Linear Switched Reluctance Machine Drives with Electromagnetic Levitation and Guidance Systems

By

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The Bradley Department of Electrical and Computer Engineering

(ABSTRACT)

Many electrically propelled, and magnetically levitated and guided actuation systems (maglev) use either linear induction or synchronous machine topologies. From the cost, reliability, fault tolerance, and phase independence points of view, linear switched reluctance topologies are attractive for transportation application. This thesis investigates a novel topology in which a linear switched reluctance machine (LSRM) propulsion drive is incorporated in the magnetically levitated and guided vehicle. Designs of the LSRM and dc electromagnet, analytical aspects of modeling and dynamics of the vehicle, and closed loop control of propulsion, levitation, and guidance systems are discussed with comprehensive simulations and experimental results.

Due to the lack of standard design procedure for LSRM, a novel design procedure is proposed using the current knowledge and design procedure of rotating switched reluctance machines. Analysis procedures for the phase winding inductance, propulsion and normal forces with translator position are developed with a lumped-parameter magnetic circuit model and the results from it are verified with two-dimensional finite element analysis. Extensive experimental correlation of inductance, propulsion and normal forces to validate the analysis and design procedure is presented.

For the stable operation of the electromagnetic levitation and guidance systems, which have inherent unstable characteristics, the air gap position and force/current control loops are designed using PID (or PD) and PI controllers, respectively, and implemented and tested. The step-by-step design procedures for each controller are systematically derived. A feedforward compensation strategy for the levitation air gap control is proposed to reject the external force disturbance
mainly caused by the normal force component generated in the LSRM propulsion drive system. The reduction of mechanical vibration and hence the enhancement of ride quality is achieved. Extensive dynamic simulations and experimental results for the integrated maglev system are presented with a 6 m long prototype system. Experimental correlation proves the validity of the controller design procedure based on the single-input and single-output model, and shows the feasibility of the LSRM-propelled electromagnetic levitation and guidance systems.

A novel maglev topology in which only two sets of LSRMs are utilized to control individually propulsion, levitation, and guidance forces is proposed. One set of the linear switched reluctance actuator produces the levitation and propulsion forces and the other set generates the propulsion and guidance forces. The proposed architecture, thereby, obviates the need for design, development, and implementation of separate actuation systems for individual control of propulsion, levitation, and guidance forces and in contrast to most of the present practice. Further, the proposed system utilizes each of the linear switched reluctance actuation system for producing the propulsion force, thereby giving an overall high force density package for the entire system. The feasibility of the proposed system by finite element analysis is demonstrated.
DEDICATION

To

my wife, Hyeong-Hwa Choi,

and my daughter, Jin-Sun,

for their patience, support, and encouragement
I would like to express my sincere gratitude to my advisor, Dr. Krishnan Ramu, for his invaluable guidance, support, and encouragement throughout my graduate studies. His impressive knowledge, creativity, and patience have been truly helpful during the course of this research work.

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<td>EMF</td>
<td>ElectroMotive Force</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>LPF</td>
<td>Low Pass Filter</td>
</tr>
<tr>
<td>LSRM</td>
<td>Linear Switched Reluctance Machine</td>
</tr>
<tr>
<td>MAGLEV</td>
<td>Electrically Propelled, and Magnetically Levitated and Guided Vehicle</td>
</tr>
<tr>
<td>MC</td>
<td>Magnetic Contact Switch</td>
</tr>
<tr>
<td>MMF</td>
<td>MagnetoMotive Force</td>
</tr>
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<td>m/s²</td>
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<tr>
<td>$a_c$</td>
<td>Cross-section area of a conductor in the winding</td>
<td>m²</td>
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a_d: Deceleration of the translator  
A_g: Cross-section area of the air gap during the pole alignment  
A_r: Cross-section area of the rotor pole  
A_s: Cross-section area of the stator pole  
A_{sp}: Specific electric loading  
A_y: Cross-section area of the stator yoke  
B_g: Flux density of the air gap  
B_s: Flux density of the stator pole  
C_{ry}: Thickness of the rotor yoke  
C_{sy}: Thickness of the stator yoke  
d_c: Diameter of a conductor in the winding  
dW_e: Incremental electric input energy  
dW_l: Incremental loss energy dissipated by heat  
dW_m: Incremental mechanical energy  
dW_s: Incremental stored magnetic energy  
D: Bore diameter of the RSRM  
D_o: Outer diameter of the RSRM stator lamination  
f_f: Form factor or packing factor  
f_x: Propulsion force of the LSRM  
f_y: Lateral force of the LSRM  
f_z: Normal force of the LSRM  
F_T: Total mmf per phase applied  
F_{a,d,f}: Instantaneous acceleration, deceleration, and friction forces  
F_{g,s,f}: MMF drop in the air gap, stator core, and translator core  
FF: Winding fill factor  
g: Air gap length  
h_r: Height of the rotor pole  
h_s: Height of the stator pole  
H_g: Magnetic field intensity in the air gap  
H_s, H_{sp}: Magnetic field intensity in the stator core, pole
H_{sy} \quad \text{Magnetic field intensity in the stator back iron} \quad \text{A/m}

H_t \quad \text{Magnetic field intensity in the translator core} \quad \text{A/m}

H_{tp} \quad \text{Magnetic field intensity in the translator pole} \quad \text{A/m}

H_{ty} \quad \text{Magnetic field intensity in the translator back iron} \quad \text{A/m}

I_p \quad \text{Peak phase winding current} \quad \text{A}

J \quad \text{Maximum allowable current density in the winding} \quad \text{A/m}^2

k \quad \text{Ratio of the stack length of core lamination to the bore diameter}

k_1 \quad \pi^2/120

k_2 \quad \text{Variable dependent on the operating point} (=1-L_u/L_{as})

k_d \quad \text{Duty cycle} (=\theta\cdot m\cdot N_t/360)

k_e \quad \text{Efficiency of SRM}

l_g \quad \text{Flux path length in the air gap} \quad \text{m}

l_s \quad \text{Flux path length in the stator core} \quad \text{m}

l_t \quad \text{Flux path length in the translator core} \quad \text{m}

L \quad \text{Stack length of the magnetic core laminations of the RSRM} \quad \text{m}

L(i,x) \quad \text{Inductance for one phase with all magnetic flux paths} \quad \text{H}

L_{as} \quad \text{Aligned saturated inductance per phase} \quad \text{H}

L_t \quad \text{Total length of the LSRM stator (track)} \quad \text{m}

L_{tr} \quad \text{Total length of the LSRM translator (vehicle)} \quad \text{m}

L_u \quad \text{Unaligned inductance per phase} \quad \text{H}

L_w \quad \text{Stack length of the magnetic core laminations of the LSRM} \quad \text{m}

m \quad \text{Number of phases}

M_t \quad \text{Maximum mass of the translator assembly or vehicle} \quad \text{Kg}

n \quad \text{Total number of stator poles of the LSRM}

N_h \quad \text{Number of horizontal layers of the phase winding}

N_r \quad \text{Rotating speed of the RSRM} \quad \text{rpm}

N_s \quad \text{Number of stator poles of the RSRM}

N_{sc} \quad \text{Number of stator sectors of the LSRM}

N_t \quad \text{Number of rotor poles of the RSRM}

N_v \quad \text{Number of vertical layers of the phase winding}

P \quad \text{Maximum power capacity of the LSRM} \quad \text{W}
\( P_{gk}(x) \) Air gap permeance in the translator position \( x \) and flux path \( k \) \( \text{Wb}/\text{AT} \)
\( t_a \) Acceleration time of the translator \( s \)
\( t_d \) Deceleration time of the translator \( s \)
\( T_{ph} \) Number of winding turns per phase \( \text{Turns} \)
\( v_m \) Maximum linear velocity \( \text{m/s} \)
\( w \) Thickness of the wedge to hold the phase windings \( \text{m} \)
\( W_s \) Stored energy in the magnetic field \( \text{Joule} \)
\( X \) Overlapped length between stator and translator poles in \( x \) \( \text{m} \)
\( Y \) Overlapped length between stator and translator poles in \( y \) \( \text{m} \)
\( \beta_s \) Stator pole angle of the RSRM \( ^\circ \) (Degree)
\( \beta_r \) Rotor pole angle of the RSRM \( ^\circ \) (Degree)
\( \phi_k(i,x) \) Magnetic flux in the path \( k \) for phase winding current \( i \) and translator position \( x \) \( \text{Wb} \)
\( \lambda \) Flux linkage of the phase winding \( \text{Wb} \cdot \text{Turn} \)
\( \theta \) Current conduction angle for each rising inductance profile \( ^\circ \) (Degree)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description (Chapter 3)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>Viscous damping coefficient</td>
<td>( \text{N/(m/s)} )</td>
</tr>
<tr>
<td>( D_l )</td>
<td>Lower freewheeling diode of the asymmetric bridge converter</td>
<td></td>
</tr>
<tr>
<td>( D_u )</td>
<td>Upper freewheeling diode of the asymmetric bridge converter</td>
<td></td>
</tr>
<tr>
<td>( f^*_x )</td>
<td>Propulsion force command</td>
<td>( \text{N} )</td>
</tr>
<tr>
<td>( f^<em>_x, f^</em>_{xk} )</td>
<td>Propulsion force feedback and command of ( k )-th phase</td>
<td>( \text{N} )</td>
</tr>
<tr>
<td>( f_{xl} )</td>
<td>External load force</td>
<td>( \text{N} )</td>
</tr>
<tr>
<td>( f_{zk} )</td>
<td>Normal force generated in the ( k )-th phase</td>
<td>( \text{N} )</td>
</tr>
<tr>
<td>( g_{sx} )</td>
<td>Rate of change of inductance with respect to ( x )</td>
<td>( \text{N/A}^2 )</td>
</tr>
<tr>
<td>( g_{sz} )</td>
<td>Rate of change of inductance with respect to ( z )</td>
<td>( \text{N/A}^2 )</td>
</tr>
<tr>
<td>( H_v )</td>
<td>Velocity feedback gain</td>
<td>( \text{V/(m/s)} )</td>
</tr>
<tr>
<td>( i_k, i^*_k )</td>
<td>Current feedback and command of the ( k )-th phase</td>
<td>( \text{A} )</td>
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<tr>
<td>( K_{pv}, K_{iv} )</td>
<td>PI velocity controller gains</td>
<td></td>
</tr>
<tr>
<td>( L_k )</td>
<td>Winding inductance of the ( k )-th phase</td>
<td>( \text{H} )</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description (Chapter 4)</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------</td>
<td>------</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Phase A winding resistance</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$S_k(x)$</td>
<td>Force distribution (or shape) function of the k-th phase</td>
<td></td>
</tr>
<tr>
<td>$T_1$</td>
<td>Lower transistor of the asymmetric bridge converter</td>
<td></td>
</tr>
<tr>
<td>$T_u$</td>
<td>Upper transistor of the asymmetric bridge converter</td>
<td></td>
</tr>
<tr>
<td>$v_c$</td>
<td>Control voltage reference for PWM</td>
<td>V</td>
</tr>
<tr>
<td>$v_o$</td>
<td>Average output voltage of each phase winding</td>
<td>V</td>
</tr>
<tr>
<td>$v_t$</td>
<td>Triangular carrier waveform</td>
<td></td>
</tr>
<tr>
<td>$v_{tm}$</td>
<td>Peak voltage magnitude of the triangular carrier waveform</td>
<td>V</td>
</tr>
<tr>
<td>$x$</td>
<td>Propulsion velocity of the translator</td>
<td>m/s</td>
</tr>
<tr>
<td>$\zeta_v$</td>
<td>Damping ratio of the velocity control loop</td>
<td></td>
</tr>
<tr>
<td>$\omega_v$</td>
<td>Bandwidth of the velocity control loop</td>
<td>rad/s</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Cross-section area of the electromagnet pole face</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between the mass centers of two electromagnets</td>
<td>m</td>
</tr>
<tr>
<td>$d_{af}$</td>
<td>Distance of the front pole face centers of phase A from the center of mass</td>
<td>m</td>
</tr>
<tr>
<td>$d_{ar}$</td>
<td>Distance of the rear pole face centers of phase A from the center of mass</td>
<td>m</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Stacking depth of U-shaped core lamination</td>
<td>m</td>
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<tr>
<td>$f_{af}$</td>
<td>Normal force generated in the front pole face of phase A in LSRM</td>
<td>N</td>
</tr>
<tr>
<td>$f_{ar}$</td>
<td>Normal force generated in the rear pole faces of phase A in LSRM</td>
<td>N</td>
</tr>
<tr>
<td>$f_d$</td>
<td>External force disturbance</td>
<td>N</td>
</tr>
<tr>
<td>$f_l$</td>
<td>External load force</td>
<td>N</td>
</tr>
<tr>
<td>$f_{mf}$</td>
<td>Normal force generated in the front electromagnet</td>
<td>N</td>
</tr>
<tr>
<td>$f_{mr}$</td>
<td>Normal force generated in the rear electromagnet</td>
<td>N</td>
</tr>
<tr>
<td>$f_n$</td>
<td>Normal force calculated in the LSRM propulsion controller</td>
<td>N</td>
</tr>
<tr>
<td>$f_n^*$</td>
<td>Feedforward normal force command</td>
<td>N</td>
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<tr>
<td>$f_{z^<em>, f_{z^</em>}}$</td>
<td>Attractive levitation force feedback and its command</td>
<td>N</td>
</tr>
<tr>
<td>$f_{za}$</td>
<td>Total normal force generated in phase A</td>
<td>N</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>g_z</td>
<td>Rate of change of inductance with respect to z</td>
<td>N/A²</td>
</tr>
<tr>
<td>G</td>
<td>Acceleration of gravity</td>
<td>m/s²</td>
</tr>
<tr>
<td>h_p</td>
<td>Height of the electromagnet pole</td>
<td>m</td>
</tr>
<tr>
<td>H_c</td>
<td>Current feedback gain</td>
<td>V/A</td>
</tr>
<tr>
<td>H_f</td>
<td>Force conversion gain</td>
<td>V/N</td>
</tr>
<tr>
<td>H_g</td>
<td>Magnetic field intensity in the air gap</td>
<td>A/m</td>
</tr>
<tr>
<td>H_p</td>
<td>Air gap position feedback gain</td>
<td>V/m</td>
</tr>
<tr>
<td>i</td>
<td>Instantaneous electromagnet winding current</td>
<td>A</td>
</tr>
<tr>
<td>I_m</td>
<td>Rated electromagnet winding current</td>
<td>A</td>
</tr>
<tr>
<td>K_pp, K_iP, K_dp</td>
<td>PID position controller gains</td>
<td></td>
</tr>
<tr>
<td>K_pc, K_ic</td>
<td>PI current controller gains</td>
<td></td>
</tr>
<tr>
<td>K_r</td>
<td>Converter gain</td>
<td>V/V</td>
</tr>
<tr>
<td>L</td>
<td>Electromagnet winding inductance</td>
<td>H</td>
</tr>
<tr>
<td>L_e</td>
<td>Winding inductance at the nominal equilibrium point</td>
<td>H</td>
</tr>
<tr>
<td>M</td>
<td>A quarter of the vehicle mass</td>
<td>Kg</td>
</tr>
<tr>
<td>M_v</td>
<td>Total mass of the vehicle</td>
<td>Kg</td>
</tr>
<tr>
<td>T_m</td>
<td>Number of winding turns per electromagnet</td>
<td>Turns</td>
</tr>
<tr>
<td>u</td>
<td>New control input for current control</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>Voltage applied to the electromagnet winding</td>
<td>V</td>
</tr>
<tr>
<td>w_p</td>
<td>Width of the electromagnet pole</td>
<td>m</td>
</tr>
<tr>
<td>w_w</td>
<td>Width of the electromagnet winding window</td>
<td>m</td>
</tr>
<tr>
<td>z</td>
<td>Air gap length or position</td>
<td>m</td>
</tr>
<tr>
<td>(\dot{z})</td>
<td>Vertical velocity of the electromagnet or vehicle</td>
<td>m/s</td>
</tr>
<tr>
<td>(\zeta_c)</td>
<td>Damping ratio of the current control loop</td>
<td></td>
</tr>
<tr>
<td>(\zeta_p)</td>
<td>Damping ratio of the position control loop</td>
<td></td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>Permeability of the free space</td>
<td>H/m</td>
</tr>
<tr>
<td>(\omega_c)</td>
<td>Bandwidth of the current control loop</td>
<td>rad</td>
</tr>
<tr>
<td>(\omega_p)</td>
<td>Bandwidth of the position control loop</td>
<td>rad</td>
</tr>
<tr>
<td>(\mathcal{I})</td>
<td>MMF of the magnetic circuit</td>
<td>AT</td>
</tr>
<tr>
<td>(\mathcal{R})</td>
<td>Reluctance of the magnetic circuit</td>
<td>AT/Wb</td>
</tr>
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</table>
1.1 Introduction

Linear motor drives are increasingly considered for machine tool drives because they reduce the need for mechanical subsystems of gears and rotary to linear motion converters such as lead screws. Positioning accuracy is improved by the absence of gears that contribute to the backlashes in the linear motor drives. Linear machine drives combined with electromagnetic levitation are strong candidates for conveyor applications in semiconductor fabrication plants and also possibly in low and high speed transit applications because of their ability to produce propulsion force on the moving part known as translator without mechanical contact and hence friction.

Systems using linear induction and linear synchronous machines are prevalent in industry. Linear switched reluctance machines are an attractive alternative due to the lack of windings on either the stator or translator structure. The windings are concentrated rather than distributed making them ideal for low cost manufacturing and maintenance. Further, the windings are always in series with a switch so that, in case of a shoot-through fault, the inductance of the winding can limit the rate of change of rising current and provide time to initiate protective relaying to isolate the faults. Moreover, the phases of the linear switched reluctance machine are independent and in case of one winding failure, uninterrupted operation of the motor drives is possible though with reduced power output [56]. These advantages enable the linear switched reluctance machine drives to operate as an economical high performance system with better fail-safe reliability. The linear switched reluctance machines are the counterparts of the rotating switched reluctance machines. In fact, the linear switched reluctance machine is obtained from its rotary counterpart by cutting along the shaft over its radius both the stator and rotor and then rolling them out. In this chapter, various linear switched reluctance machine configurations are introduced. Further, its ideal inductance profile is related to the stator and translator, lamination dimensions. A similar relationship for the rotary switched reluctance machine has been derived earlier is worth noting.
1.2 Machine Topology and Elementary Operation of Linear SRM

A linear SRM may have windings either on the stator or translator whereas in the rotary switched reluctance machine as shown in Figure 1.1 the windings are always on the stator and rotor contains no windings. Regardless of the location of phase windings, the fixed part is called either stator or track and the moving part is called translator. There are two distinct configurations of linear SRM in literature [1], [3] viz., the longitudinal flux and the transverse flux configurations. These two configurations can be obtained by unrolling both the stator and rotor of rotary SRM with radial magnetic flux path and axial magnetic flux path [1], [56], respectively. Figure 1.2 shows the longitudinal flux and transverse flux configurations for three-phase linear SRM with active (containing windings) stator and passive (with no windings) translator topology. Longitudinal magnetic flux path configuration, Figure 1.2(a), is a linear counterpart of three-phase radial flux rotary SRM, Figure 1.1(a). The flux path in this machine is in the direction of the vehicle motion. This machine is simpler to manufacture, mechanically robust and has lower eddy current losses as the flux is in the same direction as the translator motion. Transverse flux design, Figure 1.2(b), has the flux path perpendicular to the direction of vehicle motion. It allows a simple track consisting of individually mounted transverse bars. As the flux is perpendicular to the direction of motion, an emf is induced in the core resulting in high eddy current losses. Longitudinal flux and transverse flux configurations for four-phase linear SRM with active translator and passive stator structure are shown in Figure 1.3. The active stator and passive translator SRM configuration has the advantage in having the power supply and power converters being stationary resulting in reduced weight of vehicle. But that necessitates a large number of power converter sections along the track resulting in high cost. On the other hand, the structure with active translator and passive stator structure requires only one section of the power converter. But the power to the converter in the translator requires transfer by means of contact brushes that is not desirable for high speeds or by inductive transfer with additional power converter circuits with consequent complexity and higher cost. Also the linear switched reluctance machine may have either two stators or two translators or vice versa to make a double-sided linear switched reluctance machine, as shown in Figure 1.4. The double-sided linear SRM does not have as much freedom in the air gap tolerance as the single-sided linear SRM is to be noted. The single-sided linear SRM provides a net levitation force that can be
exploited in maglev systems. But the double-sided linear SRM does not produce a net levitation force and therefore is unsuitable for such applications. Its advantages are high force density and lower inductance as it has four air gaps in its flux path in contrast to two air gaps in the single-sided linear SRM.

(a) Three-phase rotary SRM with 6 stator and 4 rotor poles.

(b) Four-phase rotary SRM with 8 stator and 6 rotor poles.

Figure 1.1 Rotary switched reluctance machine configuration with radial magnetic flux path.
(a) Longitudinal magnetic flux path configuration.

(b) Transverse magnetic flux path configuration.

Figure 1.2 Three-phase linear SRM with active stator and passive translator.
Figure 1.3 Four-phase linear SRM with active translator and passive stator.
When a pair of stator windings that are connected in series is excited, the translator tends to move so as to align itself with the magnetic flux axis of the excited stator phase windings. This position is referred to as the fully aligned position and has the maximum phase inductance. The position corresponding to maximum reluctance value and hence minimum phase inductance is called the unaligned position and occurs when a corresponding pair of translator poles that eventually will be aligned is half a translator pole pitch away from the axis of the excited stator poles. The translator goes forward smoothly when the stator windings are switched in sequence. Depending on the converter topology and the mode of operation, the previously excited phase may be turned off before or after the succeeding phase is excited. Reverse motion of the translator can be achieved by reversing the excitation sequence of the stator phases.

The operation of the linear SRM is based on the inductance profile of the machine. The inductance of the machine is related to machine dimensions such as the stator and translator pole and slot widths, excitation currents and translator position. Assuming that the magnetic circuit is linear and therefore the inductance characteristics are independent of stator current excitation, a
relationship between the machine dimensions and inductance, shown in Figure 1.5, is derived. The inductance of a phase winding is its self-inductance. Five translator positions are significant to derive the inductance profile and they are given by,

\[
\begin{align*}
    x_1 &= \frac{w_{ts} - w_{sp}}{2} \\
    x_2 &= x_1 + w_{sp} = \frac{w_{ts} + w_{sp}}{2} \\
    x_3 &= x_2 + (w_{tp} - w_{sp}) = w_{tp} + \left(\frac{w_{ts} - w_{sp}}{2}\right) \\
    x_4 &= x_3 + w_{sp} = w_{tp} + \left(\frac{w_{ts} + w_{sp}}{2}\right) \\
    x_5 &= x_4 + \frac{w_{ts} - w_{sp}}{2} = w_{tp} + w_{ts}
\end{align*}
\]

where \( w_{tp} \) is the width of the translator pole, \( w_{ts} \) is the width of the translator slot, \( w_{sp} \) is the width of the stator pole, and \( w_{ss} \) is the width of the stator slot. Between \( x_2 \) and \( x_3 \), there is complete overlap between stator and translator poles, and inductance during this interval corresponds to...
the aligned value and is a maximum. As there is no change in the inductance in this region, zero propulsion force is generated in this region with an excitation current in the winding. But it is important to have this flat inductance region to give time to commutate the current and hence to prevent the generation of a negative force. The unequal stator and translator pole widths contribute to the flat-top inductance profile. On the other hand, the regions corresponding to 0−x_1 and x_4−x_5 have no overlap between stator and translator poles. These positions have the minimum phase inductance known as unaligned inductance. The rate of change of inductance is zero and hence these regions also do not contribute to propulsion force production.

The force production for motoring and regeneration is also shown in Figure 1.6. The forward direction of motion of the translator is assumed to be positive when the phase excitation sequence is abc. For forward direction of motion, regions I to III represent forward motoring operation and regions IV to VI represent forward regenerative operation for the phase sequence abc. Similarly, for reverse direction of motion, regions I to III represent reverse regenerative operation and regions IV to VI represent reverse motoring operation for the phase sequence acb. The duty cycle of each phase is only 1/3 and the induced emfs are constant between x_1 and x_2, the air gap power and hence the generated electromagnetic force can be made constant by exciting the stator phases with (x_2−x_1) wide pulses of currents. P_{em} represents the motoring back emf power and P_{er} represents the regenerative back emf power. Note that one half of the back emf power is stored in the form of magnetic field energy in phase windings and the other half of back emf power (or air gap power) is converted to mechanical power output. Ideal inductance profiles and ideal current generation are assumed in the discussion so far. But ideal currents with step rise and fall are not feasible due to the machine inductance. Therefore, compensation to obtain the desired current is advancing the energization of the windings. Note the similarity between the linear SRM and PM brushless dc machine emf, current, air gap power, and force waveforms. This clearly demonstrates that their controls are identical and hence the PM brushless dc machine controller, which is available in a chip form, can be used in the control of SRM for low cost and high volume applications.
Figure 1.6 Operation of three-phase linear SRM and its key variables.

CHAPTER 1. INTRODUCTION
1.3 Review of Previous Work

In this section, the survey of linear SRM drives with electromagnetic levitation and guidance systems is reviewed. Although the first machine operating on the variable reluctance principle was built as early as 1842, it was not until the early 1960’s that the on-going research efforts were started. The current research efforts were motivated by the invention of high power semiconductor devices and improved understanding of the principles of electromagnetic energy conversion. Since researchers started reinvestigating the rotary SRM in the 1960’s, a considerable amount of research effort on rotary SRMs covers a wide range of topics such as design of rotary SRM [22]-[24], analysis of rotary SRM drives [34]-[40], rotary SRM converter topology and its device ratings [43]-[58], and controllers for the rotary SRM converters [59]-[64]. On the contrary, few literatures are available on the research of linear SRMs [3]-[11], [13]-[21]. Nevertheless, due to the fact that linear SRMs are unrolled version of rotary SRMs, there are similarities in the electromagnetic operation principles between the linear machines and rotary machines. For this reason, the design and analysis, converter topologies, and control strategies developed for the rotary SRM can also be applied to linear SRM [18].

1.3.1 Design of a Linear Switched Reluctance Machine

Transverse flux linear SRM has been explored in detail in [4]-[9]. Takayama et al. [4]-[5] presented simple linear type SRM, which is composed of U-type stator and I-type translator, and showed total machine configuration with geometrical dimension. They focused on the measurement of propulsion force distribution on the stator and rotor poles of transverse flux linear SRM and established the method of measuring the propulsion force. Cusack et al. [6] designed and constructed the transverse flux linear SRM and demonstrated the operation by implementing hysteresis current-controlled excitation. Finite element analysis and test results were used to define the longitudinal position dependent current profile. Liu et al. [7]-[9] provide some guidance for the linear SRM design by introducing the concept of feasible polygon classification, which clarifies the physical size limits and the relation between the pole width and pole pitch of stator and translator. The developed feasible polygons were used to supply the physical constraints for linear SRM optimizations such as the arrangement of the stator poles of a
transverse flux linear SRM. All the above papers have the dimensions of their prototype linear SRM with the analytical parameter calculations and experimental results. However, the step-by-step design procedure to obtain the complete dimensions such as the pole and slot size of the stator and translator as well as the winding specifications is not described in detail although the calculation of inductances and forces by finite element analysis is covered extensively.

Deshpande et al. [10] presented a double-sided double-translator linear SRM structure, which has longitudinal magnetic flux path. An extensive theoretical analysis using algebraic lumped-parameter equivalent circuit to determine position-dependent flux linkages was developed and the potential advantage of multiple translator design to increase force density of the linear SRM was demonstrated. Corda et al. [13]-[15] proposed a linear switched reluctance actuator based on a cylindrical structure to be used for short stroke lengths using longitudinal flux configuration. The analytical calculation of inductance and phase propulsion force was also demonstrated with cylindrical prototype actuator. A longitudinal field linear SRM with multiple air gaps has been designed in [16]-[17], and the study indicated that the propulsion force per unit of active mass could be increased by properly designed multi air gap structure of linear switched reluctance actuator. Although the above papers present various experiments to verify the performance of longitudinal flux LSRM, a methodology for obtaining the machine dimensions and related material specifications is not described. The design of tubular machines has been covered in some detail in [18]. However, in general, the design of longitudinal flux single-sided machines has not been explored in literature. There has not been a standard design procedure for linear SRM so far [18].

