrid electric vehicles and a multitude of high performance space applications. Many of the benefits that AMB systems have over conventional bearings makes them ideal for use in these types of vehicular applications. However, these applications present a greater challenge to the AMB system designer because the AMB-rotor system may be subjected to external vibrations originating from the vehicle’s motion and operation. Therefore these AMB systems must be designed to handle the aggregate vibration of both the internal rotor dynamic vibrations and the external vibrations that these applications will produce.

Normally base excitation problems are not considered when designing AMB rotor systems. However these transmitted forces can be significant depending on the characteristics of the bearing and need to be considered. Only a limited amount of work has been completed involving AMB systems and base excitation. Cole, Keogh, and Burrows have written several papers detailing their work testing both classical proportional, integral and derivative (PID) control and modern state space controllers on a rotor/bearing system undergoing both direct rotor forcing and base motion [1,2,3]. The purpose of their work was to determine the best suited control method for use with a system undergoing both direct rotor
forcing and base excitation. The tests were performed on a rotor/bearing system consisting of two radial bearings, a shaft and four disks. The base-plate of the system was excited using a modal hammer to give a multiple frequency impact in the horizontal direction with a frequency content up to 27 Hz. The initial results of these studies indicate that the classical PID controller is the least suited for minimizing the effects of base excitation on the rotor/bearing system and the direct rotor forcing. These two types of excitation represent tradeoffs in the controller designs. To reduce the effects of the base excitation, low force transmissibility between the bearing and rotor is desired. However, this force transmissibility is the mechanism the bearing actuator uses to control the vibration levels resulting from direct forcing. Therefore, the best-suited controller design was concluded to be an $H_{\infty}$ type controller. This controller allows for the designer to use cost functions to weight the two sources of excitation and create an optimized controller best suited for handling both types of forcing.

Unfortunately, these papers only indirectly addressed the modeling of the base excitation on the rotor/bearing system. No comparison between modeled response due to base excitation and the actual response measured was made. Therefore, no assurance of the accuracy of the model used to simulate the plant within the control loop was given. Some of the results obtained may be a function of the sensitivity of the different control schemes to inaccuracies in the plant model. Also, the base excitation multiple frequency input was only able to excite the bearing at frequencies up to 27 Hz. This represents the lower end of the bandwidth associated with these controllers. It is also possible for AMB systems located in vehicular applications to be excited by base excitations of higher frequencies.

Hawkins [4] also investigated the effects of base excitation on a rotor supported by AMB’s. This paper outlines the non-linear analysis of an entire gas turbine system undergoing a shock test to the base of the system. The entire gas turbine was modeled and the response due to an impact that would be given by a U.S. Navy medium weight shock machine was simulated. No experimental data was taken to verify the results of this simulation and no insight into the propagation of the base excitation to the rotor was given. The results of the simulation indicated that the shock loading would be significant enough to cause collision with the retaining rings (backup bearings) on the system and would cause the rotor to become unstable for brief periods (\(\cong 25\) milliseconds) of time. The simulation indicated that after these brief periods the shaft would recover and stable levitation would reoccur.

Jagadeesh et al. [5] have described the design of a test rig that can be used to develop control schemes to control the relative vibration levels between the bearing mass and the shaft. These gentlemen have plans to build a test rig very similar to the test rig described in this paper. Two opposing electromagnets set in
the vertical plane will be used to simulate a single axis of control in a radial magnetic bearing. The electromagnets will be supported by a mass held in linear bearings which will confine the motion to the vertical direction. Similarly, a shaft mounted on separate linear bearings will be used as the rotor. The use of the linear bearings and two opposing electromagnets will create a single degree of freedom system and should provide insight into the linearity, control and base excitation of a electromagnetic actuator. However, this test rig does not represent a full two degree of freedom radial magnetic bearing due to the lack of a second opposing pair of electromagnets. Their research had not been completed at the time of this research was being conducted however the results from their work should be comparable with the results discussed in this paper.

In addition to the work that has taken place with respect to base excitation, there has been a number of studies completed which investigate the general dynamic behavior of AMB systems. Rao et al. [6] performed an analytical study of the effects of different static loads on AMB’s and their effect on the bearing’s stiffness characteristic. In general, the higher the static load, the less stiff the bearing becomes due to the decrease in available control currents. Rao points out that the actual bias currents in the top and bottom electromagnets of a radial AMB will be different to support the weight of the shaft. This is an important result that must be taken into account when creating dynamic models of the AMB systems. Rao gives a method for calculating the usable gap within the bearing for certain configurations before the rotor/system becomes marginally stable.

Knight et al. [7] has investigated the cross coupling potential of electromagnets which can also play an important role in the proper modeling of an AMB system. The results of this research obtained from both analytical and experimental work suggest that radial AMB systems could have significant cross coupling between the separate axes. This is an important result because cross coupling between the bearing axes is commonly not accounted for in AMB system modeling.

Kirk et al. [8] have performed numerous tests examining the dynamics of AMB/rotor systems undergoing power loss and subsequently dropping onto the back up bearings. This is a very important aspect of AMB operation that is not addressed in this paper. The fact that AMB systems must have backup bearings to safeguard against potential power losses is one of the most improtant limitations facing AMB systems today. The rotor drops that can occur during a power loss to AMB supporting operating rotors can lead to large transient vibration levels and/or unstable rotor vibrations as the rotor coasts down. Obviously, this can be completely destructive to the machinery in which the AMB system operates. Kirk et al. have laid a ground work for modeling the potential transient dynamics of rotor drop situation with such machinery.
His paper, “Evaluation of AMB Rotor Drop Stability”[8], investigated a rotor undergoing a rotor drop onto a rigid and soft mounted bronze bushings given different unbalance and speed conditions. The paper provides valuable insight to AMB system developers on how to design and operate AMB/rotor systems to avoid potentially destructive rotor drop events.

My research will focus on the effects of direct base excitation to a AMB/rotor system. This type of system excitation is highly possible to occur in vehicular applications. This type of excitation has been known to de-stabilize bearing rotor systems therefore this aspect of AMB system operation needs to be examined. The goal of my research was to design, build and test a test rig that has the ability to excite an AMB system with large amplitude base excitation. Results obtained from this test rig will be compared to predictions obtained from linear models commonly used for AMB analysis and determine the limits of these models. The remainder of this document details the model development, the experimental test rig design, and the experimental results and comparison with the model predictions.

1.2 Scope of Work

The purpose of my work is to take an initial look at the characterization of radial AMB systems undergoing base excitation. To simplify the analysis as much as possible, testing and modeling has been performed on a single radial AMB and a rigid non-rotating shaft in order to gain an understanding of the basic mechanisms at work. A linear model of this rotor-bearing system undergoing base excitation has been developed based on common assumptions used in AMB analysis, which will be described in more detail in Chapter Three. Under some circumstances it may be possible that base motion of the AMB system may be of large enough amplitudes to invalidate some bearing models which are based on the assumption of a linear system. The purpose of the testing performed was to simply validate or refute the use of the common assumptions used to linearize the model of the system undergoing base excitation. Therefore, the scope of this project included the development of this linearized model of a single AMB undergoing base excitation, the design and building of an experimental test rig, testing of a 2-axis, 8-pole radial magnetic bearing and stub shaft undergoing base excitation, and the comparison of the model results with experimental results. This study also lays the groundwork for future non-linear modeling and analysis of the rotor-bearing system which is considered beyond the scope of this project.
1.3 Active Radial Magnetic Bearing (AMB) Introduction

AMB systems consist of five basic components, which include the actuator, sensor, controller, amplifier, and rotor. Figure 1.1 gives a simplified schematic of these basic components as they are used in one axis of a radial AMB system.

![Active Magnetic Bearing Basic Layout](image)

**Figure 1.1:** Active Magnetic Bearing Basic Layout

The AMB system shown in Figure 1.1 works by detecting the shaft position using a position sensor. As a result, this sensor measures the relative displacement between the bearing stator and the rotor. This position signal is then sent to the controller, which adjusts the current level to the actuators through the use of a current amplifier. The change in current within the actuator coils will either increase or decrease the force placed on the rotor based on this change, which adjusts the position of the rotor within the stator.

The bearing actuators are electromagnets consisting of coils of wire wrapped around a ferromagnetic material. The force capacity of these electromagnets can be calculated based on simplified magnetic circuit analysis. Figure 1.2 shows a simplified horseshoe magnet style actuator similar to that used in radial magnetic bearings.
Currents in the wire coils induce a magnetic field within the actuator material, across both air gaps between the actuator and target (rotor), and in the rotor as shown in Figure 1.2. The magnetic field across the air gaps results in an attractive force on the rotor. This force is dependent on the geometry of the magnet, the current in the wires, and the distance between the stator and rotor. The magnetic force applied to the rotor has been derived from basic electromagnetic circuit theory [9] and is given by

\[
F = \frac{\varepsilon \mu_0 A_g N^2 I^2}{4g^2} \quad [N],
\]  

where \(\mu_0\) is the permeability of free space (\(4\pi \times 10^{-7}\) T-m/Amp), \(A_g\) is a single pole face area (m\(^2\)), \(N\) is the total number of wire coils, \(g\) is the gap distance (m) and \(I\) is the current (Amps). A dimensionless loss factor, \(\varepsilon\), is also included to account for magnetic field fringing and leakage effects along with geometric losses due to curvature.

From Equation 1.1 it is shown that the force is inversely proportional to the gap distance squared, indicating that as the rotor gets closer to the stator the stator pulls harder on the rotor. This is an important observation related to magnetic bearings. The forces in a magnetic bearing are always attractive, therefore passive magnetic bearings are inherently unstable. As a result, a control system must be used to stabilize the system. The control system accomplishes this by counteracting the potential increase in the attractive force due to the gap becoming smaller by decreasing the current supplied to the magnet. In addition, in most AMB systems, a second magnet is placed opposite the first to provide a
force input to the shaft in the opposing direction. In general the controllers will simply supply equal and opposite currents to these magnet pairs to provide the stabilizing force.

An example of a common 8-pole radial magnetic bearing is shown in Figure 1.3.

Figure 1.3: MBRetro™ 8-Pole Radial Active Magnetic Bearing

1.4 Single Degree of Freedom Base Excitation Model

Figure 1.4 gives a schematic of a single degree of freedom mass-spring-damper system.
The equation of motion describing this system undergoing base excitation is given by

\[ M\ddot{x} + C\dot{x} + Kx = Cy + Ky, \quad (\text{Eq. 1.2}) \]

where \( M \) is the Mass, \( C \) is the damping coefficient, \( K \) is the stiffness coefficient, and \( x \) and \( y \) represent the mass and the base displacement, respectively. This system is analogous to a single control axis within an AMB actuator with no cross coupling.

Equation 1.2 is the standard form of the equation for a single degree of freedom base excitation system. Notice that the left-hand side of the equation exactly describes the system regardless of whether it is undergoing base excitation or some other forcing. The extension to base excitation is reflected in the right-hand side of the equation or the input force to the system. In the case of base excitation, the driving force to the system is transmitted through the bearing’s stiffness and damping as shown in equation 1.2 and is proportional to the amplitude of base displacement and velocity. This equation can be used to model any of the forms of base excitation that could occur, which include multiple frequency shock loading and steady single frequency loads. This equation forms the basis for the common modeling which is used to analyze the response of AMB systems. Chapter Three will describe in more detail the extension of this equation to give the commonly used model of a two-degree of freedom radial AMB system.
1.5 Test Rig Conceptual Design

The test rig was designed to investigate the effects of base excitation on the shaft displacement within the bearing, specifically how the base excitation is transferred to the shaft through the bearing stiffness and damping properties. For simplicity, a single input system was designed so that the effects of base excitation could be easily detected and recorded. This led to the use of a single AMB to ensure only one base excitation input was encountered and to avoid complicated outputs due to the interaction of the shaft with a second AMB or due to shaft rotation. This then required the use of an alternative support structure for the shaft to take the place of a second bearing. A wire support system was used to provide moment control and axial support to the shaft while allowing it to vibrate freely in the vertical and horizontal planes due to the low transverse stiffness of the wires. The shaft was designed to be rigid within the operating frequency range of the electromagnetic shaker used as the base excitation source. This prevented lateral bending modes of the shaft from being excited and adding to the measured response of the system.

Figure 1.5 gives a conceptual design drawing of the test rig further describing these major components.

![Figure 1.5: Conceptual Test Rig Design](image)

Figure 1.5 shows how an electromagnetic shaker is used to excite the AMB in the vertical direction. These vibrations will be transmitted through the bearing stator and magnetic field to the shaft. The rigid stub shaft is supported in the axial direction with music wire and allowed to vibrate freely in the vertical
and horizontal directions. The details of the design and the individual components used to build the test rig based on this conceptual design are presented in Chapter Two.

This test rig has been used to obtain experimental data which represents the response of the AMB system to various forms of base excitation. These results have been compared to the results predicted by the commonly used linear model for these types of AMB systems, which is outlined in Chapter Three. The remainder of this work details the test rig design, AMB system modeling, the experimental results obtained and the comparison of these results with the models predictions.
CHAPTER 2
TEST RIG DESIGN AND COMPONENTS

2.1 Design Overview

The test rig was designed to analyze the performance of a radial magnetic bearing system undergoing base excitation. This rig consists of a single radial AMB system, a stub shaft assembly to act as the rotor, wires for moment control and an electromagnetic shaker to provide the base excitation to the bearing housing. A layout drawing of the test rig is given in Figure 2.1 with the major components of the test rig labeled.

Figure 2.1: CAD Layout View of Base Excitation Test Rig

Figure 2.1 shows how the radial AMB has been mounted directly to the shaker via a mounting plate created to provide the appropriate bolt hole locations. A single radial AMB and electromagnetic shaker have been used to isolate the characteristics of the bearing and its dynamics. In an actual system a second bearing would be present that would contribute to the response of the shaft. A stub shaft assembly has been used to act as a rigid body and rotor within the AMB. This assembly consists of a short shaft and two square end plates used to give wire mounting locations and sensor target positions. The wires were used to supply moment control to the stub shaft assembly, which would normally be supplied by a second
bearing on the shaft. The wires were used due to their high axial stiffness that supports the stub shaft assembly in the axial direction. The wires also have a very low stiffness in the transverse directions, which allows the stub shaft assembly to vibrate freely in these directions and minimizes the effects of the wires on the dynamics of the system.

The purpose of the electromagnetic shaker is to excite the bearing housing in the vertical direction. The expected result is that the stub shaft assembly will respond only in the vertical direction given a balanced stub shaft assembly. Therefore the test rig was designed to investigate the single degree of freedom base excitation of the AMB - stub shaft system. The shaker is driven by a signal generator that provides a voltage signal proportional to the force desired from the shaker. Therefore it was possible to excite the bearing with both single frequency excitation and multiple frequency excitation. When the bearing housing was excited, eddy current position probes were used to detect the motion of both the stub shaft assembly and the bearing housing. The position probes were mounted to a separate structure called the sensor mount that is shown in Figure 2.1. In this figure the sensor mount is shown transparent to reveal the details of the AMB and shaker assembly. This mounting structure was made of 10 gauge steel and mounted separately to the concrete base foundation to avoid motion of the structure while the test rig was operated. A total of nine sensors were used to fully characterize the motion of the stub shaft assembly and the bearing housing.

Figure 2.2 gives a picture of the actual test rig with the sensor mount removed to reveal the bearing shaker interface. The remainder of this chapter is dedicated to describing the major components of the test rig design in detail and verifying their properties. Appendix A includes the design drawings that were used to construct and setup the test rig.
2.2 MBRetro™ Kit

A single bearing from an MBRetro™ kit from Revolve Magnetic Bearings Incorporated was used as the AMB for this test rig. The MBRetro™ kit is designed for retrofitting small rotor kits having a shaft size of 3/8 inch in diameter. The kit includes two radial magnetic bearings, the MB350™ 5-axis digital controller, MBscope™ software, and the MBresearch™ BNC breakout box. Figure 2.3 below gives an overall equipment layout of the MBretro™ kit.

![MBRetro Kit Layout](image)

**Figure 2.3:** Equipment Layout for MBRetro™ Kit from Revolve Magnetic Bearings Incorporated

The MBRetro™ radial bearings each contain two controlled axes with a variable reluctance differential position probe dedicated to each axis. This is necessary because the bearings operate in a single plane and therefore must control the position of the shaft in two directions (2 degrees of freedom). The position sensors measure the position of the shaft in each axis and relays this information to the controller in the form of a voltage signal. The MB350™ digital controller consists of 5 separate single input single output controller channels. This allows the MB350™ to control two radial bearings (4 Axes) and a single thrust bearing (single axis). The test rig built to perform this base excitation research only utilizes two control axes due to the use of a single bearing.

In general the MB350™ controller receives position signals from the radial bearings and updates the currents in each axis based on these values. The controller parameters for each of the bearing axes are downloaded to the controller through the use of a personal computer and the MBscope™ software. This software also allows for on screen monitoring of the current and position signals from the bearings.
Revolve has also provided the MBResearch\textsuperscript{TM} BNC breakout box which allows for external monitoring of the current and position signals within the bearings.

### 2.2.1 MBRetro\textsuperscript{TM} Radial Magnetic Bearings

Each bearing is an 8 pole, 2-axis heteropolar radial bearing designed to support a 3/8 inch shaft as shown in Figure 2.4. The axes consist of two opposing horseshoe style electromagnets similar to that described in Chapter One. One of the axes is rotated 45° from the vertical plane and the other is rotated 45° from the horizontal plane so that each axis supports ½ the static weight of the shaft. This configuration allows each of the axes to contribute equally to the control of vibrations in the horizontal and vertical planes. A variable reluctance differential sensor is used to measure the position of the shaft in each axis and is located behind (axially) the bearing stator. The sensor has been placed as close as possible to the bearing stator to avoid collocation problems. However, because the stub shaft assembly design prevents bending modes, collocation between the stator and sensor should not be a problem. Figure 2.4 gives a picture of one of these bearings and depicts the axis layout discussed above.

![Figure 2.4: MBRetro\textsuperscript{TM} Radial Active Magnetic Bearing Axis Layout](image)

Figure 2.4 shows the two axes within the bearing labeled as W13 and V13, which is the convention utilized by Revolve. This notation is further broken down to describe each of the four electromagnets within the bearing. For example, W1 refers to the top magnet in the W13 axis while W3 refers to the bottom magnet of this axis. Figure 2.4 also shows that the backup bearing provided by Revolve has been removed. The backup bearing is simply a conventional roller bearing having a reduced radial clearance of ½ the AMB to provide support to the shaft during power shutdown of the AMB. This bearing has been
removed to allow for the maximum vibration levels within the stator during testing. Care was taken when removing power from the AMB so that no damage occurred to the AMB stator.

The geometric properties play an important role in the determination of the force applied to the shaft by each of the electromagnets within the AMB. The geometric properties for the electromagnets within the MBRetro™ were provided by Revolve and are given in table 2.1.

**Table 2.1: MBRetro Radial Bearing Geometric Properties**

<table>
<thead>
<tr>
<th>Description of Bearing Properties</th>
<th>Variable/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fringing, Leakage, &amp; Geometric Loss Factor</td>
<td>$\varepsilon = 0.826$</td>
</tr>
<tr>
<td>Number of turns per Actuator</td>
<td>$N = 228$ turns</td>
</tr>
<tr>
<td>Single Pole Face Area</td>
<td>$A_g = 0.1035$ in$^2$</td>
</tr>
<tr>
<td>Nominal Radial Gap Clearance (Stator ID/2 – Rotor OD/2)</td>
<td>$G_n = 0.015$ inch</td>
</tr>
<tr>
<td>Stator Outer Diameter</td>
<td>2.788 inch</td>
</tr>
<tr>
<td>Stator Inner Diameter – Bearing Clearance</td>
<td>1.380 inch</td>
</tr>
<tr>
<td>Stator Stack Length</td>
<td>0.500 inch</td>
</tr>
<tr>
<td>Rotor Lamination Inner Diameter</td>
<td>0.916 inch</td>
</tr>
<tr>
<td>Rotor Outer Diameter</td>
<td>1.350 inch</td>
</tr>
<tr>
<td>Pole Height</td>
<td>0.496 inch</td>
</tr>
<tr>
<td>Pole Centerline Angle</td>
<td>22.5°</td>
</tr>
<tr>
<td>Stator Lamination Thickness</td>
<td>0.014 inch</td>
</tr>
<tr>
<td>Rotor Lamination Thickness</td>
<td>0.005</td>
</tr>
<tr>
<td>Stator Material Grade</td>
<td>M-19,C-5</td>
</tr>
<tr>
<td>Rotor Material Grade</td>
<td>Arnon 5, C-5</td>
</tr>
<tr>
<td>Lamination Stacking Factor</td>
<td>96</td>
</tr>
<tr>
<td>Wire Coil Packing Factor</td>
<td>97</td>
</tr>
</tbody>
</table>

**2.2.2 MB350™ Controller**

The two axes, labeled V13 and W13 in Figure 2.4, each have a dedicated differential sensor measuring the displacement of the rotor within the axis. These position signals are sent to the MB350™ controller where each axis is controlled independently. The MB350™ controller has the capability to control 5 axes of motion. However, only two control axes are utilized for this test rig due to the use of a single MBRetro™ radial bearing. Each axis is controlled by a series combination of a PID, lead-lag, notch, and
low pass filters that determine the amount of control current to be sent to the actuators. Details of the controller parameters and how this system operates is discussed in detail in Chapter Three. The MB350™ controller is shown below in Figure 2.5.

![MB350™ controller](image)

**Figure 2.5:** MB350™ 5-axis digital Controller from Revolve Magnetic Bearings Incorporated (Picture Provided Courtesy Revolve Magnetic Bearings Incorporated)

### 2.2.3 MBScope™ software

The different filters discussed above within the MB350™ controller can be modified utilizing the MBscope™ software package provided by Revolve. This software package allows for the downloading of different filter parameters along with the monitoring of vital system signals. For example, the actuator currents and axis positions may be viewed using the MBscope™ software. MBScope™ is the interface between the controller and the user and has many functions that include sensor calibration, vibration monitoring and control, signal injection and orbit monitoring, which will not be discussed here. However, the functions vital to this test rig will be discussed as needed throughout this document.