1.3.2 Converter Topology for Linear Switched Reluctance Machine Drives

Linear SRM is basically a kind of linear synchronous motor, which needs power electronic converter that can change the operating frequency and the magnitude of applied voltage. The principal features of the converter for linear SRM drives generally require full four-quadrant operation, high commutation voltage for a faster current turn-off, low number of power switches and diodes, and hence low cost etc. The developments of converter circuit topologies on these lines are found in [43]-[58] and the converter topologies are mainly different in the method of regeneration and the number of switches used. Single switch per phase for bifilar winding
machines, and single switch per phase using the split dc source voltages or using dump resistor converters are proposed in [37], [49], and [50], respectively. Energy stored in the winding during the working stroke is either recovered using a bifilar winding or dissipated in a resistor. Two switches per phase configuration being the most commonly used is described in [53] and [54] for motoring and generation mode of operations of SRM with battery sources. This topology has two switches per phase. When both switches are turned off, negative dc link voltage is applied to the phase winding so that energy recovery is accelerated. Therefore, there is almost perfect phase independence. Further research efforts, [43]-[46], describe the minimization in the number of switches by increasing the utilization of switches. These topologies have one commutating switch in addition to the single-phase switches. When this switch and a phase switch are both turned off, negative dc link voltage is applied to the phase winding, as is the case in two switches per phase topology, thus accelerating energy recovery. However the commutating switch is turned on again when the next phase is excited thus slowing down the energy recovery process in much the same way as in a single switch per phase topology. A variant of more than one but less than two switches per phase converter topology is described in [55]-[58]. These converter topologies have either a buck or a buck-boost front-end converter and a machine side converter. The machine dc link voltage can be varied with machine operating speed through the step-down and step-up operation of front-end chopper power stage.

1.3.3 Controller for Linear Switched Reluctance Machine Drives

The controllers developed in early stages of the development of the drive were mainly analog [59]-[60]. Later, microprocessor-based controllers were proposed in [61]-[63]. Highly flexible personal computer-based controllers were proposed [64]. Though it is not cost effective at low power levels, the PC-based controller offers extensive flexibility, which is useful in determining the best control strategy for the drive.

1.3.4 LSRM-based Electromagnetic Levitation and Guidance Systems

Considerable efforts have been devoted to the theoretical and experimental investigations concerning the dynamics and control of the electromagnetic levitation and guidance systems
They describe the theoretical and practical aspects of the design of a single-variable or multi-variable control systems using single magnet or multi magnet configurations to provide stable levitation of a vehicle. However, the mutual interactions between the linear propulsion system and the electromagnetic levitation system are not considered in the literature. Further, none of the papers describe the effect of the normal force generated in the linear propulsion system to the electromagnetic levitation system. The normal force rejection strategy has to be considered to achieve a smooth and robust levitation air gap control. Even though Liu and Sheu [9] proposed the optimal pole arrangement design of the linear SRM which can provide both levitation and propulsion forces to electromagnetic levitation system by one electromagnetic source, the contents are restricted to the machine design issue.

1.4 Objectives and Contributions

Both low and high speed ground transit systems based on magnetically levitated and guided vehicles propelled by a linear machine drives have been proposed for future transportation requirements. Due to the absence of contact friction, the linear machine driven magnetically levitated and guided system can offer the advantages of lower noise and emissions, and better ride quality, as well as potential energy savings and economic benefits. Many electrically propelled, and magnetically levitated and guided actuation systems use either linear induction or linear synchronous machine topologies. From the cost, reliability, fault tolerance, and phase independence points of view, linear switched reluctance topologies are attractive for transportation applications. In this thesis, linear-SRM-based electromagnetic levitation and guidance systems are proposed for both low and high speed transit applications. The same systems can also be applied for the general-purpose industrial linear motion applications. This study is the first of its kind to propose and investigate the feasibility of the magnetically levitated and guided vehicle using linear switched reluctance machine. A prototype vehicle using linear SRM and controlled dc electromagnets and operating on a track has been built with the object of providing an analytical base for the design of the vehicles and to gain experience in the design and construction of electronic control and associated power converter. Designs of the linear SRM and dc electromagnet, analytical aspects of modeling and dynamics of the vehicle, and closed
loop control are discussed with comprehensive simulations and experimental results. This research work makes the following contributions.

1) A novel design procedure for linear switched reluctance machine is proposed using the current knowledge and design procedure of rotating switched reluctance machine.

2) Analytical predictions of fully aligned, two intermediate, and fully unaligned inductances using lumped-parameter magnetic equivalent circuit analysis are developed and verified with finite element analysis.

3) A 4.8 m long, three-phase longitudinal LSRM prototype with active stator and passive translator configuration is built based on the developed design procedure with 120 stator poles spread along its length and tested with a converter and controller.

4) The validity of the proposed design procedure is confirmed by experimental correlation of inductances, propulsion and normal forces. Even though the modeling of end effects are considered in the analytical calculation procedure, small discrepancies in the unaligned inductance prediction are presented. This may need an accurate three-dimensional finite element analysis.

5) A design procedure for dc electromagnet is presented using analytic relations between the design variables, and the two-dimensional finite element analysis is used to fine-tune the final design. The dc electromagnet prototype and track are built based on the final dimension, and tested with a converter and controller. To provide a linear propulsion force for the vehicle, a 6 m long, four-phase longitudinal LSRM prototype with active translator and passive stator configuration is also built and tested.

6) For the stable operation of the electromagnetic levitation system, the air gap position and force/current control loops are designed using PID and PI controllers, respectively. Modeling of each control loop is developed using single-input single-output system under the assumption that two sets of four dc electromagnets are used for the independent control of levitation and guidance systems, respectively. The design procedures for the PID position and PI force/current controllers are systematically derived. Each controller is implemented and tested using the analog control circuitry.
7) A feedforward compensation strategy for the levitation air gap control is proposed to reject the external force disturbance mainly caused by the normal force component generated in the LSRM propulsion drive system. The feedforward control command is calculated in the LSRM propulsion controller and is used in the levitation controller. The enhancement of ride quality and system efficiency for the LSRM-propelled electromagnetic levitation system is achieved.

8) Extensive dynamic simulations and experimental results for the levitation air gap position and force/current control loops are presented with the prototype system. Experimental correlation proves the validity of the controller design procedure based on the single-input single-output model, and shows the feasibility of the LSRM-propelled electromagnetic levitation system.

9) For accomplishing the independent control of the lateral vehicle motion, a set of guidance structure containing four dc electromagnets, four gap sensors, and track are built and tested with corresponding converter and controller.

10) To overcome the guideway irregularities and coupled motion dynamics between the two front magnets or two rear magnets, the air gap position command of the front or rear electromagnets is generated as a half of the sum of two front or rear air gap position feedbacks, respectively. This is implemented and tested using analog control circuitry.

11) For the stable operation of the electromagnetic guidance system, the air gap position and force/current control loops are designed using PD and PI controllers, respectively, based on the single-input single-output model. Each control loop is implemented and tested using the analog control circuitry. The design procedure for the PD position controller is systematically derived.

12) Dynamic simulations and experimental results for the guidance air gap position and force/current control loops are presented using the 6 m long prototype system. Experimental results correlate the simulation results, and it confirms the feasibility of the independent electromagnetic guidance system. Experimental results also show the validity of the air gap position command generation.
A novel approach is proposed to an LSRM-based maglev system using only two sets of actuation systems for propulsion and levitation/guidance control. This approach reduces mechanical complexity, thereby, greatly reducing the cost of the entire maglev system. This approach is founded on RSRM machine designs and verified with linear machine finite element analysis results. The analysis proves that the proposed control technique can be implemented in practice. Propulsion phases provide normal force components aiding levitation/guidance phases in this proposed maglev system. This reduces average power requirements to corresponding levitation/guidance phases. The proposed system offers high utilization of inherent normal forces for levitation and guidance. Higher reliability is made possible in the proposed system with multiple sets of lower power LSRMs.

1.5 Scope and Organization of the Thesis

The present study has the following scope: (a) The design procedure of linear switched reluctance machine is developed only for the longitudinal magnetic flux path configuration, even though the similar design procedure can be developed for the transverse magnetic flux path configuration using the axial field rotating switched reluctance machine. (b) The levitation and guidance system models and their controllers are developed using single-input and single-output system. This is based on the assumption that two sets of dc electromagnets are used for the independent control of levitation and guidance systems, and they hold the vehicle to the balanced nominal position with enough force against unevenly distributed loads, guideway irregularities, and external disturbances such as wind.

This thesis is presented in seven chapters, each of which includes an introduction and a conclusion. Chapter 2 describes the design of a linear switched reluctance machine. In Chapter 3, the linear-SRM-based propulsion drive system is explored. Chapter 4 deals with the linear-SRM-propelled electromagnetic levitation system. Chapter 5 addresses the electromagnetic guidance system. Chapter 6 proposes an integral linear-SRM-based propulsion, levitation, and guidance system. Chapter 7 provides a comprehensive summary of the major conclusions arising from the thesis work. The chapter ends with some suggestions on possible areas of further research work.
CHAPTER 2
DESIGN OF A LINEAR SWITCHED RELUCTANCE MACHINE

A standard design procedure for a single-sided and longitudinal flux-based linear switched reluctance machine (LSRM) is developed in this chapter. The proposed design procedure utilizes the rotating switched reluctance machine (RSRM) design by converting the specifications of the linear machine into an equivalent rotating machine. The machine design is carried out in the rotary domain, which then is transformed back in to the linear domain. Such a procedure brings to bear the knowledge base and familiarity of the rotary machine designers to design a linear machine effectively. This chapter contains two illustrations of the proposed design procedure for a 4.8 m long three-phase prototype with active stator and passive translator, and a 6 m long four-phase prototype with active translator and passive stator. Analysis procedures for the phase winding inductance, propulsion and normal forces with translator position are developed with a lumped-parameter magnetic circuit model and the results from it are verified with finite element analysis prior to construction of the prototype. Extensive experimental correlation in the form of inductance vs. position vs. current, propulsion force vs. position vs. current, and normal force vs. position vs. current to validate the analysis and design procedure is given in this chapter.

2.1 Introduction

There are two distinct configurations of LSRM and they are the transverse flux and longitudinal flux configurations [1], [3]. Transverse flux machines have been explored in detail in [4]-[9]. However, the design of machines is not described in detail although the calculation of inductances and force by finite element analysis is covered extensively. An extensive theoretical analysis of a double-sided double-translator longitudinal flux LSRM is described in [10]. An LSRM-based on a tubular structure to be used for short stroke lengths using a longitudinal flux configuration has been described in [13]-[15]. A longitudinal field LSRM with multiple air gaps has been designed in [16] and [17]. Although design of tubular machines has been covered in some detail in [18], in general, design of longitudinal flux single-sided machines has not been explored in literature.
There has not been a standard design procedure for LSRM so far [18]. A standard or classical design procedure begins with the power output equation relating the machine dimensions such as the bore diameter, lamination stack length, speed, magnetic loading, and electric loading. Further, the machine dimensions and their impact on performance are characterized by implicit relationships and made available in a form to enable machine design. Such a procedure allows an insight into the scaling of designs and enables the exercise of engineering judgement to select the best design with least amount of computations. These advantages are not feasible with finite element analysis. Therefore, a standard design procedure is selected for design of LSRM.

To design an LSRM using a standard design procedure, analytical expressions relating machine dimensions to output variables are required. As they are not available in literature, a slightly circuitous route is taken. Recognizing that a standard design procedure is available for the rotary SRM, the design of LSRM can proceed via the rotary SRM if the design specifications can be transformed from the linear to rotary domain and then the design is carried out in the rotary domain [22]. Then they can be recovered in the linear domain by simple algebraic transformations [19].

2.2 LSRM Configurations

Two longitudinal LSRM configurations with one three-phase LSRM with active stator and passive translator, and one four-phase LSRM with active translator and passive stator, are considered for design in this section.

2.2.1 Three-Phase LSRM with Active Stator and Passive Translator Structure

Figure 2.1 shows the three-phase LSRM structure and its winding diagram with an active stator, a passive translator, and a longitudinal flux configuration. The LSRM consists of 6 translator poles and n stator poles. This corresponds to the 6 stator and 4 rotor pole rotary SRM. One stator sector is composed of 6 stator poles and the number of stator sectors $N_{sc}$ is given by,

$$N_{sc} = \frac{n}{6}.$$ (2.1)
A rotary SRM has four rotor poles and hence the corresponding LSRM should have 4 translator poles. However, in the LSRM structure with four translator poles, there is a reversal of flux at the instant of phase current commutation. For example, consider the LSRM in Figure 2.2, which has only 4 translator poles. For the continuous forward movement of LSRM, the excitation sequence of $a_1a_2b_1b_2c_2c_3\ldots$ is necessary for motion in the increasing inductance region. However, this causes a flux reversal when the sequence is in between the excitation of $b_1b_2c_2c_3$ and so on. This results in the degradation of the LSRM performance with higher noise and increased core losses. In case this topology is used, the converter design and the switching sequence are much more complicated. Therefore, the number of translator poles is increased from 4 to 6 to prevent
the reversal of flux and to maintain the flux path in the same direction and the schematic is shown in the Figure 2.1. In case of the 8 stator and 6 rotor poles in the rotary SRM, the number of translator poles should be increased from 6 to 9 to maintain the flux in the same direction. The application of the sequence of $c_1c_1'−a_1a_1'−b_1b_1'−c_2c_2'−...$ makes the translator move in the forward direction continuously. The generated flux directions by the excitation of phase A and phase C windings are shown in Figure 2.1. Each sector operates in an independent manner, which means that there is no simultaneous excitation of poles in different sectors. For example, the simultaneous excitation of $c_1c_2$, $b_1b_2$, and $a_1a_2$, etc. is not permitted. Therefore, there is no flux flow in the back iron between $S_6$ and $S_7$, and hence the back iron portion between $S_6$ and $S_7$ can be used for other purposes such as stacking of laminations. As seen from Figure 2.1, $w_{tp}$ is the width of the translator pole, $w_{ts}$ is the width of the translator slot, $w_{sp}$ is the width of the stator pole, and $w_{ss}$ is the width of the stator slot. The stacking width of the laminations is given by $L_w$.

2.2.2 Four-Phase LSRM with Active Translator and Passive Stator Structure

Figure 2.3 shows a four-phase longitudinal LSRM with an active translator and a passive stator. The LSRM consists of 8 translator poles and this corresponds to the 8 stator and 6 rotor poles in the rotary SRM. Contrary to the active stator and passive translator structure, there is no reversal of flux at the instant of phase current commutation and the back irons of the stator and translator experience the same direction of magnetic flux regardless of the switching of the phase winding currents. The energization sequence of $aa'−bb'−cc'−dd'−aa'−...$ in the increasing inductance region makes the translator move in the forward direction continuously with the flux direction maintained the same. The flux paths and directions for phase A and phase B excitation are shown in Figure 2.3.

2.3 LSRM Design

The design of LSRM is achieved by first translating its specifications into equivalent rotary SRM specifications. Then the rotary SRM is designed from which the LSRM dimensions and design variables are recovered by inverse translation. The design procedure is derived in this section.
2.3.1 Specifications of the LSRM

The LSRM is to be designed for a machine stator length \( L_t \), with a maximum linear velocity of \( v_m \) and an acceleration time \( t_a \) required to reach the maximum velocity. The maximum mass of the translator is restricted to \( M_t \). Figure 2.4 shows the required velocity profile of the LSRM. If the deceleration time \( t_d = t_a \), the maximum acceleration is given by,

\[
a_m = \frac{v_m}{t_a}
\]

and the maximum deceleration \( a_d = -a_m \). The instantaneous acceleration force \( F_a \) is given by,

\[
F_a = M_t \cdot a_a
\]

and the instantaneous deceleration force \( F_d = -F_a \). Assuming a zero instantaneous friction force, that is, \( F_f = 0 \), the maximum power capacity of the LSRM is given by,

\[
P = F_a \cdot v_m.
\]

2.3.2 Design of Rotary SRM (RSRM)

A 6/4 RSRM is designed for a power capacity identical to that of the LSRM. The material used for the laminations is M-19 steel, which is made of non-oriented silicon steel. The RSRM
Figure 2.4 Velocity and required force profiles of LSRM.

has a stator pole angle of $\beta_s$ and a rotor pole angle of $\beta_r$. The speed of the RSRM, $N_r$, in rpm, is given by,

$$N_r = \frac{v_m \cdot 60}{D/2 \cdot 2\pi}.$$  \hspace{1cm} (2.5)

where $D$ is the bore diameter of RSRM. The power output equation of an RSRM in terms of key physical variables [22], is described by,

$$P = k_e k_d k_1 k_2 B_g A_{sp} D^2 L N_r$$ \hspace{1cm} (2.6)

where $P$ is the power output, $k_e$ is the efficiency, $k_d$ is the duty cycle determined by the current conduction angle for each rising inductance profile, $k_1 = \pi^2/120$, $k_2$ is a variable dependent on the operating point and is determined by using aligned saturated inductance and unaligned inductance, $B_g$ is the flux density in the air gap at the aligned position, $A_{sp}$ is the specific electric loading which is defined as ampere conductor per meter of stator inner periphery, $L$ is the stack length of the magnetic core, and $N_r$ is the speed. Setting the stack length as a multiple or submultiple of the bore diameter,
\[ L = kD \]
and converting the rotational angular velocity to linear velocity,
\[ P = k_e k_d k_1 k_2 B_g A_{sp} D^2 \cdot kD \cdot \left( \frac{v_m}{D/2} \cdot \frac{60}{2\pi} \right) = k_e k_d k_1 k_2 B_g A_{sp} D^2 \cdot \frac{60}{\pi} \cdot \frac{v_m}{D/2} \cdot \frac{60}{2\pi}. \]  
(2.8)

The bore diameter is obtained from the power output equation as,
\[ D = \sqrt{\frac{P\pi}{60 \cdot k_e k_d k_1 k_2 B_g A_{sp} v_m}}. \]  
(2.9)

The air gap of the LSRM is usually much larger than that of the RSRM. In the aligned portion, the B-H characteristic of the magnetic material is fairly linear and the reluctance of the steel core is small when compared to the reluctance of the air gap in the aligned position. The machine flux linkage can be calculated as,
\[ \phi = B_g A_g \]  
(2.10)

where \( A_g \) is the cross-section area of the air gap and during alignment is approximately equal to,
\[ A_g = \left( \frac{D}{2} - g \right) \left( \frac{\beta_r + \beta_s}{2} \right) L \]  
(2.11)

where \( g \) is the length of the air gap. The magnetic field intensity in the air gap can be calculated as,
\[ H_g = \frac{B_g}{\mu_0}. \]  
(2.12)

Assuming the existence of a large air gap, the ampere-turns required to produce the air gap magnetic field intensity is given by,
\[ T_{ph} I_p = H_g \cdot 2g \]  
(2.13)

where \( T_{ph} \) is the number of winding turns per phase and \( I_p \) is the peak phase winding current. Assuming a value for the peak phase winding current allowable in the machine, the number of turns per phase of the RSRM can be calculated as,
\[ T_{ph} = \frac{H_g \cdot 2g}{I_p}. \]  
(2.14)

If \( J \) is the maximum allowable current density in the winding and \( m \) is the number of phases, the cross-section area of a conductor is calculated as,
Neglecting the leakage of flux linkages, the area of the stator pole, the flux density in the stator pole, the area of the stator yoke, and the height of the stator pole can be calculated, respectively, as,

\[ A_s = \frac{D L \beta_s}{2} \]  
\[ B_s = \frac{\phi}{A_s} \]  
\[ A_y = C_{sy} \cdot L = \frac{A_s B_s}{B_y} \]  
\[ h_s = \frac{D_o}{2} - \frac{D}{2} - C_{sy} \]

where \( C_{sy} \) is the thickness of the stator yoke and \( D_o \) is the outer diameter of the stator lamination.

The rotor pole area is given by,

\[ A_r = \left( \frac{D}{2} - g \right) L \beta_r . \]

If the rotor yoke has a radius equal to the width of the rotor pole, the rotor yoke width and the height of the rotor pole are sequentially calculated as,

\[ C_{ry} = \frac{A_r}{L} \]  
\[ h_r = \frac{D}{2} - g - C_{ry} . \]

This completes the analytical relationships required for the RSRM design.

### 2.3.3 Conversion of RSRM Dimensions to LSRM Dimensions

The bore circumference of the RSRM forms the length of one sector of the LSRM. The total number of sectors of the LSRM is given by,

\[ N_{sc} = \frac{L_{scl}}{\pi D} \]  

For the number of poles \( N_s \) in the stator of the RSRM, the number of stator poles is obtained by,

\[ n = N_s N_{sc} . \]
The width of stator pole and the width of stator slot are given by,

\[
w_{sp} = \frac{A_s}{L} = \left(\frac{D}{2}\right)\beta_s \tag{2.25}
\]

\[
w_{ss} = \frac{(\pi D - N_s w_{sp})}{N_s} \tag{2.26}
\]

The translator pole width and the translator slot width are converted from the rotor pole area neglecting the air gap length and are given by,

\[
w_{tp} = \left(\frac{D}{2}\right)\beta_r \tag{2.27}
\]

\[
w_{ts} = \frac{(\pi D - N_t w_{tp})}{N_t} \tag{2.28}
\]

where \(N_t\) is the number of poles in the rotor of the RSRM. Now, the fill factor of the windings is verified to see if the slot size is sufficient to hold the windings. The fill factor is defined as,

\[
FF = \frac{\text{Stator Winding Area}}{\text{Stator Slot Window Area}} \tag{2.29}
\]

The diameter of the conductor is given by,

\[
d_c = \sqrt{\frac{4a_c}{\pi}} \tag{2.30}
\]

Assuming that a portion of the stator pole height is occupied by wedges to hold the windings in place given by \(w\), the number of vertical layers of winding is,

\[
N_v = f_f \frac{(h_s - w)}{d_c} \tag{2.31}
\]

where \(f_f\) is the form factor or packing factor. If the number is a fraction, it is rounded off to a lower integer. The number of horizontal layers of winding is given by,

\[
N_h = \frac{T_{ph}}{2N_v} \tag{2.32}
\]

The stator winding area is given by,

\[
\text{Stator Winding Area} = 2\frac{a_c N_h N_v}{f_f} \tag{2.33}
\]

Finally, the fill factor is calculated as,
Consideration has to be given to keep the two phase windings in the slot separated also. Then the normal range of the fill factor is in the range given by $0.2 \leq FF < 0.7$. The translator length for an LSRM with 6 translator poles is then calculated as,

$$L_{tr} = 6w_{tp} + 5w_{ts}.$$  

(2.35)

Since the core stack length of the LSRM equals the stator stack length of the RSRM, the core stack length is written as,

$$L_w = L = kD.$$  

(2.36)

Finally, the following condition has to be satisfied,

$$N_s (w_{sp} + w_{ss}) = N_t (w_{tp} + w_{ts}).$$  

(2.37)

The winding details of the RSRM and LSRM are identical in this design. This need not be the case as the duty cycle of a winding in the three-phase LSRM, $1/(3N_{sc})$ whereas that of the RSRM winding it is $1/3$. Therefore, the windings in the LSRM can have much lower copper volume but taking more losses.

### 2.3.4 Prototype Design Example

**Example 1: Three-Phase LSRM Prototype with Active Stator and Passive Translator**

An LSRM prototype is designed for a length of 4.8 m, with a maximum linear velocity of 1.5 m/s and acceleration time of 0.667 s. The maximum mass of translator assembly is restricted to 20 Kg. The acceleration is then given by,

$$a_a = \frac{v_m}{t_a} = \frac{1.5}{0.667} = 2.25 \text{ m/s}^2.$$ 

The force for initial acceleration is calculated as,

$$F_a = M \cdot a_a = 20 \cdot 2.25 = 45 \text{ N}.$$ 

The deceleration $a_d = -2.25 \text{ m/s}^2$ and the deceleration force $F_d = -45 \text{ N}$. The power capacity of the LSRM is $P = F_a v_m = 45 \cdot 1.5 = 67.5 \text{ W}$. The RSRM is assumed to have a stator pole angle $\beta_s =$
30° = 0.524 rads and a rotor pole angle β_r = 36° = 0.628 rads. After fine-tuning the parameters, the constants are set as follows: k_e = 0.4, k_d = 1, k_2 = 0.7, B_g = 1.1215 T, A_{sp} = 23886.5, and k = 0.655. The bore diameter is obtained as,

\[ D = \sqrt[6]{\frac{P\pi}{60 \cdot k_e k_d k_1 k_2 k B_g A_{sp} v_m}} = 76.39 \text{ mm}. \]

The stack length of the RSRM is obtained as,

\[ L = kD = 0.655 \cdot 76.39 = 50 \text{ mm}. \]

The stator yoke thickness C_{sy} is given by,

\[ C_{sy} = \frac{D\beta_s}{2} = 20 \text{ mm}. \]

Assuming D_o = 190 mm, the height of the stator pole h_s can be calculated as,

\[ h_s = \frac{D_o}{2} - \frac{D}{2} - C_{sy} = \frac{190}{2} - \frac{76.39}{2} - 20 \approx 37 \text{ mm}. \]

The rotor back iron width C_{ry} and the height of the rotor pole (the translator pole) h_r are then calculated as,

\[ C_{ry} = \left( \frac{D}{2} \right) \beta_r = \left( \frac{76.39}{2} \right) 0.628 = 24 \text{ mm} \]

\[ h_r = \frac{D}{2} - g - C_{ry} = \frac{76.39}{2} - 1 - 24 \approx 15 \text{ mm}. \]

The magnetic field intensity in the air gap is calculated as,

\[ H_g = \frac{B_g}{\mu_0} = \frac{1.1215}{4\pi \cdot 10^{-7}} = 892461.3 \text{ A/m}. \]

For a peak phase winding current of I_p = 8.5 A, the number of turns per phase is,

\[ T_{ph} = \frac{H_g \cdot 2g}{I_p} \approx 210 \text{ Turns/phase}. \]

Assuming a current density of J = 6 A/mm^2, the area of a conductor is calculated as,

\[ a_c = \frac{I_p}{J \cdot \sqrt{m}} = \frac{8.5}{6 \sqrt{3}} = 0.818 \text{ mm}^2. \]

The closest wire size for this area of cross-section of the conductor is AWG #18 and it has an area of 0.817 mm^2 and it is selected for the phase windings. The calculation of the winding turns completes the RSRM design. The conversion from rotary to linear domain follows.
The number of sectors of the LSRM and the resultant total number of stator poles are,

\[ N_{sc} = \frac{L_t}{\pi D} = \frac{4.8}{\pi \cdot 76.39 \cdot 10^{-3}} = 20 \]

\[ n = N_s N_{sc} = 6 \cdot 20 = 120. \]

The width of stator pole and the width of stator slot are given by,

\[ w_{sp} = \frac{D \beta_s}{2} = \frac{76.394 \cdot 30 \cdot \pi}{180} = 20 \text{ mm} \]

\[ w_{ss} = \frac{\pi (D - 6w_{sp})}{6} = \frac{\pi \cdot 76.394 - 6 \cdot 20}{6} = 20 \text{ mm}. \]

The width of translator pole width and the translator slot width are calculated as,

\[ w_{tp} = C_{ry} = 24 \text{ mm} \]

\[ w_{ts} = \frac{\pi (D - 4w_{tp})}{4} = \frac{\pi \cdot 76.394 - 4 \cdot 24}{4} = 36 \text{ mm}. \]

The total length of the translator is given by,

\[ L_{tr} = 6w_{tp} + 5w_{ts} = 6 \cdot 24 + 5 \cdot 36 = 324 \text{ mm}. \]

The core stack width of the LSRM is obtained from the stator stack length of the RSRM as,

\[ L_w = L = kD = 50 \text{ mm}. \]

Now, the fill factor of the windings is verified to see if the slot size is sufficient to hold the windings. The diameter of the conductor is given by,

\[ d_c = \sqrt{\frac{4a_c}{\pi}} = \sqrt{\frac{4 \cdot 0.817}{\pi}} = 1.02 \text{ mm}. \]

Assuming the width of the wedges \( w = 3 \) and packing factor \( f_t = 0.8 \), the number of vertical layers of winding and the number of horizontal layers of winding are obtained as,

\[ N_v = f_t \cdot \frac{(h_s - w)}{d_c} = 0.8 \cdot \frac{(37 - 3)}{1.02} = 26.6 \approx 26 \]

\[ N_h = \frac{T_{ph}}{2 \cdot N_v} = \frac{210}{2 \cdot 26} = 4.03 \approx 4. \]

The stator winding area is given by,

\[ \text{Stator Winding Area} = 2 \cdot \frac{a_c N_v N_h}{f_t} = 212.4 \text{ mm}^2. \]

The fill factor is calculated as,
Finally, it can be observed that the condition outlined in (2.37) is satisfied with this design. Figure 2.5 shows the finalized dimensions of the designed three-phase LSRM.

**Example 2: Four-Phase LSRM Prototype with Active Translator and Passive Stator**

An LSRM prototype is designed for a length of 6 m, with a maximum linear velocity of 1 m/s and acceleration time of 1.5 s. The maximum mass of translator assembly is restricted to 60 Kg. The acceleration is then given by,

\[
a_a = \frac{v_m}{t_a} = \frac{1}{1.5} = 0.667 \text{ m/s}^2.
\]

The instantaneous acceleration force is calculated as,

\[
F_a = M_t \cdot a_a = 60 \cdot 0.667 = 40 \text{ N}.
\]

The deceleration \(a_d = -0.667 \text{ m/s}^2\) and the instantaneous deceleration force \(F_d = -40 \text{ N}\). The power capacity of the LSRM is \(P = F_a v_m = 40 \cdot 1 = 40 \text{ W}\). The RSRM is assumed to have a stator pole angle \(\beta_s = 18^\circ = 0.314 \text{ rads}\) and a rotor pole angle \(\beta_r = 22^\circ = 0.384 \text{ rads}\). After fine-tuning the parameters, the constants are set as follows: \(k_c = 0.3\), \(k_d = 1\), \(k_2 = 0.7\), \(B_g = 0.65 \text{ T}\), \(A_{sp} = 41456.7\), and \(k = 0.8\). The bore diameter is obtained as,
The stack length of the RSRM is obtained as,
\[ L = kD = 0.8 \cdot 75 = 60 \text{ mm}. \]

The stator yoke thickness \( C_{sy} \) is given by,
\[ C_{sy} = \frac{D \beta_s}{2} \approx 12 \text{ mm}. \]

Assuming \( D_o = 190 \text{ mm} \), the height of the stator pole \( h_s \) can be calculated as,
\[ h_s = \frac{D_o}{2} - \frac{D}{2} - C_{sy} = \frac{190}{2} - \frac{75}{2} - 12 \approx 44 \text{ mm}. \]

The rotor back iron width \( C_{ry} \) and the height of the rotor pole \( h_r \) are then calculated as,
\[
C_{ry} = \left( \frac{D}{2} \right) \beta_r = \left( \frac{75}{2} \right) 0.384 \approx 15 \text{ mm}.
\]
\[ h_r = \frac{D}{2} - g - C_{ry} = \frac{75}{2} - 3 - 15 \approx 15 \text{ mm}. \]

The magnetic field intensity in the air gap is calculated as,
\[ H_g = \frac{B_g}{\mu_0} = \frac{0.65}{4\pi \cdot 10^{-7}} = 517253.6 \text{ A/m}. \]

For a peak phase winding current of \( I_p = 8.5 \text{ A} \),
\[ T_{ph} = \frac{H_g \cdot 2g}{I_p} \approx 360 \text{ Turns/phase}. \]

Assuming a current density of \( J = 6.5 \text{ A/mm}^2 \), the area of a conductor is calculated as,
\[ a_c = \frac{I_p}{J \sqrt{m}} = \frac{8.5}{6.5 \sqrt{4}} = 0.654 \text{ mm}^2. \]

AWG #19 is suitable as it has an area of 0.653 mm².

In active translator and passive stator structure of LSRM, the stator and rotor of RSRM corresponds to the translator and stator of LSRM, respectively. The widths of stator pole and stator slot are given by,
\[ w_{sp} = C_{ry} \approx 15 \text{ mm} \]
\[ w_{ss} = \frac{\pi D - 6w_{sp}}{6} = \frac{\pi \cdot 75 - 6 \cdot 15}{6} \approx 25 \text{ mm}. \]
The total number of stator poles is,
\[ n = \frac{L_t}{W_{sp} + W_{ss}} = \frac{6.0}{0.015 + 0.025} = 150. \]

The translator pole width and the translator slot width are calculated as,
\[ w_{tp} = C_{sy} \approx 12 \text{ mm} \]
\[ w_{ts} = \frac{(\pi D - 8w_{tp})}{8} = \frac{(\pi \cdot 75 - 8 \cdot 12)}{8} \approx 18 \text{ mm}. \]

The total length of the translator is given by,
\[ L_{tr} = 8w_{tp} + 7w_{ts} = 8 \cdot 12 + 7 \cdot 18 = 222 \text{ mm}. \]

The core stack width of the LSRM is obtained from the stator stack length of the RSRM as,
\[ L_w = L = kD = 60 \text{ mm}. \]

Now, the fill factor of the windings is verified to see if the slot size is sufficient to hold the windings. The diameter of the conductor is given by,
\[ d_c = \sqrt{\frac{4a_c}{\pi}} = \sqrt{\frac{4 \cdot 0.817}{\pi}} = 0.912 \text{ mm}. \]

Assuming the width of the wedges \( w = 3 \) mm and packing factor \( f_f = 0.8 \), the number of vertical layers of winding and the number of horizontal layers of winding are given by,
\[ N_v = f_f \cdot \frac{(h_s - w)}{d_c} = 0.8 \cdot \frac{44 - 3}{0.912} \approx 36 \]
\[ N_h = \frac{T_{ph}}{2 \cdot N_v} = \frac{360}{2 \cdot 36} \approx 5. \]

The stator winding area is given by,
\[ \text{Stator Winding Area} = 2 \frac{a_c N_v N_h}{f_f} = 293.9 \text{ mm}^2. \]

The fill factor is calculated as,
\[ \text{FF} = \frac{\text{Stator Winding Area}}{\text{Stator Slot Window Area}} = \frac{293.9}{(44 - 3) \cdot 18} = 0.398. \]

Note that the condition outlined in (2.37) is also satisfied with this design. Figure 2.6 shows the final dimensions of the designed four-phase LSRM.
2.4 Design Verification

The design verification process, very much similar to the rotary SRM, includes analytical calculation, and finite element analysis and experimental verification of the machine. The analytical part of the verification is in finding the performance of the machine through its flux linkage vs. current vs. translator position characteristics derived using magnetic equivalent circuit approach. The finite element analysis is made using one of the commercial software and the design is fine-tuned with these results. Then the prototype is usually built and tested. A strong correlation between the analytical, finite element, and experimental results assures confidence in the engineering analysis and design methods adopted, and enables confident scaling of the machine. A poor correlation of the results forces the designer to revisit the design methodology.

2.4.1 Analytical Inductance Calculation

Flux linkages in the machine are calculated for a given excitation and a translator position using magnetic equivalent circuit. The flux linkages can be obtained from the inductance and corresponding excitation current. In this section, the winding inductance is computed using permeance of the magnetic flux paths. The magnetic flux path of an LSRM encompasses five parts of the machine, namely, air gaps, stator poles, stator yoke, translator poles, and translator...
yoke. Each part experiences different flux densities and different lengths of flux line based on the chosen magnetic flux path. From the pattern of the flux path of the three-phase LSRM prototype shown in Figure 2.1, the lumped parameter model of the equivalent magnetic circuit of LSRM can be derived and is shown in Figure 2.7. The relevant reluctances are named for each of the flux path in one of the five parts of the machine. The mmf applied to a phase winding at any translator position is given by,

\[ F_T = F_1 + F_2 = F_g + F_s + F_t \]  

where \( F_T \) is the total mmf per phase applied, \( F_g, F_s \), and \( F_t \) are the mmf drops in the air gap, stator iron core, and translator iron core, respectively. The mmf equation can be rewritten in terms of the magnetic field intensity and flux path length using Ampere’s Circuital Law as,

\[ F_T = T_{ph}i = \sum H_g l_g + \sum H_s l_s + \sum H_t l_t \]  

where \( H_g, H_s, H_t, l_g, l_s, \) and \( l_t \) are the magnetic field intensities and flux path lengths in the air gap, stator iron core, and translator iron core, respectively.

The following assumptions are made to compute the reluctances:

1) Only one phase is excited at any given time.
2) Flux distribution is uniform over the cross-section of the core.
3) Hysteresis is negligible.

4) The flux lines enter and leave the iron surface normally.

5) The air gap sections of the flux lines consist of straight lines and concentric arc segments.

6) The flux lines in the stator and translator poles run parallel to each pole axis.

7) The flux lines in the stator and translator yokes run parallel to the longitudinal axis.

Then the flux density $B_k(i,x)$ in the path $k$ for current $i$ and translator position $x$ is given by,

$$B_k(i,x) = \frac{\phi_k(i,x)}{A_k} \tag{2.40}$$

where $\phi_k(i,x)$ is the magnetic flux, $A_k$ is the cross-section area determined by the geometry of LSRM. In the air gap, the flux density can be represented in terms of permeance equation as,

$$B_{gk}(i,x) = \frac{\phi_k(i,x)}{P_{gk}(x)} \cdot \frac{\mu_0}{l_{gk}} \cdot P_{gk}(x) = \frac{\mu_0 A_k}{l_{gk}} \tag{2.41}$$

where $P_{gk}(x)$ is the air gap permeance, $l_{gk}$ is the average air gap length of flux lines in path $k$.

Since the magnetic flux $\phi_k(i,x)$ is an unknown variable, iterative procedure such as bi-section root-finding algorithm is introduced by assuming arbitrary initial value of $\phi_k(i,x)$. The value of $B_k(i,x)$ is obtained from (2.40) and is used in the B-H characteristics of the core material to obtain the magnetic field intensity $H_k(i,x)$, except in the air gap region of linear free space.

Determination of magnetic field intensity in the iron core requires an accurate fit of the B-H nonlinear characteristic curve and cubic spline interpolation is normally used. The value of $H_k(i,x)$ thus obtained is used to test for convergence in the mmf equation (2.39). The iterative solution outlined above gives the value of magnetic flux $\phi_k(i,x)$ associated with path $k$. Finally, the inductance $L(i,x)$ for one phase with all magnetic flux paths is calculated as,

$$L(i,x) = \sum_k L_k(i,x) = \frac{T_{ph}}{i} \cdot \sum_k \phi_k(i,x) \tag{2.42}$$

A flow chart of the calculation procedure is shown in Figure 2.8.

The selection of flux paths is usually accomplished based on the flux distribution in the air gap. The air gap predominantly determines the flux distribution as most of the mmf and hence the magnetic energy is applied to the air gap as long as the core is not saturated. On this basis, 4 different translator regions are considered for the evaluation of the permeance in the air gap. The fringing flux paths in the air gap are shown in Figure 2.9. The flux paths are chosen mainly based on the experience gained through the finite element analysis (FEA). The air gap permeance path
Figure 2.8 Flow chart for the calculation of the phase winding inductance using bi-section root-finding algorithm.
(a) Aligned position region:
\((w_{ts} + w_{sp})/2 \leq x \leq (w_{ts} + w_{tp})/2\).

(b) Intermediate position region I:
\((w_{ts} + w_{sp})/2 - (h_r + g)/2 \leq x < (w_{ts} + w_{sp})/2\).

(c) Intermediate position region II:
\((w_{ts} - w_{sp})/2 \leq x < (w_{ts} + w_{sp})/2 - (h_r + g)/2\).

(d) Unaligned position region:
\(0 \leq x < (w_{ts} - w_{sp})/2\).

Figure 2.9 Magnetic flux paths and permeance calculation in the air gap for 4 different translator position regions.
for the different location of translator can similarly be inferred by these four configurations. With
careful examination of Figure 2.9 (b) and (c), it is observed that the local saturation occurs in the
pole tips of both stator and translator. The local saturation occurs when the relative position of
stator and translator is at the threshold of overlap or during partial overlap. This phenomenon is
dependent on the level of excitation and the shape of the pole tips. To accommodate the pole tip
saturation effect, the flux density in this region is assumed to be the maximum value for the iron
core material, and the effective pole width and pole height are approximated using small
rectangular area. Uniform distribution of magnetic flux density in the core is assumed for fully
aligned and unaligned positions shown in Figures 2.9(a) and (d).

The geometric parameters shown in Figure 2.9 have the following relations and they are used
in the subsequent equations for air gap permeance calculation.

\[
t = \frac{h_s}{12}, \quad f = \frac{(w_{tp} - w_{sp})}{2}
\]
\[
d_1 = x - \frac{(w_{tp} - w_{sp})}{2}, \quad d_2 = w_{sp} - d_1, \quad d_3 = w_{tp} - d_1,
\]
\[
t_1 = d_2 - g, \quad t_2 = d_3 - g
\]
\[
d_4 = x - \frac{(w_{ts} - w_{sp})}{2}, \quad d_5 = \frac{2}{\pi} \cdot (h_r + g), \quad d_6 = \frac{2}{\pi} \cdot w_{ss},
\]
\[
t_3 = d_5 - g, \quad t_4 = d_6 - g
\]
\[
d_7 = \frac{(w_{ts} - w_{sp})}{2} - x, \quad d_8 = \frac{2}{\pi} \cdot w_{ss} - d_7, \quad d_9 = \frac{2}{\pi} \cdot (h_r + g) - d_7,
\]
\[
t_5 = d_7 - g
\]

where \(x\) denotes the propulsion position of the translator which is equal to the distance between
the translator slot axis and stator pole axis.

2.4.1.1 Aligned Position Region: \((w_{ts} + w_{sp})/2 \leq x \leq (w_{ts} + w_{tp})/2\)

The air gap permeance of the aligned position region considering the end effects generated
by the end windings of the stator pole is derived in this section. The analytical estimation of the
permeance of air gap flux paths is originally developed in [10] and [12] for use in the
electromagnetic devices, and is summarized in Appendix A. The air gap has 7 different types of
flux paths as shown in Figure 2.9(a). Each flux path consists of straight lines and concentric
circular arc segments. Also, each flux path has a numeric label that is associated with the permeance expression. The paths from 1 to 3 are the main flux linkages for the air gap permeance evaluation, and the paths from 4 to 7 are included to consider the three-dimensional end effect. The fringing flux paths of pole-ends are simply indicated by dashed oblique line and a small dot because of the difficulty to show in a two-dimensional figure. Detailed equations for the permeance evaluation in the poles and back irons of the stator and translator are given in Appendix B. The derived air gap permeances are given in the following.

\[
P_{g1} = \mu_0 \frac{L_w \cdot (w_{sp} + w_{tp})/2}{g} = \mu_0 L_w \left( \frac{w_{sp} + w_{tp}}{2 \cdot g} \right)
\]  
\[P_{g2} = \mu_0 \left( \frac{L_w \cdot 0.322g}{1.22g} \right) = 0.268 \cdot \mu_0 L_w
\]  
\[P_{g31} = 0.318 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{2t}{g} \right), \quad P_{g32} = \mu_0 \frac{L_w t}{f} = \mu_0 L_w \frac{t}{f}
\]  
\[P_{g3} = \frac{P_{g31} \cdot P_{g32}}{P_{g31} + P_{g32}}
\]  
\[P_{g4} = 0.268 \cdot \mu_0 \left( \frac{w_{sp} + w_{tp}}{2} \right) = 0.134 \cdot \mu_0 (w_{sp} + w_{tp})
\]  
\[P_{g5} = 0.318 \cdot \mu_0 \left( \frac{w_{sp} + w_{tp}}{2} \right) \cdot \ln \left( 1 + \frac{2t}{g} \right) = 0.159 \cdot \mu_0 (w_{sp} + w_{tp}) \cdot \ln \left( 1 + \frac{2t}{g} \right)
\]  
\[P_{g6} = 0.076 \cdot \mu_0 g
\]  
\[P_{g7} = 0.25 \cdot \mu_0 t.
\]

By assuming the initial value for flux \( \phi_k \), the flux density of each part can be obtained from (2.40) and (2.41). The total mmf, \( F_T \), at rated phase current is calculated using (2.39) as,

\[
F_T = 2 \cdot \left( \frac{\phi_k}{P_{gk}} + H_{spk} \cdot l_{spk} + H_{syk} \cdot l_{syk} + H_{tpk} \cdot l_{tpk} + H_{tyk} \cdot l_{tyk} \right)
\]

where \( H_{spk}, H_{syk}, H_{tpk}, \) and \( H_{tyk} \) are the magnetic field intensities in the stator pole, stator back iron, translator pole, and translator back iron in path k, respectively. These values are obtained from the B-H characteristics of the lamination material, M19 steel. After the convergence of mmf equation with the designed value of \( F_T = T_{phi} = 210 \cdot 8.5 = 1785 \), the fully aligned inductance for the rated phase current is calculated as,
\[ L(i = 8.5 \text{ A}, x = 30 \text{ mm}) = L_1 + 2 \cdot (L_2 + L_3) + 4 \cdot (L_4 + L_5 + L_6 + L_7) \]
\[ = \frac{T_{ph}}{i} \cdot ((\phi_1 + 2 \cdot (\phi_2 + \phi_3) + 4 \cdot (\phi_4 + \phi_5 + \phi_6 + \phi_7)) = 32.608 \text{ mH}. \]

The flux density in the air gap is then obtained as,
\[ B_{g1} = \frac{\phi_{k1}}{P_{g1}} \cdot \frac{\mu_0}{g} = \frac{1.196 \cdot 10^{-3}}{1.382 \cdot 10^{-6}} \cdot \frac{4 \pi \cdot 10^{-7}}{0.001} = 1.087 \text{ T}. \]

2.4.1.2 Intermediate Position Region I: \((w_{ts}+w_{sp})/2-(h_r+g)\cdot 2/\pi \leq x < (w_{ts}+w_{sp})/2\)

The air gap in this region has 10 different types of flux paths and for each of these paths, the permeance is derived as,
\[ P_{g1} = \mu_0 L_w \cdot \frac{d_1}{g} \quad (2.55) \]
\[ P_{g2} = 0.5356 \cdot \mu_0 L_w \quad (2.56) \]
\[ P_{g3} = \frac{\mu_0 L_w}{\pi/2} \cdot \ln \left( \frac{g + t_1}{g} \right) = 0.637 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{t_1}{g} \right) \quad (2.57) \]
\[ P_{g4} = \frac{\mu_0 L_w}{\pi/2} \cdot \ln \left( \frac{g + t_2}{g} \right) = 0.637 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{t_2}{g} \right) \quad (2.58) \]
\[ P_{g5} = \mu_0 L_w \cdot \frac{(h_s - d_2)}{w_{ss}} = \mu_0 L_w \cdot \left( \frac{h_s - d_2}{w_{ss}} \right) \quad (2.59) \]
\[ P_{g6} = \mu_0 L_w \cdot \frac{(h_s - t_2)}{w_{ss}} = \mu_0 L_w \cdot \left( \frac{h_s - t_2}{w_{ss}} \right) \quad (2.60) \]
\[ P_{g7} = 0.268 \cdot \mu_0 d_1 \quad (2.61) \]
\[ P_{g8} = \frac{\mu_0 d_1}{\pi} \cdot \ln \left( \frac{(g + t_1 + t_2)/2}{g/2} \right) = 0.318 \cdot \mu_0 d_1 \cdot \ln \left( 1 + \frac{(t_1 + t_2)}{g} \right) \quad (2.62) \]
\[ P_{g9} = 0.152 \cdot \mu_0 d_2 \quad (2.63) \]
\[ P_{g10} = 0.152 \cdot \mu_0 d_3 \quad (2.64) \]

The computational procedure for inductance is very similar to that of the aligned inductance. The intermediate inductance for the rated phase current and translator position of 10 mm is calculated as,
\[ L(i = 8.5 \text{ A}, x = 20 \text{ mm}) = 25.280 \text{ mH}. \]
2.4.1.3 Intermediate Position Region II: \((w_{ts} - w_{sp})/2 \leq x < (w_{ts} + w_{sp})/2 - (h_r + g) \cdot 2/\pi\)

The air gap in this region is assumed to have 11 different types of flux paths and their permeances are given by,

\[
P_{g1} = \mu_0 L_w \cdot \frac{d_4}{g} 
\]

\[
P_{g2} = 0.5356 \cdot \mu_0 L_w 
\]

\[
P_{g3} = \frac{\mu_0 L_w}{(\pi/2)} \cdot \ln \left( \frac{g + t_3}{g} \right) = 0.637 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{t_3}{g} \right) 
\]

\[
P_{g4} = \frac{\mu_0 L_w}{(\pi/2)} \cdot \ln \left( \frac{g + t_4}{g} \right) = 0.637 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{t_4}{g} \right) 
\]

\[
P_{g5} = \mu_0 L_w \cdot \frac{(h_s - d_5)}{w_{ss}} = \mu_0 L_w \cdot \frac{h_s - d_5}{w_{ss}} 
\]

\[
P_{g6} = \mu_0 L_w \cdot \frac{(h_s - t_4)}{w_{ss}} = \mu_0 L_w \cdot \frac{h_s - t_4}{w_{ss}} 
\]

\[
P_{g7} = \mu_0 L_w \cdot \frac{(w_{sp} - d_4 - d_5)}{g + h_r} = \mu_0 L_w \cdot \frac{w_{sp} - d_4 - d_5}{g + h_r} 
\]

\[
P_{g8} = 0.268 \cdot \mu_0 d_4 
\]

\[
P_{g9} = \frac{\mu_0 d_4}{\pi} \cdot \ln \left( \frac{(g + t_3 + t_4)/2}{g/2} \right) = 0.318 \cdot \mu_0 d_4 \cdot \ln \left( 1 + \frac{(t_3 + t_4)/2}{g} \right) 
\]

\[
P_{g10} = 0.152 \cdot \mu_0 d_5 
\]

\[
P_{g11} = 0.152 \cdot \mu_0 d_6 
\]

The intermediate inductance for the rated phase current and translator position of 20 mm from these equations is calculated as,

\[
L(i = 8.5 \text{ A}, x = 10 \text{ mm}) = 11.680 \text{ mH}.
\]

2.4.1.4 Unaligned Position Region: \(0 \leq x < (w_{ts} - w_{sp})/2\)

The air gap in this section is assumed to have 10 different types of flux paths and their permeances are given by,

\[
P_{g1} = \mu_0 \frac{L_w \cdot t_5/\sqrt{2}}{\sqrt{2d_7}} = \mu_0 L_w \cdot \frac{t_5}{2d_7}.
\]
The mean length of the flux path 2 is equal to the length of a line drawn midway between the straight line and arc circumference, and this is equal to \(1.462 \cdot d_7\). The mean cross-section area of the flux path is estimated by dividing the entire volume of the flux path 2 by the mean length. The permeances are:

\[
P_{g2} = \mu_0 \left( \frac{0.285 \cdot d_7^2 L_w}{1.462 \cdot d_7} \right) = 0.134 \cdot \mu_0 L_w \tag{2.77}
\]

\[
P_{g3} = \frac{\mu_0 L_w}{(\pi/2)} \cdot \ln \left( \frac{d_7 + d_8}{d_7} \right) = 0.637 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{d_8}{d_7} \right) \tag{2.78}
\]

\[
P_{g4} = \frac{\mu_0 L_w}{(\pi/2)} \cdot \ln \left( \frac{d_7 + d_9}{d_7} \right) = 0.637 \cdot \mu_0 L_w \cdot \ln \left( 1 + \frac{d_9}{d_7} \right) \tag{2.79}
\]

\[
P_{g5} = \mu_0 \frac{L_w \cdot (h_s - t_5 - d_8)}{w_{ss}} = \mu_0 L_w \cdot \left( \frac{h_s - t_5 - d_8}{w_{ss}} \right) \tag{2.80}
\]

\[
P_{g6} = \mu_0 \frac{L_w \cdot (w_{sp} - 2d_9)}{h_r + g} = \mu_0 L_w \cdot \left( \frac{w_{sp} - 2d_9}{h_r + g} \right) \tag{2.81}
\]

\[
P_{g7} = \mu_0 \left( \frac{0.393 \cdot d_7^2 t_5}{1.713 \cdot d_7} \right) = 0.134 \cdot \mu_0 t_5 \tag{2.82}
\]

\[
P_{g8} = \frac{\mu_0}{\sqrt{2 \pi}} \cdot \frac{t_5}{2} \cdot \left( \frac{d_8 + d_9}{2d_7 + d_8 + d_9} \right) = 0.318 \cdot \mu_0 \cdot \frac{t_5}{1 + \frac{2d_7}{d_8 + d_9}} \tag{2.83}
\]

\[
P_{g9} = 0.268 \cdot \mu_0 (w_{sp} - 2d_9) \tag{2.84}
\]

\[
P_{g10} = \mu_0 \left( \frac{(w_{sp} - 2d_9) \cdot d_9}{\pi/2 \cdot (h_r + g + d_9)} \right) = 0.637 \cdot \mu_0 \cdot \frac{(w_{sp} - 2d_9)}{1 + \frac{(h_r + g)}{d_9}} \tag{2.85}
\]

The fully unaligned inductance for the rated phase current is calculated for the example as,

\[L(i = 8.5 \text{ A}, x = 0 \text{ mm}) = 7.490 \text{ mH}.
\]

### 2.4.2 Analytical Force Calculation

The forces in the x, z, and y directions, named as the propulsion force, normal force, and lateral force, respectively, and shown in Figure 2.10, can be calculated using the principle of conservation of energy expressed as,
Figure 2.10 Analytical force calculation using the principle of conservation of energy.

\[ \text{dW}_e = \text{dW}_l + \text{dW}_s + \text{dW}_m \] (2.86)

where \( \text{dW}_e \) is the incremental electric input energy, \( \text{dW}_l \) is the incremental loss energy dissipated by heat, \( \text{dW}_s \) is the incremental stored magnetic energy, and \( \text{dW}_m \) is the incremental mechanical energy. Assuming that the translator moves in \( x \) direction by \( dx \), the incremental mechanical energy is defined as,

\[ \text{dW}_{mx} = f_x \cdot dx \] (2.87)

where \( f_x \) is the propulsion force generated. The same expression is used if appropriate direction and the displacements in that direction is for any other force. It is clear from the conservation of energy principle that the force calculations in \( x, z, \) and \( y \) directions require incremental values of electric input energy, loss energy dissipated by heat, and stored energy in the magnetic field.