### 2.2.4 MBResearch™ BNC Breakout Box

The MBResearch™ BNC breakout box is simply a series of hardwired BNC input and output jacks that allow the measurement of the internal signals of the system. These include the individual actuator currents and the position signals from each axis. The MBScope™ software allows for on-screen monitoring of these signals, however it does not allow for the recording of these signals. Therefore, the MBResearch™ BNC breakout box must be used in conjunction with a separate data acquisition system to capture the systems internal signals. The breakout box also allows for the injection of signals into the controller to mimic either position or current signals. This allows for testing certain aspects of the controller and the bearing along with verifying the controller transfer function.
2.3 Electro-Magnetic Shaker

A VG 100G Electro-magnetic shaker from Vibration Test Systems has been used as the source of base excitation for the test rig. The bearing has been mounted directly to the shaker through the use of an aluminum plate that provides the appropriate bolt hole pattern for mounting. The shaker is supported in a trunion base which allows the shaker to be rotated angularly and which also serves as a mounting fixture between the shaker and the concrete block base. Figure 2.6 shows a schematic of this setup with the bearing displaced angularly.

![Figure 2.6: Shaker and Bearing Mounting Setup](image)

Great care has been taken when setting up this subsystem to ensure the shaker was mounted in a level and purely vertical direction. This was done by utilizing the four adjustable mounts and the pivot bolt located on the trunion to level the bearing in all three planes. Leveling the Shaker and Bearing ensured that the shaker would excite the bearing vertically. Figure 2.2 shows the shaker and bearing after it has been setup and leveled.

This type of shaker has been used due to its ability to convert voltage signals to a proportional force input to the bearing housing. This allows for the use of a simple signal generator coupled with an amplifier to produce a variety of force input signals which can be used in testing the dynamics of the rotor-bearing system. For example, single frequency signals, random noise input, and various multiple frequency signals can all be achieved using the signal generator and shaker.

Since, this shaker is a force actuator, the usable frequency range in which appreciable displacements can be achieved is limited. This occurs because at higher frequencies larger forces are required to acquire the same displacements achieved at lower frequencies given the same mass. The shaker’s power limitation of
120 Watts further reduced the usable frequency range due to the limitation placed on the voltage signal that could be sent at higher frequencies. Prior to construction of the test rig, the shaker’s displacement capabilities were tested by mounting a similar mass on the shaker and recording the highest displacements obtainable at increasing frequencies without exceeding 120 Watts to the shaker. Above approximately 100 Hz the shaker could no longer produce vertical vibrations greater than 1 mil at full power. Also the shaker could not produce sufficient input forces below 5 Hz due to the static support structure within the shaker. Therefore the usable frequency range of approximately 5-100 Hz was determined by the limitation of the shaker performance. This was deemed an acceptable range for testing base excitation because in general base excitations are low frequency signals.

### 2.4 Stub Shaft Assembly

The Stub Shaft Assembly includes a 4-inch long, 3/8-inch diameter precision ground steel shaft, a rotor lamination stack, and two square Aluminum end plates for wire mounting and sensor targets. The lamination stack (radial rotor) provided by Revolve is mounted in the center of the stub shaft and serves as the target for the magnetic field produced by the bearing stator actuators. Two Aluminum square plates are bolted on either side of the stub shaft to allow for distributed wire attachments and to serve as sensor targets. Shaft flats on either end of the stub shaft have been used in conjunction with set-screws to align the two end plates. The plates also add mass to the assembly which help reduce the rigid body resonant frequency to within the testing frequency range. The total weight of the entire assembly is 2.0 lbf. Figure 2.7 shows the assembled stub shaft assembly.

![Figure 2.7: Stub shaft Assembly](image)
The first two modes of the Stub Shaft Assembly have been determined using the software program VTFast developed by the Rotor Dynamics Laboratory at Virginia Tech. This software has allowed for the modeling and prediction of the lateral rigid body and bending modes of the stub shaft assembly through the use of the critical speed program (CRTSPD.EXE). The frequency and mode shapes of the stub shaft assembly natural frequencies are dependent on the stiffness of the MBretro radial bearing, therefore three sets of mode shapes have been determined corresponding to the three controller parameter cases used for testing. These three controller gain cases represent a low stiffness and damping case referred to as LKLC, a medium stiffness and high damping case referred to as LKHC and a high stiffness and medium damping case referred to as HKHC. The determination of the PID gains that make up each of these cases is discussed in Chapter Three. An average stiffness over 0 – 100 Hz was used to represent each of these cases to allow calculation of the mode shapes. This was done because the stiffness and damping characteristics of these bearings is frequency dependent. A detailed discussion is presented in Appendix B of the modeling of the stub shaft assembly and the mode shape plots. Table 2.2 summarizes the results from this analysis by giving the frequencies of the first two mode shapes which are the rigid body translational mode and the first bending mode respectively.

<table>
<thead>
<tr>
<th>Controller Case</th>
<th>Avg. Bearing Stiffness</th>
<th>Rigid Translational Mode</th>
<th>1st Bending Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKLC</td>
<td>219.0 lb/in.</td>
<td>31.3 Hz</td>
<td>757.1 Hz</td>
</tr>
<tr>
<td>LKHC</td>
<td>237.0 lb/in.</td>
<td>32.5 Hz</td>
<td>757.2 Hz</td>
</tr>
<tr>
<td>HKHC</td>
<td>460.0 lb/in.</td>
<td>45.2 Hz</td>
<td>758.6 Hz</td>
</tr>
</tbody>
</table>

The results shown in Table 2.2 verify that the design of the stub shaft assembly should only act as a rigid body below 100 Hz and the first bending mode of the assembly occurs well above the frequency range of interest.

2.5 Wires and Tuner Mounts

Steel music wires with a diameter of 0.046 inch were attached to the stub shaft assembly to provide axial support and moment control to the shaft while levitated in the single bearing. Figure 2.9 shows how a total of four music wires were attached to each end of the stub shaft assembly to control the axial plane of the assembly. The wires are attached to the shaft plates simply by threading the wires through a hole and having a knotted end of the wire pull against the plate. Guitar strings have been used as the wire supports because they are manufactured with one end wrapped around a small metal ball, which acts as a knot.
The wires are tightened through the use of guitar tuning mechanisms mounted 15 inches away on the tuner mounts shown in Figure 2.1.

The tuner mounts are simple structures built to support the tuning mechanisms and also allow for the alignment of the stub shaft assembly and bearing. There are two tuner mounts located on either side of the shaker-bearing assembly as seen in Figure 2.1 and 2.2. Each tuner mount is built using a hat shaped section of 10-gauge steel that provides the appropriate geometry to mount the four tuning machines. This hat section is bolted to a flat aluminum plate whose height can be adjusted using four threaded rods that are secured to T-slots in the concrete base. After the shaker and bearing assembly has been leveled, the tuner mounts are adjusted to the appropriate height to align the stub shaft assembly within the bearing.

The wires serve as an important fine adjustment to position the stub shaft assembly properly within the bearing. Once the shaft is levitated within the bearing, the wire tensions are adjusted to equalize the top and bottom bias currents in either axis. This ensures that both axes are supporting the static mass of the stub shaft assembly equally and therefore each axis will contribute equally to the control of the mass. The MBScope™ Snapshots software tool provided by Revolve gave an instantaneous view of the current levels in all four magnets within the two axes. Therefore this software was used to verify that the top and bottom bias currents were equalized.

Once the tension of the wires was set to deliver the desired equalized bias currents, the stiffness of the wire support system was measured to ensure that its effects on the system were negligible compared to the bearing stiffness. Appendix C describes the process used to measure the wire stiffness in the lateral direction. The approximate stiffness of the wires was determined to be less than 2 lbf/inch throughout the entire range of motion of the stub shaft-wire assembly. Table 2.3 summarizes the results of the wire stiffness tests as compared to the bearing stiffness.

<table>
<thead>
<tr>
<th>Controller Case</th>
<th>Approximate Bearing Stiffness (lb/in.)</th>
<th>Wire Support Stiffness (lb/in.)</th>
<th>$% \frac{K_{wire}}{K_{Bearing}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>219.0</td>
<td>$\approx 1.7$</td>
<td>0.77 %</td>
</tr>
<tr>
<td>H</td>
<td>237.0</td>
<td>$\approx 1.7$</td>
<td>0.717 %</td>
</tr>
<tr>
<td>K</td>
<td>460.0</td>
<td>$\approx 1.7$</td>
<td>0.37 %</td>
</tr>
</tbody>
</table>
Table 2.3 shows that the wire stiffness is less than 1% the bearing stiffness therefore the stiffness of the wire support system has a negligible effect on the systems dynamics.

### 2.6 Concrete Base

The entire test rig is mounted to a large 15-foot deep concrete block located in the Modal Analysis Laboratory at Virginia Tech. The block has T-slots along its surface, which have been utilized for mounting of the test rig. The concrete block serves to isolate the entire test rig from any external vibrations from other machinery or vibrations within the building. The block also prevents the propagation of vibrations from the shaker to the sensor mounting structure.

### 2.7 Position Sensors

A total of nine Bentley-Nevada® eddy current position probes have been used to measure the positions of the bearing and the stub shaft assembly during testing. These probes measure the relative displacement between the probe tips and a target and therefore must be mounted to a separate stationary structure. This structure is referred to as the sensor mount and can be seen in Figure 2.1. The sensors operate about a nominal gap distance from their targets of approximately 35 mils to maximize their linear dynamic range. At this gap distance the probes output a negative voltage of approximately -7.5 VDC and their total output range spans from 0 VDC to –14 VDC. Therefore, when collecting dynamic position signals the output from these probes must be AC coupled to shunt the negative DC component of the output. The sensors are also well suited for the measurement of the position signals from the test rig because they are capable of measuring DC position signals up to signals of 10,000 Hz.

Each of the 9 different position probes has been calibrated using a dial micrometer attached to a linear slide to which the position probe was attached. A stationary target was used while the sensor position was incremented away from the target in 2 mil increments and the corresponding output voltage was measured. This data was then taken and fit with a least squares linear curve fit of the form

\[ Y = Ax + B, \]  

(Eq. 2.1)

where \( Y \) is the measured displacement in mils, \( x \) is the output voltage of the sensors, \( A \) is the sensor sensitivity in mils/Volt and \( B \) is simply an offset displacement based on an offset voltage. The offset is included in the curve fit to allow for a closer approximation of the actual sensitivity. This offset voltage is simply the voltage the sensors would read if the sensor were touching the target. This is actually not a real quantity because the sensors are unable to measure position differences less than 2 mils, however the
offset voltage was included to arrive at a more accurate sensor sensitivity during the linear least squares curve fit. The offset voltage does not affect the position signals recorded during testing because the position channels are AC coupled therefore this component is removed. The sensitivity calculated from the least squares curve fits for each sensor was used during data collection to convert the voltage signals to position signals. Figure 2.8 gives an example of a calibration curve recorded for a single proximity probe.

Figure 2.8: Example Calibration Curve for Eddy Current Position Probe A

Figure 2.8 shows that the sensors are very linear within our range of interest as reflected in the $R^2$ value. The linear range falls between 10 to 65 mils and the sensitivities for each probe vary slightly about 200 mV/mil. Appendix D gives all of the calibration curves along with the curve fits, individual probe sensitivities and uncertainties based on a 95% confidence interval.

### 2.8 Sensor Positions and Targets

The placement of the position sensors on the rig were important in order to properly characterize the motion of both the bearing and stub shaft assembly. A total of nine Bentley-Nevada® position probes were used, with six reading positions from the stub shaft assembly and three from the bearing stator. Figure 2.9 shows the location of the detection points for these sensors and their corresponding label. Four
position sensors were used to detect the motion in the horizontal plane at two locations on either end of the stub shaft assembly, which are labeled H, I, J, and K. These sensors were used to detect both horizontal motion and angular motion at either end. Two additional sensors were used to detect the centered vertical displacement of the assembly, which are labeled B & C in Figure 2.9. With these six signals and the shaft acting as a rigid body it was possible to reconstruct the complete motion of the stub shaft assembly. Appendix E describes the method for calculating the orbits of the stub shaft assembly in each plane from the position signals recorded from these probes.

A similar approach was taken to characterize the motion of the bearing to fully characterize the system input. Figure 2.9 shows the five bearing stator detection points of A, D, E, F, and G. However points G and F were determine to be redundant and therefore only points D and E were used to detect the horizontal motion of the bearing. A single probe was positioned to detect the vertical motion at point A, on top of the bearing housing. This signal is considered the input to the system because it measures the vertical displacement of the bearing housing which the shaker controls directly. An experimental modal analysis performed on the bearing housing ensured there were no bending modes below 100 Hz which could distort the input signal.

![Sensor Target Material](image)

**Figure 2.9:** Position Sensor Targets on Stub Shaft Assembly and Bearing
In addition 1040 steel targets have been placed on the bearing and the end plates of the shaft at the sensor detection points because the sensors achieve the best performance when used in conjunction with this material.

### 2.9 Signal Processing and Data Acquisition

In addition to the nine position signals discussed in the previous section, the four magnet currents and two axis position signals were recorded utilizing the MBResearch™ breakout box. Therefore a total of 15 channels of data were recorded. Two types of tests have been performed, frequency response testing and single frequency multiple amplitude testing, which each required a separate data acquisition setup. Frequency response testing was used to measure the displacement transmissibilities between the output position signals (probes B-K) and the input base excitation given by probe A along with the frequency response functions (FRFs) of the magnet currents and internal position signals with respect to the base excitation.

![Displacement Transmissibility](image)

**Figure 2.10:** Displacement Transmissibility of a Single Degree of Freedom Base Excitation System

Displacement transmissibility is the measure of an input displacement to an output displacement of a system over a specific frequency range. Figure 2.10 depicts a single degree of freedom base excitation system and the corresponding displacement transmissibility equation. An important difference between this system shown and the AMB system is that the equivalent stiffness and damping are also functions of frequency. Therefore the results obtained from the displacement transmissibility measurements may not depict the traditional displacement transmissibility responses. Measuring the displacement transmissibility of the stub shaft assembly/bearing system in several different locations allowed for the direct comparison of the modeled displacement transmissibilities to the measured transmissibilities which is presented in Chapter Three. The single frequency tests were performed to verify that the model
predicts the response of the stub shaft undergoing base excitation throughout the entire range of motion possible within the stator.

The displacement transmissibilities were measured using the shaker to excite the bearing with a multiple frequency burst chirp and the position response of the stub shaft was recorded using an HP 35665A Digital Signal Analyzer. The analyzer was only capable of measuring a single position response and the corresponding input excitation, therefore multiple tests were performed to capture the position response of all the probes. The HP analyzer allowed for the averaged measured displacement transmissibilities to be saved directly along with the coherence function associated with them. Capturing the transmissibilities of all the position signals allowed for the reproduction of the orbits of the stub shaft assembly and bearing housing occurring at specific frequencies which will be presented in Chapter Four. The test setup used to measure the transmissibilities is shown in Figure 2.11.

The bearings vertical position signal (location A of Figure 2.9) was used as the input for all testing because it directly represents the base excitation of the bearing. The remaining position signals were all referenced to this input to give displacement transmissibilities. The current signals and internal position signals were also recorded using the MBResearch™ breakout box. The HP analyzer was used as the signal source, which was amplified and then sent to the shaker to excite the bearing. Table 2.4 gives the signal processing setup used with the HP analyzer when collecting these transmissibilities.

![Figure 2.11: Test Setup for Measuring Displacement Transmissibilities](image)
Table 2.4: Signal Processing Setup for Measurement of Displacement Transmissibilities

<table>
<thead>
<tr>
<th>DSA Signal Processing Configuration for Measurement of Displacement Transmissibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Source</strong></td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
</tr>
<tr>
<td><strong>Window</strong></td>
</tr>
<tr>
<td><strong>Input Channel 1</strong></td>
</tr>
<tr>
<td><strong>Output Channel 2</strong></td>
</tr>
<tr>
<td><strong>Trigger</strong></td>
</tr>
<tr>
<td><strong>Averages</strong></td>
</tr>
</tbody>
</table>

A burst chirp was used to give a multiple frequency input up to 100 Hz to the shaker so that the transmissibility frequency response of all the output signals could be recorded. The burst chirp allowed for a uniform window to be used because the response of the stub shaft died out before the end of the collection time for one block of data. Five averages were used to cancel out any noise in the signals.

The HP analyzer has the ability to trigger the collection time for the data block, which made it possible to use the burst chirp input to obtain the transmissibilities from the different signals. This triggering was not needed for the acquisition of single frequency response data therefore a different setup was used for these tests. Single frequency tests were taken using the National Instruments 16-Channel PC based data acquisition system. This system allowed for the simultaneous data collection from all 9-position signals, the four current signals, and two internal position signals from the bearing. The HP analyzer was again used as the signal source to the amplifier and shaker while the MBResearch™ breakout box was used to extract the current and internal position signals. A layout of the data acquisition system is shown in Figure 2.12.
The sensor signals are sent to National Instruments SCXI modules 1140 and 1141 within a SCXI 1000DC chassis, which AC coupled and applied an anti-aliasing filter to each signal. The position signals are AC coupled to remove the approximate 7.5-Volt DC offset of the position probes. The SCXI modules also simultaneously sample and hold the signals then a National Instruments PCI-MIO-16E-4 16-channel data acquisition card was used to acquire the signals. The card was placed in a laptop computer running LabVIEW® to process and save the data. The card can except ±10 Volts, therefore no amplification or attenuation of the sensor signals was needed before sending to the data acquisition system.

The LabVIEW® code used to process the data and save the complete time record along with averaged auto-spectrums of each channel. The code also saved the cross-spectrum of each channel referenced to the input channel, which was allocated to the vertical bearing position (probe location A). This allowed for the convenient calculation of any displacement transmissibilities between the input signal and an output signal by using the $H_1$ frequency response estimator given as

$$H_1(\omega) = \frac{G_{xy}(\omega)}{G_{ss}(\omega)}, \quad \text{(Eq. 2.2)}$$

where $G_{xy}(\omega)$ is the single sided cross-spectrum between the input signal of position probe A and an output position or current signal and $G_{ss}(\omega)$ is the single sided auto-spectrum of the input signal of position probe A [10]. The coherence function between the input signal and any of the output signals can also be calculated using the following relationship;

$$\gamma^2 = \frac{|G_{xy}(\omega)|^2}{G_{ss}(\omega)G_{yy}(\omega)}, \quad \text{(Eq. 2.2)}$$

where $\gamma^2$ is the coherence function and $G_{yy}(\omega)$ is the auto-spectrum of the output signal [10].

The test setup used to measure stub shaft response to single frequency multiple amplitude base excitation is shown in Figure 2.13.
The signal processing setup used for measuring the response of the stub shaft to single frequency inputs is given in Table 2.5.

**Table 2.5:** Signal Processing Setup for Single Frequency Measurements

<table>
<thead>
<tr>
<th>DSA Signal Processing Configuration for Single Frequency Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Source</strong></td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
</tr>
<tr>
<td><strong>Window</strong></td>
</tr>
<tr>
<td><strong>Input Channel 1</strong></td>
</tr>
<tr>
<td><strong>Output Channels 2-14</strong></td>
</tr>
<tr>
<td><strong>Trigger</strong></td>
</tr>
<tr>
<td><strong>Averages</strong></td>
</tr>
<tr>
<td><strong>Anti-Aliasing Filter</strong></td>
</tr>
</tbody>
</table>
To ensure accurate measuring of the signal amplitudes a flat top window was used when acquiring steady state single frequency data. This was done to prevent spectral leakage from degrading the amplitudes measured at the desired frequency. This also allowed for a higher sampling rate of 2048 Hz to be used since the frequency resolution could be increased using the flat top window. A total of 2048 data points were collected per time block resulting in a 1 Hz frequency resolution. The driving frequencies sent to the shaker were known, therefore the correct amplitudes could be measured simply by measuring the largest amplitude close to this driving frequency.
CHAPTER 3
Base Excitation Modeling and Validation

3.1 Introduction

A model of the MBRetro™ radial magnetic bearing undergoing base excitation has been developed using stiffness and damping properties derived from the bearing geometry and its controller transfer function. Therefore, a discussion of the control system tuning procedure is also given along with the resulting gain settings. Three gain settings were selected to give three different stiffness and damping cases for testing. The associated stiffness and damping values were determined corresponding to these three gain settings and used in conjunction with the base excitation model presented in Chapter One to create an overall two degree of freedom model of the base excitation system. These stiffness and damping calculations were verified by measuring the controller transfer function and comparing them to modeled results. Finally, the base excitation model was used to predict the vertical displacement transmissibility. The results of which are compared to the measured displacement transmissibilities in Chapter Four.

3.2 Base Excitation Modeling

The bearing actuator can be represented by a horizontal and vertical axis, each with an associated stiffness and damping as shown in Figure 3.1. Cross coupling terms between the two planes of motion have been assumed to be negligible which is a standard assumption in the modeling of active magnetic bearing systems [Allure]. Therefore, the bearing actuator can be reduced to two single degree of freedom systems acting in the horizontal and vertical directions.

![Figure 3.1: Stiffness and Damping Representation of Bearing Actuator](image-url)
The equation of motion for a mass undergoing single degree of freedom base excitation is
\[ M\ddot{x} + C\dot{x} + Kx = C\ddot{y} + Ky, \]  
(Eq. 3.1)
where \( M \) is the mass, \( C \) and \( K \) are the stiffness and damping coefficients, \( x \) is the mass displacement and \( y \) is the base displacement. A schematic of such a system is given in Figure 3.2.

![Figure 3.2: Single Degree of Freedom Base Excitation System](image)

This single degree of freedom system was used to describe both the horizontal and vertical axes shown in Figure 3.1. Figure 3.3 gives a simplified diagram of this system.

![Figure 3.3: Simplified Two degree of Freedom Model for the MBRetro™ radial AMB](image)

A two-degree of freedom model was created in matrix form which describes the motion in each direction.
This is given as

\[
\begin{bmatrix}
M & 0 \\
0 & M
\end{bmatrix}\begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
C_x & 0 & 0 & K_x \\
0 & C_y & 0 & K_y
\end{bmatrix}\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} + \begin{bmatrix}
K_x & 0 \\
0 & K_y
\end{bmatrix}\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
C_x & 0 & 0 & K_x \\
0 & C_y & 0 & K_y
\end{bmatrix}\begin{bmatrix}
\dot{x}_B \\
\dot{y}_B
\end{bmatrix} + \begin{bmatrix}
K_x & 0 \\
0 & K_y
\end{bmatrix}\begin{bmatrix}
x_B \\
y_B
\end{bmatrix},
\]  
(Eq. 3.2)

where \( M \) is the rotor mass, \( C_x \) and \( C_y \) are the damping coefficients, \( K_x \) and \( K_y \) are the stiffness coefficients, \( x \) and \( y \) are the rotor displacements and \( x_B \) and \( y_B \) are the base displacements in the vertical and horizontal planes.

This model was used to represent the motion in each plane within the actuator of the radial magnetic bearing, however to utilize this model, the stiffness and damping terms for each direction had to be determined. These linearized values are derived in the next section in detail.

### 3.3 Determination of Linearized Stiffness and Damping terms

#### 3.3.1 Position and Current Stiffness for a single axis

To begin, the force equation for a single horseshoe type electro-magnet is revisited. Figure 1.2, which gives a schematic of such a magnet is given here as Figure 3.4 for convenience.

![Simple Horseshoe Electro-Magnet](image)

**Figure 3.4:** Simple Horseshoe Electro-Magnet

The attractive magnetic force applied to the mass has been derived from basic electro-magnetic theory [9] and is given by

\[
F = \frac{\mu_0 A_n N^2 I^2}{4g^2} \quad \text{[lb]}
\]  
(Eq. 3.3)
where $\mu_o$ is the permeability of the air gap ($2.825 \times 10^7 \text{ lb/ft} \cdot \text{Amp}$), $A_g$ is a single pole face area ($\text{in}^2$), $N$ is the total number of wire coils, $g$ is the gap distance (in.) and $I$ is the current (Amps). A dimensionless loss factor, $\varepsilon$, is also included to account for magnetic field fringing and leakage effects. The remaining variables simply describe the physical configuration of the magnet and they remain constant. Equation 3.3 shows that the applied force can change either with a change in position or a change in current. As a result, a position stiffness and current stiffness must be determined to fully characterize the forces applied to the mass.

A single axis within the bearing actuator actually consists of two identical opposing horseshoe magnets as shown in Figure 3.5.

![Figure 3.5: Single Axis Layout of a Radial Magnetic Bearing](image)

Figure 3.5 shows the force from each opposing magnet, $F_1$ and $F_2$, acting on the mass within a single axis. The equation of motion describing this system is given by

$$M_r \ddot{x} + F_2 - F_1 = F_i \ [\text{lb}], \quad \text{(Eq. 3.4)}$$

where $F_i$ is an external force applied to the mass and $M_r$ is the mass of the rotor. The stiffness and damping for the axis are derived from the net force applied to the mass by the two opposing magnets. This net force is given by

$$F_{\text{net}} = F_2 - F_1 \ [\text{lb}] \quad \text{(Eq. 3.5)}$$

Substituting equation 3.3 into equation 3.5 gives

$$F_{\text{net}} = F_2 - F_1 = \frac{\varepsilon \mu_o A_g N^2}{4} \left( \frac{I_2^2}{g_2^2} - \frac{I_1^2}{g_1^2} \right), \quad \text{(Eq. 3.6)}$$
where $I_1$ and $I_2$ refer to the currents in magnets 1 and 2 and $g_1$ and $g_2$ refer to the gap distance between these magnets and the mass.

The gap terms $g_1$ and $g_2$ can be replaced by the following:

$$g_1 = g_o - x \quad \text{(Eq. 3.7)}$$

and

$$g_2 = g_o + x \quad \text{(Eq. 3.8)}$$

where $g_o$ is the nominal gap distance with the mass centered in the axis and $x$ represents a perturbation in the position of the rotor measured from the center. Equations 3.7 and 3.8 reflect how a positive increase in the $x$ direction will decrease $g_1$ and increase $g_2$.

Similarly, the currents $I_1$ and $I_2$ include a bias current $I_B$ and a perturbation current $I_p$. An equal bias current is applied to both magnets while the perturbation current is added to $I_2$ and subtracted from $I_1$. This occurs because as the mass moves closer to magnet 1 and further from magnet 2 the force from magnet 1 must be reduced and the force from magnet 2 must be increased to maintain equilibrium. This allows for the currents $I_1$ and $I_2$ to be replaced by the following:

$$I_1 = I_B - I_p \quad \text{(Eq. 3.9)}$$

and

$$I_2 = I_B + I_p \quad \text{(Eq. 3.10)}$$

The relationships given in equations 3.7 – 3.10 have been substituted into equation 3.6 to yield

$$F_{\text{net}} = \frac{\varepsilon \mu_o A_z N^2}{4} \left( \frac{(I_B - I_p)^2}{(g_o - x)^2} - \frac{(I_B + I_p)^2}{(g_o + x)^2} \right). \quad \text{(Eq. 3.11)}$$

To linearize the net force given in Equation 3.11, the perturbation current $I_p$, and the perturbation position $x$, are assumed to be small compared to the bias current $I_B$ and the nominal gap $g_o$, respectively. This allows for Equation 3.11 to be expanded and simplified by excluding higher order terms of the perturbation current $I_p$, and the perturbation position $x$ to give the linearized net force as

$$F_{\text{net}} = \frac{\varepsilon \mu_o A_z N^2 I_B}{g_o^2} I_p - \frac{\varepsilon \mu_o A_z N^2 I_B^2}{g_o^3} x. \quad \text{(Eq. 3.12)}$$
The position stiffness for the actuator is calculated by taking the partial of the net force equation (Eq. 3.12) with respect to perturbation position $x$ and evaluating this expression at the nominal gap value ($g_o$) and bias current ($I_B$). Similarly, the current stiffness is determined by taking the partial of the force equation with respect to the perturbation current, $I_P$, and evaluating the partial at the bias current ($I_B$) and nominal gap ($g_o$) of the axis. The resulting position stiffness is given by

$$K_p = \frac{\partial F_{net}}{\partial x} \bigg|_{g_o, I_B} = -\frac{\epsilon \mu_0 A g^2 B^2}{g^4} [\text{lb/in.}],$$

(Eq. 3.13)

and the resulting current stiffness is given by

$$K_i = \frac{\partial F_{net}}{\partial I_P} \bigg|_{I_B, g_o} = \frac{\epsilon \mu_0 A g^2 B^2}{g^4} [\text{lb/Amp}].$$

(Eq. 3.14)

These stiffness values can now be used to represent the net force in the equation of motion for the mass within the axis. Substituting the position stiffness and current stiffness into equation 3.4 gives

$$M_i \ddot{x} + K_p x + K_i I_P = F_i [\text{lb/in.}],$$

(Eq. 3.15)

and expanding yields

$$M_i \ddot{x} - \frac{\epsilon \mu_0 A g^2 B^2}{g^4} x + \frac{\epsilon \mu_0 A g^2 B^2}{g^4} I_P = F_i [\text{lb/in.}].$$

(Eq. 3.16)

An important result, as discussed earlier in Chapter One, is that the position stiffness is negative. This reflects the attractive nature of the electro-magnets. As the mass gets closer to one magnet, the force applied from that magnet increases. This results in the bearing actuator being inherently unstable and brings forth the need for an active control system to induce stability.

### 3.3.2 Equivalent Stiffness and Damping

The addition of a feedback controller takes advantage of the positive current stiffness to provide stability to the actuator. The controller adjusts the perturbation current, $I_P$, to the magnets to counteract the change in position detected by a sensor within the bearing. Therefore the controller transfer function simply gives a ratio of output perturbation current to input position as discussed in Chapter One. The perturbation current, $I_P$, is also referred to as the control current, $I_C$, because it is produced by the control
system and associated electronics. The controller transfer function also contains phase information relative to the input position signal therefore it is a complex valued function and can be written as

\[ G(i\omega) = a_G(i\omega) + ib_G(i\omega) \text{ [Amps/in.],} \]  

(Eq. 3.17)

where \( a_G \) represents the real part of the transfer function, \( b_G \) represents the imaginary part of the transfer function, and \( \omega \) is the forcing frequency. This transfer function multiplied by the position \( x \) yields the control current, \( I_C \), which can then be substituted into equation 3.16 as \( I_P \) to give

\[ -M_x x\omega^2 + (K_P + K_I(a_G + ib_G))x = F_i \text{ [lb],} \]  

(Eq. 3.18)

Equation 3.18 assumes a harmonic forcing function therefore the mass acceleration has been represented as \(-x\omega^2\). The stiffness and damping can now be determined from the net force produced by the axis position stiffness, current stiffness and controller transfer function by equating a force produced by an equivalent stiffness and damping. Equating these two forces gives

\[ (K_{eq} + C_{eq}i\omega)x = (K_P + K_I(a_G + ib_G))x \text{ [lb],} \]  

(Eq. 3.19)

and equating similar terms on both sides of the equation yields the equivalent axis stiffness as

\[ K_{eq} = K_P + K_Ia_G \text{ [lb/in.],} \]  

(Eq. 3.20)

and equivalent axis damping as

\[ C_{eq} = \frac{K_Ib_G}{\omega} \text{ [lb/s/in.].} \]  

(Eq. 3.21)

Equations 3.20 and 3.21 represent the single axis linearized stiffness and damping values. These values change with frequency due to their dependence on the real and imaginary parts of the controller transfer function. To successfully model the entire radial magnetic bearing system, the controller transfer function must be known. The next section deals with modeling and verification of the controller transfer function.

### 3.4 Controller Modeling and Tuning for the Base Excitation Test Rig

The MB350™ controller from Revolve Magnetic Bearings Incorporated, shown in Figure 3.6, was used as the active control system for the bearing rotor system. The MB350™ is a 5-axis digital controller with a sampling rate of 10,000 Hz. The five separate axes allow it to control two MBRetro™ radial magnetic bearings and a single MBRetro™ thrust bearing. A single MBRetro™ radial magnetic bearing was used during this research, therefore only two control axes were utilized. Figure 3.7 gives a block diagram of a single radial bearing control axes within the MB350™ controller.
**Figure 3.6:** MB350™ 5-axis digital controller from Revolve Magnetic Bearings Incorporated (Picture courtesy of Revolve Magnetic Bearings Incorporated)

**Figure 3.7:** Single Axis Closed Loop Control Block Diagram for MBretro Radial Bearing (Diagram courtesy of Revolve Magnetic Bearings Incorporated)
There are several operations that take place within the controller that must be considered to properly model the system and arrive at the appropriate controller transfer function.

Upon setting up the AMB system, the reference signal was set to zero to center the rotor within the axes. Therefore this input into the system could be ignored in the controller model. Centering the rotor also resulted in the largest gap clearance on either side of the rotor, which allowed for the largest vibration levels to be obtained during testing.

The controller is a digital device. Therefore, the position signal (voltage) is digitized through an analog to digital converter before being sent to the plant controller. The digital control signal produced by the plant controller along with any open loop control, bias current signals and signal injection are transformed back to analog voltage signals through a digital to analog converter before being sent to the power amplifiers for each magnet. The sample rate of the controller is 10,000 Hz, which is two orders of magnitude higher than bandwidth of the tests performed (0-100 Hz). Therefore the digitization process had a negligible effect on the control of the system and was ignored.

The block diagram in Figure 3.7 can be simplified for several reasons. Open loop control and signal injection were not used for the tests performed. Therefore, these blocks can be ignored along with the D/A and A/D converters as discussed above. The reference signal was set to zero, therefore this input to the system can be ignored and the position input is only considered. The stiffness and damping characteristics are determined from the feed through transfer function, G(s), therefore the block diagram used to determine the axis stiffness and damping characteristics simplifies to what is shown in Figure 3.8.

**Figure 3.8:** Simplified Block Diagram of Single Radial Control Axis
Notice that both the top and bottom amplifiers receive the same control voltage and therefore, output the same control current differing only in sign. Note that the perturbation current, $I_p$, discussed in the previous section is now replaced by the control current, $I_c$, from the controller. The top actuators receive a reduction in current when the rotor position detects a positive displacement toward the top magnets in the stator and the bottom actuators receive an increase in current to pull the rotor back into the centered position.

The plant controller has the option to incorporate a combination of a PID filter, two lead-lag filters, four notch filters, and two low pass filters. Figure 3.9 shows the MBScope™ plant controller software program provided by Revolve Magnetic Bearings Incorporated where the parameters for these different filters are entered.

![Filter Parameter Input Screen within MBScope™ Software Program](image)

**Figure 3.9:** Filter Parameter Input Screen within MBScope™ Software Program

The PID filter, a single notch filter and a low pass filter were necessary to levitate and control the stub shaft assembly within the bearing and are the basis of the plant controller. Figure 3.10 gives a block diagram where these three filters have replaced the plant controller and only a single power amplifier is considered due to the equivalence previously discussed.

![Bottom Actuator Open Loop Control Block Diagram](image)

**Figure 3.10:** Bottom Actuator Open Loop Control Block Diagram
The elements of this block diagram describe the controller transfer function G(s) described previously in section 3.3.2. The complete transfer function is given by

\[ G(s) = SS(s)LP(s)PID(s)N(s)AMP(s) \text{ [Amps/in.]} \]  \hspace{1cm} (Eq. 3.22)

Recall s is the complex frequency variable and can be replaced with \( i\omega \). The modeling of each individual component of this diagram is discussed in the following section.

### 3.4.1 Differential Sensor

A differential sensor is used to detect the change in position of the rotor within the axis from the set point of the controller. The set point was set to zero, therefore the sensor simply measured the position change of the rotor from the center of the axis. The differential sensor was modeled simply as a gain relating output sensor voltage to the measured differential position. This gain or sensor sensitivity was user defined and set to 0.1572 Volts/mil. This value was determined by measuring the inside diameter of the stator and recording the corresponding output voltage of the stator when the rotor was against one magnet. The sensitivity was determined by dividing the output voltage by the measured radial clearance. Obviously the sensor has been assumed to behave linearly throughout the range of motion within the stator. The sensor transfer function simply becomes

\[ SS(s) = \frac{V_X}{X} = 0.1572 \text{ [Volts/mil]} \]  \hspace{1cm} (Eq. 3.23)

### 3.4.2 Low Pass Filter

The low pass filter is used within the plant controller to reduce the controller’s high frequency gain above the bandwidth of the hardware and at least one must be used for the bearing to operate properly. The low pass filter also allows for the bearing to operate quietly by attenuating high frequency electrical noise, however by doing so high frequency resonance’s may not be controlled properly. A second order low pass filter of the form

\[ LP(s) = \frac{V_{LP}}{V_X} = \frac{\omega_{LP}^2}{s^2 + 2\xi_{LP}\omega_{LP}s + \omega_{LP}^2} \text{ [Volt/Volt]} \]  \hspace{1cm} (Eq. 3.24)

was used to model the low pass filter where \( V_{LP} \) is the output voltage of the filter, \( V_X \) is the input position voltage, \( \omega_{LP} \) is the filter cutoff frequency, \( \xi_{LP} \) is the filter damping ratio and s is the complex frequency variable. The damping ratio (\( \xi_{LP} \)) and filter cutoff frequency (\( \omega_{LP} \)) are software selectable as seen in Figure 3.9.
3.4.3 Proportional, Integral, and Derivative (PID) Filter

PID control is the most commonly used control method for magnetic bearings and is also the method of control used by the MB350™ controller. The standard continuous PID form is given by

$$\text{PID}(s) = \frac{V_{\text{PID}}}{V_{\text{LP}}} = \frac{K_T}{s} \left( K_D s^2 + K_P s + K_I \right)$$ [Volt/Volt], \hspace{1cm} (Eq. 3.25)

where $K_T$ is the total gain, $K_D$ is the derivative gain, $K_P$ is the proportional gain, $K_I$ is the integral gain and $s$ is the complex frequency variable. $V_{\text{PID}}$ is the output voltage of the filter and $V_{\text{LP}}$ is the input voltage from the low pass filter. In general, the proportional gain directly effects the bearing stiffness because it is multiplied by the position signal directly. Similarly, the derivative gain directly effects the damping of the axis because it is multiplied by the derivative of the position signal. The integral gain acts on steady offsets within the axis and provides a control signal to eliminate the offset. The total gain is simply a multiplier on all three gains simultaneously. The four gains are software selectable through the use of the MBscope software program provided with the MBRetro kit shown in Figure 3.9.

3.4.4 Notch Filter

Notch filters are used to damp out problematic high frequency resonances above the operating speed or frequency bandwidth of the system. These filters act over a narrow frequency band and highly attenuate the gain of the controller at the notch frequency. This is done by introducing two damped zeros and two similar undamped poles into the system to attenuate the gain at the notch frequency. This also results in phase lead after the notch frequency, which is the real benefit of these filters. Normally, a notch filter is placed slightly before a problematic high frequency resonance during the tuning process to damp out the peak with the phase lead following the notch filter. This process is called phase stabilization. Equation 3.26 gives the model used to represent the notch filter.

$$\text{N}(s) = \frac{V_N}{V_{\text{PID}}} = \frac{s^2 + 2\xi_N\omega_N s + \omega_N^2}{s^2 + 2\xi_N\omega_N s + \omega_N^2}$$ [Volt/Volt]. \hspace{1cm} (Eq. 3.26)

This form was chosen because the software characterized the notch filters using a frequency ($\omega_N$) and a filter damping ($\xi_N$). Again, $s$ is the complex frequency variable and $V_N$ is the filter output voltage and $V_{\text{PID}}$ is the input voltage from the PID filter.
3.4.5 Power Amplifier

The power amplifiers used by the MB350™ controller are switching amplifiers with a switching frequency of 20,000 Hz. Although the effects of these amplifiers in the low frequency range (0-100Hz) are negligible, a model of the frequency response of these amplifiers has been included for completeness and to incorporate the gain associated with the amplifiers. The amplifiers were modeled as Butterworth second order low pass filters having the following form;

\[
\text{AMP}(s) = \frac{I_C}{V_N} = K_a \frac{\omega_A^2}{s^2 + \sqrt{2}\omega_A s + \omega_A^2} \text{[Amp/Volt]}, \quad \text{(Eq. 3.27)}
\]

where \(I_C\) is the control current to the individual magnets, \(V_N\) is the input notch filter voltage, and \(s\) is the complex frequency variable. The switching frequency of 20,000 Hz was taken to be the filter cutoff frequency \(\omega_A\) and the Amplifier gain, \(K_a\), was experimentally determined to be 760.

3.4.6 Tuning for the Base Excitation Test Rig

The parameters for the low pass filter, notch filter, and PID filter were set during the initial tuning of the controller after the test rig had been set up. Bias currents for each of the magnets within the bearing actuator were set to 0.75 Amps. Normally, the bias currents for the bearing magnets are set to \(\frac{1}{2}\) the magnet’s saturation current, which is 3.0 Amps, however a stable control system could not acquired using bias currents of this level. Therefore the bias currents for the top and bottom magnets were reduced to 0.75 Amps, which more closely mimics a power-limited system. A low pass filter with a cutoff frequency of 800 Hz and a damping ratio of 0.707 was used along with a PID filter with very low gains to initially levitate the system. A notch filter then had to be used at 1000 Hz with a damping ratio of 0.10 to dampen out a high frequency resonance of the system. The peak occurred at approximately 1140 Hz therefore the phase lead of the notch filter effectively damped the resonant peak.

With a successful notch filter and low pass filter in place, it was then possible to adjust the gains of the PID filter to acquire the three gain cases used for testing. The PID filter gains provide the most direct method of changing the bearing stiffness and damping properties. Therefore, the integral, derivative and proportional gains were incremented until the limits of stability were reached for high and low values of these gains. These values were then used to create three controller parameter cases shown in Table 3.1.
Table 3.1: Controller Parameter Cases Used for Testing

<table>
<thead>
<tr>
<th>Gain</th>
<th>Case LKLC</th>
<th>Case LKHC</th>
<th>Case HKHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_I$</td>
<td>80</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>$K_P$</td>
<td>40</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>$K_D$</td>
<td>.025</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>$K_T$</td>
<td>.00006</td>
<td>.00006</td>
<td>.00006</td>
</tr>
</tbody>
</table>

Case LKC (low stiffness, low damping) refers to the gain case utilizing the lowest gain settings possible to maintain stability of the stub shaft assembly within the bearing. These low gain settings correspond to the lowest stiffness and damping characteristics that can be achieved for this system. This case also represents the most power-limited system because the low gain settings minimize the amount of current requested by the control system. The LKHC (low stiffness, high damping) case refers to a low stiffness and high damping gain case. This case is similar to the LKLC case, except the derivative gain has been increased to reduce the vibration levels to determine the effects of higher damping on the system. The final case, HKHC (high stiffness, high damping), has incorporated the highest combination of gain settings to achieve a higher stiffness and damping than what was achieved with the other two cases.

Figure 3.9 shows the MBScope software screen where the tuning parameters have been entered into the controller for case LKLC. With all the controller parameters set, the model of the controller transfer function was used to predict the stiffness and damping of the bearing. The next section provides the results of this modeling and the validation from measured transfer functions, which verify the modeling used here.