The flux density in the air gap is given by,

\[ B_g = \frac{\phi}{XY} \] (2.88)

where \( X \) and \( Y \) are the overlapped lengths in a particular position of the translator with respect to the stator in the \( x \) and \( y \) directions, respectively. As described in Section 2.3, assuming a large air gap in the linear machine structure, (2.10) can be written as,
\[ \phi = B_g \cdot XY = \mu_0 H_g \cdot XY = \mu_0 \frac{T_{ph} i}{Z} \cdot XY \Rightarrow T_{ph} i = \frac{\phi Z}{\mu_0 XY}. \]  

(2.89)

The incremental electric input energy \( dW_e \) is given by the multiplication of flux linkage \( \lambda \) and constant excitation current \( i \) as,

\[ dW_e = i \cdot d\lambda = i \cdot d(T_{ph} \phi) = T_{ph} i \cdot d\phi = \frac{\phi Z}{\mu_0 XY} \cdot d\phi. \]  

(2.90)

The stored energy in the magnetic field \( W_s \) is,

\[ W_s = \frac{B_g^2 \cdot XYZ}{2\mu_0} = \left( \frac{\phi}{XY} \right)^2 \cdot \frac{XYZ}{2\mu_0} = \frac{\phi^2 Z}{2\mu_0 XY}. \]  

(2.91)

The incremental stored magnetic energy \( dW_{sx} \) during the incremental displacement \( dx \), which causes the propulsion force, is given by,

\[ dW_{sx} = -\frac{\phi^2 Z}{2\mu_0 X^2 Y} \cdot dx + \frac{\phi Z}{\mu_0 XY} \cdot d\phi. \]  

(2.92)

Ignoring incremental energy due to losses such as copper loss, core loss, and mechanical friction loss, the incremental mechanical energy \( dW_{mx} \) during the incremental displacement \( dx \) can be written as,

\[ dW_{mx} = dW_e - dW_{sx} = \frac{\phi Z}{\mu_0 XY} \cdot d\phi + \frac{\phi^2 Z}{2\mu_0 X^2 Y} \cdot dx - \frac{\phi Z}{\mu_0 XY} \cdot d\phi = \frac{\phi^2 Z}{2\mu_0 X^2 Y} \cdot dx. \]  

(2.93)

From which the propulsion force is now obtained as,

\[ f_x = \frac{dW_{mx}}{dx} = \frac{\phi^2 Z}{2\mu_0 X^2 Y} = \frac{(B_g XY)^2 Z}{2\mu_0 X^2 Y} = \frac{B_g^2}{2\mu_0} \cdot YZ. \]  

(2.94)

The incremental stored magnetic energy \( dW_{sz} \) during the incremental displacement \( dz \), which is the cause of the normal force, is given by,

\[ dW_{sz} = \frac{\phi^2 Z}{2\mu_0 XY} \cdot dz + \frac{\phi Z}{\mu_0 XY} \cdot d\phi. \]  

(2.95)

Ignoring incremental energy due to losses, the incremental mechanical energy \( dW_{mz} \) during the incremental displacement \( dz \) can be written as,

\[ dW_{mz} = dW_e - dW_{sz} = \frac{\phi Z}{\mu_0 XY} \cdot d\phi - \frac{\phi^2}{2\mu_0 XY} \cdot dz - \frac{\phi Z}{\mu_0 XY} \cdot d\phi = -\frac{\phi^2}{2\mu_0 XY} \cdot dz. \]  

(2.96)

The normal force is then obtained as,
The negative sign indicates that the force generated tends to decrease the length of the air gap, which in this case is the downward force experienced by the translator. Similarly the lateral force can be calculated as,

\[ f_y = \frac{B_g^2}{2\mu_0} \cdot XZ. \]  

In the prototype design described in Example 1, with \( L_w = 50 \text{ mm}, w_{sp} = 20 \text{ mm}, g = 1 \text{ mm}, \) and \( B_g = 1.087 \text{ T}, \) the propulsion force due to excitation of a single pole is,

\[ f_{x1} = \frac{B_g^2}{2\mu_0} \cdot YZ = \frac{B_g^2}{2\mu_0} \cdot L_w g = 23.506 \text{ N}. \]

The propulsion force due to the excitation of a phase is \( f_x = 2 \cdot f_{x1} = 47.01 \text{ N}. \) The normal force due to excitation of a single pole is calculated as,

\[ f_{z1} = -\frac{B_g^2}{2\mu_0} \cdot XY = -\frac{B_g^2}{2\mu_0} \cdot w_{sp} L_w = -470.13 \text{ N}. \]

The total downward force experienced by the translator is given by \( f_z = 2 \cdot f_{z1} = -940.26 \text{ N}. \) The normal force can be calculated at different translator positions by changing the overlap length \( X. \)

### 2.4.3 FEA Verification

In this section, two-dimensional finite element analysis (FEA) is used to verify the design procedure of the LSRM and the accuracy of the analytical method. The flux linkages and the inductances of the stator phase windings, the propulsion and normal forces developed by the machine for various excitation currents and translator positions are determined. Finite element analysis, in general, provides more accurate results than the magnetic analytical method with equivalent circuit method because it considers a large number of flux paths compared to the magnetic equivalent circuit method.

To determine the magnetic field distribution of the LSRM, the following assumptions are made. (a) The magnetic field distribution is constant along the longitudinal direction of the LSRM. (b) The magnetic field outside the LSRM periphery is negligible and zero magnetic
vector potential as Dirichlet boundary condition is assigned to the outer periphery of the LSRM surface. (c) In the two-dimensional analysis, the current density vector has component only in the y direction. For this reason, the flux density vector has components only in the x and z directions. (d) Hysteresis effects are neglected under the assumptions that the magnetic materials of the stator and translator are isotropic and the magnetization curve is single-valued. (e) The end effects are neglected.

The field solution is obtained using Flux-2D software package of Magsoft Corporation. The entire problem region is subdivided into triangular finite elements, with 14916 nodes and 7149 elements. The LSRM translator is moved from an unaligned position with respect to the stator to an aligned position for different excitation currents and corresponding to each translator position, inductance, propulsion force, and normal force are obtained. Figure 2.11 shows the flux distributions inside the LSRM for the fully aligned position, two intermediate positions of 10 mm and 20 mm shifted from the fully aligned position, and fully unaligned position of the translator. These plots are obtained for a current of 8.5A in one phase winding. It is observed that the leakage flux lines cross the stator slot air gap to the adjacent stator poles and complete their path through the stator back irons. The flux densities in the excited stator poles and the force producing translator poles are high for a given excitation. The flux densities in other parts of the machine are very low.

Figure 2.13 shows the inductance profile at rated phase current for various translator positions. The force in a given direction is obtained by differentiating the magnetic coenergy of the system with respect to a virtual displacement of the translator in this direction using the calculated flux densities in each triangular element. Based on this approach, the propulsion force at various currents for various translator positions are calculated and shown in Figure 2.16. The normal force vs. current for different translator positions is shown in Figure 2.17. Solid curves in each figure represent the finite element analysis results.

2.4.4 Experimental Setup and Measurements

Figure 2.12 shows the schematic diagram for the measurement of phase winding inductance. After the translator is locked mechanically to the desired position, a step voltage is applied to the test phase winding through a power semiconductor switch and hence a constant current is applied
(a) Fully aligned position.

(b) Intermediate position I: 20 mm shifting of translator from fully unaligned position.

(c) Intermediate position II: 10 mm shifting of translator from fully unaligned position.

(d) Fully unaligned position.

Figure 2.11 Flux distribution of the three-phase LSRM prototype for rated phase current.
Figure 2.12 Schematic diagram for the measurement of phase winding inductance.

to the phase winding. To apply the rated current to the phase winding, only small voltage is required to overcome the machine winding resistance that is very small. Once the current reaches the steady state, the power switch is turned off and the current is diverted through the diode and the current starts to decay and the falling current profile is captured using the triggering mode of the oscilloscope. The time constant is measured from the profile and hence the inductance is calculated. A MOSFET power switch with fast turn-off characteristic is used for disconnecting the dc source from the phase winding. A freewheeling diode of the fast recovery type has to be connected across the test phase winding to provide a path for the stored energy after the winding is disconnected from the source. The forward voltage drop of the freewheeling diode during the measurement period degrades the accuracy of the inductance measurement. For this reason, additional external resistor is used to improve the measurement accuracy by overwhelming the effect of forward voltage drop in the freewheeling diode. Note that a large capacitance is required as much as 30 mF to filter the rectified supply voltage and to have negligible variations in the machine current.

Figure 2.13 shows the measured results along with finite element analysis results and show correlation to the finite element analysis results. The inductance profiles at currents lower than rated value are almost identical to the profile at rated current. It is due to the fact that the air gap is usually very large in the linear machine and hence machine does not saturate at all. There is high correlation in the aligned inductance region, but there are small discrepancies in the
Table 2.1 Comparison of winding inductance results by three methods at rated phase current

<table>
<thead>
<tr>
<th>Translator position</th>
<th>Analytical value without end-effect</th>
<th>Analytical value with end-effect</th>
<th>Finite element analysis</th>
<th>Measured value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully aligned position (x = 30 mm)</td>
<td>31.459 mH</td>
<td>32.608 mH</td>
<td>31.961 mH</td>
<td>32.375 mH</td>
</tr>
<tr>
<td>Intermediate position I (x = 20 mm)</td>
<td>24.408 mH</td>
<td>25.280 mH</td>
<td>23.301 mH</td>
<td>23.100 mH</td>
</tr>
<tr>
<td>Intermediate position II (x = 10 mm)</td>
<td>11.392 mH</td>
<td>11.680 mH</td>
<td>11.073 mH</td>
<td>12.110 mH</td>
</tr>
<tr>
<td>Fully unaligned position (x = 0 mm)</td>
<td>7.014 mH</td>
<td>7.490 mH</td>
<td>6.382 mH</td>
<td>9.380 mH</td>
</tr>
</tbody>
</table>

Figure 2.13 Measured and finite element analysis results of inductance at rated phase current of 8.5 A.
unaligned inductance region that may be due to the end effects. Note that they are not considered in the two-dimensional electromagnetic field analysis. A comparison of inductances by three different methods at fully aligned, two intermediate, and fully unaligned positions is given in Table 2.1 at rated phase current. In the fully aligned translator position, both the analytical and finite element results are very close to the measured inductance values with less than 2% error. On the other hand, in the fully unaligned translator position, both the analytical and finite element results have the errors of 20.1% and 31.9%, respectively. The error is attributable to the end effects, and the distortion of the magnetic properties of the core material due to punching stresses, inexact B-H characteristics provided by the steel manufacturers and non-uniformity of the air gap to a smaller degree. End leakage effects are severe in the fully unaligned position because the reluctance of the end paths becomes comparable to that of the air gap paths. Even though the end effect has been included in the analytical method, the accurate modeling of the three-dimensional geometry of leakage flux tubes is difficult. It is seen that the analytical results are closer to the measurements than the results of the finite element analysis.

Figure 2.14 and Figure 2.15 show the experimental setup for the measurement of the propulsion and normal forces, respectively. They are measured after locking the translator at the desired given position and energizing one phase winding with a constant current. When the current reaches steady state, the load cell sensor output is recorded. This test procedure is repeated for various currents and translator positions. The value is measured using an S-type load cell, which is used where the load is pull or pushed in a straight line without a side load. The translator and its mechanical support are connected through the screws at each end of S-type load cell to minimize any off-center pulling or pushing. In case of the normal force measurement, two load cells of same capacity are used as shown in Figure 2.15. For this reason, two read outputs should be added to obtain the total normal force.

Figure 2.16 shows the measured propulsion forces at different translator positions and phase winding currents together with the finite element analysis results. At smaller currents, there is high correlation between the finite element analysis and measured values, but at higher currents there is a minor discrepancy between the two sets of values. Figure 2.17 shows the measured and finite element analysis-computed normal forces at different translator positions and phase winding currents. The discrepancy between the results by different methods is attributed to the side load acting on the S-type load cell. In the increasing inductance region, the translator
Figure 2.14 Experimental setup for the propulsion force measurement.

Figure 2.15 Experimental setup for the normal force measurement.
Figure 2.16 Measured and finite element analysis results of propulsion force at different phase currents.

Figure 2.17 Measured and finite element analysis results of normal force at different phase currents.
receives not only the downward normal force but also the propulsion force. This propulsion force generated affects the S-type load cell as a side load, which is undesirable during the measurement of normal force. Figure 2.18 shows the analytical and finite element analysis results for various translator positions and phase winding currents. High correlation is observed between the two methods.

**2.5 Conclusions**

The following are considered to be original contributions arising out of this study:

1) A novel design procedure for linear switched reluctance machine has been proposed using the current knowledge and design procedure of rotating switched reluctance machine.
2) Analytical predictions of fully aligned, two intermediate, and fully unaligned inductances using lumped-parameter magnetic equivalent circuit analysis are developed and verified with finite element analysis.

3) A 4.8 m long, three-phase LSRM prototype has been built based on the developed design procedure with 120 stator poles spread along its length and tested with a converter and controller discussed in a paper [20].

4) Experimental correlation of inductances, propulsion and normal forces has confirmed the validity of the proposed design procedure.

5) Even though the modeling of end effects is considered in the analytical calculation procedure, small discrepancies in the unaligned inductance prediction are presented. This may need an accurate three-dimensional finite element analysis and will be recommended for a future work.
CHAPTER 3
LINEAR SWITCHED RELUCTANCE MACHINE BASED
PROPULSION DRIVE SYSTEM

A single-sided longitudinal flux LSRM is proposed as a frictionless propulsion drive system for a magnetically levitated and guided vehicle. Low and high speed transits are the potential applications. Based on the design results described in Chapter 2, the four-phase longitudinal flux LSRM prototype with passive stator and active translator configuration has been constructed and tested. This chapter describes the implementation of the converter, control strategy, and their integrated drive performance. Further, the effect of normal force generated in the LSRM propulsion drive system and the ways it can be usefully incorporated in the electromagnetic levitation control system are also investigated. Dynamic simulations and experimental correlation of the LSRM propulsion drive system is presented with a 6 m long LSRM prototype.

3.1 Introduction

No original contribution is being claimed in this chapter. It describes the implementation of the converter, control strategies for the velocity and force/current control system, and their integrated propulsion drive performance. Although LSRM configurations and finite element results have been described in literature, very little material describes the control of the LSRM. Basic converter configurations have been described in [3], [6], [16]-[18], but the control procedure is not described extensively. There is, however, extensive material covering the various converter topologies for an RSRM as summarized in [56], as well as various control strategies described in literature. As it will be seen, the existing knowledge base for rotary machines can be easily adapted to linear machine control. This chapter intends to describe the converter topology as well as the control strategy chosen for the four-phase LSRM prototype designed in Chapter 2. The converter topology is chosen to minimize the manufacturing effort so that the same topology can be applied for the control of levitation and guidance dc electromagnets. It is a traditional asymmetric bridge converter with two switches and two diodes per phase. This is also known as a two-quadrant class-D chopper.
The propulsion drive system is an indispensable unit providing traction force for driving the magnetically levitated and guided vehicle. One important aspect to be observed is that the LSRM produces not only the propulsion force but also a normal force perpendicular to the direction of motion. This normal force affects the motion dynamics of the levitating vehicle as it is propelled. The magnitude and shape of the normal force produced in the LSRM are very much dependent on the excited phase winding current and translator position, as well as the converter topology and force/current control strategy. Acquiring the knowledge of the effects caused by the converter and control strategy used is essential. This is the starting point for obtaining better control performance of the levitation control system. In the search to achieve the desired performance, two control strategies are implemented and tested: single-phase excitation and two-phase excitation. It is shown that the two-phase excitation scheme [20]-[21] is better in that it reduces propulsion force ripple as well as the normal force magnitude and ripple. A performance comparison is conducted between the two control strategies. The characteristics of the normal force generated by using those control strategies are discussed. Dynamic simulations and experimental results for the velocity and force/current control system are also included.

3.2 LSRM Topology

A four-phase LSRM with single-sided longitudinal flux configuration is designed as a propulsion subsystem of the magnetically levitated and guided vehicle. The vehicle has a rated velocity of 1 m/sec, rated force of 40 N, and rated mass of 60 kg. Detailed design parameters are presented in Appendix C.2. The machine structure and phase winding diagram are shown in Figure 3.1. The LSRM has an active translator and a passive stator configuration. In other words, the excitation windings are located only on the translator laminations and no windings exist on the stator laminations. Note that the machine translator is fixed in the movable vehicle and the stator is fixed in the track, and both core laminations are built up using M-19 electric steel sheet. In the prototype LSRM, the translator pole width, \( w_{tp} \), and slot width, \( w_{ts} \), are 12 mm and 18 mm, and the stator pole width, \( w_{sp} \), and slot width, \( w_{ss} \), are 15 mm and 25 mm, respectively. For this reason, one excitation period which is equal to the stator pole pitch, \( (w_{sp} + w_{ss}) \), repeats at intervals of 40 mm. The LSRM consists of 8 translator poles with phase windings and 150 stator poles spread over 6 m of track. The zero reference position required for exciting and
commutating the phase winding current is defined as the interpolar axis located between two stator poles, say $S_1$ and $S_2$. At this position, the polar axis of the phase A translator pole, $T_1$, is aligned with the interpolar axis of those stator poles.

In Figure 3.1, phase C winding and its associated translator poles are located at the maximum inductance position, that is, the fully aligned position. For this reason, the energization sequence $dd' - aa' - bb' - cc' - dd' - ...$ in the increasing inductance region propels the translator in the forward direction continuously. Similarly, the excitation sequence $bb' - aa' - dd' - cc' - bb' - ...$ in the decreasing inductance region can be deduced for a backward direction of motion. In an LSRM, excitation of a phase winding produces a propulsion force along the $x$ direction and a normal force along the $z$ direction. Similar to an RSRM, inductance values can be obtained as the translator moves from the unaligned to the aligned position and back to the unaligned position.

Figure 3.2 shows the profiles of the phase winding inductance $L_k$, the propulsion force $f_{xk}$, and normal force $f_{zk}$ at rated phase winding current of 8.5 A, and these values are obtained from finite element analysis. The values are calculated only for the phase A winding and the values of the other phases can be obtained by shifting the curves by one fourth of the stator pole pitch, that is, 10 mm. Note that the phase inductance profiles for the various excitation currents maintain the same shape as that of rated current in Figure 3.2, unless the excitation current exceeds the rated value. This is because the air gap of LSRM (3 mm in this application) is very large compared to that of a typical RSRM (0.25 mm), this insures the translator and stator cores are not saturated. If the excitation current exceeds the rated current value, the aligned inductance
Figure 3.2 Inductance, propulsion force, and normal force profiles of the four-phase LSRM prototype at the rated current of 8.5 A.
gradually begins to decrease because of the core saturation effect. It is also noticed that the magnitude of the generated normal force is six to seven times larger than that of the propulsion force and this force affects the motion dynamics of the levitation control system.

Figure 3.3 shows the rate of change of inductance with respect to x, $\frac{\partial L_k}{\partial x}$, and the rate of change of inductance with respect to z, $\frac{\partial L_k}{\partial z}$ for $k = a, b, c, d$ at rated phase current of 8.5 A. This can be analytically calculated from the propulsion and normal force profiles using the following relations between the phase winding current and generated electromagnetic force.

\[
\begin{align*}
    f_{xk} &= \frac{1}{2} \frac{\partial L_k}{\partial x} \cdot i_k^2 = \frac{1}{2} g_{xk} \cdot i_k^2, \quad k = a, b, c, d \\
    f_{zk} &= \frac{1}{2} \frac{\partial L_k}{\partial z} \cdot i_k^2 = \frac{1}{2} g_{zk} \cdot i_k^2, \quad k = a, b, c, d.
\end{align*}
\]
where $g_{sk}$ and $g_{zk}$ represent $\partial L_k/\partial x$ and $\partial L_k/\partial z$ for notational convenience, respectively. The rate of change of inductance with respect to $x$ and $z$, $g_{sk}$ and $g_{zk}$, are used as parameters for the force/current control strategy in Section 3.4.

### 3.3 Converter Topology

The phase winding current of the LSRM is unidirectional, namely, positive and the phase winding voltage of LSRM can be either positive or negative. These unidirectional current and bipolar voltage characteristics of the LSRM drives motivate the use of the asymmetric bridge converter, which is also known as two-quadrant class-D chopper. The converter circuit topology is shown in Figure 3.4. One current sensor, two switches, and two diodes per phase are required and the switches are connected in series with the phase winding of LSRM. Note that the same

![Asymmetric bridge converter topology with dc link power circuit](image-url)
converter topology can also be applied for the control of the levitation and guidance dc electromagnets. It is due to the fact that both the LSRM and dc electromagnet have independent excitation windings and two-quadrant operation with unipolar winding current and bipolar winding voltages. Isolating the modes of the machine phase switches and diodes derives various operational modes of the converter circuit. This stratagem works very well in determining the modes of the converter circuit and they are assembled in Table 3.1, assuming that the phase A is in the active region of operation and there are no other freewheeling phases. Note that the first four modes are the distinct conduction modes and the latter three modes are accompanied by the existence of freewheeling current for that phase. These four distinct conduction modes are referred to as the energization mode, regeneration mode, and two freewheeling modes, respectively. Therefore, modes I to IV are sufficient to fully understand the operation and to find analytical relationships between the converter and machine variables for the design of the converter. The operation principle and simple equivalent circuit modeling of the four distinct main modes are described. Figure 3.5 shows the modes of operation of this converter circuit.

(i) Mode I: \( T_{ua} \) on, \( T_{la} \) on

The energization mode is initiated by turning on the two switches, \( T_{ua} \) and \( T_{la} \). The dc link voltage \( V_{dc} \) is applied to the phase A winding to inject the current through the filter capacitor \( C_f \). Both the winding voltage and current are positive in this mode. The machine dc link voltage, and phase A winding current during this mode can be expressed as,

\[
\frac{di_a}{dt} = \frac{1}{L_a} \cdot (v_a - R_a i_a - e_a), \quad i_a \geq 0, \quad v_a = V_{dc}, \quad e_a = \frac{\partial L_a}{\partial x} \cdot \frac{dx}{dt} \cdot i_a = g_{xa} \cdot \dot{x} \cdot i_a \quad (3.1)
\]

where \( L_a \) is the phase winding inductance which is a function of translator position and phase winding current, \( R_a \) is the phase winding resistance, \( e_a \) is the back emf generated.

(ii) Mode II: \( T_{ua} \) off, \( T_{la} \) off

If both \( T_{ua} \) and \( T_{la} \) are turned off when the current exceeds the reference or the current has to be commutated, the fixed negative dc link voltage, \(-V_{dc}\), is applied to the phase A winding. The stored energy in the machine winding keeps the current in the same direction, resulting in the
Table 3.1 Modes of operation

<table>
<thead>
<tr>
<th>Mode</th>
<th>$T_{ua}$</th>
<th>$D_{ua}$</th>
<th>$T_{la}$</th>
<th>$D_{la}$</th>
<th>$i_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>&gt;0</td>
</tr>
<tr>
<td>II</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>&gt;0</td>
</tr>
<tr>
<td>III</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>&gt;0</td>
</tr>
<tr>
<td>IV</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>&gt;0</td>
</tr>
<tr>
<td>V</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>0</td>
</tr>
<tr>
<td>VI</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>0</td>
</tr>
<tr>
<td>VII</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3.5 Modes of operation of the asymmetric bridge converter topology.
forward bias of the two diodes. Phase A current is routed through the lower diode $D_{la}$, dc link filter capacitor $C_f$, upper diode $D_{ua}$, and the phase A winding. Phase current is rapidly reduced to zero and the energy is transferred from the machine winding to the dc link capacitor in this mode. Equivalent circuit expression is given below.

$$\frac{d i_a}{d t} = \frac{1}{L_a} \cdot (v_a - R_a i_a - e_a), \quad i_a \geq 0, \quad v_a = -V_{dc}, \quad e_a = \frac{\partial L_a}{\partial x} \cdot \frac{dx}{dt} \cdot i_a = g_{xa} \cdot \dot{x} \cdot i_a. \quad (3.2)$$

(iii) Mode III ($T_{ua}$ on, $T_{la}$ off) and Mode IV ($T_{ua}$ off, $T_{la}$ on)

Turning off either $T_{ua}$ or $T_{la}$ initiates the freewheeling mode, which routes the winding current through one of the freewheeling diodes, either $D_{ua}$ or $D_{la}$, respectively. This causes the winding current to decrease and zero voltage is applied to the phase winding if the voltage drops in switches and diodes are neglected. This particular fact reduces the ripple contents of the phase winding current and hence the generated electromagnetic force by effectively increasing the switching frequency. State space equation of the phase winding current is given below.

$$\frac{d i_a}{d t} = \frac{1}{L_a} \cdot (v_a - R_a i_a - e_a), \quad i_a \geq 0, \quad v_a = 0, \quad e_a = \frac{\partial L_a}{\partial x} \cdot \frac{dx}{dt} \cdot i_a = g_{xa} \cdot \dot{x} \cdot i_a. \quad (3.3)$$

The above expressions for the four distinct modes are derived by assuming that only one phase is in the active region of operation at a time and there are no other freewheeling phases. This is not true at the instants of commutation where two phase currents are in the active region of operation. It is also not true if the regenerative operation is required in the middle of commutation period for motoring operation or vice versa when three phase currents are in the active region of operation. However, (3.1) to (3.3) still can be used on the grounds that the mutual inductance between machine windings is small compared to the winding self inductance and the converter guarantees independent phase current control. Therefore, the situations with two or three active phase currents can be solved with a combination of the four distinct modes.

A number of different switching strategies, which are mainly determined by the combination of four main operating modes, can be applied to the control of the asymmetric bridge converter. For instance, the switching strategy using only modes I and II is referred to as a bipolar voltage switching strategy due to the fact that the output voltage waveform swings from $+V_{dc}$ to $-V_{dc}$ or
vice versa without utilizing two zero voltage modes. On the other hand, unipolar switching strategy allows zero voltage modes in the output voltage. In this thesis, the unipolar voltage switching strategy proposed in [20]-[21], [30], and [33] is applied. It leads to a better current control performance with reduced current and voltage ripples in the phase winding and better frequency response of the LSRM propulsion drives. Figure 3.6 shows the pulse width modulation with unipolar voltage switching strategy with three commonly existing conditions. Four operating modes are incorporated in this strategy. A triangular carrier waveform, \( v_t \), and its inverted triangular carrier waveform, \(-v_t\), are compared to the control voltage, \( v_c \), which is usually the output of the proportional and integral current controller. This comparison determines the on/off status of the upper and lower switches in the asymmetric bridge converter as follows.
Note that the on/off status of two diodes is automatically determined by the on/off status of two switches, assuming the existence of the phase winding current. In other words, the two diodes, D_{ua} and D_{la}, have the opposite status of the two switches, T_{ua} and T_{la}, respectively. If magnetic energy stored in the phase winding is depleted (no current in the phase winding), the diodes are turned off regardless of the on/off status of the switches. The status of the switches decides the operating modes to be used and hence the phase winding voltage to be applied, and the average output voltage of each phase winding, v_o, can be calculated as,

\[
v_o = \frac{V_{dc}}{v_{tm}} \cdot v_c
\]

where \( v_{tm} \) is the peak voltage magnitude of triangular carrier waveform.

### 3.4 Control Strategies

The LSRM propulsion drive system is composed of three control loops, that is, position, velocity, and force/current control loops. Position control loop forms the outermost control loop and force/current control loop constitutes the innermost control loop. The control system is composed of three nested loops because of the easiness to obtain stable operation. The three loops can be designed independently due to the fact that the crossover frequency of inner control loop is much higher than that of outer control loop. Furthermore, the limitation of velocity or current in the inner control loop can be easily achieved. In this section, the design and control strategies of velocity and force control loops are described. The position control loop is not considered in this thesis due to the fact that the implementation of the position control loop is relatively simple if the velocity and force/current control loops are properly designed and implemented. The similarity of the voltage equations for LSRM propulsive drives and the electromagnetic levitation system allows the same control design procedure to be used for the design of the levitation current control loop. This procedure is described in Chapter 4.
3.4.1 Force Control

Figure 3.7 shows the block diagram of the force/current control loop. It is composed of a force distribution function, a current command generator, a current controller, and propulsion force and normal force calculators. Assuming that instantaneous phase currents are controlled accurately, the force control strategy is based on tracking the optimum force distribution function of each phase, which is generated using the required force command obtained from the velocity control loop. In order to achieve the required force command, the force distribution function should satisfy the following condition: the sum of the force distribution functions for all phases should be equal to unity at all translator positions. The relations between the force command and force distribution function are given as follows:

\[
\begin{align*}
    f_{xk}^* &= f_{xe}^* \cdot S_k(x) \quad \text{where} \quad k = a, b, c, d \quad \text{and} \quad \sum_{k=a}^{d} S_k(x) = 1 \quad \text{for all translator positions} \quad x \\
    f_{xe}^* &= \sum_{k=a}^{d} f_{xk}^* = \sum_{k=a}^{d} f_{xe}^* \cdot S_k(x) = f_{xe}^* \cdot \sum_{k=a}^{d} S_k(x) = f_{xe}^* \cdot 1 = f_{xe}^* 
\end{align*}
\]

where \( f_{xe}^* \) is the required force command obtained from the velocity control loop, \( f_{xk}^* \) is the force command of \( k \)-th phase, and \( S_k(x) \) is the force distribution function of \( k \)-th phase. When \( S_k(x) \) is equal to zero, the corresponding phase is turned off instead of regulating the phase current \( i_k \) to avoid unnecessary switching [20]. From the phase force command \( f_{xk}^* \), the phase current command \( i_k^* \) can be computed using (3.1). Note in Figure 3.7 that the magnitude of the normal force generated in the LSRM is also calculated in the force/current control loop. This

![Figure 3.7 Block diagram of the force/current control with normal force computation, \( k = a, b, c, \) and \( d \) phases.](image-url)
normal force of each phase is utilized in the electromagnetic levitation control system by adding it to the force command obtained from the air gap position control loop in feedforward manner, resulting in better disturbance rejection performance for the levitation air gap control.