### 3.5 Measured Controller Transfer Function and Model Validation

To validate the controller model discussed in the previous section, the transfer functions of the different bearing magnets were measured and compared to the model predictions. This was done utilizing the MBresearch™ breakout box provided by Revolve Magnetic Bearings Incorporated which allows for the measurement of the position signals from each axis and the control currents to each magnet. Figure 3.11 shows the block diagram of a single axis of control and the measurement points shown by the dashed boxes.
Two transfer functions were measured between the input position signal and the control current to both magnets in an axis. The shaker and bearing assembly of the base excitation test rig described in Chapter 2 was used to excite the bearing and create a position signal due to the relative motion between the stator and rotor within the bearing. A Hewlett Packard 35665A Digital Signal Analyzer was used to record both the position signals and the current signals taken from the MBResearch™ BNC breakout box while providing a voltage source signal to the shaker. The MB350™ controller received signals from the bearing sensors and sent these signals to the MBResearch™ BNC breakout box. Figure 3.10 gives the test setup utilized to measure the controller transfer functions and Table 3.2 gives a description of the signal-processing configuration used with the Digital Signal Analyzer.
A transfer function was measured for each of the 4 magnets within the test bearing and for each set of control parameter settings (LKLC, LKHC, & HKHC) as described in section 3.4.6. Therefore, a total of twelve transfer functions were measured using this setup. Figure 3.13 is a comparison of the measured transfer functions of the top magnets in the bearing compared to the controller model for case LKHC.
Figure 3.13: Top Magnets (V13 & W13 Axis) Measured and Modeled Transfer Functions

Figure 3.13 shows how the model matches the measured transfer function for the top magnets of the MBRetro™ radial magnetic bearing. A single model simulation was prepared and plotted because both magnets are controlled identically based on the parameters loaded for each axis. The poor coherence in the low frequency range explains the deviation of the measured transfer functions from the modeled results, which is most likely due to poor shaker performance below 5 Hz. The phase plot shows the top magnet control currents 180 degrees out of phase with a positive displacement within the axis. This is congruent with the modeling of the bearing where the top magnet current is reduced when a positive displacement within the axis is detected by the position sensor.

The transfer functions could only be measured over the usable bandwidth available from the shaker. Therefore, the effects of the notch filter placed at 1000 Hz and the low pass filter cannot be seen in Figure 3.13. This is acceptable because the model is shown to match within the bandwidth possible during
testing. Figure 3.14 is a comparison of the measured transfer functions of the bottom magnets in the bearing compared to the controller model for case LKHC.

![Case LKHC - Bottom Magnet-Controller Transfer Function Comparison](image)

**Figure 3.14:** Bottom Magnets (V13 & W13 Axis) Measured and Modeled Transfer Functions

Figure 3.14 shows that the bottom magnet of axis V13 contains some noise, which shows up as oscillations about the modeled values for all cases. This is most likely due to electrical noise in this magnet’s power amplifier. However, the measured transfer function still matches very closely with the modeled values above 5 Hz. Notice that there is no phase difference between the input position signals and the output control currents for the bottom. This again coincides with the modeling performed in section 3.3.1. Figures 3.12 and 3.13 serve as an example to the accuracy to which the model matches the measured transfer functions of the controller cases. The remaining comparison plots for cases LKLC and HKHC are provided in Appendix E. Based on these comparisons, the model is considered sufficient for use in deriving the overall stiffness and damping characteristics for each controller gain case (LKLC, LKHC & HKHC).
3.6 Equivalent Stiffness and Damping Determination

The equivalent AMB stiffness and damping values can now be determined for each controller gain case based upon the transfer function model described in the previous section. Recall from section 3.3.2 the equivalent axis stiffness is

$$K_{eq} = K_p + K_I a_G \text{ [lb/in.], (Eq. 3.28)}$$

and equivalent axis damping is

$$C_{eq} = \frac{K_I b_G}{\omega} \text{ [lb-s/in.], (Eq. 3.29)}$$

where $K_p$ is the axis position stiffness, $K_I$ is the axis current stiffness, $a_G$ is the real part of the controller transfer function, and $b_G$ is the imaginary part of the controller transfer function. Using the model of the controller transfer function for each axis and the position and current stiffness discussed in section 3.3.1, the axes stiffness and damping values were calculated as a function of frequency and are provided in Figure 3.15.

**Figure 3.15:** Stiffness and Damping Characteristic's for each axis for all Controller Gain Cases
Figure 3.15 shows that the gain case LKLC produces the lowest stiffness and damping values throughout the frequency range as expected. The average stiffness for this case over 0 to 100 Hz was determined to be 217 lbf/in. and the damping values approach 0.01 lb·s/in. asymptotically. Table 3.3 summarizes the average stiffness calculated from 0 – 100 Hz for each gain case and the damping values each case approaches asymptotically.

Table 3.3: Equivalent Stiffness and Damping Results

<table>
<thead>
<tr>
<th>Gain Case</th>
<th>Stiffness (K_{eq}) lbf/in.</th>
<th>Damping (C_{eq}) lb·s/in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKLC</td>
<td>217</td>
<td>0.01</td>
</tr>
<tr>
<td>LKHC</td>
<td>247</td>
<td>0.30</td>
</tr>
<tr>
<td>HKHC</td>
<td>478</td>
<td>0.16</td>
</tr>
</tbody>
</table>

An interesting result shown in Figure 3.15 and Table 3.3 is that the gain case HKHC produced less damping than the gain case LKHC. The gain case HKHC has higher integral (K_i), derivative (K_d) and proportional gain (K_p) than case LKLC and LKHC. Therefore, it was expected that this case would produce both higher stiffness and damping throughout the frequency range. Case HKHC did produce higher damping values than case LKHC because the integral gain (K_i) was increased. The integral gain tends to reduce the damping of the system because it reduces the imaginary part of the controller transfer function. Recall from section 3.4.3 that the transfer function for a PID controller takes the form

\[
PID(s) = \frac{V_{OUT}}{V_{IN}} = \frac{K_T (K_D s^2 + K_p s + K_i)}{s}, \quad (\text{Eq. 30})
\]

which can be rewritten as

\[
PID(s) = \frac{V_{OUT}}{V_{IN}} = K_T K_D s + K_p + \frac{K_i}{s}. \quad (\text{Eq. 31})
\]

This equation can be further simplified by substituting the complex frequency \(i\omega\) for the Laplace variable \(s\) and simplifying to arrive at

\[
PID(i\omega) = \frac{V_{OUT}}{V_{IN}} = K_T K_p i\omega + K_p - \frac{K_i}{\omega}. \quad (\text{Eq. 32})
\]

Equation 32 shows how increasing the integral gain, \(K_i\), will tend to reduce the imaginary part of the controller transfer function due to the negative sign on the integral gain term.
The equivalent stiffness and damping calculated represent the stiffness and damping for a single control axis within the AMB. Recall from Chapter Two that the two axes within the bearing labeled, V13 and W13, are rotated 45° from the horizontal and vertical planes. It can be shown that the stiffness and damping properties calculated and shown in Figure 3.15 for a single axis also represent the vertical and horizontal axis as well. This can be proven by performing a potential energy balance of four springs and dampers placed in the bearing actuator in the configuration of axis V13 and W13 shown in Figure 2.4. However, it is intuitive that the properties are the same in the horizontal and vertical directions due to the layout of the bearing axes. The control axes within the bearing (V13 & W13) are rotated 45° from the horizontal and vertical planes, therefore each axis is responsible for ½ the bearing load in the vertical and horizontal planes. Because both stiffness and damping properties are assumed equal in the two control axes, the stiffness and damping in the horizontal and vertical directions are equal to the single axis properties given in Figure 3.15.

### 3.7 Base Excitation Model

With the stiffness and damping characteristics known, Equation 3.1 can now be revisited and used to predict the frequency response of the rotor-bearing system undergoing base excitation. Equation 3.1 is repeated here as Equation 3.33 were K and C have been replaced by $K_{eq}$ and $C_{eq}$.

$$M_r\ddot{y}_R + C_{eq}\dot{y}_R + K_{eq}y_R = C_{eq}\dot{y}_B + K_{eq}y_B \ [\text{lb}], \quad \text{(Eq. 3.33)}$$

Equation 3.33 can be used to derive the displacement transmissibility of the system. The general form of the displacement transmissibility in the frequency domain is given as

$$\frac{Y_B(s)}{Y_R(s)} = \frac{C_{eq}s + K_{eq}}{M_r s^2 + C_{eq}s + K_{eq}}, \quad \text{(Eq. 3.34)}$$

where $s$ represents the complex frequency variable. It is important to remember at this point that the bearing equivalent stiffness ($K_{eq}$) and equivalent damping ($C_{eq}$) are a function of frequency as shown in Figure 3.15 due to there reliance on the control system. This is a departure from the classical displacement transmissibility normally associated with mass-spring-damper systems. Recall from Chapter Two that the stub shaft assembly weighs 2 lbm, therefore the rotor Mass, $M_r$, is equal to 0.00518 lb-ft/s^2/in. Figure 3.16 shows a schematic of this system in equivalent form describing the base excitation of the rotor-bearing system in the vertical direction.
Equation 3.34 has been used in conjunction with the stiffness and damping values determined for the three controller gain cases in section 3.6 to calculate the associated displacement transmissibilities. Matlab® was used to create the model results and this code is provide in Appendix F. The results are plotted in Figures 3.17-3.19, respectively.
Figure 3.17 gives the displacement transmissibility for case LKLC, which has an averaged stiffness value of 217 lb/in. and a damping of approximately 0.01 lb\cdot s/in. This damping value has been used to calculate the relative damping ratio of this system to be 0.0046 using Equation 3.35:

\[ \xi = \frac{C}{2\omega_n M_r}, \] (Eq. 3.35)

where C is the damping value (0.01 lb\cdot s/in), M_r is the stub shaft assembly mass (0.00518 lb\cdot s^2/in.) and \( \omega_n \) is the resonant frequency (33 Hz) in radians. As a result of this case's low damping, a very sharp peak occurs in the displacement transmissibility at the resonant frequency, which is approximately equal to 33 Hz. The low damping is also apparent in the associated phase plot, which gives a sharp drop in phase at the resonant frequency of 33 Hz. This response does not represent a typical displacement transmissibility plot due to the extremely low damping ratio of 0.0046 that this case represents. For example, a displacement transmissibility plot for a system with appreciable damping will reflect the phase approaching 90 degrees after slight dip (approx. 150 degrees) in phase and in fact all displacement transmissibility phase plots must ultimately approach 90 degrees due to the system characteristics. This was verified for this case (LKLC) by increasing the frequency range to 10,000 Hz which shows the phase plot approaching 90 degrees. This figure is given in Figure 3.18 below.

**Figure 3.18:** Modeled Displacement Transmissibility for Case LKLC (frequency to 10,000 Hz)
Figure 3.19 gives the displacement transmissibility for case LKHC, which reflects an increase in the derivative gain, $K_D$, of the controller from 0.025 to 0.05. This case has an average stiffness within this frequency range of approximately 247 lb/in. and a damping of approximately 0.30 lb-s/in. ($\zeta = 0.14$). The change in the average stiffness of the bearing demonstrates how the increase in the derivative gain also effects the stiffness of the bearing. However, the stiffness was only increased from 217 lb/in. to 247 lb/in., which is an increase of 13.8% while the damping was increased from 0.01 lb-s/in. to 0.30 lb-s/in., which is an increase of 2900%. Therefore, the derivative gain has a much greater effect on the damping than on the stiffness as assumed. The increase in damping is apparent in the reduced peak shown in Figure 3.19 occurring at an apparent resonant frequency of approximately 32.5 Hz in comparison to that of case LKLC. The phase plot also reflects this increase in damping through the decrease in slope during the transition from $0^\circ$ to $180^\circ$ in the phase. Similar to Case LKLC the phase plot does ultimately approach 90 degrees as expected.
Figure 3.20 gives the displacement transmissibility for case HKHC, which shows an increase in the resonant frequency to approximately 48 Hz from 33 Hz due to the increase in stiffness of the bearing. The average stiffness of the bearing within this frequency range is 478 lb/ft and the average damping is approximately 0.16 lb-f s/ft ($\xi = 0.05$). This increase in stiffness is due to the increase in the proportional gain, $K_p$, from 60 to 80. The damping of the bearing has also been increased due to the increase of the derivative gain, $K_d$, from 0.025 to 0.05. The integral gain, $K_i$, was also increased from 80 to 100 which did not allow the damping of the bearing to increase above 0.16 lb-f s/ft, as it did with the controller gain case LKHC. This is reflected in the sharper peak and increase in the slope in the phase plot shown in Figure 3.20.

Figures 3.17-3.20 represent the linear displacement transmissibilities for each of the controller gain cases (LKLC, LKHC, & HKHC) used in the experimental testing. This chapter has described the steps taken to arrive at the linear models for these three cases. First, a single degree of freedom base excitation model was used to describe a single control axis within the AMB. This model was then extended to represent the two degrees of freedom within the AMB. To complete this model the stiffness and damping for each
axis had to be determined, which included using the elementary force equation for two opposing horseshoe magnets to arrive at the dynamic model of a single axis within the AMB. This also included modeling the controller transfer function to understand the relationship between the detected position and the resulting control current. Because the controller gains were user set, a brief discussion of tuning process and the resulting gains for each case was also given. Finally, the model was completed by incorporating the stiffness and damping values determined by the controller and magnet modeling into the original base excitation model. The results are given in the form of displacement transmissibilities, which give the ratio of the absolute vertical displacement of the stub shaft assembly with respect to the absolute vertical base excitation. Chapter Four describes the experimentally determined displacement transmissibilities obtained from the test rig and compares them with the modeled transmissibilities given in this chapter.
CHAPTER 4
Experimental Testing and Results

4.1 Introduction

This Chapter presents the results from the experimental testing performed using the base excitation test rig and the data acquisition setups described in Chapter Two. The test rig was used to measure the displacement transmissibilities from each of the nine probe locations as a result of vertical base excitation. This allowed for a complete characterization of the response of the stub shaft-bearing system. The displacement transmissibilities give the ratio of the stub shaft vibration in the direction of the measuring probe relative to the base excitation input over the frequency range of interest. Figure 4.1 gives the probe locations used to determine the displacement transmissibilities from the test rig.

![Figure 4.1: Probe Target Locations on Stub Shaft and Magnetic Bearing](image)

The locations shown in Figure 4.1 represent the targets for the non-contact position probes that were mounted on a separate structure called the sensor mount described in Chapter Two. The sensors measured a relative displacement between the sensor mount and the stub shaft or bearing. To ensure the
position probes gave absolute displacements of the bearing housing and the stub shaft assembly, the sensor mount was fixed directly to the concrete base to isolate it from motion.

The response of the stub shaft assembly and the bearing housing were measured from position probes A – K while exciting the bearing with the electromagnetic shaker. A multiple frequency voltage signal was used to activate the shaker and excite the bearing vertically while measuring the response of the system from all the position probes. With the response from each probe, displacement transmissibilities were then constructed by referencing one position signal to another. The vertical transmissibilities were determined using the responses measured from probes B and C and referencing them to the measured response of the bearing at position A. The displacement transmissibilities from the remaining probe locations allowed for the development of the mode shapes of the stub shaft and the characterization of the bearing motion.

The transmissibilities were obtained utilizing a 10 millivolt, 0-100 Hz burst chirp input signal, which resulted in stub shaft vibrations of less than 1 mil. Therefore, the measured transmissibilities only gave an indication of how well the model matched the stub shaft response for low amplitude base excitation levels. Recall from Chapter Two that the backup bearing has been removed from the magnetic bearing allowing for vibrations up to 15 mils (zero to peak) throughout the stator clearance. To verify that the model predicts the response of the stub shaft undergoing base excitation throughout this entire range of motion, single frequency tests have also been used. Specific frequencies have been selected for each controller gain case based on the measured transmissibilities and the response of the stub shaft has been recorded for varying amplitudes of base excitation at each frequency.

These tests have allowed for a closer examination of the stub shaft response while pushing the bearing to the limits of its performance. Recall from Chapter Three that the bias currents for each magnet have been reduced from 1.5 Amps to 0.75 Amps and the controller gain cases represent the highest and lowest values achievable to maintain stability. Also, with the backup bearing removed, the stub shaft assembly has been allowed to vibrate very close to the bearing stator. The single frequency tests allow for examination of the linearity of the response of the stub shaft undergoing base excitation at very high amplitudes in a power-limited configuration. These tests also allow for the investigation of the limits of stability within the bearing stator in relation to the amplitude of vibration and control current levels.
Chapter 4  Experimental Testing and Results

4.2 Measured and Modeled Vertical Displacement Transmissibilities

4.2.1 Case LKLC
As presented in Chapter three, this controller gain case represents the lowest PID gain values, and therefore the lowest bearing stiffness and damping, which could be used to obtain stable levitation of the stub shaft assembly. This case resulted in an averaged equivalent stiffness between 0 and 100 Hz of 217 lbf/in. and a damping ratio of 0.0046. Averaged values are used to describe the system due to the relationship between frequency, stiffness and damping. The notch filters cutoff frequency and damping ratio remained constant for all three gain cases along with the low pass filters cutoff frequency and damping ratio. The PID gain values used for this case are given in Table 4.1.

Table 4.1:  PID Gains used for case LKLC

<table>
<thead>
<tr>
<th>Gain</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_I</td>
<td>Integral Gain</td>
<td>60</td>
</tr>
<tr>
<td>K_p</td>
<td>Proportional Gain ((\propto) Stiffness)</td>
<td>40</td>
</tr>
<tr>
<td>K_D</td>
<td>Derivative Gain ((\propto) Damping)</td>
<td>0.025</td>
</tr>
<tr>
<td>K_T</td>
<td>Total Gain – Multiplier on all Gains</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Figure 4.2 gives the measured displacement transmissibilities recorded from the test rig along with the results of model discussed in Chapter Three for case LKLC. The results from the testing show two closely spaced modes measured at 30.5 Hz and 34 Hz giving transmissibility ratios of 13 mils/mil and 67 mils/mil, respectively. However, the model developed in Chapter Three has predicted a single peak occurring at 32.75 Hz giving a transmissibility ratio of 520 mils/mil due to the low damping of the system. This indicates that the actual system has more damping than the model predicts which could be due to a separation in the bearings axes properties. Recall from Chapter Two that the two controller axes, V13 and W13, were assumed to have the same stiffness and damping values due to the equivalence of their control parameters. This assumption dictated that both axes would resonate at the same frequency. Therefore a single peak would represent the resonant frequency for both axes. The measured transmissibilities indicate that the stiffness and damping characteristics of the V13 and W13 axis are not equal. The separate peaks most likely represent the resonant frequencies of each of the control axes due to slight differences between the two axes.
Figure 4.2: Measured and Modeled Vertical Displacement Transmissibilities for Case LKLC

The coherence for this case and all the measured vertical transmissibilities was greater than 0.9999, therefore coherence has not been shown here in Figure 4.2 for brevity. The measured transmissibility plots with the associated coherence plots are provided in Appendix G. To verify that the two peaks seen in Figure 4.2 represent the modes occurring in each axis, orbit plots of the stub shaft have been calculated using the response of the stub shaft recorded by the position probes. The orbit plots have been determined by utilizing the response signals recorded at the various positions on the stub shaft. A detailed description of the calculation method used is given in Appendix H.

Figure 4.3 gives the orbits of both the center of the stub shaft within the bearing stator and the bearing orbit at 30.5 Hz. The axes are labeled on the orbit plot to show how the stub shaft orbit is tilted slightly toward the W13 axis. This occurs because 30.5 Hz is the resonant frequency of the W13 axis and therefore it has a greater contribution to the response of the stub shaft. However, the mode occurring in the V13 axis also contributes to the response of the stub shaft. This is evident because the major axis of the stub shaft’s orbit does not fall on the W13 axis. The contribution from the V13 axis rotates the orbit towards the V13 axis. If the stiffness and damping properties in both the V13 and W13 axes were equal, the response would be vertical.
The bearing orbit seen in Figure 4.3 shows a relatively large horizontal motion which could be the result of the stiffness and damping differences between the axes. The shaker inputs a force to the bearing base in the vertical direction. The orbit plot of the stub shaft verifies this because the response is dominated by vertical motion. Therefore the horizontal motion of the bearing stator could simply be a response to the stub shafts horizontal component of displacement, which is directly related to the difference in stiffness and damping properties within the axes.

Figure 4.4 and 4.5 give the orbit plots for the center of the stub shaft and the bearing at 32 Hz and 34 Hz for this case. These figures show how the stub shaft orbit rotates toward the V13 axis as the frequency of excitation is increased. Figure 4.4 shows the stub shaft orbit at 32.0 Hz is rotated toward the V13 due to the relative high contribution of the 34.0 Hz resonant frequency of the V13 axis. The orbit of the stub shaft at 32.0 Hz is wider due to the contribution of the mode along the W13 axis and the mode along the V13 axis. This frequency is between the two resonant frequencies therefore the mode shapes contribute more evenly than on the specific frequencies which results in a wider orbit as seen in Figure 4.4.
Figure 4.4: Stub Shaft Orbit and Bearing Orbit for Case LKLC at 32.0 Hz

Figure 4.5: Stub Shaft and Bearing Orbits for Case LKLC at 34.0 Hz
The orbit of the stub shaft at 34.0 Hz tilts towards the V13 axis because this is the resonant frequency for this axis. The contribution of the W13 axis is again apparent in that the orbit does not lie on the V13 axis, however it’s contribution is much less significant than the contribution the V13 axis played at 30.5 Hz. This is apparent in the width of the orbit. Frequency response plots derived from the two internal bearing position probes showed that the W13 axis had higher damping than the V13 axis, which reduces the motion of the stub shaft in this direction resulting in a very narrow stub shaft orbit. Figure 4.4 showed a wider orbit that verifies the damping in the V13 direction is much less than that in the W13, which allows for higher vibration levels in this direction at 30.5 Hz. The vertical displacement of the bearing is very small in comparison to the stub shaft response that indicates a resonance is occurring at this frequency. As a result, the bearing has a large horizontal motion in comparison to its vertical motion, however no amplification in the horizontal displacement of the stub shaft takes place. This again is due to the force being input into the bearing in the vertical direction. The bearings horizontal motion is simply a response to the horizontal component of the stub shaft motion.

To estimate the amount of stiffness and damping separation between the two axes the model from Chapter Three has been re-visited. Recall from Chapter Three that each control axis within the bearing can be represented by an equivalent mass-spring-damper system as shown in Figure 4.6.

![Figure 4.6: Equivalent Mass-Spring-Damper Systems in Bearing axes V13 and W13](image)
The equations of motion in each axis are given by

\[ M_i \ddot{x}_V + C_V \dot{x}_V + K_V x_V = C_V \dot{y}_V + K_V y_V \ [lb] \]  
(Eq. 4.1)

and

\[ M_i \ddot{x}_W + C_W \dot{x}_W + K_W x_W = C_W \dot{y}_W + K_W y_W \ [lb] , \]  
(Eq. 4.2)

where \( C_V \) and \( C_W \) are the V13 and W13 axis equivalent damping, \( K_V \) and \( K_W \) are the V13 and W13 axis equivalent stiffness, \( x_V \) and \( x_W \) are the displacements of the stub shaft in the V13 and W13 axis, \( y_V \) and \( y_W \) are the stator displacements along the V13 and W13 axis and \( M_i \) is the stub shaft mass. A Laplace transformation of Equations 4.1 and 4.2 has been used to give the corresponding displacement transmissibilities as

\[ \frac{X_V(s)}{Y_V(s)} = \frac{C_V s + K_V}{M_i s^2 + C_V s + K_V} \]  
(Eq. 4.3)

and

\[ \frac{X_W(s)}{Y_W(s)} = \frac{C_W s + K_W}{M_i s^2 + C_W s + K_W} , \]  
(Eq. 4.4)

where \( s \) is the complex frequency variable. The electromagnetic shaker excited the bearing vertically. Therefore, these displacement transmissibilities were used to obtain the transmissibility in the vertical direction. The base excitation resulting in each of the two axes within the bearing stator is given by

\[ Y_W = Y_V = Y_B \cos 45^\circ , \]  
(Eq. 4.5)

where \( Y_W \) and \( Y_V \) are the components of the vertical base excitation, \( Y_B \), in each control axis as shown in Figure 4.6. The stub shaft displacement in the vertical and horizontal planes can be determined by summing the components of the axis displacements giving

\[ Y_r = (x_w + x_v) \sin 45^\circ \]  
(Eq. 4.6)

and

\[ X_r = (x_w - x_v) \cos 45^\circ , \]  
(Eq. 4.7)

where \( Y_r \) is the vertical stub shaft displacement and \( X_r \) is the horizontal stub shaft displacement. The relationships in Equations 4.4 – 4.7 can be used to determine the vertical displacement transmissibility from the two independent axis transmissibilities.
Substituting Equation 4.5 into Equations 4.3 and 4.4 gives

\[
\frac{X_v(s)}{Y_b(s)} = \frac{C_v s + K_v}{M_v s^2 + C_v s + K_v} \cos 45^\circ \quad \text{(Eq. 4.8)}
\]

and

\[
\frac{X_w(s)}{Y_b(s)} = \frac{C_w s + K_w}{M_w s^2 + C_w s + K_w} \cos 45^\circ \quad \text{(Eq. 4.9)}
\]

Equation 4.6 can now be used to solve for the displacement transmissibility in the vertical direction.

\[
\frac{Y_v(s)}{Y_b(s)} = \left( \frac{X_v(s)}{Y_b(s)} + \frac{X_w(s)}{Y_b(s)} \right) \sin 45^\circ = 0.5 \left( \frac{C_v s + K_v}{M_v s^2 + C_v s + K_v} + \frac{C_w s + K_w}{M_w s^2 + C_w s + K_w} \right) \quad \text{(Eq. 4.10)}
\]

The case where the stiffness and damping of the V13 and W13 are equal is a special case where equation 4.10 simplifies to the model given in equation 3.34 of Chapter Three. In this case, the stiffness and damping in any direction within the bearing stator are equivalent and the response in any direction will yield only one resonant peak corresponding to the mode in that direction. However, when the stiffness and damping parameters for each axis differ, this symmetry breaks down and equation 4.10 must be used to model the system. This model will give two resonant peaks in the displacement transmissibility representing the two degrees of freedom within the bearing stator.

An averaged equivalent stiffness for case LKLC below 100 Hz was determined in Chapter Three to be 217 lbf/in. assuming the axes stiffness and damping properties were equal. Equation 4.10 was used to approximate the separation in stiffness and damping between the V13 and W13 axis by utilizing an averaged axis stiffness and damping value for each axis. These averaged values were varied to arrive at a model that approximated the measured transmissibilities given in Figure 4.2. Figure 4.7 gives this model and the measured transmissibilities from probes A, B and C of the test rig. Table 4.2 summarizes the changes in stiffness and damping of each axis relative to the modeled equivalent stiffness and damping determined in Chapter Three.
Chapter 4  Experimental Testing and Results

Figure 4.7: Modeled Stiffness Separation Displacement Transmissibility for Case LKLC

Table 4.2: Stiffness and Damping Separation Values for Case LKLC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Equivalent Axis Value</th>
<th>Approximated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kw</td>
<td>W13 averaged equivalent stiffness</td>
<td>217 lb/in.</td>
<td>191 lb/in.</td>
</tr>
<tr>
<td>Kv</td>
<td>V13 averaged equivalent stiffness</td>
<td>217 lb/in.</td>
<td>236 lb/in.</td>
</tr>
<tr>
<td>Cw</td>
<td>W13 averaged equivalent damping</td>
<td>0.01 lb-r s²/in.</td>
<td>0.025 lb-r s²/in.</td>
</tr>
<tr>
<td>Cv</td>
<td>V13 averaged equivalent damping</td>
<td>0.01 lb-r s²/in.</td>
<td>0.01 lb-r s²/in.</td>
</tr>
</tbody>
</table>

The damping term for the W13 axis of approximately 0.025 lb-r s²/in is significantly higher than the modeled value of 0.01 lb-r s²/in. This controller gain case, LKLC, yields very low damping due to the low derivative gain of 0.025 used in the control of the axes. The damping ratio, $\xi$, for each axis has been calculated using the following relationship

$$
\xi = \frac{C}{C_{CR}} = \frac{C}{\frac{2\omega_n M_r}{}}.
$$

(Eq. 4.11)
where $C$ is the average damping value, $C_{CR}$ is the critical damping value, $\omega_n$ is the resonant frequency for each axis, and $M_r$ is the mass of the stub shaft. For both axes the damping is below 1% of critical damping, therefore a slight increase in damping in the W13 axis has resulted in a large change between the modeled and measured damping value.

This modified model of the system dynamics within the bearing stator matches the measured transmissibilities from the test rig very well up to approximately 55 Hz. Above 55 Hz a mode occurring at approximately 70 Hz begins to contribute to the response of the system and this model becomes invalid. The model also deviates slightly from the measured transmissibilities around 41 Hz where a less significant mode occurs. The phase plot of Figure 4.4 shows that at 41 Hz and 70 Hz a separation in the phase of either end of the stub shaft (B and C) occurs. This indicates a rocking of the stub shaft about the shaft center where the bearing is located. As a result, both of these modes are unaffected by the bearing stiffness and damping properties. These modes are most likely a result of the stub shaft configuration and wire support system that has placed the bearing at the stub shaft center. The transmissibilities measured from the remaining probe locations verify that the mode occurring at 70 Hz is the rigid body conical mode shape. The measured horizontal (top) and vertical (side) plane mode shapes, given in Figure 4.8, clearly show the stub shaft vibrating in the rigid body conical mode. This mode is well separated from the rigid body resonant frequencies of 30.5 Hz and 34.0 Hz and therefore does not significantly contribute to the response of the stub shaft below 55 Hz.

A small peak also occurs at approximately 20 Hz, which the model does not predict. The orbit plot for the response at 20.75 Hz is given in Figure 4.9, which shows the stub shaft responding to the vertical component of the bearing orbit. This indicates a resonance in the horizontal plane near this frequency, which could be due to the shaker exciting the base of the magnetic bearing horizontally at this frequency. The exact cause of this mode has not been determined, however the total vertical response of the stub shaft within this range is less than 4% of the overall response at 34.0 Hz. Therefore no further consideration was given to this peak.
Figure 4.8: Stub Shaft Measured Mode Shape for Case LKLC at 70 Hz

Figure 4.9: Orbits for Case LKLC at 20.75 Hz
4.2.2 Case LKHC

The model developed in Chapter Three estimates an average equivalent stiffness value of 247 lb/in and an average equivalent damping value of 0.30 lb-f-s/in. ($\xi = 0.14$) for the system operating with controller gain case LKHC loaded. The PID gain values used for this case are given in Table 4.3.

Table 4.3: PID Gains used for case LKHC

<table>
<thead>
<tr>
<th>Gain</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_I$</td>
<td>Integral Gain</td>
<td>60</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Proportional Gain ($\propto$ Stiffness)</td>
<td>40</td>
</tr>
<tr>
<td>$K_D$</td>
<td>Derivative Gain ($\propto$ Damping)</td>
<td>0.050</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Total Gain – Multiplier on all Gains</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

The low damping resulting from the LKLC gains gave a very high displacement transmissibility of 67 mils/mil at 34.0 Hz. This case differs from the LKLC case in that the derivative gain used within the PID controller has been increased from 0.025 to 0.05 resulting in an increase in the damping ratio, $\xi$, from 0.0046 to 0.14. This was done to give an increase in the system damping while minimizing the change in the stiffness characteristics of the bearing. The vertical displacement transmissibility for this case, measured from probes B and C with probe A as the input, are given in Figure 4.10. These transmissibilities are also compared to the modeled displacement transmissibility given in Chapter Three. The coherence plot for this case can be found in Appendix G.

Figure 4.10 shows that the model developed in Chapter Three, which assumes that the stiffness and damping properties of each axis are equivalent, has matched the measured transmissibilities very well up to approximately 55 Hz. The effects of the differences between the two axes has been reduced due to the introduction of higher gains into the system. The higher gains have increased the damping of the system and therefore has reduced the influence of both of the modes occurring in each axis. Therefore, the overall response of the stub shaft in this frequency range has been reduced resulting in a single well-damped peak of approximately 3.5 mils/mil at 32 Hz. This is a significant reduction in the displacement transmissibility amplitudes that resulted from the previous case, LKLC, which were approximately 13 mils/mil at 30.5 Hz and 67 mils/mil at 34 Hz.
As discussed in the previous section, the two modes occurring at approximately 41 Hz and 70 Hz are unaffected by the damping increase associated with this controller gain case. This is due to the bearing being placed at the node points of these modes.

### 4.2.3 Case HKHC

Case HKHC describes the controller gain case where the highest possible PID gains were used for which stability was maintained when levitating the stub shaft. These PID gains resulted in the highest stiffness of all three gain cases, however the gains did not provide the highest absolute damping value. This is due to the increase in the integral gain that tends to reduce the damping of the system. Recall from Chapter Three that the damping is calculated from the imaginary part of the controller transfer function. The integral gain is inversely proportional to the complex frequency term \( s (j\omega) \), therefore an increase in the integral gain actually reduces the imaginary part of the controller transfer function thereby reducing the equivalent damping. While the increase in the derivative gain has increased the critical damping ratio from 0.0046 (Case LKLC) to 0.05, the increase in the integral gain has prevented the damping value from reaching the critical damping ratio of 0.14, which was possible using the gains provided in case LKHC. The PID gains used for this case are given in Figure 4.4.
**Table 4.4:** PID Gains used for case HKHC

<table>
<thead>
<tr>
<th>Gain</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_I$</td>
<td>Integral Gain</td>
<td>80</td>
</tr>
<tr>
<td>$K_P$</td>
<td>Proportional Gain ($\propto$ Stiffness)</td>
<td>60</td>
</tr>
<tr>
<td>$K_D$</td>
<td>Derivative Gain ($\propto$ Damping)</td>
<td>0.050</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Total Gain – Multiplier on all Gains</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

The measured vertical transmissibilities for this case are given in Figure 4.11 along with the modeled transmissibility assuming the axis stiffness and damping parameters are equivalent.

![Displacement Transmissibility for Case HKHC](image)

**Figure 4.11:** Measured Vs. Modeled Displacement Transmissibility for Case HKHC

Similar to the transmissibilities measured for case LKHC, Figure 4.11 also does not show the effects of the separation of the axis parameters, which was apparent with low damping case. The damping ratio for this case of 0.05 is approximately 10 times greater than the damping ratio of 0.0046 associated with case LKLC, where the double modes are apparent. This increased damping has decreased the response corresponding to each axis.
The increase in the system damping has decreased the maximum response of the stub shaft to give a peak displacement transmissibility ratio of 10 mils/mil at approximately 47.75 Hz. The increase in stiffness of the bearing axes has resulted in this increase in the resonant frequency of the system. As a result, the response is affected more by the contribution of the conical mode occurring at 70 Hz, which accounts for the small deviations between the modeled transmissibility and the measured transmissibilities in this frequency range.

4.3 Single Frequency Linearity Tests

As mentioned in the introduction, the transmissibilities discussed in the previous section were measured utilizing a 10 millivolt multiple frequency input to the shaker. As a result, the energy exciting each frequency was low in comparison to that which could be obtained using a single frequency excitation. This was evident in the low amplitude displacement response (less than 1 mil displacement of the stub shaft) achieved using the multiple frequency input. In this range we expect the bearing to behave linearly. This assumption allowed for the removal of the higher order non-linear terms in the model discussed in Chapter Three and as shown resulted in a good match to the experimental data. However, as the vibration levels increased within the bearing stator, the displacement of the shaft becomes more significant in comparison to the nominal gap distance and the perturbation current levels increase to control these vibration levels. These conditions can lead to a non-linear or unstable response of the stub shaft. Therefore, single frequency tests were performed using a sinusoidal input to the shaker at multiple amplitudes to test the linearity of the rotor-bearing system to increases in amplitude of the base excitation.

During this testing it was discovered that the bottom bias currents clipped when the relative displacement detected within the bearing stator increased past certain levels. Originally the bias currents were to be set at 1.5 Amps (1/2 the max current of 3.0 Amps) however stable levitation was unable to be achieved at this bias current level. Therefore the bias current settings were reduced to 0.75 Amps where stable levitation could be achieved. This reduced bias current level allowed for the exploration of this system in a situation that may be dictated by a power limited configuration. The bias current setting of 0.75 Amps resulted in actual bottom bias currents of 0.365 Amps and top bias currents of 1.12 Amps. This occurred because the controller had to apply a steady increase in current to the top magnets and a steady decrease in current to the bottom magnets to support the static weight of the stub shaft. The control currents for the bottom magnets operate about this reduced bias current which limited the control current amplitude that could be used to counter any vibrations within the stator before clipping. Current clipping occurred when the requested control current was greater than 0.365 Amps which resulted in an aggregate current (Bias –
Control Current) of less than zero. When this occurred the bottom currents would simply clip and remain zero until the control current requested fell below 0.365 amps.

It was possible to predict when the bottom control currents would begin to clip based upon the measured controller transfer functions given in Chapter 3. Clipping would occur when the control current needed to control the vibrations within the bearing were equal to or greater than the 0.365 Amp bias current. The transfer functions give the ratio of controller current requested as a function of the measured relative displacement within the bearing. Therefore the clipping threshold was determined by calculating the relative displacement that corresponds to the bottom bias current level of 0.365 Amps. Equation 4.12 describes this operation:

\[
\frac{I_c(s)}{X(s)} = G(s) \Rightarrow X(s) = \frac{I_c}{G(s)},
\]

(Eq. 4.12)

where \( I_c \) has been set equal to 0.365 Amps and \( G(s) \) is known from Chapter Three. The transfer function \( G(s) \) is frequency dependent, therefore, the clipping threshold is also a function of frequency. The clipping thresholds for each gain case have been determined and are provided in Figure 4.12.

![Theoretical Bottom Current Clipping Thresholds](image)

**Figure 4.12:** Bottom Control Current Clipping Thresholds for all Gain Cases.

Figure 4.12 shows how the bottom control currents will clip when the relative displacement detected within the bearing stator increases above 3 mils for gain cases LKLC and LKHC and 2 mils for gain case HKHC. The initial low bias current setting used in this power-limited configuration has caused this clipping effect to occur at relatively low vibration levels, however this effect is still present with higher
bias current level configurations. For example, the maximum bias current setpoint that should be used for this particular bearing is 1.5 amps. The bottom bias currents would drop to 1.26 Amps to support the same static load of the stub shaft assembly. The bottom magnet control currents would then clip when the relative vibration levels increased above 10 mils for cases LKLC and LKHC and above 7 mils for case HKHC. This indicates that the control current clipping could still be an issue in bearings operating at higher bias currents under extreme loading cases.

Figure 4.13 gives an example of the bottom bias currents clipping for case LKHC at 30.5 Hz where the relative vibration was at a maximum of 15 mils.

Figure 4.13 shows the response of the stub shaft at 30.5 Hz for case LKHC as measured by the four internal position probes where Vt is the top current from V axis, Vb is the bottom current of the V axis and Wt and Wb are top and bottom currents for the W axis. This plot gives a case where the bottom currents are significantly clipped. Clipping occurred on a large number of data points taken at each frequency due to the low clipping thresholds shown in Figure 4.12. Only at the low base excitation levels did this clipping not occur. Section 4.2 of this chapter discussed in detail the linear displacement transmissibility model between the stub shaft and bearing stator and compared its output with the measured displacement transmissibilities. This clipping introduces another source for non-linearity in the system, which was not incorporated in the model developed in the previous section.
Single frequency excitation was used with varying amplitudes to examine the response of the stub shaft throughout the range of motion within the stator. This was done to determine the effects of the large vibrations, higher control currents and current clipping on the system response. A plot of the base excitation amplitude versus the amplitude of the vertical response of the stub shaft was used to determine the linearity of the system at specific frequencies. Horizontal motion of the bearing and stub shaft assembly was also monitored for indications of cross-coupling. The ratio of the base excitation amplitude and the stub shaft assembly amplitude reflects the displacement transmissibility value at the particular frequency. Therefore, if the system responds linearly, the plot should reveal a line with the slope given by the displacement transmissibility at that frequency as determined in the previous sections. The setup described in Chapter Two describing the single frequency test setup was used to excite the bearing and record the response of the stub shaft and the bearing at the different amplitudes.

### 4.3.1 Case LKLC

Single frequency linearity tests were performed at 25 Hz, 30.5 Hz, 32 Hz, 34 Hz and 45 Hz for case LKLC based upon the measured displacement transmissibilities. Frequency 30.5 Hz and 34 Hz represent the resonant frequencies of the system and 25 Hz and 45 Hz give a low and high frequency non-resonant response. At each frequency the base excitation amplitude was incremented and the stub shaft response and base excitation signals were recorded. The base excitation vibration level was increased until the vibration limit within the bearing clearance was reached. Figure 4.14 gives an example of the time response of the stub shaft (Rotor) and the time history of the base excitation input (Stator) for case LKLC at 25.0 Hz.

Figure 4.14 shows how a 3.78 mil amplitude, 25 Hz sinusoidal base excitation resulted in an 8.08 mil amplitude sinusoidal response giving a measured displacement transmissibility ratio of 2.14 mils/mil. This transmissibility was determined from a single point of data resulting from a single base excitation amplitude. A better estimation of the measured displacement transmissibility at this frequency has been determined by performing a least squares curve fit of all the data points representing the full range of motion within the stator. Each data point was determined by finding the amplitudes of the base excitation and the stub shaft response from the frequency spectrum of each signal. Figure 4.14 also shows an example of the frequency spectrum of the base excitation and stub shaft response for case LKLC at 25.0 Hz. Similar plots for the other tested frequencies (30.5 Hz, 32 Hz & 45 Hz) and the other gain cases are given in Appendix K which reveal the phase information about the stub shaft response at each frequency.
Figure 4.14: Response of Stub Shaft and Bearing for Case LKLC at 25.0 Hz

Figure 4.15 gives the plot consisting of all the points tested at this frequency and the linear squares curve fit.

Figure 4.15: Base Excitation Amplitude versus Stub Shaft Response for Case LKLC at 25.0 Hz
Recall that the slope of the line formed by the data points represents the displacement transmissibility because this is the ratio of the stub shaft response amplitude to the base excitation amplitude. The linear least squares curve fit also allows for the calculation of the R-squared value which is a measure of the linearity of the data points. A perfect line would give an R-squared value of one, while purely random data should give an R-squared value of zero. The curve fit gave a displacement transmissibility value of 2.205 mils/mil and an R-squared value of 0.999 for the data collected for Case LKLC at 25 Hz given in Figure 4.15. The R-squared value and visual inspection of Figure 4.15 indicate that the system is highly linear at this frequency. The uncertainty of the estimated displacement transmissibility has also been calculated based on the residuals of the measured response versus the fitted response values. The results indicate that the displacement transmissibility falls between 2.165 mils/mil and 2.245 mils/mil with 95% confidence. Appendix I describes the method for determining this uncertainty based on a 95% confidence interval.

Although the R-squared value for the single frequency test indicated a highly linear result, a slight non-linearity is present. This is evident in Figure 4.15 from the data points which have a definite pattern about the least squares fit line. This indicates that a slight non-linear trend does exist in the data. This can be seen more easily from a residual plot between the least squares curve fit line and the data points. Figure 4.16 gives a plot showing the residuals determined from the linear least squares curve fit line and the data points measured for case LKLC at 25.0 Hz.

![Residual Plot for Case LKLC at 25.0 Hz](image)

**Figure 4.16:** Residual Plot for Single Frequency Test for Case LKLC at 25.0 Hz
These same single frequency test was performed at the resonant frequencies of 30.5 Hz and the results are presented in Figure 4.17. The slight non-linearity that was apparent at 25.0 Hz is more pronounced at this resonant frequency which is reflected in the lower R-squared value of 0.9720. The curve fit estimates the displacement transmissibility at this frequency to be 6.51 mils/mil with a 95% confidence bound of ±0.5728 mils/mil. The single axis model predicts the displacement transmissibility at this frequency to be 7.58 mils/mil. However these results are misleading at this frequency due to the separation of axis parameters apparent for this gain case (LKLC), which was demonstrated earlier in this chapter. It should be assumed coincidental that the two displacement transmissibility values are relatively close because the value determined from the single frequency test represents a peak in the stub shaft response whereas the value determined by the original model (equal axis parameters) represents a point prior to the resonant frequency of that system.