According to the allocation algorithm of the force distribution function of each phase, there exist two different force control strategies, that is, single-phase excitation and two-phase excitation. These force control strategies are described in the following sections.

3.4.1.1 Strategy I: Single-Phase Excitation

As shown in Figure 1.5, if current flows in a phase winding at the increasing inductance region, a motoring positive force is generated and if current flows in a phase winding at the decreasing inductance region, a regenerating negative force is produced. The electromagnetic force is not affected by the polarity of the phase winding current and is dependent only on the translator position and the phase winding current magnitude. A single-phase excitation strategy is based on the above principle, and the sequences for forward and backward motions and their corresponding force distribution functions for the four-phase LSRM prototype are shown in Tables 3.2 and 3.3, respectively. Four subsequent excitations of each phase winding make the translator move forward or backward by one stroke length which is equal to the stator pole pitch, \((w_{sp} + w_{ss}) = (15 + 25) = 40 \text{ mm}\). Only one phase winding is energized during one-fourth of the stroke length, that is, 10 mm. The excitation cycles shown in Tables 3.2 and 3.3 repeat every 40 mm, respectively. From Figure 3.2, the excitation and commutation position of each phase winding for forward or backward motions can be found from the viewpoint that the peak current magnitude of each phase winding is minimized. For forward movement of the translator, the conduction period of phase A winding can be found to be 5.8 to 15.8 mm. Similarly, for backward movement of the translator, the conduction period of phase A winding can be found to be 24.4 to 34.4 mm. Note that the excitation and commutation positions are dependent on the magnitude of peak phase winding current as well as the speed. Advance excitation and advance commutation of phase winding current may be required at high speed operation range.
3.4.1.2 Strategy II: Two-Phase Excitation

For the purpose of reducing the high force ripple and audible noise produced in the single-phase excitation strategy, a two-phase excitation scheme is proposed in [20]-[21]. If the motoring propulsion force in Figure 3.2(b) is carefully examined, both adjoined phase windings are generating the motoring propulsion force and the combination of the adjoined two phases is altered at intervals of 10 mm positions. A similar tendency can be found in the regenerative
propulsion force region. For this reason, by allocating the required force command to two adjacent phase windings with an appropriate force distribution function, both phase windings can contribute to produce propulsion force of the same polarity. This reduces the rate of change of phase currents and the peak phase current magnitude. The force distribution function also helps to reduce the peak and the rate of change of the normal force, compared to those characteristics in a single-phase excitation strategy. The motion sequences for forward or backward operations and their corresponding force distribution functions are shown in Tables 3.4 and 3.5, respectively. The excitation cycle for the four-phase LSRM prototype repeats every 40 mm and two adjacent phase windings are energized at the same time during one-fourth of the stroke length, that is, 10 mm. Each phase is conducting 20 mm, which is a half of one stroke length.

3.4.2 Velocity Control

Figure 3.8 shows the block diagram of the velocity control loop of the LSRM propulsion drive system. The force control loop is modeled as unity gain when the velocity control loop is considered, by assuming that the bandwidth of the force control loop is much higher than that of the velocity control loop and the actual phase current follows the phase current command accurately. Proportional and integral (PI) controller is used to obtain the required transient and steady state characteristics. In Figure 3.8, \( K_{pv} \) and \( K_{iv} \) are the gains of the PI controller, \( M_t \) is the translator assembly mass, \( B \) is the viscous damping coefficient, \( H_v \) is the velocity feedback gain, \( f_{xl} \) is the external load force, \( \dot{x} \) is the translator velocity, and * represents the command of each quantity. The appropriate gains of the proportional and integral controller can be derived by finding the transfer function of the velocity control loop and from Figure 3.8, the transfer function between velocity input and output is expressed as,

\[
G_v(s) = \frac{\dot{x}(s)}{\ddot{x}^*(s)}\bigg|_{f_{xl}=0} = \frac{1}{H_v s^2 + \left(\frac{B + H_v K_{pv}}{M_t}\right)s + \frac{H_v K_{pv} K_{iv}}{M_t}}
\]

\[
= \frac{1}{H_v} \cdot \frac{H_v K_{pv}}{M_t} \cdot \frac{s}{s^2 + \frac{H_v K_{pv}}{M_t}} \cdot \frac{M_t}{H_v K_{pv} K_{iv}}
\]

\[
= \frac{1}{H_v} \cdot \frac{M_t}{s^2 + \frac{H_v K_{pv}}{M_t}} \cdot \frac{M_t}{H_v K_{pv} K_{iv}} \quad \text{for } H_v K_{pv} >> B.
\]
Table 3.4 Forward motion sequence and force distribution function in two-phase excitation strategy ($f_{x_e}^* \geq 0$)

<table>
<thead>
<tr>
<th>Translator position (x, \text{mm})</th>
<th>Excited phase</th>
<th>Force distribution function (S_k(x), k = a, b, c, d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \leq x &lt; 10)</td>
<td>D, A</td>
<td>(S_d(x) = \frac{g_{xa}(x)^2}{g_{xd}(x)^2 + g_{xa}(x)^2}, S_a(x) = \frac{g_{xa}(x)^2}{g_{xd}(x)^2 + g_{xa}(x)^2})</td>
</tr>
<tr>
<td>(10 \leq x &lt; 20)</td>
<td>A, B</td>
<td>(S_a(x) = \frac{g_{xa}(x)^2}{g_{xa}(x)^2 + g_{xb}(x)^2}, S_b(x) = \frac{g_{xb}(x)^2}{g_{xa}(x)^2 + g_{xb}(x)^2})</td>
</tr>
<tr>
<td>(20 \leq x &lt; 30)</td>
<td>B, C</td>
<td>(S_b(x) = \frac{g_{xb}(x)^2}{g_{xb}(x)^2 + g_{xc}(x)^2}, S_c(x) = \frac{g_{xc}(x)^2}{g_{xb}(x)^2 + g_{xc}(x)^2})</td>
</tr>
<tr>
<td>(30 \leq x &lt; 40)</td>
<td>C, D</td>
<td>(S_c(x) = \frac{g_{xc}(x)^2}{g_{xc}(x)^2 + g_{xd}(x)^2}, S_d(x) = \frac{g_{xd}(x)^2}{g_{xc}(x)^2 + g_{xd}(x)^2})</td>
</tr>
</tbody>
</table>

Table 3.5 Backward motion sequence and force distribution function in two-phase excitation strategy ($f_{x_e}^* < 0$)

<table>
<thead>
<tr>
<th>Translator position (x, \text{mm})</th>
<th>Excited phase</th>
<th>Force distribution function (S_k(x), k = a, b, c, d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \leq x &lt; 10)</td>
<td>B, C</td>
<td>(S_b(x) = \frac{g_{xb}(x)^2}{g_{xb}(x)^2 + g_{xc}(x)^2}, S_c(x) = \frac{g_{xc}(x)^2}{g_{xb}(x)^2 + g_{xc}(x)^2})</td>
</tr>
<tr>
<td>(10 \leq x &lt; 20)</td>
<td>C, D</td>
<td>(S_c(x) = \frac{g_{xc}(x)^2}{g_{xc}(x)^2 + g_{xd}(x)^2}, S_d(x) = \frac{g_{xd}(x)^2}{g_{xc}(x)^2 + g_{xd}(x)^2})</td>
</tr>
<tr>
<td>(20 \leq x &lt; 30)</td>
<td>D, A</td>
<td>(S_d(x) = \frac{g_{xd}(x)^2}{g_{xd}(x)^2 + g_{xa}(x)^2}, S_a(x) = \frac{g_{xa}(x)^2}{g_{xd}(x)^2 + g_{xa}(x)^2})</td>
</tr>
<tr>
<td>(30 \leq x &lt; 40)</td>
<td>A, B</td>
<td>(S_a(x) = \frac{g_{xa}(x)^2}{g_{xa}(x)^2 + g_{xb}(x)^2}, S_b(x) = \frac{g_{xb}(x)^2}{g_{xa}(x)^2 + g_{xb}(x)^2})</td>
</tr>
</tbody>
</table>
From the given specifications of bandwidth $\omega_v$ and damping ratio $\zeta_v$ of velocity control loop, the gains of the PI controller are analytically derived and given as,

$$K_{pv} = \frac{2\zeta_v \omega_v}{\sqrt{(1 + 2\zeta_v^2)^2 + 1}} \cdot \frac{M_t}{H_v} \quad (3.10)$$

$$K_{iv} = \frac{\omega_v}{2\zeta_v \sqrt{(1 + 2\zeta_v^2)^2 + 1}} \cdot \frac{1}{M_t \cdot s + B} \quad (3.11)$$

In the velocity control loop, the system crossover frequency and hence the bandwidth is normally designed using 100 Hz and the damping ratio is given as 1.

### 3.5 Dynamic Simulations and Experimental Results

#### 3.5.1 Control Block Diagram

Figure 3.9 shows the experimental control block diagram of the LSRM propulsion drive system. The available feedback signals for control are the four phase currents, discrete translator position signals from the pulse generator to generate the commutation pulses for the phase switches, and translator velocity signal from the position signal. The signal from pulse generator is processed through the digital pulse-counting circuits and the resulting modified velocity signal is compared with the velocity reference to produce the velocity error. The force command signal is obtained from the velocity error signal through a proportional and integral controller and
Texas instrument TMS320C31 floating point DSP-based digital controller board (dSPACE) for the velocity and force/current control

Figure 3.9 Experimental control block diagram of the LSRM propulsion drive system.
The corresponding phase force command is generated using the force distribution function of each phase, which is determined using three inputs, that is, force command magnitude reference, discrete translator position signal, and the polarity of the force command. The on-phase selection and commutation procedure are implemented as shown in Tables 3.2 to 3.5. The corresponding phase current command is generated using the phase force command. The current error between the reference and actual phase current is processed through a proportional and integral controller and limiter to yield the reference voltage command for the comparison with triangular carrier wave and hence the gating signals for the phase switches. Note that there may be one or two proportional and integral current controllers according to the control strategy: single-phase excitation or two-phase excitation strategies, respectively. The normal force allocation block calculates feedforward compensation forces for the dc electromagnets in the levitation control system and the allocated compensation forces are added to the force commands of the individual air gap controller for levitation. The algorithm for how to allocate the normal forces of the LSRM to the dc electromagnets of levitation system is described in Chapter 4.

3.5.2 Dynamic Simulations

Figure 3.10 shows the dynamic simulation of the force/current control system with a single-phase excitation strategy. The machine parameters used are given in Appendix C.2. The operating speed and propulsion force command of the LSRM are fixed to 0.5 m/s and 30 N, which correspond to half of the rated speed and three-quarters of the rated force of the machine, respectively. The responses of the position $x$, velocity $\dot{x}$, force distribution function $S_k(x)$, phase currents $i_k$ and their references $i_k^*$, actual propulsion forces $f_{x_k}$ and their reference $f_{xe}^*$, voltages $v_k$ and flux linkages $\lambda_k$ of the phase windings, and the generated normal forces $f_{zk}$ are sequentially presented in the figure. Only one phase is energized during a certain period of time as shown in the force distribution function. The sharp rate of change of phase current command is generated in the stepwise manner. Because of the time required for the actual current to follow the current command in a highly inductive circuit, there is a delay in tracking the current command, resulting in the force dips of the actual propulsion force in the machine at the instants of commutation from one phase to the next. Even though judicious selection of firing moments...
Figure 3.10 Dynamic simulation of the force-controlled LSRM drive with single-phase excitation strategy.
of the phases can be applied to reduce the force ripple, it is not easy to achieve a flat-topped force response in single-phase excitation strategy. This disadvantage results in the increased audible noise and fatigue of the core material of both the stator and translator. It is observed that highly fluctuating normal force in the order of 55 to 225 N is produced and this force makes the air gap control performance of the electromagnetic levitation system worse, even though the total system efficiency is increasing with the help of attractive normal force.

Figure 3.11 shows the dynamic simulation of force/current control system with two-phase excitation strategy. The simulation conditions and the displaying sequence of each figure are the same as those of Figure 3.10. Two phases are energized at the same time at intervals of 0.02 s and hence 10 mm positions in this case as shown in the force distribution function. In two-phase excitation strategy, the current command of each phase starts and ends with zero. The rate of change of current command is reduced. For this reason, the actual phase current can more easily track the phase current reference, resulting in the notable reduction in the propulsion force ripple. Further, the rate of change and the peak-to-peak magnitude of the produced normal force are also reduced. The produced normal force magnitude is in the order of 90 to 140 N. The reduction in the magnitude of normal force fluctuation contributes to the better air gap control of the electromagnetic levitation system and hence the better ride quality of the vehicle compared to those of single-phase excitation strategy. One disadvantage of two-phase excitation strategy might be a lower efficiency compared to single-phase excitation due to the fact that two phase currents are always conducting.

Figure 3.12 shows the dynamic simulation of velocity control system with single-phase excitation strategy. The propulsion force magnitude is restricted to 30 N. The velocity command of 0.5 m/s is initially applied and switched to −0.5 m/s after 2 s so that both the motoring and regenerative operation can be observed. Sequentially presented in the figure are the responses of the position $x$, actual velocity $\dot{x}$ and its reference $\dot{x}^*$, actual propulsion force $f_{sk}$ and its reference $f_{xe}^*$, actual phase currents $i_k$ and their references $i_k^*$, voltage $v_k$ and flux linkages $\lambda_k$ of phase windings, and the generated normal force $f_{zk}$. The winding voltage of only phase A is shown in the figure for clarity. At the periods of acceleration and deceleration, highly fluctuating normal force is generated and gradually decreased as the actual velocity of the translator reaches its commanded value.
Figure 3.11 Dynamic simulation of the force-controlled LSRM drive with two-phase excitation strategy.

CHAPTER 3. LINEAR SWITCHED RELUCTANCE MACHINE BASED PROPULSION DRIVE SYSTEM
Figure 3.12 Dynamic simulation of the velocity-controlled LSRM drive with single-phase excitation strategy.
Figure 3.13 shows the dynamic simulation of velocity control system with two-phase excitation strategy. The simulation conditions and the displaying sequence of each figure are the same as those of Figure 3.12. The actual force and phase currents are tracking the references very well and almost ripple-free operation in the force control is achieved.

3.5.3 Experimental Results

For the implementation of the velocity and force/current control systems, a TMS320C31 DSP-based digital controller board manufactured by dSPACE Inc. is adopted along with an interface board. The dSPACE controller board, DS1102, is a single board solution which includes TMS320C31 and TMS320P14 DSPs, two 12-bit and two 16-bit ADCs, four 12-bit DACs, and two sets of incremental encoder interface (24-bit position counter). It processes the control blocks shown in the dotted lines of Figure 3.9. The interface board generates gate drive signals for converter boards from the unipolar voltage switching PWM using the control voltage obtained from the DS1102 digital controller board. The triangular waveform PWM carrier frequency is 20 kHz. A magnetic sensor strip that has a resolution of 10 μm runs alongside the stator giving propulsion position feedback signal. A sampling time of 500 μs is used for the digital implementation of the force/current control loop. Velocity feedback is calculated every 500 μs using moving average method, and the velocity control loop is also digitally implemented with a sampling time of 2 ms.

Figure 3.14 shows the experimental waveforms of the force-controlled LSRM drive with single-phase excitation strategy. The propulsion force command of the LSRM is fixed to 30 N. Phase current command and its feedback, generated electromagnetic propulsion and normal forces are sequentially shown in the figure. The propulsion and normal force plots are reproduced using phase currents and translator position in the dSPACE digital controller board. As described in the simulation results, the current feedback cannot instantaneously follow the step change in the current command in R-L magnetic circuit. This delay in tracking the current command is shown in Figure 3.14(a) along with the current feedbacks of phase A and B windings. The force dips caused by the tracking delay in current feedback are shown in Figure 3.14(b) along with the force feedbacks of phase A and B. They generate the increased audible noise, mechanical vibration, and fatigue of the machine structure. It is observed from Figure
Figure 3.13 Dynamic simulation of the velocity-controlled LSRM drive with two-phase excitation strategy.

CHAPTER 3. LINEAR SWITCHED RELUCTANCE MACHINE BASED PROPULSION DRIVE SYSTEM
Figure 3.14 Experimental waveforms of the force-controlled LSRM drive with single-phase excitation strategy.

(a) Phase current command and feedback (10 ms/div).

(b) Generated electromagnetic propulsion force (10 ms/div).

(c) Generated electromagnetic normal force (10 ms/div).

Figure 3.14 Experimental waveforms of the force-controlled LSRM drive with single-phase excitation strategy.
3.14(c) that highly fluctuating normal force in the order of approximately 55 to 225 N is produced, and this force affects the electromagnetic levitation control system.

Figure 3.15 shows the experimental waveforms of the force-controlled LSRM drive with two-phase excitation strategy. The experimental conditions and the displaying sequence of each figure are the same as those of Figure 3.14. From Figure 3.15(a), the current command starts and ends with zero, and the current command is gradually increased and decreased. For this reason, the actual phase winding current tracks the phase current command closely as shown in the figure. It is observed that phase B is turned on when phase A is still in conducting mode. Note also that there is a sampling time delay between the current command and actual current feedback due to the fact that the force/current control is digitally implemented. Figure 3.15(b) shows a nearly flat-topped force feedback compared to that of single-phase excitation strategy. It is observed from Figure 3.15(c) that the produced normal force magnitude is in the order of 90 to 140 N, and both the magnitude and the rate of change of normal force are reduced.

Figure 3.16 shows the experimental waveforms of the velocity-controlled LSRM drive with single-phase excitation strategy. The propulsion force magnitude is restricted to 30 N. The velocity command is switched periodically in a range between 0.5 and −0.5 m/s in stepwise manner so that the four-quadrant machine operation can be observed. Velocity command and its feedback, phase A current command and its feedback, generated electromagnetic propulsion and normal forces are sequentially shown in the figure. It is observed from Figures 3.16(a) and (b) that the velocity and phase current feedbacks are tracking their references very closely. At the periods of acceleration and deceleration, highly fluctuating normal force is generated and gradually decreased as the actual velocity of the translator reaches its commanded value, as shown in Figure 3.16(c). It is also noticed that the propulsion force feedback contains high ripple component.

Figure 3.17 shows the experimental waveforms of the velocity-controlled LSRM drive with two-phase excitation strategy. The experimental conditions and the displaying sequence of each figure are the same as those of Figure 3.16. It is observed from Figures 3.17(a) and (b) that the velocity and phase current feedbacks are tracking their references very well. Ripple-free propulsion force feedback and reduced normal force variation can be observed in Figure 3.17(c).
Figure 3.15 Experimental waveforms of the force-controlled LSRM drive with two-phase excitation strategy.

(a) Phase current command and feedback (10 ms/div).

(b) Generated electromagnetic propulsion force (10 ms/div).

(c) Generated electromagnetic normal force (10 ms/div).
Figure 3.16 Experimental waveforms of the velocity-controlled LSRM drive with single-phase excitation strategy.

(a) Velocity command and feedback (2 s/div).

(b) Current command and feedback of phase A winding (1 s/div).

(c) Generated electromagnetic propulsion and normal forces (2 s/div).
Figure 3.17 Experimental waveforms of the velocity-controlled LSRM drive with two-phase excitation strategy.

(a) Velocity command and feedback (2 s/div).

(b) Current command and feedback of phase A winding (1 s/div).

(c) Generated electromagnetic propulsion and normal forces (2 s/div).

Figure 3.17 Experimental waveforms of the velocity-controlled LSRM drive with two-phase excitation strategy.
3.6 Conclusions

A single-sided longitudinal flux LSRM is proposed as a frictionless propulsion drive system of the magnetically levitated and guided vehicle. The implementation of the converter, control strategies for the velocity and force/current control system, and their integrated performance for the LSRM propulsion drives are described in this chapter. Even though no original contribution is claimed, the following results are obtained in this study.

1) A 6 m long, four-phase LSRM prototype with active translator and passive stator configuration has been built based on the developed design procedure described in Chapter 2 and tested with a converter and controller.

2) Asymmetric bridge converter with two switches and two diodes per phase is chosen and implemented using unipolar voltage switching PWM strategy to obtain better current control performance with reduced current and voltage ripples in the phase winding.

3) Two different force control strategies, that is, conventional single-phase excitation and two-phase excitation, are implemented and compared under the same operation conditions. Two-phase excitation strategy shows a flat-topped force response while single-phase excitation strategy shows force dips of the actual propulsion force. Reduced audible noise and vibration, and low fluctuation in the generated normal force are obtained using two-phase excitation strategy. The system efficiency with the two-phase excitation is estimated to be lower than that with single-phase excitation due to the fact that two phase currents are always conducting.

4) The PI controllers for the velocity and current control as well as two force control strategies are designed, implemented, and tested with a TI TMS320C31 DSP-based dSPACE digital controller board.

5) Experimental results of the force-controlled and velocity-controlled LSRM drives correlate the simulation results, and it confirms the effectiveness of the LSRM drives for the use in the frictionless propulsion drive system of the magnetically levitated and guided vehicle.
CHAPTER 4
LINEAR SWITCHED RELUCTANCE MACHINE PROPELLED
ELECTROMAGNETIC LEVITATION SYSTEM

4.1 Introduction

Among the several electromagnetic methods of lifting moving masses, there are two basic forms of magnetic levitation, that is, the electrodynamic repulsion system and the electromagnetic attraction system. The former uses the superconducting repulsion principle while the latter is based on the ferromagnetic attraction principle. The superconducting repulsion system is applicable to only high speed applications because of the minimum velocity requirement to generate the lifting force. On the other hand, the ferromagnetic attraction system is suitable for both low and high speed applications. Further, it provides the lift force even at zero speed when the vehicle is stationary [65]-[66].

The attractive electromagnetic levitation system is described in this chapter. First, a design procedure for dc electromagnet is presented using analytic relations between the design variables. Magnet dimensions and wire sizing are derived in detail using given system design specifications such as the vehicle weight, working air gap, lift force, and winding current. Then finite element analysis of the dc electromagnet design is used to fine-tune the final design. Second, the mathematical model of the electromagnetic levitation system is developed using single-input single-output system modeling. The modeling is based on the assumption that two sets of dc electromagnets are used for the independent control of levitation and guidance systems and they hold the vehicle to the balanced nominal position with enough force against unevenly distributed loads, guideway irregularities, and external wind disturbances. This ensures that only vertical motion can be considered for the levitation system modeling and the vehicle levitation system can be modeled as four independent single-input single-output systems. Third, for stabilizing the electromagnetic levitation system which has inherent unstable characteristic, position and force/current control loops are designed using proportional-integral-derivative (PID) and proportional-integral (PI) controllers, respectively. They are implemented using analog control circuitry. The design procedures for the PID position and PI force/current controllers are
systematically derived. Fourth, a feedforward controller for the levitation air gap control is proposed to reject the external force disturbance caused by the normal force component generated in the LSRM propulsion drive system. The feedforward control input is calculated in the LSRM propulsion controller and is used in the levitation controller. The enhancement of ride quality and system efficiency for the LSRM-propelled electromagnetic levitation system is achieved. Finally, extensive dynamic simulations and experimental results for the position and force/current control system of a 6 m long prototype are given in this chapter.

4.2 Configuration of the LSRM-Propelled Electromagnetic System

Figure 4.1 shows a configuration of the attractive electromagnetic levitation and guidance systems with LSRM propulsion drive system. Detailed views of the internal structure are shown in Figures 4.2 and 4.3, which are the front view and side view of the attractive electromagnetic systems, respectively. The electromagnetic system comprises of three independent subsystems, that is, propulsion, levitation, and guidance subsystems, and each subsystem is described in detail in the following. The propulsion subsystem consists of four-phase LSRM and magnetic linear position sensor. The LSRM has a longitudinal magnetic flux path with an active translator and passive stator configuration. Both the stator and translator cores are laminated using M-19 electric steel sheet to reduce the eddy current loss and they are installed under the track to utilize the normal force component for levitation purposes. The magnetic linear position sensor provides the translator position required for exciting and commutating the phase winding currents, and it has non-contacting head and scale to offer resistance against vibration and shock.

The levitation subsystem consists of four dc electromagnets, four gap sensors, and two levitation rails running the full length on both sides of the track. Four dc electromagnets and four gap sensors are positioned at each corner of the vehicle as shown in Figure 4.3. Levitation rails which are constructed with ferromagnetic steel (can also be laminated to reduce eddy currents) have even surface and are not shown in Figure 4.3 for the sake of clarity. The dc electromagnet has the longitudinal magnetic flux configuration, that is, the magnetic flux path has the same direction as vehicle motion. The magnet core is laminated using M-19 electric steel sheet. To control the position of levitation dc electromagnets, feedback signals provided by inductive gap sensors are used. In order to control the guidance system independently, it is necessary to use
four additional dc electromagnets and four gap sensors as well as two guidance rails. The structure of the guidance subsystem is similar to that of the levitation subsystem except that the guidance components are installed on the lateral surface perpendicular to the levitation surface.

Brushes and copper strips are installed through the I-beam for collecting the input power. The brushes slide on the surface of the copper strip as the vehicle is propelled, thereby transferring power to the electronic power circuit placed on the top surface of the vehicle. When the vehicle is in a stationary state, the landing wheels support its weight. They are also used for the forward or backward movement of the LSRM translator to adjust the initial position of the magnetic linear position sensor. In order to prevent the damage of the dc electromagnets and gap sensors caused by the accidental vibration of the vehicle, shock protectors are attached alongside
Figure 4.2 Front view of the LSRM-propelled electromagnetic system.

Figure 4.3 Side view of the LSRM-propelled electromagnetic system.
the dc electromagnets of levitation and guidance systems. The three subsystems are tightly supported using the I-beam and several guideway supporters.

4.3 Design of the DC Electromagnet

4.3.1 Design Procedure

The design procedure presented in this section is based on the assumption that the levitation and guidance systems are independent. This allows for simplicity of the design procedure and overall high reliability of the system. U-shaped electromagnet which gives less eddy current losses compared to E-shaped geometry [66] is also assumed. Figure 4.4 shows the configuration of the longitudinal flux dc electromagnet, which consists of flat steel bar (track) and the U-shaped electromagnet. The dc electromagnet prototype is to be designed for the air gap flux density $B_g$ and rated winding current $I_m$. The maximum mass of vehicle is restricted to $M_v$. In general, the electromagnet is designed starting from the equation of attractive levitation force acting on the track and U-shaped core lamination [65]-[66]. This equation can be obtained from the normal force equation of the LSRM derived in Chapter 2 due to the fact that the electromagnet configuration is similar to that of the LSRM located in the fully aligned translator.
position, which has only the normal force component. Noting that there are two electromagnet pole face areas that contribute to the levitation force, the attractive force equation is given by,

\[ f_z = \frac{B_g^2}{2\mu_0} \cdot 2A_p = \frac{B_g^2}{2\mu_0} \cdot 2w_pd_p = \frac{B_g^2}{\mu_0} \cdot w_pd_p \quad (4.1) \]

where \( f_z \) is the attractive levitation or guidance force, \( \mu_0 \) is the permeability of the free space, \( A_p \) is the cross-section area of pole face, \( w_p \) is the width of pole, and \( d_p \) is the stacking depth of U-shaped core lamination. Equation (4.1) is derived by neglecting the leakage flux and iron reluctance, and by assuming the constant flux density over the cross-section area. Assuming that the air gap flux density, \( B_g \), is known, the pole face area of electromagnet is determined by,

\[ A_p = w_p d_p = \frac{\mu_0 f_z}{B_g^2} \quad (4.2) \]

The levitation force required for one electromagnet, \( f_z \), is expressed in terms of a quarter of the vehicle mass, \( M_v \), and is given by,

\[ f_z = \frac{M_v \cdot G}{4} \quad (4.3) \]

where \( G \) is the acceleration due to gravity. By choosing either the width of pole, \( w_p \), or the stacking depth of U-shaped core lamination, \( d_p \), the dimensions of the pole face area can be determined. The magnetomotive force (mmf) required to produce the magnetic flux density in the air gap is derived from Ampere’s Circuital Law and is given by,

\[ \text{mmf} = T_m I_m = 2z \cdot H_g = \frac{2zB_g}{\mu_0} \quad (4.4) \]

where \( T_m \) is the number of winding turns per electromagnet (per two poles), \( z \) is the air gap length, and \( H_g \) is the magnetic field intensity in the air gap. Then, the number of turns per electromagnet can be calculated as,

\[ T_m = \frac{2zB_g}{\mu_0 I_m} \quad (4.5) \]

If \( J \) is the maximum allowable current density in the winding, the cross-section area, \( a_c \), and diameter, \( d_c \), of a conductor are calculated, respectively as,

\[ a_c = \frac{I_m}{J} \quad (4.6) \]
\[ d_c = \sqrt{\frac{4a_c}{\pi}}. \quad (4.7) \]

Winding window area is determined in relation to the winding fill factor and is given by,

\[ FF = \frac{\text{Magnet Winding Area}}{\text{Winding Window Area}} = \frac{\pi(d_c/2)^2T_m}{w_wh_p} \Rightarrow w_wh_p = \frac{\pi(d_c/2)^2T_m}{FF} \quad (4.8) \]

where FF is the winding fill factor, \( w_w \) is the width of winding window, and \( h_p \) is the height of electromagnet pole. The normal range of the fill factor is in the range given by \( 0.2 \leq FF < 0.7 \).

The results obtained from the lumped-parameter magnetic circuit analysis provide the preliminary data for the accurate calculation using finite element analysis. The fine-tuned dimensions of the electromagnet are obtained through the finite element analysis.

### 4.3.2 Prototype Design

The dc electromagnet prototype is designed with a maximum vehicle mass of 60 Kg and a rated winding current of 3.5 A. The reasonable assumption of the air gap flux density is the order of 0.5 T because of the heat generation in the magnet winding. If the air gap flux density is assumed on the order of 0.7 T to 1 T, this will require forced cooling of the magnet winding [66].

In this prototype design, an air gap flux density of 0.528 T is assumed. The nominal air gap length is 3 mm and the maximum air gap length is restricted to 6 mm. The required nominal levitation force per one electromagnet is given by,

\[ f_z = \frac{M_v \cdot G}{4} = \frac{60 \cdot 9.8}{4} = 147 \text{ N}. \]

Assuming that \( w_p = 30 \text{ mm} \), the stacking depth of U-shaped core lamination, \( d_p \), is calculated as,

\[ d_p = \frac{\mu_0 f_z}{w_p B_g^2} = \frac{4\pi \cdot 10^{-7} \cdot 147}{0.03 \cdot 0.528^2} = 22.1 \text{ mm}. \]

With 10% margin in the levitation force, \( d_p \) is selected as 25 mm. The mmf required to produce the air gap magnetic flux density and the number of turns per electromagnet are given respectively by,

\[ \text{mmf} = \frac{2zB_g}{\mu_0} = \frac{2 \cdot 0.003 \cdot 0.528}{4\pi \cdot 10^{-7}} = 2521 \text{ AT} \]
\[ T_m = \frac{\text{mmf}}{I_m} = \frac{2521}{3.5} = 720 \text{ Turns / Electromagnet} (= 360 \text{ Turns / Pole}) . \]

If the maximum allowable current density \( J \) is 5.36 A/mm\(^2\), the cross-section area and diameter of a conductor are calculated respectively as,

\[ a_c = \frac{I_m}{J} = \frac{3.5}{5.36} = 0.653 \text{ A / mm}^2 \]
\[ d_c = \sqrt{\frac{4a_c}{\pi}} = \sqrt{\frac{4 \times 0.653 \times 10^{-6}}{\pi}} = 0.912 \text{ mm}. \]

This coil size corresponds to AWG #19. By assuming \( FF = 0.33 \), the winding window area is calculated as,

\[ w_w h_p = \frac{\pi (d_c/2)^2 T_m}{FF} = \frac{\pi \times (0.912 \times 10^{-3}/2)^2 \times 720}{0.33} = 1425 \text{ mm}^2. \]

From this result, the width and height of the winding window are selected as \( w_w = 40 \text{ mm} \) and \( h_p = 35 \text{ mm} \), respectively. Figure 4.5 shows the fine-tuned final dimensions of dc electromagnet prototype.

![Figure 4.5 Dimensions of the designed dc electromagnet prototype.](image)
4.3.3 Characteristics of the DC Electromagnet Prototype by Finite Element Analysis

Figure 4.6 shows the winding inductance, $L$, rate of change of inductance, $g_z$, and generated attractive electromagnetic force, $f_z$, of the dc electromagnet prototype. These results are obtained using finite element analysis with 22,172 nodes and 10,683 elements. The winding inductance profiles in Figure 4.6(a) maintain the same shapes in the vicinity of the nominal air gap length regardless of the various excitation currents. Because of the magnet core saturation at the smaller air gap regions (less than 2 mm air gap), the winding inductance is decreased as the winding excitation current is increased. From Figure 4.6(a), it is observed that the attractive electromagnetic force is inversely proportional to the square of the air gap length. It is also noticed from the Figure 4.6(b) that a generated attractive electromagnetic force up to 800 N is proportional to the square of the winding excitation current. Magnetic core saturation is established beyond 800 N. The rate of change of inductance with respect to the air gap length $z$, $\frac{\partial L}{\partial z}$, can be analytically calculated from the attractive normal force profiles using the similar relation applied in Chapter 3, (3.2), and is given by,

$$ f_z = \frac{1}{2} \cdot \frac{\partial L}{\partial z} \cdot i^2 = \frac{1}{2} \cdot g_z \cdot i^2. $$

(4.9)

where $g_z$ represents $\frac{\partial L}{\partial z}$ for notational convenience, $i$ is the winding current. At the nominal air gap of 3 mm and rated winding current of 3.5 A, the values of inductance, rate of change of inductance, and attractive electromagnetic force are given by 108.08 mH, 29.04 N/A², 177.94 N, respectively.

4.4 Mathematical Modeling and Open Loop Instability of the Electromagnetic Levitation System

4.4.1 Mathematical Modeling

A magnetically levitated and guided vehicle is a rigid body in three-dimensional space. Therefore, the vehicle has six degrees of freedom, that is, three translational motions (propulsion $x$, levitation $z$, and guidance $y$) and three rotational motions (roll $\phi$, yaw $\psi$, and pitch $\theta$). In this thesis, however, the mathematical modeling of the electromagnetic levitation and guidance
Figure 4.6 Inductance, rate of change of inductance, and attractive electromagnetic force of the dc electromagnet prototype.

(a) Inductance, rate of change of inductance, and attractive electromagnetic force for various winding currents.

(b) Attractive electromagnetic force for various airgap lengths.

Figure 4.6 Inductance, rate of change of inductance, and attractive electromagnetic force of the dc electromagnet prototype.
systems is developed using single-input single-output system. This is based on the assumption that two sets of dc electromagnets are used for the independent control of levitation and guidance systems and they hold the vehicle to the balanced nominal position with enough force against unevenly distributed loads, guideway irregularities, and external wind disturbance. This ensures that only vertical motion can be considered for the levitation system modeling and only lateral motion can be considered for the guidance system modeling. This kind of vehicle modeling using independent single-input single-output system endows simplicity and reliability to the control system as well as provides easy tuning of the controller gains for stabilizing the controller operation.

The configuration of the single electromagnet and track is shown in Figure 4.7. To derive the mathematical model of the electromagnetic levitation system, the following assumptions are made. (a) The mmf drops in the track iron bar and magnet core lamination are negligible compared to that of the air gap region. (b) There are no leakage and fringing flux linkages. From the above assumptions, the flux linkage and inductance of the magnet winding can be expressed respectively as,

\[ \lambda(z,i) = T_m \cdot \phi(z,i) - T_m \cdot \frac{3}{R} = T_m \cdot \frac{T_m i}{R} = \frac{\mu_0 A_p T_m^2 i}{2z} \]  

\[ L(z,i) = \frac{\lambda(z,i)}{i} = \frac{\mu_0 A_p T_m^2}{2z} \]  

Figure 4.7 Configuration of the single electromagnet and track.
where $\lambda$ is the flux linkage, $\phi$ is the magnetic flux, $\mathcal{I}$ is the mmf, $\mathcal{R}$ is the reluctance of the magnetic circuit, and $L$ is the winding inductance. Since the winding inductance is inversely proportional to the air gap length, the levitation system has a nonlinear time-varying characteristic. If the electromagnet winding has electrical resistance $R$, the voltage equation can be written as,

$$v = Ri + \frac{d\lambda(z,i)}{dt} = Ri + \frac{d}{dt}(L(z,i) \cdot i) = Ri + L(z,i) \frac{di}{dt} + \frac{\partial L(z,i)}{\partial z} \cdot \frac{dz}{dt} \cdot i$$

$$= Ri + L(z,i) \cdot \frac{di}{dt} + g_z \cdot \dot{z} \cdot i$$  (4.12)

where $v$ is the applied voltage, $i$ is the instantaneous winding current, and $\dot{z}$ represents $dz/dt$ which is the vertical velocity of the vehicle. In (4.12), the three terms in the right hand side represent the resistive voltage drop, inductive voltage drop, and back emf ($e$), respectively. Equation (4.12) can be rearranged in the form of state space representation and is given by,

$$\frac{di}{dt} = -\frac{R}{L} \cdot i - \frac{g_z}{L} \cdot \dot{z} \cdot i + \frac{1}{L} \cdot v = -\frac{2z}{\mu_0 A_p T_m^2} \cdot i + \frac{1}{z} \cdot \dot{z} \cdot i + \frac{2z}{\mu_0 A_p T_m^2} \cdot v$$

$$= -a_1 \cdot i - a_2 \cdot \dot{z} \cdot i + a_0 \cdot v$$  (4.13)

where $a_0 = 1/L$, $a_1 = R/L$, $a_2 = g_z/L$.

The generated electromagnetic force, which is attractive, is expressed as,

$$f_z = \frac{1}{2} \cdot \frac{\partial L}{\partial z} \cdot i^2 = \frac{1}{2} \cdot g_z \cdot i^2 = \frac{\mu_0 A_p T_m^2}{4} \left( \frac{i}{z} \right)^2.$$  (4.14)

From (4.14), it is seen that the generated electromagnetic force is proportional to the square of winding current and inversely proportional to the square of air gap length. The larger the air gap length, the smaller the generated electromagnetic force. For this reason, the electromagnetic levitation system has an open-loop-unstable characteristic.

Figure 4.8 shows a free body diagram of the single electromagnet system alongside the forces acting on a quarter of the vehicle mass. The vertical motion dynamics of the dc electromagnet can be derived from the free body diagram and is given by,

$$M \cdot \ddot{z} + B \cdot \dot{z} = -f_z + M \cdot G + f_d = -f_z + M \cdot G + (f_1 - f_n)$$  (4.15)

where $M$ is a quarter of the vehicle mass, $B$ is the viscous damping coefficient, $G$ is the acceleration of gravity, $f_d$ is the external force disturbances which include the load disturbance $f_1$ and normal force disturbance $f_n$ generated in the LSRM propulsion drive system. Note that the
normal force disturbance will be compensated in the levitation controller using feedforward control strategy to have better air gap control performance. A complete block diagram of the single electromagnet levitation system obtained by combining (4.13)-(4.15) is shown in Figure 4.9.

4.4.2 Open Loop Instability

The open loop instability of the electromagnetic levitation system has conceptually been shown using the generated electromagnetic force equation, (4.14). In this section, a mathematical analysis to confirm the open loop instability of the electromagnetic levitation system is attempted using linearization technique [65], [70], [78], which gives the qualitative behavior of a nonlinear
system near a nominal equilibrium point. Further, the linearized equations give insight to the design requirements of the closed loop control system for stabilizing the electromagnetic levitation system.

The states of the electromagnetic levitation system are the air gap position, $z$, vertical velocity of the vehicle, $\dot{z}$, and winding current, $i$. Equations (4.13) to (4.15) can be rewritten in the following form using these three state variables:

$$x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T = [z(t) \ \dot{z}(t) \ i(t)]^T$$

$$u(t) = [u_1(t) \ u_2(t)]^T = [v(t) \ f_d(t)]^T$$

$$\dot{x}(t) = \begin{bmatrix} x_2 \\ \frac{B}{M} \cdot x_2 - \frac{\mu_0 A_p T_m^2}{4M} \cdot \left(\frac{x_3}{x_1}\right)^2 + G + \frac{1}{M} \cdot u_2 \\ -\frac{2R}{\mu_0 A_p T_m^2} \cdot x_1 x_3 + \frac{x_2^2 x_3}{x_1} + \frac{2}{\mu_0 A_p T_m^2} \cdot x_1 u_1 \end{bmatrix} = \begin{bmatrix} f_1(x, u, t) \\ f_2(x, u, t) \\ f_3(x, u, t) \end{bmatrix} = f(x, u, t)$$

where superscript $T$ represents the transpose of the matrix. Nominal equilibrium points can be found by solving the nonlinear algebraic equations obtained by equating (4.18) to zero and assuming no external force disturbance. They are derived as the following.

$$x_e = [x_{e1} \ x_{e2} \ x_{e3}]^T = [z_e \ \dot{z}_e \ i_e]^T = \begin{bmatrix} \frac{V}{R} \\ \sqrt{\frac{\mu_0 A_p T_m^2}{4MG}} \ 0 \ V \end{bmatrix}^T$$

$$u_e = [u_{e1} \ u_{e2}]^T = [v_e \ f_{de}]^T = [V \ 0]^T$$

where $V$ is the applied constant voltage and subscript $e$ represents the nominal equilibrium point. Perturbing the system around the nominal equilibrium point with small signals, the new system states and inputs are,

$$x(t) = x_e(t) + \delta x$$

$$u(t) = u_e(t) + \delta u$$

$$\dot{x}(t) = \dot{x}_e(t) + \delta \dot{x} = f(x_e + \delta x, u_e + \delta u, t)$$

$$= f(x_e, u_e, t) + \left[ \frac{\partial f}{\partial x} \right]_e \cdot \delta x + \left[ \frac{\partial f}{\partial u} \right]_e \cdot \delta u + \text{H.O.T.}$$

where $\delta$ represents the small signals, and H.O.T. stands for higher-order terms of the Taylor series expansion. Since the nominal equilibrium point satisfies (4.18), the first term in the right-
hand side of (4.23) is cancelled. Assuming a sufficiently small deviation of the equilibrium point, the higher-order terms are negligible, resulting in the linearized equation as,

\[ \delta \dot{x} = \left[ \frac{\partial f}{\partial x} \right]_e \delta x + \left[ \frac{\partial f}{\partial u} \right]_e \delta u = A \cdot \delta x + B \cdot \delta u \]

\[ A_{ij} = \frac{\partial f_i}{\partial x_j} \bigg|_{x_e, u_e}, \quad B_{ik} = \frac{\partial f_i}{\partial u_k} \bigg|_{x_e, u_e} \quad \text{for } i = 1, 2, 3, \quad j = 1, 2, 3, \quad \text{and } k = 1, 2 \]

where \( A \) and \( B \) represent \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial u} \), respectively, subscript \( i \) denotes the row index of the matrix, and subscripts \( j \) and \( k \) denote the column indices of the matrix. Each component of the matrices \( A \) and \( B \) are derived as follows:

\[ \frac{\partial f_1}{\partial x_1} = 0, \quad \frac{\partial f_1}{\partial x_2} = 1, \quad \frac{\partial f_1}{\partial x_3} = 0 \]

\[ \frac{\partial f_2}{\partial x_1} = \frac{\mu_0 A_p T_m^2}{2M} \cdot \frac{i_e^2}{z_e^2}, \quad \frac{\partial f_2}{\partial x_2} = -\frac{B}{M}, \quad \frac{\partial f_2}{\partial x_3} = -\frac{\mu_0 A_p T_m^2}{2M} \cdot \frac{i_e}{z_e} = -\frac{L_e}{M} \cdot \frac{i_e}{z_e} \]

\[ \frac{\partial f_3}{\partial x_1} = 0, \quad \frac{\partial f_3}{\partial x_2} = \frac{i_e}{z_e}, \quad \frac{\partial f_3}{\partial x_3} = -\frac{2R}{\mu_0 A_p T_m} \cdot z_e = -\frac{R}{L_e} \]

\[ \frac{\partial f_1}{\partial u_1} = 0, \quad \frac{\partial f_1}{\partial u_2} = 0, \quad \frac{\partial f_2}{\partial u_1} = \frac{1}{M}, \quad \frac{\partial f_2}{\partial u_2} = 1, \quad \frac{\partial f_3}{\partial u_1} = \frac{2 \cdot z_e}{\mu_0 A_p T_m} = \frac{1}{L_e}, \quad \frac{\partial f_3}{\partial u_2} = 0 \]

where \( L_e \) is the winding inductance at the nominal equilibrium point. The following is the set of linearized voltage and motion equations obtained from the above derivations.

\[ \delta \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} \frac{B}{M} & \frac{L_e}{M} & \frac{i_e}{z_e} \end{bmatrix} \delta x(t) + \begin{bmatrix} 0 & 0 \end{bmatrix} \delta u(t) \]

The stability of the electromagnetic levitation system in the neighborhood of \( x_e \) can be determined by the eigenvalues which are the solutions of the characteristic equation of the linearized system matrix \( A \). The characteristic equation of (4.26) is given as,

\[ \Delta(s) = |sI - A| = s^3 + \left( \frac{B}{M} + \frac{R}{L_e} \right) s^2 + \frac{B}{M} \cdot \frac{R}{L_e} s - \frac{R}{M} \cdot \left( \frac{i_e}{z_e} \right)^2 = 0 \]
where $\Delta$ denotes the determinant of the matrix, $s$ is the Laplace operator, and $I$ is the identity matrix. Using the Routh-Hurwitz criterion for stability, there is one sign change in the first column of the Routh table, meaning that there is one right-half plane pole which makes the electromagnetic levitation system unstable. If the parameters of the electromagnet prototype shown in Appendix C.3 are substituted in (4.27), the characteristic equation is expressed by,

$$\Delta(s) = s^3 + 30.09 \cdot s^2 + 0 \cdot s - 2.9491 \times 10^5$$

$$= (s - 57.89)(s + 43.99 + j56.20)(s + 43.99 - j56.20) = 0.$$  \hspace{1cm} (4.28)

One of the open loop eigenvalues is positive and hence the electromagnetic levitation system is inherently unstable, which requires closed loop feedback control for stability. Because of the positive eigenvalue in the open loop system, a compensator including at least one zero in classical control strategy is indispensable for the stable operation of the electromagnetic levitation system [65]. Proportional-derivative (PD), proportional-integral-derivative (PID), lead, and lead-lag compensators can be considered as appropriate candidates from the above analysis.

### 4.5 Control of the Electromagnetic Levitation System

In this section, the configuration of the control loops for stabilizing the electromagnetic levitation system is described and the controller design procedure of each control loop is also presented using the described prototype electromagnetic levitation system.

#### 4.5.1 Configuration of the Control Loops

The control loops are composed of a position control loop with feedforward compensation strategy and a force/current control loop. Figure 4.10 shows the block diagrams of the position and force/current control loops of electromagnetic levitation system with unity feedback gains. These are the theoretical control block diagrams and each control variable is represented by using its physical unit such as $A$ for the winding current, $m$ for the air gap position, $m/s$ for the vertical velocity, and $N$ for the electromagnetic force generated. In the implementation of each control loop, however, the control variables have their own conversion factors due to the use of sensors for the feedback of control signals. Figure 4.11 shows the implemented control block diagrams with the non-unity feedback gains that are the conversion factors between the physical
Figure 4.10 Control block diagrams of the electromagnetic levitation system with unity feedback gains.

(a) Position control loop.

(b) Force/current control loop.

Figure 4.10 Control block diagrams of the electromagnetic levitation system with unity feedback gains.
Figure 4.11 Control block diagrams of the electromagnetic levitation system with non-unity feedback gains.

(a) Position control loop.

(b) Force/current control loop.

Figure 4.11 Control block diagrams of the electromagnetic levitation system with non-unity feedback gains.

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quantities and the implemented quantities. In Figures 4.10 and 4.11, the lowercase letters represent the physical quantities, the uppercase letters represent the implemented quantities whose units are voltages obtained from the sensor feedback gains, $H_p$ is the air gap position feedback gain, $H_c$ is the current feedback gain, $H_v$ is the velocity feedback gain, $H_f$ is the force conversion gain, and $K_r$ is the converter gain. All gains are defined by the ratio of maximum value of the implemented quantity to the maximum value of its physical quantity.

The outer loop is a position control loop and the inner loop is a force/current control loop. Proportional-integral-derivative (PID) and proportional-integral (PI) controllers are selected for the precise control of the air gap position and force/current, respectively. The force/current control loop is approximated as unity and its dynamic characteristics are ignored when the position control loop is designed. This is due to the fact that the bandwidth of the force/current control loop is at least ten times higher than that of position control loop and hence the speed of response of the current control loop is much faster than the response of the position control loop. The time delay of the converter is neglected here due to the assumption that the switching frequency is at least ten times greater than that of the electrical time constant in the current loop and therefore the converter is modeled as a simple gain with $K_r$. The feedforward normal force command, $f_n^*$, in the position control loop is generated in the LSRM propulsion drive system and is connected to the output of the PID position controller to generate the force command, $f_z^*$, required for the levitation. This feedforward control strategy is proposed to enhance the ride quality and system efficiency of the LSRM-propelled electromagnetic levitation system.

4.5.2 Force/Current Control [20]-[21]

Assuming that the actual winding current of the dc electromagnet follows the required current commands with accuracy, the performance of force control depends on the appropriateness of the current command generation. From this point of view, force/current control loop is composed of the current command generator, PI current controller, and feedforward back EMF compensator. These functions are designed and implemented using analog control circuit in this thesis. Figures 4.10(b) and 4.11(b) show the block diagrams of the force/current control loop with unity feedback gains and non-unity feedback gains, respectively.
4.5.2.1 Force/Current Control Loop with Unity Feedback Gain

Current Command Generator

The required instantaneous current command is calculated using (4.14) as a function of a square-root and is given by,

\[ i^* = \sqrt{\frac{2 \cdot f^*_z}{g_z}} \]  \hspace{1cm} (4.29)

where \( f^*_z \) is the force command required for the levitation of the vehicle and \( g_z \) is the rate of change of winding inductance. The force command \( f^*_z \) is obtained from the sum of the PID position controller output and the feedforward normal force command \( f^*_n \) generated by the LSRM propulsion controller. Note that the instantaneous value of the rate of change of winding inductance, \( g_z \), is continuously varying for the air gap length and winding current of the dc electromagnet.

Current Controller

The dynamic characteristic of the electromagnet winding current is represented by the nonlinear time-varying equation (4.13) and this can be linearized by defining the control input \( v \) as,

\[ v = a_2 \cdot \dot{z} \cdot i / a_0 + u \]  \hspace{1cm} (4.30)

where \( a_0 = 1/L \), \( a_2 = g_z/L \), and \( u \) is a new control input to be designed. Substituting (4.30) into (4.13) gives the linearized state space expression for the electromagnet winding current and is given by,

\[ \frac{di}{dt} = -a_1 \cdot i + a_0 \cdot u \]  \hspace{1cm} (4.31)

where \( a_1 = R/L \). The new control input \( u \) is specified by a PI controller to obtain the required transient and steady state performance and expressed as,
where $K_{pc}$ and $K_{ic}$ are the gains of the PI controller. Figure 4.12 shows the block diagram of the current control loop with unity feedback gain. The appropriate gains of the PI controller can be derived by finding the transfer function of the current control loop and from Figure 4.12, the transfer function between the current input and output is expressed as,

$$G_c(s) \equiv \frac{i(s)}{i^*(s)} = \frac{a_0 K_{pc} \cdot s + a_0 K_{pc} K_{ic}}{s^2 + (a_0 K_{pc} + a_1) \cdot s + a_0 K_{pc} K_{ic}}$$

(4.33)

From a given set of bandwidth $\omega_c$ and damping ratio $\zeta_c$ of current control loop, the PI controller gains are analytically derived as,

$$K_{pc}(z) = \frac{2\zeta_c \omega_c}{\sqrt{(1 + 2\zeta_c^2) + \sqrt{(1 + 2\zeta_c^2)^2 + 1}}} \cdot \frac{1}{a_0(z)}$$

(4.34)

$$K_{ic} = \frac{\omega_c}{2\zeta_c \sqrt{(1 + 2\zeta_c^2) + \sqrt{(1 + 2\zeta_c^2)^2 + 1}}}$$

(4.35)

Note that $a_0(z)$ is a function of electromagnet winding inductance and hence it varies according to the air gap length. From (4.34) and (4.35), $K_{pc}(z)$ is inversely proportional to $a_0(z)$ and hence it is directly proportional to the electromagnet winding inductance. $K_{ic}$ is a constant regardless of the operating conditions for a given set of $\omega_c$ and $\zeta_c$. Therefore, by adjusting $K_{pc}(z)$.
instantaneously according to the variation of air gap length instead of keeping it constant, the identical transient and steady state characteristics of the current control loop can be obtained at all positions. In current control loop, \(a_0(z)\) can be regarded as a slowly changing variable or a constant compared to the fast response of the current loop if high bandwidth of current control loop is implemented [20]-[21].

4.5.2.2 Implementation of the Force/Current Control Loop with Non-unity Feedback Gains

Current Command Generator

Current command generation is implemented using an analog multiplier which can calculate the square-root function. The transfer function of a square-rooter implemented by an analog multiplier is given by \(\sqrt{10 \cdot (\bullet)}\). In Figure 4.11(b), the relation between the force command and the current command is derived as,

\[
\begin{align*}
    i^* &= \sqrt{\frac{2f_z^*}{g_z}}, \quad F_z^* = H_f \cdot f_z^*, \quad I^* = H_c \cdot i^* \quad \Rightarrow \quad I^* = \frac{H_c}{\sqrt{5g_zH_f}} \cdot \sqrt{10 \cdot F_z^*} \\
\end{align*}
\]

where the lowercase letters represent the physical quantities, the uppercase letters represent the implemented quantities, \(H_f\) is the force conversion gain and \(H_c\) is the current feedback gain.

Shown in Figure 4.13 is a block diagram of the current command generator. The current feedback gain \(H_c\) is determined by the amplifying gain of the current sensor signal. The generated electromagnetic force is proportional to the square of winding current and inversely proportional to the square of air gap length. For this reason, even though the rated current of 3.5 A is required for maintaining the nominal air gap of 3 mm in the steady state, twice the rated

\[\text{Figure 4.13 Block diagram of the current command generator.}\]
current is required for the levitation of the vehicle at initial operating air gap of 6 mm. With the assumption of 150% margin to twice the rated current, the current feedback gain is calculated as,

\[
H_c = \frac{V_{\text{sensor, max}}}{i_{\text{max}}} = \frac{10 [V]}{10 [A]} = 1 [V/A].
\] (4.37)

The force conversion gain \(H_f\) can be calculated from (4.36) so as to satisfy the following condition that the current command output becomes the full range of 10 V when the force command is the full range of 10 V and \(g_z\) is the minimum value.

\[
H_f = \frac{2H_c^2 F_z^*}{(1^*)^2 g_{zk}} = \frac{2 \cdot 1^2 \cdot 10}{10^2 \cdot 7.74} = 0.0258 [V/N].
\] (4.38)

The gain \(K\) in Figure 4.13 varies according to the rate of change of electromagnet winding inductance \(g_z\) and hence the air gap length. For this reason, the gain \(K\) is continuously adjusted using a digital potentiometer in the analog circuit implementation to obtain the correct relationship between the force command and the current command with respect to the air gap length. Figure 4.14 shows the variation of the gain \(K\) to the air gap length.