![Figure 4.17](image)

**Figure 4.17** Base Excitation Amplitude versus Stub Shaft Response for Case LKLC at 30.5 Hz

Figure 4.18 gives the results obtained from the same single frequency tests performed at 32.0 Hz which lies between the two resonant frequencies of this system.
Figure 4.18:  Base Excitation Amplitude versus Stub Shaft Response for Case LKLC at 32.0 Hz

This figure again shows a highly non-linear response of the stub shaft throughout the range of motion in the stator due to its proximity to both resonant frequencies. The linear least squares curve fit determined a displacement transmissibility value of 3.09 mils/mil at this frequency, however the $R^2$ value of 0.9191 and the data points show the non-linear response of the system. Recall that at this frequency the original model predicted a displacement transmissibility of  21.98 mils/mil due to the equivalent axis assumption. However the separation of axis parameters (stiffness and damping) have reduced the displacement transmissibility at this frequency. The general model created to mimic the separation of axis parameters predicted a transmissibility ratio of 1.99 mils/mil while the multiple frequency testing gave a ratio of 3.87 mils/mil. The non-linearity of the system explains the discrepancies in these values because the models are based on linear assumptions which have been violated in this frequency range.

An attempt to perform the multiple amplitude test at 34.0 Hz failed due to the shakers inability to produce a pure sinusoidal tone at this frequency. This frequency represents a resonant peak giving displacement transmissibility ratio of approximately 56 mils/mil. This is a highly un-damped resonant peak that is believed to have caused significant reaction forces which the shaker could not overcome. Based upon the results obtained from the 25.0 Hz, 30.5 Hz, and 32.0 Hz data it is expected that this frequency is also highly non-linear.

The final frequency tested for this case was 45.0 Hz. The results of this test are given in Figure 4.19.
The results from this frequency gave similar results as recorded at 25.0 Hz. This occurred because this frequency is removed from the resonant frequencies of the system and therefore the non-linear effects on the system are reduced. The measured transmissibility from this test was determined to be 1.28 mils/mil from the linear least squares curve fit with a 95% confidence interval of ±0.0166 mils/mil. The equivalent axis model predicted a transmissibility ratio of 1.15 mils/mil and the low amplitude multiple frequency test resulted in a 1.21 mils/mil transmissibility.

Table 4.5 Summarizes the comparisons made in this section which compare the modeled and measured transmissibility ratios determined for each frequency for this case (LKLC). The last column gives the percent difference between the original model, which assumed equivalent axis parameters, and the single frequency test results.

<table>
<thead>
<tr>
<th>Case LKLC Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>30.5</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>45</td>
</tr>
</tbody>
</table>
The differences between the modeled transmissibility ratios and the measured is related both to the non-linearities in the system and the separation of the axis parameters. The separation of the axis parameters affects the frequencies of 30.5 Hz and 32 Hz greatly because these frequencies are on or near the resonant peaks of the system.

### 4.3.2 Case LKHC

Case LKHC incorporated an increase in the derivative gain in the PID controller to increase the damping of the system in comparison to gain case LKLC. The damping was increased 30 times over case LKLC to give an increase from 0.01 lb·s/in to 0.30 lb·s/in. This corresponds to an increase in the critical damping ratio, $\xi$, from 0.0046 to 0.14. This damping and stiffness increase has reduced the measured displacement transmissibility values at 30.5 Hz and 34.0 Hz for case LKLC from 13.4 mils/mil and 56 mils/mil to a single peak at 34 Hz of 3.5 mils/mil. This significant reduction in the displacement transmissibilities prevented the shaker limits from being reached during testing as believed to have occurred at 34.0 Hz for case LKLC.

**Figure 4.20:** Base Excitation Amplitude versus Stub Shaft Response for Case LKHC at 30.5 Hz

Figure 4.20 gives the results of the multiple amplitude tests performed at 30.5 Hz for case LKHC. A linear least squares curve fit has been performed on the data and gives a displacement transmissibility ratio of 3.09 mils/mil with an uncertainty of 0.0786 mils/mil and an R-squared value of 0.9979. Both the figure and the R-squared value indicate that the response of the system at this frequency is highly linear. The displacement transmissibility predicted by the equivalent axis stiffness model is 3.53 mils/mil. The
The difference between the modeled value and the curve fit value is approximately 11.4%. The equivalent axis model more accurately describes the response of the system due to the higher gains within the system. The increased gains made the system less sensitive to the slight differences in the axis parameters and resulted in the expected single peak in the displacement transmissibility. Also, similar to gain case LKLC, a slight non-linearity in the response in the stub shaft is apparent both from Figure 4.20 and the R-squared value.

Figure 4.21 gives the results of the single frequency testing performed at 34.0 Hz for case LKHC.

Figure 4.21 gives the curve fit displacement transmissibility as 3.72 mils/mil and an R-squared value of 0.999. The figure and the R-squared value both indicate the response of the system at this frequency is also highly linear. The equivalent axis stiffness model predicts the displacement transmissibility to be 3.725 mils/mil giving nearly zero error between the modeled and measured displacement transmissibility. Table 4.6 gives the results obtained from the single frequency tests and summarizes the differences between the modeled and measured displacement transmissibilities for this case.
Table 4.6: Results for Case LKHC

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Modeled (mils/mil) Equal Axis</th>
<th>Measured (mils/mil) Single frequency</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.5</td>
<td>3.53</td>
<td>3.09 ± 0.078</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>3.72</td>
<td>3.72 ± 0.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>3.46</td>
<td>11.4 %</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>3.56</td>
<td>0 %</td>
</tr>
</tbody>
</table>

It appears that the increase in damping that is associated with this gain case (LKHC) has resulted in better agreement between the experimental results and the linear model of the system.

4.3.3 HKHC

Case HKHC incorporated an increase in all the gains (proportional, integral and derivative) in the PID controller, which increased both the stiffness and damping of the system in comparison with case LKLC. The damping was increased 16 times over case LKLC to give an increase in the critical damping ratio, $\xi$, from 0.0046 to 0.05 and the stiffness was increased 2.2 times from 217 lb/in to 478 lbf/in. This stiffness increase is accomplished by the increase in proportional gain which causes the controller to request higher currents for a given relative displacement. As a result the bottom control currents clip at a lower relative displacement level as shown earlier in Figure 4.12. The increased stiffness and damping has also increased the resonant peak of the system to 47.75 Hz and gave a modeled displacement transmissibility ratio at this frequency of 9.78 mils/mil. Single frequency tests were performed at 40.75 Hz, 47.75 Hz and 52 Hz for this gain case to test the linearity of the system at these frequencies. Figure 4.22 gives the results of this test for case HKHC at 40.75 Hz.

![Figure 4.22: Base Excitation Amplitude versus Stub Shaft Response for Case HKHC at 40.75 Hz](image)
A linear least squares curve fit was performed on the results obtained from the multiple amplitude test performed at 40.75 Hz for case HKHC as can be seen in Figure 4.22. The figure and the R-squared value of 0.9987 both indicate that the system is highly linear. However, only the first 10 data points corresponding to a maximum base excitation amplitude of 2.28 mils and relative displacement of approximately 10 mils could be used due to an instability within the system at higher base excitation inputs.

The system became unstable when the amplitude of the control current requested grew to greater than 0.88 Amps in amplitude. Within the MB350™ controller a maximum current limitation of 2.0 amps had been set to avoid integrator windup and saturation of the bearing. The top bias current for the bearing was set for 1.12 Amps, therefore only a 0.88 Amp current was needed to reach this 2.0 Amp limit. Figure 4.23 shows how the vibration levels and high stiffness requirement resulted in higher current requests for this amplitude of base excitation and resulted in the top currents clipping at the 2.0 Amp level.

![Figure 4.23: Instability at 40.75 Hz for case HKHC](image)

Figure 4.23 shows the time history recorded by the internal bearing probes when the instability initially began. The control current clipping is also shown in this figure where Vt is the top current from V axis, Vb is the bottom current of the V axis and Wt and Wb are top and bottom currents for the W axis. The bottom currents were already being significantly clipped when the top control currents began clipping at
the 2.0 Amp limit. With both the top and bottom control currents clipped the bearing lost control of the stub shaft and the system went unstable when the base excitation amplitude increased above approximately 2.3 mils and the stub shaft vibration increased above approximately 9.8 mils. This instability is a direct result of the increase in stiffness associated with this case. Higher control currents are requested to achieve this higher stiffness level, therefore the clipping of the top control current becomes a stability issue at lower vibration levels.

The displacement transmissibility determined from the least square curve fit of the stable response data was 4.06 mils/mil with a 95% confidence bound of ±0.088 mils/mil while the equivalent axis stiffness model gave a displacement transmissibility ratio of 3.98 mils/mil. This value falls within the confidence bound of the measured transmissibility and indicates the system behaves linearly and the model is very accurate for this frequency and gain case (HKHC) prior to the instability.

The multiple amplitude test was also performed at 47.75 Hz and 52.0 Hz for case HKHC with very similar results. At both these frequencies, when the requested control current amplitude increased above 0.88 Amps the system became unstable due to the 2.0 Amp current windup limit. Figure 4.24 gives the multiple amplitude test results obtained at 47.75 Hz.

![Figure 4.24: Base Excitation Amplitude versus Stub Shaft Response for Case HKHC at 47.75 Hz](image)
Figure 4.24 shows the curve fit displacement transmissibility ratio as 8.54 mils/mil ± 0.144 mils/mil and the R-squared value of 0.9996. The equivalent axis stiffness model predicted a displacement transmissibility ratio of 9.75 mils/mil which yields an error of approximately 12% from the curve fit value. Both the figure and the R-squared value indicate that the system response is highly linear at this frequency below the instability margin. The instability occurs when the base excitation amplitude increases greater than 0.8 mils and the corresponding stub shaft vibration levels increase greater than approximately 6.7 mils. This again corresponds with the level at which the control currents clip on both the top and bottom (0.88 Amp control current amplitude) of their waveform as shown in Figure 4.25.

Figure 4.25: Instability at 47.75 Hz for case HKHC

Figure 4.26 gives the results of the multiple amplitude test at 52.0 Hz for case HKHC. Figure 4.26 shows the curve fit displacement transmissibility ratio as 4.74 mils/mil ± 0.253 mils/mil and the R-squared value of 0.9955. The modeled transmissibility for this frequency was calculated to be 5.02 mils/mil giving a difference of 0.5% between the modeled and measured transmissibility ratio. At this frequency the system remains highly linear as indicated by the R-squared value of 0.9955 until the instability margin is reached. At this frequency the system becomes unstable when the base excitation amplitude becomes greater than approximately 1.2 mils and the corresponding stub shaft amplitude becomes greater than approximately 5.6 mils. Figure 4.27 gives the time history of the internal position signals and the control currents at the instability margin.
Figure 4.26: Base Excitation Amplitude versus Stub Shaft Response for Case HKHC at 52.0 Hz

Figure 4.27: Instability at 52.0 Hz for case HKHC

Figure 4.27 shows how the top currents begin to clip along with the bottom control currents creating the loss of stability within the system. Unfortunately the initial moment when the system became unstable
was not captured in the time record due to the data acquisition system reaching the end of a data block, which is apparent in Figure 4.27 at 3.0 seconds.

Table 4.7 summarizes the results obtained for case HKHC.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Modeled (mils/mil)</th>
<th>Measured (mils/mil)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equal Axis</td>
<td>Unequal Axis</td>
<td>Single frequency</td>
</tr>
<tr>
<td>40.75</td>
<td>3.52</td>
<td>NA</td>
<td>4.06 ± 0.088</td>
</tr>
<tr>
<td>47.75</td>
<td>9.77</td>
<td>NA</td>
<td>8.54 ± 0.144</td>
</tr>
<tr>
<td>52.0</td>
<td>5.02</td>
<td>NA</td>
<td>4.74 ± 0.253</td>
</tr>
</tbody>
</table>

4.4 Experimental Measurements Overview

Two distinct sets of measurements were discussed in this chapter which included the multiple frequency system response presented in section 4.2 and the single frequency system response presented in section 4.3. Both these measurement techniques proved to be important in uncovering unique system attributes and limits. The multiple frequency tests uncovered the separation of axis parameters which occurred while utilizing gain case LKLC with the AMB controller. Also, the single frequency tests were able to indicate the non-linear behavior for specific frequencies and unstable behavior.

In general the model predictions have been compared with the transmissibilities determined from the single frequency tests. The results from the single frequency testing are expected to be more accurate due to the numerous samples taken at one frequency and over the multiple range of input amplitudes. The multiple frequency tests were used simply as a guide by giving a map of the displacement transmissibility over the entire frequency range. This was helpful in determining the specific frequencies at which to focus on more closely.
Chapter 5
Discussion and Conclusions

5.1 Conclusions

The following is a listing of the major findings and conclusions resulting from the experimental measurements and the modeling results. The rest of this chapter is dedicated to a discussion and future work as it relates to this research.

- Well tuned AMB systems are highly linear systems.
- Non-Linear modeling of an the AMB system is an excellent baseline for the design of well-tuned systems.
- AMB systems of similar design to that utilized in this testing are highly insensitive to bottom current clipping.
- Differences between two axes in a radial bearing will always exist which can lead to split resonant peaks when the tuning of the system does not provide appreciable damping or stiffness.
- Hardware limitations tend to be the limiting element for AMB systems (i.e. Current Clipping, power requirements, axis equivalence …)

5.2 Overview of Work Completed

A preliminary investigation into the effects of base excitation of a radial AMB has been performed. This investigation incorporated the design and construction of a test rig to experimentally measure the response of a rigid stub shaft due to base excitation propagating through a radial AMB bearing housing. A linear model of this system was also created and the results from both the experimental measurements and the model predictions have been compared for three distinct sets of bearing parameters. These three cases were chosen to give varying stiffness and damping levels to determined how this affected the response of the system.

The test rig was designed to excite the base of a single MBRetro™ (of Revolve Magnetic Bearings) AMB with a variety of input signal types originating from an electromagnetic shaker. The AMB supported a non-rotating, rigid stub shaft which was also supported by music wires on either end in the axial direction. This configuration allowed the stub shaft to vibrate freely in the vertical and horizontal planes while and
providing moment control and axial support to the shaft. A total of nine positions were recorded which allowed for the motion of both the bearing and stub shaft assembly to be fully characterized.

Several steps were taken to ensure the test rig performed as designed. The stiffness of the music wire support system was measured and determined to be negligible in comparison to the vertical and horizontal stiffness of the bearing for all three gain cases. The stub shaft assembly was also modeled in the VTFastae software program (Virginia Tech Rotor Dynamics Laboratory) to ensure the bending modes of the shaft were out of the frequency range of interest. A free-free modal analysis was also performed on the MBRetro™ radial AMB to ensure it acted as a rigid body in the desired frequency range of interest (0-100Hz). During testing, two modes were detected within the desired frequency range of 0 – 100 Hz which were determined to be due to the wire support system. These modes were located at approximately 40 Hz and 70 Hz. The resonant frequencies which resulted from the use of all three gain cases (LKLC, LKHC, and HKHC) were well below these frequencies and therefore these modes did not interfere with the testing.

This test rig was used to perform two distinct tests on the AMB/stub shaft system for each gain case. For each case the electromagnetic shaker was used to determine the frequency response function between the vertical position of the bearing and the remaining positions probes. This information was used to construct the displacement transmissibility of the system between the vertical bearing position and the vertical position of the stub shaft at the bearing location. These displacement transmissibilities were used to understand the overall response of the system and locate the resonant frequencies for each gain case. However, these displacement transmissibilities could provide no insight into the linearity of the system because they were determined while exciting the bearing with a low level multiple frequency input signal. Therefore, the test rig was also used to excite the system at specific frequencies with varying amplitudes to test the linearity of the system.

In addition to the experimental testing, a linear model of the displacement transmissibility between the vertical bearing position and the vertical stub shaft position was also developed. The model incorporated the MB350™ controller with the associated gains, the bearing axes, and the stub shaft to arrive at a complete model of the displacement transmissibility. The force equation for each of the bearing axes was linearized and no cross coupling between the bearing axes was assumed, which is a common practice in the modeling of AMB systems. The model of the MB350™ controller channels was compared to measured transfer functions from the controller and determined to match very well. The models for each
gain case were then compared to the experimental results obtained from the single frequency testing to determine the limitations of the linearized models.

### 5.3 Discussion of Results

Before an assessment of the model’s performance can be made a discussion of the experimental data and the results must be discussed to better explain the complexities of the system that were not consider in the model described in Chapter Three. For example, the bottom control current clipping and the separation of the axes parameters, which were discovered during testing, were unexpected system complexities that were not considered in the model described in Chapter Three.

Originally the comparison between the experimental results and the modeled results were envisioned to indicate the limits of the linearized net force equation used in the development of the model. However, the experimental testing revealed that the bottom control currents clipped when the base excitation levels increased beyond pre-determined limits based upon the controller gain case. This clipping occurred for all three gain cases but was of large enough magnitude for case HKHC that it created an instability in the system due to the top currents also clipping. The magnitude of the clipping increased as the stiffness of the bearing increased due to the increased demand for current. For the high stiffness case, HKHC, the stiffness was high enough to cause the top control currents to reach the integrator wind up limit, which had been set to 2.0 Amps. At this point both the top and bottom control currents were clipped which resulted in loss of control of the stub shaft and the instabilities presented in Chapter Four occurred. The other gain cases did not have this instability due to the reduced stiffness and therefore avoided clipping the top current. In these cases, the tests were halted when mechanical contact between the bearing stator and stub shaft assembly was detected.

The experimental test setup for which the testing was performed had a reduced bias currents, which contributed to the early clipping of the control currents. The bias currents for this testing were set to 0.75 Amps. In general the bias currents should be set to $\frac{1}{2}$ the total current range available to the AMB. For the MBRetro™ AMB this is 3.0 Amps, therefore the ideal bias current set points should be 1.5 Amps. However stable levitation of the stub shaft rotor was unable to be accomplished with the bias currents set at this level. As a result the operating bottom bias current (after the addition of the stub shaft weight) was 0.365 Amps and the operating top bias current was 1.12 Amps. Therefore the bearing could only give a maximum control current of amplitude 0.365 Amps before a net current of zero resulted by the control current canceling out the steady bias current level. This reduction in the bias currents also represents the
potential of a power limited configuration within a AMB/rotor system that may be present in a vehicular application. This represents an extreme case where power conservation would be performed. The results from the testing for this case indicate that the current clipping does not present a serious problem until the top control currents clip along with the bottom control currents. In the case of the experimental testing, instability occurred only in the highest stiffness case, HKHC, and the instability was caused by the integrator windup limit.

Suprisingly, the current clipping did not seem to have a large affect on the linearity of the response of the system as long as stability was maintained. For example, for case LKLC at 25 Hz the bottom control current did not clip until the base excitation level increased above 3.2 mils. Figure 5.1, which gives the response of the system at this frequency, shows that no significant change in the system linearity appears to have occurred after the bottom current begins to clip.

![Figure 5.1](image)

**Figure 5.1:** Example of System Insensitivity to Current Clipping for Case LKLC at 25.0 Hz

The bottom control current clipping certainly is an added mechanism for non-linearity in the system, however the measured responses from the system seem to indicate a relatively low sensitivity to this occurrence.

The experimental testing also revealed that a difference existed in the stiffness and damping properties of the two separate control axes within the AMB. This was most apparent from the displacement...
transmissibility measured for case LKLC with the low level multiple frequency input. This test revealed a split peak where the resonant frequency of the system was predicted by the model. The split peak indicates a difference in the stiffness and damping properties of the two separate control axes within the bearing.

The exact cause of the differences between the two axes has not been determined. However, there are several reasons why this may have occurred. Slight differences in the physical configuration of the two sets of actuators within the bearing can produce this result due to the dependence on geometry for the equal force application to the rotor. It is believed that the control system was most likely the source for this separation in this system. Because this system is electrical, the two channels which independently control the axes may have slightly different noise levels and/or offset errors. The measured transfer functions of the controller channels did show some noise and this noise level was not equal in all channels. Appendix E gives the measured transfer functions of the two controller channels responsible for the axes, V13 and W13, within the AMB for all three gain cases. These figures clearly show that the V13 axis within the bearing contained more noise than did the W13 axis in general and is more pronounced for case LKLC where the separate peaks are the most pronounced.

Case LKLC demonstrated non-linear behavior of the system at and near the resonant frequencies of the system. Figure 5.3 summarizes the responses of the system to the multiple amplitude inputs operating with gain case LKLC. Both frequencies 25 Hz and 45 Hz show a highly linear response whereas the frequencies 30.5 Hz and 32 Hz show a non-linear response. The linear model does a poor job of modeling the response at the frequencies of 30.5 Hz and 32 Hz due to the non-linear behavior of the response at these frequencies. At this point the exact cause of the non-linear response of the system has not been determined and the determination of the cause of this non-linearity is considered out of the scope of this project.
Figure 5.2: Measured and Single Axis Modeled Vertical Displacement Transmissibilities for Case LKLC

Both the separation of axis parameters and the non-linear response do not present a significant problem for this test set-up if the bearings are tuned with appreciable damping as in cases LKHC and HKHC. The
damping ratio utilized for case LKLC is potentially unrealistic for the operation of an actual system and was simply used to investigate the properties of radial AMB systems undergoing base excitation with extremely low damping. This case does give an indication of what limitations are presented if the derivative gain of an AMB system where to be lowered in an effort to lower power consumption of the bearing. It is important to be aware of the benefit of damping in the bearings to negate the effects of the axis differences. This also gives a general restriction on the use of the model developed in Chapter 3, which assumed identical axes within the bearing and a linear response. The model appears to break down with tunings that do not produce sufficient damping levels at some point dependent on the magnitude and impact of the differences between the two control axes.

Case LKHC was tested as a nominal setting that provides an increase of damping over the LKLC case and approximately the same stiffness. The original SDOF model performed very well when applied to this gain case. Figures 5.4 and 5.5 show both the frequency response and the single frequency results corresponding to this case. The model was able to successfully predict the resonant peak of the system within 3.5% and the displacement transmissibility values within 13% of the measured value. The response of the system was highly linear at both frequencies tested.