![Figure 4.14 Variation of the gain K for the current command generation to the airgap length.](image)
Current Controller

Figure 4.15 shows the block diagram of the implemented current control loop with non-unity feedback gains. E is back emf compensation input, a feedforward, to linearize the current dynamic equation by a nonlinear feedback linearization scheme. The appropriate gains of the PI controller can be derived by finding the transfer function of the current control loop and from Figure 4.15. The transfer function between the current input and output is expressed as,

\[ G_c(s) \equiv \frac{i(s)}{i^*(s)} = \frac{a_0H_cK_rK_{pc} \cdot s + a_0H_cK_rK_{pc}K_{ic}}{s^2 + (a_0H_cK_rK_{pc} + a_1) \cdot s + a_0H_cK_rK_{pc}K_{ic}} \]

\[ \equiv \frac{a_0H_cK_rK_{pc} \cdot s + a_0H_cK_rK_{pc}K_{ic}}{s^2 + a_0H_cK_rK_{pc} \cdot s + a_0H_cK_rK_{pc}K_{ic}} \text{ for } a_0H_cK_rK_{pc} \gg a_1 \]

(4.39)

where \( H_c \) is the current feedback gain, and \( K_r \) is the converter gain. From a given set of bandwidth, \( \omega_c \), and damping ratio, \( \zeta_c \), the gains of the PI controller are analytically derived as,

\[ K_{pc}(z) = \frac{2\zeta_c \omega_c}{\sqrt{(1 + 2\zeta_c^2) + \sqrt{(1 + 2\zeta_c^2)^2 + 1}}} \cdot \frac{1}{a_0(z)H_cK_r} \]

(4.40)

\[ K_{ic} = \frac{\omega_c}{2\zeta_c \sqrt{(1 + 2\zeta_c^2) + \sqrt{(1 + 2\zeta_c^2)^2 + 1}}} \]

(4.41)

Figure 4.15 Block diagram of the implemented current control loop with non-unity feedback gains.
Note that $K_{pc}(z)$ is a function of winding inductance, current feedback gain, and converter PWM gain. As described in the control loop with unity feedback gain, $K_{pc}(z)$ is a function of winding inductance and hence it is varying according to the air gap length. In the implementation, $K_{pc}(z)$ is adjusted using a digital potentiometer to achieve the identical transient and steady state characteristics of the current control loop at all positions [20]-[21].

**Design Example of the PI Controller Gains**

If $\omega_c = 2\pi(2000 \text{ Hz}) = 4000\pi \text{ rad/s}$, $\zeta_c = 2$, $L_e = 108.08 \text{ mH}$ where $L_e$ is the winding inductance at the nominal equilibrium point, $K_r = V_{dc}/V_{cm} = 120\sqrt{2}/10 \equiv 16.97 \text{ V/V}$ where $V_{dc}$ is the dc link voltage and $V_{cm}$ is the maximum control voltage, and $H_c = 1 \text{ V/A}$, the gains of the PI controller are calculated as,

$$K_{pc} = \frac{2 \cdot 2 \cdot 4000\pi}{\sqrt{(1 + 2 \cdot 2^2)} + \sqrt{(1 + 2 \cdot 2^2)^2 + 1}} \cdot \frac{108.08 \cdot 10^{-3}}{1 \cdot 16.97} \equiv 75.26$$

$$K_{ic} = \frac{4000\pi}{2 \cdot 2 \cdot \sqrt{(1 + 2 \cdot 2^2)} + \sqrt{(1 + 2 \cdot 2^2)^2 + 1}} \equiv 739.34$$

$$G_{PI}(s) = \frac{K_{pc}(s + K_{ic})}{s} = \frac{75.26(s + 739.34)}{s}.$$  

**Analog Circuit Implementation of the PI Current Controller**

The PI current controller is realized using the operational amplifier as shown in Figure 4.16 and the transfer function between input and output is derived as,

$$G_{PI}(s) \equiv \frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1} \cdot \left( s + \frac{1}{R_2C} \right) = -\frac{K_{pc}(s + K_{ic})}{s}. \quad (4.42)$$

Since there are three unknowns and two equations, a practical value for one of the elements is arbitrarily selected. With the selection of $C$, the remaining values are found to be,

$$R_2 = \frac{1}{K_{ic}C}, \quad R_1 = \frac{R_2}{K_{pc}}. \quad (4.43)$$
Using the PI controller gains from the previous design example, if $C = 6800 \ \text{pF}$, $R_1$ and $R_2$ are determined as $2.7 \ \text{K}\Omega$ and $200 \ \text{K}\Omega$, respectively. Note that $K_{pc}$ varies with the winding inductance variation and hence the component $R_1$ varies. This varying gain of PI controller in the analog circuit implementation can be adjusted using the digital potentiometer.

![Analog circuit implementation of the PI current controller.](image)

**4.5.3 Position Control**

Position control loop is composed of the position controller, feedforward normal force command, and air gap position feedback. A number of different approaches can be applied to design a position controller. In this thesis, the classical PID controller is selected to stabilize the air gap position control of the electromagnetic levitation system. The PID controller is a cascaded composite of the PI controller and PD controller. The PI controller acts to eliminate the steady state error to the position command and provides some robustness to parameter variations. The PD controller provides the necessary phase lead to stabilize the system and to improve the transient response [77]. The feedforward control strategy to effectively reject the normal force disturbance coming from the propulsion drive system is described in Section 4.5.4.
4.5.3.1 Position Control Loop with Unity Feedback Gain

For improving both the steady state error and transient response, the PD controller is designed first to meet the transient response requirement, and the PI controller is designed with the PD compensated system to yield the required steady state error. A disadvantage of this approach is the slight decrease in the speed of response when the steady state error is improved [77]. Figure 4.17 shows a block diagram of the PD position control loop with unity feedback gain.

Assuming that \( i^* \equiv i \) and hence \( f_z^* \equiv f_z \) and there are no feedforward disturbance inputs, the transfer function between the position input and output is derived as,

\[
G_p(s) \equiv \frac{z(s)}{z^*(s)} = -\frac{K_{dp}}{s^2 + \left(\frac{B + K_{dp}}{M}\right)s + \frac{K_{dp}K_{pp}}{M}}
\]

(4.44)

where \( K_{dp} \) and \( K_{pp} \) are the gains of the PD position controller. From a given set of bandwidth \( \omega_p \) and damping ratio \( \zeta_p \) of the position control loop, the PD controller gains are analytically derived as,
\begin{align*}
K_{dp} &= \frac{2\zeta_p \omega_p}{\sqrt{(1 + 2\zeta_p^2) + \sqrt{(1 + 2\zeta_p^2)^2 + 1}}} \cdot M \tag{4.45} \\
K_{pp} &= \frac{\omega_p}{2\zeta_p \sqrt{(1 + 2\zeta_p^2) + \sqrt{(1 + 2\zeta_p^2)^2 + 1}}} \tag{4.46}
\end{align*}

This completes the design of the PD controller. Next, the PI controller is designed to reduce the steady state error to zero for a given position command. Any zero of the PI controller will work as long as the zero is placed close to the origin due to the fact that the angular contribution of the zero and the pole of the PI controller is cancelled out by each other [77]. Assuming \( K_{ip} \) is close to the origin, the transfer function of the PI controller is given by,

\[ G_{PI}(s) = \frac{s + K_{ip}}{s} \tag{4.47} \]

Cascading the transfer functions of the PD and PI controllers results in the following transfer function of the PID controller.

\[ G_{PID}(s) = G_{PD}(s) \cdot G_{PI}(s) = \frac{K_{dp}(s + K_{pp})(s + K_{ip})}{s} = \frac{K_{dp} \cdot s^2 + K_{pp}(K_{ip} + K_{ip}) \cdot s + K_{dp}K_{pp}K_{ip}}{s} \tag{4.48} \]

### 4.5.3.2 Implementation of the Position Control Loop with Non-unity Feedback Gains

#### Position Controller

Figure 4.18 shows a block diagram of the implemented PD position control loop with non-unity feedback gains. The transfer function between the position input and output is derived as,

\[ G_p(s) \equiv \frac{z(s)}{z^*(s)} = \frac{H_{p}K_{dp}}{H_{f}M} \cdot (s + K_{pp}) \]

\[ = \frac{H_{p}K_{dp}}{H_{f}M} \cdot s + \frac{H_{p}K_{dp}K_{pp}}{H_{f}M} \]

\[ \equiv \frac{H_{p}K_{dp}}{H_{f}M} \cdot s + \frac{H_{p}K_{dp}K_{pp}}{H_{f}M} \quad \text{for} \quad H_{p}K_{dp} \gg H_{f}B . \tag{4.49} \]
From a given set of bandwidth, $\omega_p$, and damping ratio, $\zeta_p$, of the position control loop, the PD controller gains are analytically derived as,

\[
K_{dp} = \frac{2\zeta_p \omega_p}{\sqrt{(1 + 2\zeta_p^2) + \sqrt{(1 + 2\zeta_p^2)^2 + 1}}} \cdot \frac{H_f M}{H_p} \tag{4.50}
\]

\[
K_{pp} = \frac{\omega_p}{2\zeta_p \sqrt{(1 + 2\zeta_p^2) + \sqrt{(1 + 2\zeta_p^2)^2 + 1}}} \tag{4.51}
\]

After completing the design of the PD controller, the PI controller is designed to reduce the steady state error to zero for a given position command. The transfer functions of PI and PID controllers can similarly be obtained as (4.47) and (4.48), respectively.

**Design Example of the PID Controller Gains**

If $\omega_p = 2\pi \times 25 \text{ Hz} = 50 \pi \text{ rad/s}$, $\zeta_p = 1.5$, $H_p = (4 \text{ V})/(3 \text{ mm})$, $M = M_g/4 = 15 \text{ Kg}$, and $H_f = 0.0258 \text{ V/N}$, the gains of PD controller is calculated as,

\[
K_{dp} = \frac{2 \cdot 1.5 \cdot 50\pi}{\sqrt{(1 + 2 \cdot 1.5^2) + \sqrt{(1 + 2 \cdot 1.5^2)^2 + 1}}} \cdot \frac{0.0258 \cdot 15}{4/0.003} \approx 4.11 \cdot 10^{-2}
\]

\[
K_{pp} = \frac{50\pi}{2 \cdot 1.5 \cdot \sqrt{(1 + 2 \cdot 1.5^2) + \sqrt{(1 + 2 \cdot 1.5^2)^2 + 1}}} \approx 15.72
\]

\[
G_{PD}(s) = K_{dp}(s + K_{pp}) = 4.11 \cdot 10^{-2} \cdot (s + 15.72)
\]
Cascading the PD and the PI controllers results in the following PID controller transfer function.

\[ G_{\text{PID}}(s) = G_{\text{PD}}(s) \cdot G_{\text{PI}}(s) = \frac{4.11 \cdot 10^{-2} \cdot s^2 + 0.670 \cdot s + 0.387}{s}. \]

### Analog Circuit Implementation of the PID Position Controller

The PID position controller is realized using operational amplifiers. Figure 4.19 shows the analog circuit implementation of the PID position controller with a low pass filter (LPF). The derivative component of the PID controller is acting on the signal noise of the air gap position sensor, resulting in the high frequency noise of the PID controller output and hence the levitation force command. This produces the uncomfortable sound noise and vibration in the electromechanical system as well as the eddy current in the iron cores. For this reason, the low pass filter is attached to the PID controller output so that the high frequency noise contained in the PID controller output can be removed. The transfer function between input and output is derived as,

\[ G \equiv G_{\text{PID}}(s) \cdot G_{\text{LPF}}(s) = \frac{V_c(s)}{V_i(s)} \cdot \frac{V_o(s)}{V_c(s)} = \frac{R_2 C_1 \cdot s^2 + \left(\frac{C_1}{C_2} + \frac{R_2}{R_1}\right) \cdot s + \frac{1}{R_1 C_2}}{s} \cdot \frac{R_4}{R_3} \cdot \frac{R_3}{R_4 C_3 \cdot s + 1}. \] (4.52)

Under the assumption that \( G_{\text{PID}}(s) = (a \cdot s^2 + b \cdot s + c)/s, \)

\[ R_2 C_1 = a, \quad \frac{C_1}{C_2} + \frac{R_2}{R_1} = b, \quad \frac{1}{R_1 C_2} = c. \] (4.53)

Since there are four unknowns and three equations, a practical value for one of the elements is arbitrarily selected. With the selection of \( C_2, \) the remaining values are derived as,

\[ R_1 = \frac{1}{c C_2}, \quad R_1 C_1^2 - b R_1 C_2 C_1 + a C_2 = 0, \quad C_1 = \text{Min. (Two Roots)}, \quad R_2 = \frac{a}{C_1} \] (4.54)

Using the PID controller gains from the previous design example, if \( C_2 = 4.7 \mu F, R_1, R_2, \) and \( C_1 \) are determined as 550 K\( \Omega, \) 355 K\( \Omega, \) and 0.12 \( \mu F, \) respectively. Note that the adjustment of the
Figure 4.19 Analog circuit implementation of the PID position controller with a low pass filter.

circuit components may be necessary in the experimentation stage. The components of the low pass filter can be found using the assumption that the dc gain of the filter is equal to unity and the bandwidth of the filter is three times magnitude of the position loop bandwidth. These assumptions result in,

\[ R_3 = R_4 , \quad \frac{1}{R_4 C_3} = 3\omega_p . \]  

(4.55)

If \( R_3 = R_4 = 20 \text{ K}\Omega \) and \( \omega_{LPF} = 3\omega_p = 3 \cdot 50 \cdot \pi = 150 \cdot \pi \text{ rad/sec} \), \( C_3 \) is approximately determined as 0.1 \( \mu \text{F} \).

4.5.4 Feedforward Compensation Strategy for the Normal Force Disturbance Rejection

The LSRM propulsion drive system generates not only the propulsion force but also the attractive normal force. The normal force is advantageous if it is used for the levitation, resulting in the improvement of the total system efficiency. On the other hand, the normal force is an external disturbance input that causes the system to provide an inaccurate air gap control performance accompanied by jerky motion and vibration of the vehicle. For this reason, a feedforward compensation strategy is proposed in this section to reject the normal force disturbance coming from the LSRM propulsion drive system, and thereby enhancing the ride quality and total efficiency of the LSRM-propelled electromagnetic levitation system. Both
Figures 4.10(a) and 4.11(a) show the control block diagrams of the position control loop with the feedforward compensation strategy.

**Evaluation of the Instantaneous Normal Force of the LSRM**

The evaluation of the instantaneous normal force of the LSRM is a prerequisite for implementing the feedforward compensation strategy. The normal force profile of the LSRM to the translator position and phase winding current can be calculated and measured through finite element analysis and experiment, respectively. This result is stored in the memory in the form of look-up table and is used to calculate the feedforward normal force $f_n^*$ in Figure 4.10(a) using the feedback signals obtained from the propulsion position sensor and winding current sensor in an instantaneous manner. The look-up table approach can be replaced by an intelligent scheme such as neural-network-based interpolation with two inputs, that is, the translator position and phase winding current. The feedforward normal force can also be calculated using (4.9) alongside the parameter $g_x$ obtained at the rated winding current.

**Feedforward Compensation strategy**

Figure 4.20 shows the force configuration acting on the LSRM-propelled electromagnetic levitation system. As shown in Figures 4.1 to 4.3, there are two sets of propulsion and levitation systems one on each side. Only one side is shown in the Figure 4.20 owing to the symmetry of the mechanical structure. Further, the weight of the vehicle is expressed as one half of the total weight for the same reason. Phase A winding, $aa'$, is excited and the two pole faces of phase A are generating the normal forces: $f_{ar}$ that is in rear and $f_{af}$ that is in front of the pole pairs. These two forces have the same magnitude and the center of the forces is different from the center of mass. In the forward movement of the translator with the excitation sequence of $bb'−cc'−dd'$, the center of the forces moves toward the front electromagnet in a discrete manner and the magnitude of the normal force also varies. Therefore, the levitation control system is affected by the magnitude variation of the normal force as well as the location change of the center of the normal force. The principle of the feedforward compensation strategy is based on the adequate distribution of the LSRM normal force to the front and rear electromagnets such that the
following two conditions are satisfied: (a) The sum of the forces acting on the vehicle is equal to zero, (b) The sum of the moments about the center of mass of the forces acting on the vehicle is equal to zero. The above two conditions can be expressed by writing the equations, respectively, as,

\[
\sum F = f_{mf} + f_{mr} + f_{af} + f_{ar} - \frac{M_v \cdot G}{2} = 0
\]

(4.56)

\[
\sum M_o = f_{mf} \cdot \frac{d}{2} - f_{mr} \cdot \frac{d}{2} + f_{af} \cdot d_{af} - f_{ar} \cdot d_{ar} = 0
\]

(4.57)

where \( M_v \) is the total mass of the vehicle, \( G \) is the acceleration of gravity, \( f_{mf} \) and \( f_{mr} \) are the normal forces generated in the front and rear electromagnets, respectively, \( f_{af} \) and \( f_{ar} \) are the forces generated in the front and rear pole faces of phase A of LSRM, respectively, \( d \) is the distance between the mass centers of two electromagnets, \( d_{af} \) and \( d_{ar} \) are the distances of the front and rear pole face centers of phase A from the center of mass, respectively. By using Cramer’s rule, the forces of the front and rear electromagnets are derived respectively as,

\[
f_{mf} = \frac{M_v \cdot G}{4} + \frac{f_{af} \cdot (-2d_{af} - d) + f_{ar} \cdot (2d_{ar} - d)}{2d}
\]

(4.58)
Because $f_{af}$ and $f_{ar}$ have the same magnitude which is a quarter of the total normal force generated in phase A, (4.58) and (4.59) can be rewritten as,

$$f_{mf} = \frac{M_v \cdot G}{4} - \frac{f_{za}}{4} \left( \frac{d + d_{af} - d_{ar}}{d} \right)$$

$$f_{mr} = \frac{M_v \cdot G}{4} - \frac{f_{za}}{4} \left( \frac{d - d_{af} + d_{ar}}{d} \right)$$

where $f_{za}$ is the total normal force generated in phase A of LSRM. The first term in (4.60) and (4.61) stands for the nominal force of an electromagnet required for the levitation of the vehicle. The second term denotes the normal force disturbances coming from phase A of LSRM, which have to be compensated by the feedforward control strategy in the levitation controller. Finally, the feedforward compensation commands for the controllers of the front and rear electromagnets to reject the normal force disturbances are expressed respectively as,

$$f^*_{naf} = \frac{f_{za}}{4} \left( \frac{d + d_{af} - d_{ar}}{d} \right)$$

$$f^*_{nar} = \frac{f_{za}}{4} \left( \frac{d - d_{af} + d_{ar}}{d} \right)$$

These feedforward compensation commands are added to the PID position controller output to generate the final force command of the levitation controller. The normal force disturbance generated in the other phases of LSRM can similarly be obtained as that of phase A. If two or more phases are excited, the feedforward compensation force is given by the sum of the disturbance normal forces of the excited phases.

Figure 4.21 shows the dynamic simulation of the normal force disturbances for the feedforward compensation scheme for two force control strategies of the LSRM. The distances $d$, $d_{af}$, and $d_{ar}$ of the prototype system are 604, 15, and 105 mm, respectively. The distances between each pole face center of the other phases and the center of mass can algebraically be evaluated using the translator pole pitch of 30 mm, $d_{af}$, and $d_{ar}$. The same simulation conditions applied for plotting the Figures 3.10 and 3.11 have been used for calculating the feedforward compensation command. In other words, the LSRM is operating as forward motoring mode, and the operating speed and propulsion force command of LSRM are fixed to 0.5 m/s and 30 N, respectively, which are corresponding to a half of the rated speed and three-quarters of the rated
Figure 4.21 Dynamic simulation of the normal force disturbances for the feedforward compensation scheme for two force control strategies of the LSRM.

(a) Single-phase excitation strategy.

(b) Two-phase excitation strategy.

Figure 4.21 Dynamic simulation of the normal force disturbances for the feedforward compensation scheme for two force control strategies of the LSRM.
Figure 4.22 Experimental waveforms of the feedforward compensation commands in the force-controlled LSRM drive (10 ms/div).

(a) Single-phase excitation strategy.  
(b) Two-phase excitation strategy.

Figure 4.23 Experimental waveforms of the feedforward compensation commands in the velocity-controlled LSRM drive (1 s/div).

(a) Single-phase excitation strategy.  
(b) Two-phase excitation strategy.
force of the machine. Sequentially shown in the figure are the profiles of the normal force $f_{zk}$ generated in each phase, the sum of the normal forces, the feedforward compensation force $f_{nf}^*$ of the front electromagnets, the feedforward compensation force $f_{nr}^*$ of the rear electromagnets. The peak value of $f_{nf}^*$ is gradually increasing as the translator of the LSRM moves in the forward direction while the peak value of $f_{nr}^*$ is gradually decreasing as the translator of the LSRM moves in the forward direction. These profiles are repeated at intervals of one excitation period of the LSRM phase windings.

Figures 4.22 and 4.23 show the experimental waveforms of the feedforward compensation command in the force-controlled and velocity-controlled LSRM drives, respectively. The experimental conditions for plotting each figure are the same as those of Figures 3.14 to 3.17. Dynamic simulation results, Figure 4.21, show high correlation to the experimental results, Figure 4.22. At the periods of acceleration and deceleration in Figure 4.23, highly fluctuating feedforward compensation command is generated and gradually decreased as the actual velocity of the translator reaches its steady state value. The correlation of Figure 4.23 to the simulation results can be inferred from the Figure 4.21 and the normal force profiles in Figures 3.12 and 3.13 for single-phase and two-phase excitation strategies, respectively.

4.6 Dynamic Simulations and Experimental Results

4.6.1 Control Block Diagram

The experimental control block diagram of the LSRM-propelled electromagnetic levitation system is shown in Figure 4.24. The available feedback signals for control are the winding current of the dc electromagnet and air gap position signal from the analog gap sensor. The position signal from the analog gap sensor is processed through a low pass filter whose crossover frequency is three times larger than that of the position control loop. The high frequency noise contained in the measured air gap position signal is filtered in this stage. The filtered air gap position signal is compared with the air gap position command to produce the position error. This error goes through a PID position controller, limiter, and another low pass filter whose crossover frequency is also three times larger than that of the position control loop. This low pass
Figure 4.24 Experimental control block diagram of the LSRM-propelled electromagnetic levitation system.

Analog hardware implementation of the position and force/current control

\[ F^*_n = \frac{1}{1 + \tau_{\text{pid}s}} (F^*_n - F^*_r) \]

\[ F^*_r = F^*_n \] for the front electromagnet,

\[ F^*_r = F^*_n \] for the rear electromagnet

Propulsion system controller

PWM signal

\[ T_u, T_i \]

Unipolar voltage switching PWM

Triangular carrier wave

PID position controller + Limiter

\[ F^*_{\text{PID}} \]

Low pass filter

\[ I \]

Feedforward compensation

PI current controller + Limiter

\[ F^*_Z = I^* + \]

Current sensor

Asymmetric bridge converter with 160V dc link

Inductive analog gap sensor

Position signal processor with low pass filter

Levitation dc electromagnet

\[ Z \]

\[ Z' \]

\[ v_c \]
filter is necessary to remove the high frequency noise of the PID position controller output, which is mainly caused by the derivative controller. The force command signal is obtained as the difference between the filtered PID controller output and the normal force compensation command, which comes from the propulsion system controller. In principle, even though the current command is calculated using the force command as a square-root function, the square-root function is not implemented in the experiment and the force command is directly used as a current command in this study. The current error between the reference and actual winding current is processed through a PI controller and limiter to yield the reference voltage command for comparison with the triangular carrier wave. The gating signals for the converter switches are obtained using unipolar voltage switching PWM. Note that the same converter topology applied to the LSRM propulsion drive system is used for the control of the electromagnetic levitation system due to the fact that both the LSRM and dc electromagnet have independent winding requiring two-quadrant operation with unipolar winding current and bipolar winding voltages.

4.6.2 Dynamic Simulations

It is assumed in the simulation that the air gap position feedback from the analog gap sensor is corrupted by the noise whose probability density function follows the normal (Gaussian) distribution function with zero mean and 0.1 mm standard deviation. The low pass filter with the crossover frequency of 75 Hz is used to filter out the high frequency component of the air gap position feedback.

Figure 4.25 shows the dynamic simulation of the position and force/current control responses in the electromagnetic levitation system without load variation. The system parameters used in the simulation are given in Appendix C.3. The initial air gap position feedback of the electromagnet is 6 mm. The total weight applied to the electromagnet is assumed to be 23 Kg, even though the electromagnet has been designed for the rated weight of 15 Kg. This assumption is established owing to the unevenly distributed control electronic equipments on the top surface of the vehicle such as power converters, power supplies, and digital control boards, and can be estimated from the measured winding current value. The air gap position reference is fixed to 3 mm as a step command and it is maintained constant. Sequentially presented in the figure are the responses of the applied load $f_{\text{load}}$, air gap position feedback $z$ and its reference $z^*$, winding
Figure 4.25 Dynamic simulation of the position and force/current control responses in the electromagnetic levitation system without load variation.
current $i$ and its reference $i^*$, actual electromagnetic levitation force $f_z$, voltage $v$ and flux linkage $\lambda$ of the electromagnet winding, and vertical velocity $\dot{z}$ of the vehicle. It is noticed that the actual air gap position experiences an initial transient state and reaches its final value in steady state with settling time of 0.3 s. The actual air gap position does not change until enough levitation force is generated. The winding current is initially increased to approximately twice the rated value in short period and is reduced to the required value in the steady state. Both the winding current command and feedback are drawn in the same plot and they are closely matching each other, which demonstrates the current controller performance. The actual winding voltage is continuously changing in a range between the negative and positive dc link voltage with high switching frequency. The actual winding voltage shown in the figure is a filtered signal using a low pass filter. Note that when the system reaches the steady state, the winding voltage is mainly applied to overcome the voltage drop in the winding resistance, and back emf generated is very low because of the low vertical velocity of the vehicle as well as the low rate of change of winding inductance.

Figure 4.26 shows the dynamic simulation of the electromagnetic levitation system with load variation. The system is assumed to be in the steady state with operation conditions identical to those of Figure 4.25. Initially there is no external load and at time 1.5 s a load of 40 N is applied (approximately 30 % of the rated force of a dc electromagnet). The load is removed at 3.5 s. The displaying sequence of each figure is the same as that of Figure 4.25. The actual air gap position and winding current follow their references well. At the instants of the addition and removal of the load, the air gap position changes and quickly comes back to the reference demand. The transient performance of this load disturbance rejection is determined by the gains of the PID position controller. Both the electromagnetic force and the winding current are increased as soon as the load is applied and go back to the previous values after the load is removed.