![Figure 5.4: Measured Vs. Modeled Displacement Transmissibility for Case LKHC](image)

Figure 5.4: Measured Vs. Modeled Displacement Transmissibility for Case LKHC
The original SDOF model also performed very well for the high stiffness case of HKHC. Similarly to case LKHC, the response of the system was highly linear and the model was able to predict the resonant frequency of the system prior to the system instability within 2.8% and estimated the displacement transmissibility within 13%. The system instability occurred due to the relatively high stiffness required by this gain case. The increased stiffness resulted in the increase in current requested to control the vibration levels within the bearing stator. An integrator windup limit had been set at 2.0 Amps which resulted in the top control current clipping along with the bottom currents when the input base excitation amplitude increased beyond a base excitation level as small as 0.8 mils. The clipping of both the top and bottom control currents caused the loss of stability within the bearing. Figures 5.6 and 5.7 summarize the results for this case.
Figure 5.7:  Single Frequency System Response Summary for Case HKHC

The results from all the testing verify the expectation that the gain cases used has a large effect on the amounts of base excitation the bearing/rotor system can handle. For example, the highest gain case HKHC, which would consume the greatest amount of power during operation, resulted in instability at very low base excitation amplitudes. This was due to the high stiffness and damping which tended to draw more current and violated the integrator windup current level limit. The power limited configuration, LKLC, could only handle approximately a 1.2 mil base excitation at the resonant frequency of the system before the stub shaft would contact the bearing stator. This was due to the low stiffness and damping this case applied to the stub shaft that allowed higher response vibration of the stub shaft. The “balanced” gain case, LKHC, was the most forgiving as expected because this case represented a well tuned controller for the system. This case could handle approximately 3mils of base excitation at the resonant frequency of the system before contact between the stub shaft and the bearing stator occurred. Table 5.1 summarizes the base excitation limits each gain case imposed on the system due to either stub shaft to bearing contact or instability.

Table 5.1:  Base Excitation Upper Limits for Each Gain Case

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Excitation Amplitude Maximum (mils)</th>
<th>Resonant Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKLC</td>
<td>0.8</td>
<td>47.75</td>
</tr>
<tr>
<td>LKHC</td>
<td>3</td>
<td>32</td>
</tr>
<tr>
<td>HKHC</td>
<td>1.2</td>
<td>3</td>
</tr>
</tbody>
</table>
5.4 Conclusions and Future Work

The purpose of this project was to take an initial look at the characterization of radial AMB systems undergoing base excitation and understand the limits of a common linear modeling method. The test rig was designed to specifically exam the properties of a single AMB undergoing base excitation. The intent was to understand the features and limitations of this important element undergoing this specific excitation. It is hoped that this knowledge can then be used as a useful reference in the design and development of a rotor/bearing system undergoing base excitation such as in a power-limited vehicular application.

In general the model developed in Chapter Three proved to be an excellent baseline guide to the behavior of the AMB undergoing base excitation. The model however should only be used as a guide for designing and developing an AMB rotor/bearing system. As shown in Chapter Four the model does have some obvious limitations and cannot properly model the system under certain cases. For example, these tests indicate that systems with extremely low damping have a greater potential for having a non-linear response and a separation of axis parameters, which the model does not compensate for. The model also does not take into account the hard limits such as the integrator current wind up limit and the bottom current clipping. These limits could be incorporated into the system model but are beyond the scope of this project. In general the data gathered shows that the model performs very well when operating with a nominal gain case that will yield appreciable stiffness and damping values for this system. In addition the model also performed well with the bottom control currents clipping, which occurred for all three gain cases.

In the selection or tuning of an actual system the limitations encountered here should be taken into consideration. In the case where an extremely stiff bearing is required the hard limits for currents must be large enough to prevent instabilities. The instabilities discovered during testing of the high stiffness case proved to be very abrupt and gave no prior warning. In the extreme case of a power-limited configuration, where gains and bias currents may be reduced, non-linear responses may be encountered. In particular, asymmetries between control axes could become appreciable resulting in split natural frequencies. Also, low gains may result in a regime of non-linear response as amplitudes are increased.

Future work could examine this low damping case and attempt to verify the non-linear mechanism that may be causing the response. Evidence from the experimental results obtained during this testing indicate the assumptions made to obtain a linearize bearing stiffness in Chapter Three were good for two of the
three parameter cases tested. During the testing of these cases the vibration level of the stub shaft assembly was pushed all the way to the stator where the vibration level is exactly the static gap distance. In most cases the system response remained linear all the way out to this extremity, indicating that the system has a low sensitivity to large motion in the gap. Similarly, the same can be stated about the assumption that the control currents would be small in comparison to the bias currents. In fact, during testing the bottom bias currents actually clipped with no apparent affect on the linearity of the system. Again this indicates a low sensitivity to the control current level and validates the assumption that control currents are much smaller made in Chapter Three in obtaining a linearized stiffness.

Future work could also include the modeling and testing of an actual rotor/bearing system undergoing rotor forcing and different types of base excitation to the bearing housings. The base excitation could take several forms such as impact loading, steady sub-synchronous, steady synchronous, and steady super-synchronous. All of these are possible forcing configurations that could occur in vehicular applications and others. Modeling for the aggregate system would include a similar linear model to what has been presented here.
References


Figure A.1: Front Assembly View of Test Rig
Figure A.2: Side Assembly view of Test Rig
Appendix B
Stub Shaft Assembly VTFast\textsubscript{se} Modeling

This Appendix details the rotor modeling performed on the stub shaft assembly described in section 2.4 of Chapter 2. The software program VTFast\textsubscript{se} from the Virginia Tech Rotor Dynamics Laboratory has been used to model the assembly and determine the systems’ first two mode shapes and corresponding frequencies. A conservative model of the assembly has been developed which consists of a simple cylindrical shaft with external weights added to the ends of the shaft to represent the wire mounting plates and a distributed load applied to the center to represent the rotor lamination stack. No additional stiffness has been incorporated due to these components. Figure B.1 shows the rotor model used to describe the Stub Shaft Assembly.

<table>
<thead>
<tr>
<th>VIRGINIA TECH ROTOR LAB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Length of Rotor</strong>: 4.00 in.</td>
</tr>
<tr>
<td><strong>Maximum Weight Diameter</strong>: 0.38 in.</td>
</tr>
<tr>
<td><strong>Maximum Stiff. Diameter</strong>: 0.38 in.</td>
</tr>
<tr>
<td><strong>Maximum External Load</strong>: 0.34 lbf</td>
</tr>
<tr>
<td><strong>Magenta</strong> : Ext. Load at Station</td>
</tr>
</tbody>
</table>

![Figure B.1: Stub Shaft Assembly VTFast\textsubscript{se} Rotor Model](image)

A single bearing was placed in the center of the rotor model to represent the MB\textsuperscript{Retro} radial magnetic bearing. An additional bearing had to be used in order for the program to run properly. Therefore a bearing was added on the right end of the shaft with a stiffness of 0.1% the magnetic bearing stiffness. Three stiffness cases were considered for the magnetic bearing which correspond to the stiffness values obtained when using the three different controller parameters as detailed in Chapter 3. Table B.1 gives the controller case and the corresponding stiffness values used to model the stub shaft assembly.
Table B.1: Stiffness Cases for Stub Shaft Assembly Model

<table>
<thead>
<tr>
<th>Controller Case</th>
<th>Stiffness (lb/in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKLC</td>
<td>217.0</td>
</tr>
<tr>
<td>LKHC</td>
<td>247.0</td>
</tr>
<tr>
<td>HKHC</td>
<td>478.0</td>
</tr>
</tbody>
</table>

Figures B.2 – B.4 give the mode shapes and the corresponding frequencies for each of the stiffness cases given in Table B.1. This was done by using the program CRTSPD.EXE of the VTFastse software and utilizing the nospin option to calculate the lateral modes of the shaft.

Figure B.2: Mode Shapes for Stub Shaft Assembly for Stiffness Case L
Figure B.3: Mode Shapes for Stub Shaft Assembly for Case H

Figure B.4: Mode Shapes for Stub Shaft Assembly for Case K
Figures B.2 - B.4 all show the first mode being the rigid body bounce mode as desired for the test rig design with resonant frequencies all well below 100 Hz. The second modes for each case are all the first bending mode of the shaft which occur well above 100 Hz as was needed. Figures B.2-B.4 give the corresponding frequencies for each of the mode shapes in RPM, which have been converted to Hertz in Table B.2.

<table>
<thead>
<tr>
<th>Controller Case</th>
<th>1st Mode</th>
<th>2nd Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKLC</td>
<td>31.3 Hz</td>
<td>757.1 Hz</td>
</tr>
<tr>
<td>LKHC</td>
<td>32.5 Hz</td>
<td>757.2 Hz</td>
</tr>
<tr>
<td>HKHC</td>
<td>45.2 Hz</td>
<td>758.6 Hz</td>
</tr>
</tbody>
</table>
Appendix C
Wire Support Stiffness Measurement

Figure C.1 shows the experimental setup used to measure the stiffness of the wire support system.

The wire support stiffness was measured utilizing a spring-loaded force gauge and braided aircraft wire used to connect to the stub shaft assembly. The bearing was removed from the assembly to allow access to the stub shaft assembly and also allow room for an eddy current position probe to be placed between the stub shaft assembly and the shaker. The spring loaded force gauge was attached to an overhead crane that was used to incrementally load the stub shaft assembly while measuring the static deflection. Figure C.2 gives the results obtained from this testing.
Figure C.2: Wire Support Stiffness Measurement Results

The results from the stiffness test was curve fit with a linear least squares curve fit and the resulting stiffness from this fit was determined to be approximately 1.7 lbs/in.
Appendix D

Sensor Calibration Curves and Sensitivities

D.1 Measured Calibration Curves

**Eddy Current Probe A Calibration Curve**

\[ \text{mils} = -5.0588 \text{Volts} + 9.6415 \]

Sensitivity Uncertainty (95% CI) = 0.023132 mils/Volt

**Figure D.1:** Calibration Curve for Probe A

**Eddy Current Probe B Calibration Curve**

\[ \text{mils} = -5.2343 \text{Volts} + 9.7281 \]

Sensitivity Uncertainty (95% CI) = 0.023761 mils/Volt

**Figure D.2:** Calibration Curve for Probe B
Appendix D

Sensor Calibration

Figure D.3: Calibration Curve for Probe C

\[
mils = -5.2075\text{Volts} + 7.7267
\]

Sensitivity Uncertainty (95% CI) = 0.032414mils/Volt

Figure D.4: Calibration Curve for Probe D

\[
mils = -5.223\text{Volts} + 0.3844
\]

Sensitivity Uncertainty (95% CI) = 0.02863mils/Volt
**Figure D.5:** Calibration Curve for Probe E

**Figure D.6:** Calibration Curve for Probe F
Appendix D

Sensor Calibration

Figure D.7: Calibration Curve for Probe G

Figure D.8: Calibration Curve for Probe H
Eddy Current Probe I Calibration Curve

\[ \text{mils} = -4.9726 \text{Volts} - 0.1984 \]

Sensitivity Uncertainty (95% CI) = 0.11573 mils/Volt

Figure D.9: Calibration Curve for Probe I

Eddy Current Probe J Calibration Curve

\[ \text{mils} = -4.6823 \text{Volts} - 0.85344 \]

Sensitivity Uncertainty (95% CI) = 0.094753 mils/Volt

Figure D.10: Calibration Curve for Probe J
D.2 Sensor Sensitivity Calculations

The individual sensor uncertainties were determined by examining the distribution of the residuals from the fitted values from the plots given in this Appendix. The following equation [11] was applied to determine the 95% confidence interval for the sensitivity (fitted line slope) based on the residuals of the fitted values from the actual measured values.

\[
\hat{\beta}_1 \pm t(\alpha/2; n-2) \sqrt{ \frac{\sum (y_i - \hat{y}_i)^2 / (n-2)}{\sum (x_i - \bar{x})^2} },
\]

(Eq. D.1)

Where \( \hat{\beta}_1 \) is the fitted slope from the least squares curve fit in mils/Volt, \( y_i \) are each of the actual displacements used to during the calibration in mils, \( x_i \) are the measured voltage from the sensors during the calibration, \( \hat{y}_i \) are the fitted displacement values, \( \bar{x} \) is the mean of the voltage values, \( n \) is the number of samples and \( \alpha \) represents the desired confidence intervals. Matlab\textsuperscript{®} was used to determine the
Confidence intervals for each individual probe and is provided in the plots given in this Appendix. Table D.1 also summarizes the results of the calibration exercise.

Table D.1  Sensor Calibration Summary

<table>
<thead>
<tr>
<th>Probe</th>
<th>Sensitivity (mils/Volt)</th>
<th>Sensor Uncertainty (mils/Volt)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-5.0588</td>
<td>0.0231</td>
<td>0.46</td>
</tr>
<tr>
<td>B</td>
<td>-5.2343</td>
<td>0.0238</td>
<td>0.45</td>
</tr>
<tr>
<td>C</td>
<td>-5.2075</td>
<td>0.0324</td>
<td>0.62</td>
</tr>
<tr>
<td>D</td>
<td>-5.223</td>
<td>0.0286</td>
<td>0.55</td>
</tr>
<tr>
<td>E</td>
<td>-5.0979</td>
<td>0.0308</td>
<td>0.60</td>
</tr>
<tr>
<td>F</td>
<td>-4.7549</td>
<td>0.0257</td>
<td>0.54</td>
</tr>
<tr>
<td>G</td>
<td>-4.8359</td>
<td>0.0446</td>
<td>0.92</td>
</tr>
<tr>
<td>H</td>
<td>-5.1221</td>
<td>0.0872</td>
<td>1.70</td>
</tr>
<tr>
<td>I</td>
<td>-4.9726</td>
<td>0.1157</td>
<td>2.33</td>
</tr>
<tr>
<td>J</td>
<td>-4.6823</td>
<td>0.0948</td>
<td>2.02</td>
</tr>
<tr>
<td>K</td>
<td>-4.8262</td>
<td>0.0729</td>
<td>1.51</td>
</tr>
</tbody>
</table>
Appendix E
Measured Controller Transfer Functions

Figure E.1: Case LKLC Top Magnets Controller Transfer Functions

Figure E.2: Case LKLC Bottom Magnets Controller Transfer Functions
Figure E.3:  Case LKHC Top Magnets Controller Transfer Functions

Figure E.4:  Case LKHC Bottom Magnets Controller Transfer Functions
Appendix E

Measured Controller Transfer Functions

Figure E.5:  Case HKHC Top Magnets Controller Transfer Functions

Figure E.6:  Case HKHC Bottom Magnets Controller Transfer Functions
clear all
close all

% parameter input%
Ki=80; % Integral gain of PID
Kp=40; % Proportional gain of PID
Kd=.025; % Derivative gain of PID
Kt=.00006; % Total gain of PID
ib1=1.12; % ib1 refers to top magnet
ib2=.365; % ib2 refers to bottom magnet
c1=381*10^-6; % c1 refers to top gap distance (must be determined from calibration)
c2=381*10^-6; % c2 refers to bottom gap distance (must be determined from calibration)
ss=161.53*10^-6; % Sensor Sensitivity (microns/V)

Sensor=1/ss; % Sensor output in Volts

% PID
numpid=Kt*[Kd Kp Ki]; % numerator of PID controller
denpid=[1 0]; % denominator of PID controller
syspid=tf(numpid,denpid);

% filter parameter inputs
fc=800; % cutoff frequency of low pass filter (Hz)
lpskiggly=.707; % damping ratio of low pass filter
fn=1000; % Notch frequency (Hz)
skiggly=.1; % Notch damping

wc=2*pi*fc; % Low pass cutoff frequency in radians/s
wn=2*pi*fn; % Notch frequency in radians/s

numlp=[wc^2];
denlp=[1 2*lpskiggly*wc wc^2];
numN=[1 2*nskiggly*wn wn^2];
denN=[1 2*wn wn^2];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%Calculating Pade (2,2) Approximation %%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Td=1/10000;                         % time delay determined experimentally
(fs=3508 Hz)

%numpade=[(Td^2)/12 -Td/2 1];        % Pade approximation for time delay
%denpade=[(Td^2)/12 Td/2 1];

numpade=[1];
denpade=[1];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%% Power Amplifier TF %%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fca=20000;
wca=2*pi*fca;
skigglya=.707;
Bias=1000/2.5;               % Amplifier
sensitivity and Gain

numamp=Bias*[wca^2];
denamp=[1 2*skigglya*wca wca^2];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%   
%%%%% controller Open Loop Transfer Function %%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

num1=conv(numpid,numlp);           % multiplying lpnum with pidnum
den1=conv(denpid,denlp);           % multiplying lpden with pidden

num2=conv(num1,numN);           % multiplying PID and LP with delay-numerator
den2=conv(den1,denN);           % multiplying PID and LP with delay-
denominator

numolc=conv(num2,numamp);           % multiplying PID, LP, delay and amp-
umerator
denolc=conv(den2,denamp);           % multiplying PID, LP, delay and amp-
denominator

sysolc=tf(numolc,denolc)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%% Magnetic Bearing Plant open loop %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

eps=.826;      % geometric correction factor
mu=(4*pi)*10^-7; % permeability of free space (N/Amp^2)
N=228;         % Number of turns per coil (2*114/pole)
Ag=6.677e-5;   % Pole face area (m^2)

%%% Position stiffnesses
Kx1 = -\(\text{eps} \times \mu \times (N^2) \times (ib1^2) \times \text{Ag}/(2 \times c1^3)\); % position stiffness for bottom Magnet (N/m)
Kx2 = -\(\text{eps} \times \mu \times (N^2) \times (ib2^2) \times \text{Ag}/(2 \times c2^3)\); % position stiffness for top Magnet (N/m)
Kxx = Kx1 + Kx2 % Total position stiffness of single axis (N/m) = -149e5
Kx1e = Kx1 * 0.00571;
Kx2e = Kx2 * 0.00571;

%% Current stiffnesses
Ki1 = \(\text{eps} \times (\mu \times \text{Ag} \times (N^2) \times (ib1))/(2 \times c1^2)\); % Current stiffness for bottom Magnet (N/Amp)
Ki2 = \(\text{eps} \times (\mu \times \text{Ag} \times (N^2) \times (ib2))/(2 \times c2^2)\); % Current stiffness for top Magnet (N/Amp)
Ki = Ki1 + Ki2 ; % Total Current Stiffness of single axis (N/m) = 38
Kile = Ki1 * 0.22481;
Kil2 = Ki2 * 0.22481;
DCgain = \(\sqrt{(Ki1 + Ki2)^2/(Kx1 + Kx2)^2}\);

%% Approximating controlled mass as 1/2 Total shaft mass %
Meq = 0.90687; % 2 lbf in kg

%% Open loop Plant transfer function %
numplant = [Ki1 + Ki2]; % Open loop plant transfer function
denplant = [Meq 0 Kx1 + Kx2];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%% Comparison Plots
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%% Loading actual measured transfer functions
load Lvrfrf.txt
load Lvrcoh.txt
load Lvrbfrefrftxt
load Lvrbcin Coh
load Lvrfrfrefrftxt
load Lvrbcin Coh

load Lwtrfrf.txt
load Lwtcoh.txt
load Lwtrbfrefrftxt
load Lwtrbcin Coh
load Lwtrfrfrefrftxt
load Lwtrbcin Coh

f = Lvrfrf(:,1);
w = 2 * pi * f;
Model = 119.5062 * f 
Modelt = -119.5062 * f; % Converting files to complex frf form and to Amps/in units
\[ Lvrfrf = 62.898 \times 1.9 \times (Lvrfrf(:,2) + i \times Lvrfrf(:,3)) \]
\[ Lvrbfrf = 62.898 \times 1.9 \times (Lvrbfrf(:,2) + i \times Lvrbfrf(:,3)) \]
\[ Lwtfrf = 62.898 \times 1.9 \times (Lwtfrf(:,2) + i \times Lwtfrf(:,3)) \]
\[ Lwtbfrf = 62.898 \times 1.9 \times (Lwtbfrf(:,2) + i \times Lwtbfrf(:,3)) \]

```matlab
figure(1)
subplot(311)
plot(f,abs(Lwtbfrf),'b--',f,abs(Lvrbfrf),'r--',f,abs(Model),'k'), zoom on
title('Case LKLC - Bottom Magnet-Controller Transfer Function Comparison')
ylabel('|Amps/inch|')
axis([0 100 0 300])

subplot(312)
plot(f,(180/pi)*angle(Lwtbfrf),'b--',f,(180/pi)*angle(Lvrbfrf),'r--',f,(180/pi)*angle(Model),'k'), zoom on
ylabel('Phase (degrees)')
legend('exp. W13 bot','exp. V13 bot','Model')
axis([0 100 -200 200])

subplot(313)
plot(f,Lwtbcoh(:,2),'b--',f,Lvrbcoh(:,2),'r--'), zoom on
ylabel('Coherence')
xlabel('frequency (Hz)')
```

```matlab
figure(2)
subplot(311)
plot(f,abs(Lwtfrf),'b--',f,abs(Lvrfrf),'r--',f,abs(Modelt),'k'), zoom on
title('Case LKLC - Top Magnet-Controller Transfer Function Comparison')
ylabel('|Amps/inch|')
axis([0 100 0 300])

subplot(312)
plot(f,(180/pi)*angle(Lwtfrf),'b--',f,(180/pi)*angle(Lvrfrf),'r--',f,(180/pi)*angle(Modelt),'k'), zoom on
ylabel('Phase (degrees)')
legend('exp. W13 top','exp. V13 top','Model')
axis([0 100 -200 200])

subplot(313)
plot(f,Lwtcoh(:,2),'b--',f,Lvrcoh(:,2),'r--'), zoom on
ylabel('Coherence')
xlabel('frequency (Hz)')
```

```
%%%%%%%%%%%%%% Model Matching %%%%%%%%%%%%%%%%%%%%%
K1=Kx1e+Ki1e.*real(Model);
C1=Ki1e.*imag(Model)./(w);
K2=Kx2e+Ki2e.*real(Model);
C2=Ki2e.*imag(Model)./(w);
K=K1+K2;
C=C1+C2;
```

```matlab
figure(4)
subplot(211)
plot(f,K), zoom on, grid on
title('Stiffness and Damping for Case KKLC')
```
ylabel('Stiffness (lbf/in.)')

axis([0 100 0 500])

subplot(212)
plot(f,C),zoom on, grid on
ylabel('Damping (lbf-s/in.)')
xlabel('frequency (Hz)')
axis([0 100 -1 1])

M=2/386;
for k=1:size(f);

Transmit(k,:)=((C(k,:)*i*w(k,:)+K(k,:))/(M*(i*w(k,:))^2+C(k,:)*(i*w(k,:))+K(k,:));
end

M=2/386;
C1=.0475;
%C1=0;
K1=191.0;

num1=[C1 K1];
den1=[M C1 K1];

K2=236.0;
C2=.0185;
%C2=0;
um2=[C2 K2];
den2=[M C2 K2];

fT=0:.125:100;
wT=2*pi.*fT;
Tf1=freqs(num1,den1,wT);
Tf2=freqs(num2,den2,wT);
T=Tf1+Tf2;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%% Actual experimental transfer function from stator to Rotor %%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

load hpfrfl

%%% Determining FRF sensitivity from probe sensitivities %%%% sensetime=[-5.0592 5.2348 -5.2083 -5.2244 -5.0995 -5.1262 -4.9801 -4.6877 -4.8293 6.36 6.36 .4 .4 .4 .4];
for k=2:15;
    frfsense(k-1)=sensetime(k)/sensetime(1);
end
frfsense=diag(frfsense);

%% Determining Voltage FRF's and Coh's from Auto-Spec's and Cross-Spec's

for pp=1:14;
    frf(pp,:)=Gab(pp,:)./Gaa(pp,:);
    coh(pp,:)=abs(Gab(pp,:)).^2./(Gaa(pp,:).*Gbb(pp,:));
end

%% Converting Voltage FRF's to Unit FRF's

frf=frfsense*frf;

figure(7)
subplot(311)
semilogy(freq,abs(frf(1,:)), 'r' ,f,abs(Transmit), 'b' ),zoom on
title('Displacement Transmissibility for Case LKLC')
ylabel('$|\text{mils/mils}|$')
%xlabel('frequency (Hz)')
legend('Rotor/Stator','Model')
%axis([0 100 1e-5 1e3])

subplot(312)
plot(freq,(180/pi)*unwrap(angle(frf(1,:)))-180, 'r' ,f,(180/pi)*unwrap(angle(conj(Transmit))), 'b' ),zoom on
ylabel('Phase')

subplot(313)
plot(freq,coh(1,:), 'r' ),zoom on
ylabel('Coherence')
xlabel('frequency (Hz)')
axis([0 100 0 1])
Figure G.1: Measured Vertical Displacement Transmissibilities for Case LKLC
Figure G.2: Measured Vertical Displacement Transmissibilities for Case LKHC
Vertical Displacement Transmissibility for Case HKHC

Figure G.3: Measured Vertical Displacement transmissibilities for Case HKHC
Appendix H
Orbit Plots
Measured from Single Frequency Data

H.1 Orbit Calculation Method

The orbit plots presented in Chapter Four and also in this appendix were determined by using the position signals recorded from positions A-K on the stub shaft rotor and the bearing housing. Figure H.1 shows the location of these probes on the stub shaft assembly and the bearing housing.

Two orbits were determined for each frequency and each case representing the motion of the stub shaft rotor and the bearing housing in the bearing’s center line plane shown in Figure H.1. Because probes could not be positioned on the stub shaft assembly in the bearing center line plane, the position signals gathered from the probes located at the ends of the shaft were used to determine the motion of the stub shaft in the center line plane. It was also beneficial to place the probes at these locations to detect any rocking of the stub shaft assembly. To determine the orbit of the stub shaft assembly and the bearing housing a horizontal and vertical motion in the center line plane had to be determined based on the position signals gathered. The method of similar triangles was used to combine the position signals located at the various positions on the stub shaft assembly and bearing housing to arrive at the motions of
these structures in the center line plane. For example the position response recorded for probes located at D and E were combined in the following manner to obtain the horizontal displacement of the bearing housing.

\[ X_{bh} = \frac{1}{2} (D - E) + E , \quad \text{(Eq. H.1)} \]

This formula was derived from the geometry shown in Figure H.2.

**Figure H.2:** Similar triangle Center line position diagram

This technique was used to determine the displacements \( X_1, X_2, X_s, X_{bh} \) and \( Y_s \) shown in Figure H.3.

**Figure H.3:** Calculated position responses using similar triangle method.
Figure H.3 shows the position signals which were calculated to give the center line horizontal and vertical position response of both the stub shaft assembly and the bearing housing in the center line plane. The vertical displacement of the stub shaft assembly was determined from position signals from probes A and C, while the horizontal stub shaft assembly response was determined from X₁ and X₂ which were determined respectively from probes H and I and J and K. The vertical displacement of the bearing housing was taken directly from the signal recorded from probe A while the horizontal position, Xₗₗ, was determined from the signals from probes D and E.

The following sections give the orbits determined for each case and several frequencies using the method described here.

**H.2 Orbits from Case LKLC**

![Orbits for Case LKLC at 20.75 Hz](image)
Figure H.5: Orbits for Case LKLC at 30.5 Hz

Figure H.6: Orbits for Case LKLC at 32.0 Hz
Figure H.7: Orbits for Case LKLC at 34.0 Hz

Figure H.8: Orbits for Case LKHC at 20.75 Hz
Figure H.9: Orbits for Case LKHX at 30.5 Hz

Figure H.10: Orbits for Case LKHC at 34.0 Hz
H.3 Orbits from Case HKHC

Figure H.11: Orbits for Case HKHC at 40.75 Hz

Figure H.12: Orbits for Case HKHC at 47.75 Hz
Figure H.13: Orbits for Case HKHC at 52.0 Hz.
Appendix I
Uncertainty Determination for Single Frequency Testing

The uncertainty analysis for the experimental results obtained from the single frequency testing was performed by examining the residuals of the measured output versus the predicted values from a least squares curve fitted line. This same technique was used in determining the sensor uncertainties in Appendix D. The same technique was applied here because a linear least squares curve fit was applied to the measured data. The data points measured for each of the single frequency displacement transmissibilities are assumed to have error based on the propagation of the sensor uncertainty, the A/D converter and the associated calculations which followed. Therefore each data point has an associated error bar associated with that particular point. However, due to the least squares curve fit line only the measured points were considered and these points were assumed to be the center of the confidence intervals associated with each point. The following equation [11] was applied to determine the 95% confidence interval for the sensitivity (fitted line slope) based on the residuals of the fitted values from the actual measured values.

\[ \hat{\beta}_1 \pm t(\alpha/2; n-2) \sqrt{\frac{\sum (y_i - \hat{y}_i)^2 / (n-2)}{\sum (x_i - \bar{x})^2}}, \]  

(Eq. I.1)

Where \( \hat{\beta}_1 \) is the fitted slope from the least squares curve fit in mils/mils, \( y_i \) are the measured vertical response amplitudes of the stub shaft rotor, \( x_i \) are the measured vertical amplitudes of the base excitation to the bearing \( \hat{y}_i \) are the fitted response values, \( \bar{x} \) is the mean of the base excitation amplitudes, \( n \) is the number of samples and \( \alpha \) represents the desired confidence intervals. Matlab® was used to determine the Confidence intervals for each frequency and gain case. Table I.1 also summarizes the results of the uncertainty analysis.

<table>
<thead>
<tr>
<th>Case</th>
<th>Frequency (Hz)</th>
<th>Displacement Transmissibility 95% CI (mils/mil)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LKLC</td>
<td>25</td>
<td>2.20 ± 0.040</td>
</tr>
<tr>
<td>LKLC</td>
<td>30.5</td>
<td>6.51 ± 0.573</td>
</tr>
<tr>
<td>LKLC</td>
<td>32</td>
<td>3.09 ± 0.641</td>
</tr>
<tr>
<td>LKLC</td>
<td>45</td>
<td>1.28 ± 0.017</td>
</tr>
<tr>
<td>LKHC</td>
<td>30.5</td>
<td>3.09 ± 0.078</td>
</tr>
<tr>
<td>LKHC</td>
<td>34</td>
<td>3.72 ± 0.022</td>
</tr>
<tr>
<td>HKHC</td>
<td>40.75</td>
<td>4.06 ± 0.088</td>
</tr>
<tr>
<td>HKHC</td>
<td>47.75</td>
<td>8.54 ± 0.144</td>
</tr>
<tr>
<td>HKHC</td>
<td>52</td>
<td>4.74 ± 0.253</td>
</tr>
</tbody>
</table>
Appendix J
Matlab® Code for Model Vs. Measured Comparison

J.1 Code

The following code was used to calculate the complete model of the AMB system given the gain values determined for case HKHC. However, this same code was also used for gain cases LKLC and LKHC by utilizing the appropriate gain values and importing the corresponding data file.

```matlab
%% Case HKHC
%% Controller Modeling and multiple frequency experimental measurements

clear all
close all

%% parameter input%%
Ki=100;                          % Integral gain of PID
Kp=60;                           % Proportional gain of PID
Kd=.05;                          % Derivative gain of PID
Kt=.00006;                       % Total gain of PID
ib1=1.12;        % ib1 refers to top magnet
ib2=.365;        % ib2 refers to bottom magnet
c1=381*10^-6;          % c1 refers to top gap distance (must be
determined from calibration)
c2=381*10^-6;          % c2 refers to bottom gap distance (must be
determined from calibration)
ss=161.53*10^-6;                 % Sensor Sensitivity (microns/V)

Sensor=1/ss;       % Sensor output in Volts

numpid=Kt*[Kd Kp Ki];            % numerator of PID controller
denpid=[1 0];                    % denominator of PID controller
syspid=tf(numpid,denpid);

% filter parameter inputs
fc=800;                      % cutoff frequency of low pass filter
lpskiggly=.707;              % damping ratio of low pass filter
fn=1000;         % Notch frequency (Hz)
nskiggly=.1;       % Notch damping
```
wc=2*pi*fc;                      % Low pass cutoff frequency in radians/s
wn=2*pi*fn;        % Nothc frequency in radians/s

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%% Low pass filter %%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

numlp=[wc^2];        % Numerator of Low Pass filter transfer
function

denlp=[1 2*lpskiggly*wc wc^2];  % Denominator of Low Pass filter transfer
function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% Notch Filter %%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

numN=[1 2*nskiggly*wn wn^2];   % Numerator of Notch filter transfer
function
denN=[1 2*wn wn^2];      % Denominator of Notch filter transfer
function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%Calculating Pade (2,2) Approximation %%%%%%%%% NOT USED!!!
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Td=1/10000;                         % time delay determined experimentally
(fs=3508 Hz)

%numpade=[(Td^2)/12 -Td/2 1];        % Pade approximation for time delay
%denpade=[(Td^2)/12 Td/2 1];

numpade=[1];        % PADE APPROXIMATION FOR TIME DELAY NOT USED
denpade=[1];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%% Power Amplifier TF %%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fca=20000;         % Amplifier Switching frequency (Hz)
wca=2*pi*fca;        % Amplifier Switching frequency (rad/s)
skigglya=.707;        % Amplifier damping ratio (butterworth 2nd order)
Bias=760;        % Amplifier sensitivity and Gain

numamp=Bias*[wca^2];      % Numerator of Amplifier transfer
function
denamp=[1 2*skigglya*wca wca^2];  % Denominator of Amplifier transfer
function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% controller Open Loop Transfer Function %%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

num=conv(numpid,numlp);            % multiplying lpnum with pidnum
den1=conv(denpid,denlp);            % multiplying lpden with pidden
num2=conv(num1,numN); % multiplying PID and LP with delay-
numerator

den2=conv(den1,denN); % multiplying PID and LP with delay-
denominator

numolc=conv(num2,numamp); % multiplying PID, LP, delay and amp-
numerator

denolc=conv(den2,denamp); % multiplying PID, LP, delay and amp-
denominator

sysolc=tf(numolc,denolc) % Complete Open Loop controller

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
%%%%%% Magnetic Bearing Plant open loop %%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  

eps=.826; % geometric correction factor
mu=(4*pi)*10^-7; % permeability of free space (N/Amp^2)
N=228; % Number of turns per coil (2*114/pole)
Ag=6.677e-5; % Pole face area (m^2)

%% Position stiffnesses

Kx1=-eps*mu*(N^2)*(ib1^2)*Ag/(2*c1^3); % position stiffness for bottom Magnet (N/m)
Kx2=-eps*mu*(N^2)*(ib2^2)*Ag/(2*c2^3); % position stiffness for top Magnet (N/m)
Kxx=Kx1+Kx2 % Total position stiffness of single axis (N/m)=-149e5

Kx1e=Kx1*.00571; % Position stiffness for bottom Magnet (lbf/in)
Kx2e=Kx2*.00571; % Position stiffness for top Magnet (lbf/in)

%% Current stiffnesses

Ki1=eps*(mu*Ag*(N^2)*(ib1))/(2*c1^2); % Current stiffness for bottom Magnet (N/Amp)
Ki2=eps*(mu*Ag*(N^2)*(ib2))/(2*c2^2); % Current stiffness for top Magnet (N/Amp)
Ki=Ki1+Ki2 % Total Current Stiffness of single axis (N/m)=38

Kile=Ki1*.22481; % Current stiffness for bottom Magnet (lbf/in.)
Kile=Ki2*.22481; % Current stiffness for top Magnet (lbf/in.)
DCgain=sqrt((Ki1+Ki2)^2/(Kx1+Kx2)^2);

%% Approximating controlled mass as 1/2 Total shaft mass %

Meq=.90687; % 2 lbf in kg - Mass of stub shaft assembly

%% Open loop Plant transfer function %

numplant=[Ki1+Ki2]; % Open loop plant transfer function
denplant=[Meq 0 Kx1+Kx2];
% Comparison Plots

f=0:0.125:100;  % frequency vector for plotting (Hz)
w=2*pi.*f;      % frequency vector for plotting (rad/s)
Model=62.898.*freqs(numolc,denolc,w);  % frequency response of model converted to Amps/in. from

K1=Kx1e+Ki1e.*real(Model);      % Calculating top and bottom equivalent stiffness
Cl=Ki1e.*imag(Model)./(w);      %   and damping from modeled values
K2=Kx2e+Ki2e.*real(Model);
C2=Ki2e.*imag(Model)./(w);

K=2*(.5*K1+.5*K2);        % Combining each axes stiffness and damping to give
C=2*(.5*C1+.5*C2);        %   aggregate stiffness and damping values in vertical
%     direction.  Notice the assumption of equivalent axis
% figure(4)                         % parameters results in the
% subplot(211)                     % vertical direction equal to
% plot(f,K),zoom on               % in either of the axes alone
due to symetry.
title('Stiffness and Damping for Case HKHC')
ylabel('Stiffness (lbf/in.)')      % Plotting Modeled Stiffness and damping
%axis([0 100 0 500])

subplot(212)
plot(f,C),zoom on
ylabel('Damping (lbf-s/in.)')
xlabel('frequency (Hz)')
axis([0 100 -1 1])

M=2/386;

for k=1:size(f);        % Calculating the frequency
response of the modeled
% system
Transmit(k,:)=(C(k,:)*(i*w(k,:))+K(k,:))/(M*(i*w(k,:))^2+C(k,:)*(i*w(k,:))+K(k,:));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%% Actual experimental transfer function from stator to Rotor
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

load hpfrfk  % Loading experimental frf data for Case HKHC

%%% Determining FRF sensitivity from probe sensitivities %%%%%%%%%%%%%%%
sensetime=[-5.0592 5.2348 -5.2083 -5.2244 -5.0995 -5.1262 -4.9801 -4.6877 -4.8293 6.36 6.36 .4 .4 .4 .4];
for k=2:15;
    frfsense(k-1)=sensetime(k)/sensetime(1); % Creating sensor sensitivity matrix
end
frfsense=diag(frfsense); % Creating diagonalized matrix of sensor sensitivities.

%%% Determining Voltage FRF's and Coh's from Auto-Spec's and Cross-Spec's %%%%%
for pp=1:14;
    frf(pp,:)=Gab(pp,:)./Gaa(pp,:);
    coh(pp,:)=abs(Gab(pp,:)).^2./(Gaa(pp,:).*Gbb(pp,:));
end

%%% Converting Voltage FRF's to Unit FRF's
frf=frfsense*frf;

figure(7) % Plotting modeled displacement transmissibility vs. actual (measured)
subplot(311) % actual (measured)
transmissibility w/ Coherence function
plot(freq,abs(frf(1,:)),'r',freq,abs(frf(2,:)),'b',freq,abs(Transmit),'k'),zoom on
title('Displacement Transmissibility for Case HKHC')
ylabel('[mils/mils]')
%xlabel('frequency (Hz)')
axis([0 100 1e-1 1e1])

subplot(312)
plot(freq,(180/pi)*unwrap(angle(frf(1,:))-180,'r',freq,(180/pi)*unwrap(angle(frf(2,:))),'b',freq,(180/pi)*unwrap(angle(Transmit)),'k'),zoom on
Appendix J

Matlab® Code for Model Vs. Measured

```
ylabel('Phase')
legend('Rotor B/Stator','Rotor C/Stator','Model',3)
axis([0 100 -600 200])

subplot(313)
plot(freq,coh(1,:),'r',freq,coh(2,:), 'g'), zoom on
ylabel('Coherence')
xlabel('frequency (Hz)')
axis([0 100 0 1])

xcliplast=(abs(Model)).*0.0148;
figure(24)          % Plotting Theoretical Clipping
threshold based upon
plot(f,xcliplast,'r'), zoom on    % bias current levels and modeled
transfer function
title('Theoretical Bottom Current Clipping Max for LKHC')
xlabel('frequency (Hz)')
ylabel('Amps')
axis([0 100 0 15])

figure(9)          % Plotting modeled vs. measured
% Plotting modeled vs. measured
% without coherence function
subplot(211)
plot(freq,abs(frf(1,:)), 'r', freq,abs(frf(2,:)), 'b--
', f, abs(Transmit), 'k'), zoom on
xlabel('frequency (Hz)')
%ylabel('Amps/meter')
axis([0 100 1e-1 1.1e1])

subplot(212)
plot(freq,(180/pi)*unwrap(angle(frf(1,:)))-
180, 'r', freq,(180/pi)*unwrap(angle(frf(2,:))),'b--
', f,(180/pi)*unwrap(angle(Transmit)),'k'), zoom on
ylabel('Phase')
axis([0 100 -270 100])
xlabel('frequency (Hz)')
legend('Stub Shaft B/Stator A','Stub Shaft C/Stator A','Model',3)
```

J.2 Resulting Modeled Controller Transfer functions

The modeled controller transfer functions for each gain case were determined by combining the individual filter models (i.e sensor, PID filter, low pass filter …) and incorporating the appropriate PID gains for each case. These transfer functions were used to derive the stiffness and damping of the bearing because they relate the stub shaft position to the current (i.e Force) obtained from the control system. These transfer functions are provided here for reference.

Case LKLC (Amps/meter)
\[
\frac{I_c(s)}{X(s)} = \frac{2.394e14s^4 + 6.839e17s^3 + 9.933e21s^2 + 1.512e25s + 3.0214e25}{s^7 + 1.974e5s^6 + 3.387e14s^4 + 2.54e18s^3 + 9.622e21s^2 + 1.575e25s},
\] (Eq. J.1)

Case LKHC (Amps/meter)

\[
\frac{I_c(s)}{X(s)} = \frac{6.268e16s^4 + 1.289e20s^3 + 2.537e24s^2 + 1.98e27s + 3.959e27}{s^7 1.974e5s^6 + 1.944e10s^5 + 3.387e14s^4 + 2.54e18s^3 + 9.622e21s^2 + 1.575e25s},
\] (Eq. J.2)

Case HKHC (Amps/meter)

\[
\frac{I_c(s)}{X(s)} = \frac{9.097e14s^4 + 2.235e18s^3 + 3.729e22s^2 + 4.31e25s + 7.183e25}{s^7 + 1.974e5s^6 + 1.944e10s^5 + 3.387e14s^4 + 2.54e18s^3 + 9.622e21s^2 + 1.575e25s},
\] (Eq. J.3)
Appendix K
Single Frequency Time History and Spectrum Example Plots

K.1 Case LKLC

Figure K.1  Case LKLC, 25 Hz Time History and Spectrum Example Plot
Figure K.2  Case LKLC, 30.5 Hz Time History and Spectrum Example Plot

Figure K.3  Case LKLC, 32 Hz Time History and Spectrum Example Plot
Figure K.4  Case LKLC, 45 Hz Time History and Spectrum Example Plot
K.2 Case LKHC

Figure K.5  Case LKHC, 30.5 Hz Time History and Spectrum Example Plot

Figure K.6  Case LKHC, 34 Hz Time History and Spectrum Example Plot
K.3 Case HKHC

Figure K.7  Case HKHC, 40.75 Hz Time History and Spectrum Example Plot

Figure K.8  Case HKHC, 47.75 Hz Time History and Spectrum Example Plot
Figure K.9  Case HKHC, 52 Hz Time History and Spectrum Example Plot
Josh Clements was born in Richmond Virginia and was raised by James and Cynthia Clements of Beach, Virginia, which is located in Chesterfield County. He originally attended Virginia Tech from August of 1992 to June of 1996, where he earned a B.S. in Mechanical Engineering. He then worked as process development engineer for Babcock & Wilcox Naval Nuclear Fuel Division in Lynchburg, Virginia until August of 1997. During his time in Lynchburg Josh met his wife to be, Kelly Hamlett, and they were soon married. While working at Babcock & Wilcox Josh made the decision to return to Virginia Tech to further pursue Mechanical Engineering and to acquire a Masters degree in this field. While pursuing his Master’s degree Kelly gave birth to his first daughter Baileigh. Prior to the completion of his degree Josh accepted a position with Cummins Engine Company of Columbus Indiana. Josh, Kelly and Baileigh all moved to Columbus where Josh continued to work on this thesis. Josh currently works as a Power Cylinder Development Engineer for Cummins and has a second daughter Barrett.