Figures 4.27 and 4.28 show the simulation results of the electromagnetic levitation system without and with the feedforward compensation strategy, respectively. As shown in the Figures 4.21 and 4.22, the feedforward compensation command of the front electromagnets is slightly different from that of the rear electromagnets in its magnitude and shape. Even though both commands can be used to simulate single-input and single-output electromagnetic levitation system, the feedforward compensation command for the front side is selected in this study. The electromagnetic levitation system is initially assumed to be in the steady state. The LSRM
Figure 4.26 Dynamic simulation of the electromagnetic levitation system with load variation.
Figure 4.27 Dynamic simulation of the electromagnetic levitation system with velocity-controlled LSRM drive using single-phase excitation (Without feedforward compensation).
Figure 4.28 Dynamic simulation of the electromagnetic levitation system with velocity-controlled LSRM drive using single-phase excitation (With feedforward compensation).
operates in velocity control mode, and single-phase excitation strategy is used for the LSRM force/current control. In the velocity-controlled LSRM drive, the magnitude of the propulsion force is restricted to 30 N. The velocity command of 0.5 m/s is applied at 0.5 s and switched to −0.5 m/s at 2.5 s. Sequentially shown in the figure are the responses of the propulsion velocity feedback $\dot{x}$ and its reference $x^*$, LSRM normal force generated, feedforward compensation command $f_{nf}^*$, air gap position feedback $z$ and its reference $z^*$, winding current $i$ and its reference $i^*$, actual electromagnetic levitation force $f_z$, voltage $v$ and flux linkage $\lambda$ of the electromagnet winding, and vertical velocity $\dot{z}$ of the vehicle. From Figure 4.27, it is observed that the air gap length is decreased as soon as the propulsion drive system is turned on. Further, mechanical vibration can be noticed from the vertical velocity profile. The peak-to-peak magnitude of the normal force disturbance varies with the motoring and regenerative operation of the LSRM and this degrades the air gap control performance. On the other hand, in Figure 4.28, the effect of the normal force disturbance is fairly rejected by applying the feedforward compensation strategy. That is, the air gap position feedback is maintained within small error and the mechanical vibration of the vehicle is reduced. Small discrepancy in the air gap position feedback may be due to the current command generation scheme whose square-root function is not implemented. Note that both the electromagnetic force and the winding current of the levitation system are reduced when the LSRM propulsion drive system operates owing to the attractive normal force of the LSRM.

Figures 4.29 and 4.30 show the simulation results using the same simulation conditions as in Figures 4.27 and 4.28, respectively, except that two-phase excitation strategy is used for the force/current control, instead of single-phase excitation. In Figure 4.29, similar fluctuation in the air gap position is observed, even though the mechanical vibration is reduced compared to that of the single-phase excitation strategy. The reduction of the mechanical vibration as compared to single-phase excitation is due to the reduction of the rate of change as well as the peak-to-peak magnitude of the produced normal force of the LSRM.
Figure 4.29 Dynamic simulation of the electromagnetic levitation system with velocity-controlled LSRM drive using two-phase excitation (Without feedforward compensation).
Figure 4.30 Dynamic simulation of the electromagnetic levitation system with velocity-controlled LSRM drive using two-phase excitation (With feedforward compensation).
4.6.3 Experimental Results

The position and force/current control of the electromagnetic levitation system is implemented using analog hardware circuit. The gap sensor used in this study is the model BAW-018-PF-1-K-5 made by BALLUFF INC. It is an inductive analog type sensor whose principle is based on the inductance variation between the approaching ferromagnetic target and the sensor surface. The maximum sensing distance is 8 mm and the linearity range is in between 1.75 and 6 mm, which is good for this application that has the nominal air gap length of 3 mm. Each gap sensor is placed just beside the corresponding dc electromagnet as shown in Figure 4.3. To provide a smooth and soft levitation of the vehicle, a ramp circuit with a resistor and a capacitor is applied for the generation of the air gap position command. This ramp circuit controls the rate of change of position command to achieve good ride quality without high vertical acceleration. The time constant of the RC circuit adjusts the rate of change of the air gap position command, and the settling time of 2 s is implemented in this study. The current command generator is treated as unity in this experiment. In other words, the square-root function and gain K in Figure 4.13 are not implemented in the analog hardware circuit and therefore the force command is considered as the current command. This may generate the performance degradation of the transient characteristic, but it will not affect the steady state. The triangular waveform PWM carrier frequency is 20 kHz.

Figure 4.31 shows the experimental waveforms of the electromagnetic levitation system without external load and LSRM propulsion drives. It is to demonstrate the position and current control performances of the electromagnetic levitation system. Subscripts in the figures, fl, fr, rl, and rr, stand for front-left, front-right, rear-left, and rear-right corners of the vehicle, respectively. Experimental verification of the air gap position and current controllers is shown in Figure 4.31(a). The air gap position command is fixed to 3 mm as a step command. The initial air gap position feedback of the electromagnet is approximately 7.5 mm and it follows the commanded value with settling time of about 0.3 s after the converter gate driver is turned on. It is also seen that the actual electromagnet winding current is tracking the current command very closely. These experimental results correlate the simulation results shown in Figure 4.25. Figure 4.31(b) shows the air gap position feedbacks of the four dc electromagnets. The initial air gap positions are different from each other owing to the track irregularities and machine part
Figure 4.31 Experimental waveforms of the position and current control responses in the electromagnetic levitation system without external load and LSRM propulsion drives.

(a) Verification of the air gap position and current controllers (0.2 s/div).

(b) Air gap position feedbacks of the four dc electromagnets (0.2 s/div).

(c) Winding current feedbacks of the four dc electromagnets (0.2 s/div).

Figure 4.31 Experimental waveforms of the position and current control responses in the electromagnetic levitation system without external load and LSRM propulsion drives.
assembly errors, but the final air gap position feedbacks follow the position reference of 3 mm. Winding current feedbacks of the four dc electromagnets are shown in Figure 4.31(c). There are differences in the magnitude of the four winding current feedbacks due to the fact that the weight of the vehicle is not uniformly distributed to the four dc electromagnets. The heaviest weight of the vehicle is distributed to the rear-right corner while the lightest weight of the vehicle is distributed to the front-right corner. Note that regardless of the unbalanced weight distribution and the track irregularities, the air gap position feedbacks are tracking their references very closely. This proves the effectiveness of the independent electromagnet control based on the single-input single-output model.

Figure 4.32 shows the experimental waveforms of the load disturbance rejection with an external load of 40 N. The air gap position command is 3 mm. Initially there is no load, and then a load of 40 N is applied (approximately 30% of the rated force of a dc electromagnet) only to the front-left corner of the vehicle after the machine operation has reached the steady state. The load is removed after 2 s. The air gap position and current feedbacks are shown in the figure. At the instants of the addition and removal of the load, the air gap position changes and then immediately follows the air gap position command. The electromagnetic winding current is increased as soon as the load is applied and goes back to the previous value after the load is removed. Correlation can be observed between Figure 4.26 (simulation results) and Figure 4.32 (experimental results).

Figure 4.33 shows the experimental waveforms of the electromagnetic levitation system with the velocity-controlled LSRM propulsion drive and feedforward compensation strategy. In the velocity-controlled LSRM drive, the propulsion force magnitude is restricted to 30 N. The velocity command is switched periodically in a range between 0.5 and –0.5 m/s in stepwise manner so that the four-quadrant machine operation can be observed. The air gap position and winding current feedbacks are shown in the figure. As can be seen, the air gap position feedbacks in both single-phase and two-phase excitation strategies are maintained constant through the four-quadrant propulsion operation, even though there are minor transients caused by the guideway irregularities. It can be noticed that the winding current is reduced during the vehicle acceleration and deceleration regions owing to the attractive normal force of the LSRM and this fact correlates the simulation results shown in Figures 4.28 and 4.30 for single-phase and two-phase excitation strategies, respectively.
Figure 4.32 Experimental waveforms of the load disturbance rejection with load of 40 N.

(a) Single-phase excitation strategy (2 s/div).

(b) Two-phase excitation strategy (2 s/div).

Figure 4.33 Experimental waveforms of the electromagnetic levitation system with the velocity-controlled LSRM propulsion drive and feedforward compensation strategy.

(a) Single-phase excitation strategy (2 s/div).

(b) Two-phase excitation strategy (2 s/div).
4.7 Conclusions

The attractive electromagnetic levitation system with LSRM propulsion drive is described in this chapter. The following are considered to be the contributions arising out of this study:

1) A design procedure for dc electromagnet is presented using analytic relations between the design variables. The two-dimensional finite element analysis is used to fine-tune the final design. The dc electromagnet prototype and track have been built based on the final dimension, and tested with a converter and controller.

2) Mathematical modeling of the electromagnetic levitation system is systematically presented using single-input single-output system under the assumption that two sets of four dc electromagnets are used for the independent control of levitation and guidance systems, respectively. Open loop instability of the electromagnetic levitation system is also demonstrated.

3) For the stable operation of the electromagnetic levitation system, which has inherent unstable characteristic, the air gap position and force/current control loops are designed using PID and PI controllers, respectively, and implemented and tested using the analog control circuit. The design procedures for the PID position and PI force/current controllers are systematically derived.

4) A feedforward compensation strategy for the levitation air gap control is proposed to reject the external force disturbance mainly caused by the normal force component generated in the LSRM propulsion drive system. The feedforward control command is calculated in the LSRM propulsion controller and is used in the levitation controller. The enhancement of ride quality and system efficiency for the LSRM-propelled electromagnetic levitation system is achieved.

5) Extensive dynamic simulations and experimental results for the air gap position and force/current control loops taken from the 6 m long prototype system are presented. Experimental correlation proves the validity of the controller design procedure based on the single-input single-output model, and shows the feasibility of the LSRM-propelled electromagnetic levitation system.
CHAPTER 5
ELECTROMAGNETIC GUIDANCE SYSTEM

5.1 Introduction

In the previous chapters, the vehicle propulsion and levitation systems were described. It should be noted that the vehicle also experiences lateral disturbances caused by winds, cornering, and track irregularities. In spite of the maximum anticipated lateral disturbance force, the vehicle should remain on the track providing a safe and comfortable ride. The propulsion and levitation systems do not provide enough guidance force to maintain the dynamic position of the vehicle against the lateral disturbances because they have been designed under the assumption that three subsystems are controlled independently. This chapter describes the electromagnetic guidance system.

5.2 Configuration of the Electromagnetic Guidance System

The configuration of the guidance system has been shown in the Figures 4.1 to 4.3. It is also realized using the attractive electromagnetic principle. Therefore, the mechanical structure and specification of the guidance system are exactly same as those of the levitation system except that the guidance components are installed on the lateral surface perpendicular to the levitation surface of the vehicle and track. The guidance system has four dc electromagnets, four gap sensors, and two guidance rails for independent control. The four dc electromagnets and gap sensors are mounted at each corner of the vehicle, and two guidance rails are located on both lateral surfaces of the track, as shown in Figures 4.2 and 4.3. The same prototype of the dc electromagnet designed in section 4.3 is used for the guidance electromagnet. Shock protectors are also attached alongside the guidance electromagnets to prevent the damages of the electromagnets and gap sensors caused by the accidental vibration of the vehicle. The lateral air gap of 3 mm is selected as a nominal operating point, even though it may vary owing to the track irregularities.
5.3 Control of the Electromagnetic Guidance System

The mathematical modeling of the attractive electromagnetic system has been developed in Chapter 4 using single-input single-output system. Based on the mathematical modeling, the configuration of the control loops for stabilizing the electromagnetic guidance system is described in this section. Because the guidance system is basically using the attractive dc electromagnets, the configuration and design procedure of each control loop developed in Chapter 4 can also be applied to the guidance control system. In the viewpoint of the mechanical structure of both systems, however, it should be clear that the pole surfaces of each pair of guidance electromagnets, say in the front part of the vehicle are respectively facing each other while all the pole surfaces of the levitation electromagnets are facing upward. For this reason, the motion dynamics of the two pairs of guidance electromagnets in the front and rear part of the vehicle are respectively coupled to each other while the levitation electromagnets can be controlled independently. From this point of view, the modification of the position control loop is necessary and is described in the following. Note that the configuration and design procedure of force/current control loop developed for the levitation system can directly be applied to the guidance system.

5.3.1 Position Control

Position Command Generator

The air gap position commands of the levitation electromagnets are usually constant and independent from each other in single-input single-output system, even though they are adjusted according to the propulsion velocity of the vehicle. On the other hand, the air gap position commands of the guidance electromagnets continuously vary because of the coupled motion dynamics, that is, the opposing action of the front (or rear) pair of electromagnets, and the track irregularities. Therefore, the air gap position command of the guidance electromagnet should be generated so that the vehicle can always be returned to its desired central position, regardless of the coupled motion dynamics and the track irregularities. Air gap position command is generated and applied independently for each pair of electromagnets. It is simply obtained as a half of the
Figure 5.1 Airgap position command generation for the guidance electromagnet control.

The sum of two air gap position feedbacks. This strategy ensures that the opposing actions of the electromagnets fight against each other, and the desired air gap on both sides is achieved. The analog implementation of the position command generator using operational amplifier is shown in Figure 5.1. The relation between the input and output is derived as,

$$Y_{fl,fr}^* (or \ Y_{rl,rr}^*) = - \frac{R_2}{R_1 C} \cdot (Y_{fl} + Y_{fr}) = - \frac{0.5}{R C \cdot s + 1} \cdot [Y_{fl} (or \ Y_{rl}) + Y_{fr} (or \ Y_{rr})] \quad (5.1)$$

where $Y^*$ and $Y$ are the air gap position command and feedback of the guidance electromagnet, the subscripts, fl, fr, rl, and rr represent the front-left, front-right, rear-left, and rear-right corners of the vehicle, respectively.

**Position Controller**

PID position controller has been used for stabilizing the levitation electromagnetic system. Especially, the integral controller has been included to eliminate the steady state error to the position command and provide some robustness to parameter variations. However, the integral controller cannot be used for the position control of the guidance electromagnetic system unless the properties of the analog circuit components and gap sensors are ideal and uniform. The characteristics of the analog gap sensors and operational amplifiers used in the implementation practically vary product by product and their output signals contain a random noise component.
Therefore, if the integral controller is applied for the position control of the guidance electromagnet, the continuous integration of the spurious error caused by the minor differences between the conflicting two feedback signals of the front/rear side can lead the guidance control system to be destroyed by the large winding current. For this reason, a PD instead of PID controller is selected for stabilizing the position loop of the electromagnetic guidance system.

Figure 5.2 shows a block diagram of the PD position control loop. The transfer function between the position input and output is derived as,

\[
G_p(s) = \frac{y(s)}{y^*(s)} = \frac{H_p K_{dp}}{s^2 + \left( \frac{H_f B + H_p K_{dp}}{H_f M} \right) s + \frac{H_p K_{dp} K_{pp}}{H_f M}} \cdot (s + K_{pp})
\]

\[(5.2)\]

From a given set of bandwidth \( \omega_p \) and damping ratio \( \zeta_p \) of the position control loop, the PD controller gains are analytically derived as,

\[
K_{dp} = \frac{2\zeta_p \omega_p}{\sqrt{(1 + 2\zeta_p^2) + \sqrt{(1 + 2\zeta_p^2)^2 + 1}}} \cdot \frac{H_f M}{H_p}
\]

\[(5.3)\]

\[
K_{pp} = \frac{\omega_p}{2\zeta_p \sqrt{(1 + 2\zeta_p^2) + \sqrt{(1 + 2\zeta_p^2)^2 + 1}}}
\]

\[(5.4)\]
where $K_{dp}$ and $K_{pp}$ are the gains of the PD position controller.

**Design Example of the PD Controller Gains**

If $\omega_p = 2 \pi (25 \text{ Hz}) = 50 \pi \text{ rad/sec}$, $\zeta_p = 1$, $H_p = (4 \text{ V})/(3 \text{ mm})$, $M = M_v/2 = 30 \text{ Kg}$, and $H_f = 0.0258 \text{ V/N}$, the gains of PD controller is calculated as,

$$
K_{dp} = \frac{2 \cdot 1 \cdot 50\pi}{\sqrt{(1 + 2 \cdot 1^2)} + \sqrt{(1 + 2 \cdot 1^2)}^2 + 1} \cdot \frac{0.0258 \cdot 30}{4/0.003} \approx 7.34 \cdot 10^{-2}
$$

$$
K_{pp} = \frac{50\pi}{2 \cdot 1 \cdot \sqrt{(1 + 2 \cdot 1^2)} + \sqrt{(1 + 2 \cdot 1^2)}^2 + 1} \approx 31.64
$$

$$
G_{PD}(s) = K_{dp}(s + K_{pp}) = 7.34 \cdot 10^{-2} \cdot (s + 31.64).
$$

**Analog Circuit Implementation of the PD Position Controller**

The PD position controller is realized using an operational amplifier. Figure 5.3 shows the analog circuit implementation of the PD position controller with a low pass filter. The purpose of the low pass filter is the same as that of the levitation position controller. The transfer function between input and output is derived as,

$$
G \equiv G_{PD}(s) \cdot G_{LPF}(s) = \frac{V_c(s)}{V_i(s)} \cdot \frac{V_o(s)}{V_c(s)} = R_2 C_1 \cdot \left( s + \frac{1}{R_1 C_1} \right) \cdot \frac{R_4}{R_3} \cdot \frac{R_4}{R_2 C_2} \cdot s + 1.
$$

(5.5)

Under the assumption that $G_{PD}(s) = K_{dp}(s + K_{pp})$,

$$
R_2 C_1 = K_{dp}, \quad \frac{1}{R_1 C_1} = K_{pp}.
$$

(5.6)

Since there are three unknowns and two equations, a practical value for one of the elements is arbitrarily selected. With the selection of $C_1$, the remaining values are derived as,

$$
R_1 = \frac{1}{K_{pp} C_1}, \quad R_2 = \frac{K_{dp}}{C_1}.
$$

(5.7)

Using the PD controller gains from the previous design example, if $C_1 = 0.13 \mu\text{F}$, $R_1$ and $R_2$ are determined as $243 \text{ K\Omega}$ and $565 \text{ K\Omega}$, respectively. Note that the adjustment of the circuit
components may be necessary in the experimentation stage. The components of the low pass filter can similarly be found as in section 4.5.3.2.

5.4 Dynamic Simulations and Experimental Results

5.4.1 Control Block Diagram

The experimental control block diagram of the electromagnetic guidance system is shown in Figure 5.4. This is the same configuration as that of the electromagnetic levitation control system except that the air gap position command is generated using two air gap position feedbacks in the front or rear side and there is no feedforward compensation input for rejecting the normal force disturbance generated by the LSRM. The functions and specifications of the other control blocks are the same as that of the levitation control system and hence Section 4.6.1 can be referred for the description of each control block.

5.4.2 Dynamic Simulations

As in the simulation of the electromagnetic levitation system, it is also assumed that the air gap position feedback from the analog gap sensor is corrupted by the noise whose probability
Figure 5.4 Experimental control block diagram of the electromagnetic guidance system.
density function follows the normal (Gaussian) distribution function with zero mean and 0.1 mm standard deviation. The low pass filter with the crossover frequency of 75 Hz is used to filter out the high frequency component of the air gap position feedback.

Figure 5.5 shows the dynamic simulation of the position and force/current control responses in the electromagnetic guidance system. The same dc electromagnet parameters applied to the electromagnetic levitation system are used in the simulation. It is assumed that there are no lateral and vertical load forces caused mainly by wind and tilted guideway owing to cornering and guideway irregularities. The mass of 30 Kg is applied to the electromagnet and it corresponds to a half of the rated mass of the vehicle. The sum of two air gap position feedbacks in the left and the right sides is assumed to be 6 mm and hence the air gap position reference is obtained as a half of the value, that is, 3 mm. The air gap position feedback of the left electromagnet is initially placed at 5 mm position and hence that of the right electromagnet is given as 1 mm. Sequentially shown in the figure are the responses of the lateral air gap position feedback \( y \) and its reference \( y^* \), winding current \( i \) and its reference \( i^* \), actual electromagnetic guidance force \( f \), voltage \( v \) and flux linkage \( \lambda \) of the electromagnet winding, and lateral velocity \( \dot{y} \) of the vehicle. It is noticed that the actual air gap positions experience initial transient state with oscillation and then reaches their final references in steady state. The oscillatory position response is controlled by the PD controller gains. In the electromagnetic guidance system, the winding current flows only if the air gap position feedback is larger than its commanded value, as shown in the current response. The currents of the left and right electromagnet windings flow alternately according to the air gap position error. The winding current is initially increased to maximum limit value with short pulse shape and is decreased to zero when the air gap position feedback reaches the reference value. Both the winding current command and feedback are drawn in the same plot and they are closely matching each other, which demonstrates the current controller performance. The actual winding voltage shown in the figure is a filtered signal using a low pass filter.

5.4.3 Experimental Results

Figure 5.6 shows the experimental waveforms of the electromagnetic guidance system. Verification of the air gap position and current controllers, air gap position feedbacks and

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Figure 5.5 Dynamic simulation of the position and force/current control responses in the electromagnetic guidance system.
Figure 5.6 Experimental waveforms of the position and current control responses in the electromagnetic guidance system.

(a) Verification of the air gap position and current controllers (0.2 s/div).

(b) Air gap position feedbacks of the four dc electromagnets (0.2 s/div).

(c) Winding current feedbacks of the four dc electromagnets (0.2 s/div).

Figure 5.6 Experimental waveforms of the of the position and current control responses in the electromagnetic guidance system.
winding current feedbacks of the four dc electromagnets are sequentially shown in the figure. From Figure 5.5(a), the air gap position command is given at about 4 mm step command. The initial air gap position feedback of the electromagnet is approximately 5.1 mm and it follows the commanded value after the transient. The reason why the air gap position command is larger than 3 mm is due to track irregularity, that is, the implemented width of the track is narrower than the track design specified. It is observed that the actual electromagnet winding current is tracking the current command very closely. The actual winding current flows only if the air gap position feedback is larger than its commanded value, as shown in the Figure 5.5(a). The residual winding current in the steady state is attributable to a slight tilt to the left of the guideway. Figure 5.5(b) shows the air gap position feedbacks of the four dc electromagnets. The initial air gap positions of the dc electromagnets are different from each other, and there is a difference in the guideway width between the front and rear sides of the vehicle owing to the guideway irregularities and machine part assembly errors. It is also noticed that the air gap position feedbacks of the front-left and rear-left electromagnets are slightly larger than those of the front-right and rear-right electromagnets, respectively. This is also caused by the tilted guideway, which results in the residual winding currents of the front-left and rear-left electromagnets in the steady state and this is shown in the Figure 5.5(c). Note that the steady state winding current of the front-right and rear-right electromagnets are almost zero.

Figure 5.7 shows the experimental waveforms of the air gap position and current responses for arbitrary lateral disturbance force (approximately maximum 107 N in this experiment). When the two electromagnets in the front part of the vehicle are in the steady state, an arbitrary lateral disturbance is applied to the electromagnets from the right to the left side and then the opposite disturbance is applied after 2 s. Two air gap position feedbacks show the coupled dynamic characteristic and only one winding current flows for each disturbance.

5.5 Conclusions

The attractive electromagnetic guidance system is described in this chapter. The following results are obtained in this study.
1) For accomplishing the independent control of the lateral vehicle motion, a set of guidance structure containing four dc electromagnets, four gap sensors, and track have been built and tested with corresponding converter and controller.

2) To overcome the guideway irregularities and coupled motion dynamics between the two front magnets or two rear magnets, the air gap position command of the front or rear electromagnets is generated as a half of the sum of two front or rear air gap position feedbacks, respectively. This is implemented and tested using an analog control circuit.

3) For stable operation of the electromagnetic guidance system, the air gap position and force/current control loops are designed using PD and PI controllers, respectively. The controllers are based on the single-input single-output model developed in Chapter 4. Each control loop is implemented and tested using the analog control circuit. The design procedure for the PD position controller is systematically derived.

4) Dynamic simulations and experimental results for the air gap position and force/current control loops are presented using the 6 m long prototype system. Experimental results correlate the simulation results, and it confirms the feasibility of the independent electromagnetic guidance system. Experimental results also show the validity of the air gap position command generation.

Figure 5.7 Experimental waveforms of the air gap position and current responses for arbitrary lateral disturbance force (0.5 s/div).
CHAPTER 6
INTEGRAL LINEAR SWITCHED RELUCTANCE MACHINE
BASED PROPULSION, LEVITATION, AND GUIDANCE SYSTEM

Many electrically propelled, and magnetically levitated and guided actuation systems use either induction or synchronous machine topologies. From the cost, reliability, fault tolerance, and phase independence points of view, linear switched reluctance topologies are attractive for transportation application. This chapter contains a novel topology in which only two sets of linear switched reluctance machines are utilized to control individually propulsion, levitation, and guidance forces. One set of the linear switched reluctance actuator produces the levitation and propulsion forces and the other set generates the propulsion and guidance forces. The proposed architecture, thereby, obviates the need for design, development, and implementation of separate actuation systems for individual control of propulsion, levitation, and guidance forces and in contrast to most of the present practice. Further, the proposed system utilizes each of the linear switched reluctance actuation system for producing the propulsion force, thereby giving an overall high force density package for the entire system. Even though the control complexity increases in the proposed system, the hardware requirement decreases by going from three actuation systems to two actuation systems resulting in huge savings in cost. In this chapter, the proposed system concept will be systematically presented with various arrangements of the proposed concept, their merits and demerits, and performance analysis through simulation. The feasibility of the proposed system by finite element analysis is demonstrated.

6.1 Introduction

LSRMs endow large attractive normal force [3], [10], [19]. The normal force is highly dependent on the translator position and the phase winding current. The magnitude of normal force is the order of five to twenty times the magnitude of propulsion force and very much dependent on the ratio of the air gap length to stator pole width. The normal force of LSRM propulsion system is advantageous if it is used for the levitation and guidance purposes.
Combining levitation and propulsion, as well as, guidance and propulsion allows the LSRM drives to be both electrically and mechanically efficient. Mechanical efficiency derives from the fact that this proposed system would require fewer components on both the vehicle and track.

This chapter presents a possible mechanical implementation of the system and discusses the control of the vehicle. Two commonly used LSRM topologies capable of providing both propulsion and levitation actuation are examined. The same LSRM structure would also be used to combine propulsion and guidance control over the magnetically levitated and propelled (maglev) vehicle. Two stator and rotor pole combinations of 6/4 and 8/6 are examined using their inductance profiles that could be used to support both levitation and propulsion controls. The system is presented with various arrangements of the proposed concept, their merits and demerits. The feasibility of the proposed system through finite element analysis is demonstrated.

6.2 Proposed Maglev System

The proposed maglev system design was based on a longitudinal flux configuration of LSRM. This configuration is based on the radial flux path of the rotating switched reluctance machine (RSRM) of Figure 6.1. The isometric view, Figure 6.2, shows the relative sizes of the stator and translator cores of propulsion/levitation actuator as well as propulsion/guidance actuator. In this application the track is passive, meaning no coils are embedded on the track, and the vehicle is active with phase windings excited sequentially. Notice that this view shows an LSRM equivalent of an 8/6 RSRM. There are two vehicle supports one on each side and only one side is shown here for brevity.

The general mechanical configuration of the maglev system is shown in Figure 6.3. The propulsion/levitation actuators are placed at the four corners of the moving surface hereafter referred to as vehicle. This maximizes the pitch and roll control over the vehicle. The propulsion/guidance actuators are also placed at the four corners of the length edges to maximize yaw control over the vehicle. It should be understood that these actuators provide propulsion to the vehicle simultaneously as they respectively levitate and guide. The mechanical implementation shown in Figure 6.3 involves the track enclosing the moving surface of the maglev vehicle. The vehicle moves inside the slot of the track. This configuration allows for future development in changing lanes within a given maglev transit system. Mechanical
Figure 6.1 Four-phase rotating switched reluctance machine with 8/6 pole combination and wider rotor pole arc for generating larger aligned inductance region.

Figure 6.2 Isometric view of the proposed integral linear switched reluctance machine based propulsion, levitation, and guidance system.
Figure 6.3 Mechanical configuration of the proposed LSRM-based maglev system (front view).

Figure 6.4 Four-phase linear switched reluctance machine with wider stator pole width for generating larger aligned inductance region (side view).
configurations that enclose the track do not allow the vehicle to change tracks. Shown in Figure 6.4 is an 8/6 LSRM with longitudinal magnetic flux path topology and its phase winding configuration. Here $a_{a′}, b_{b′}, c_{c′},$ and $d_{d′}$ constitute phase windings A, B, C, and D, respectively. They are electrically independent. Unlike the ordinary LSRM design, two phases are always excited with the rated phase winding current. One phase is excited for the generation of propulsion force and the other phase is excited for the generation of levitation force. For this reason, twice the magnetic flux of the ordinary LSRM flows through the back iron steel core and hence the width of the back iron steel should be increased as twice as that of ordinary LSRM.

6.3 Design Considerations of Integral LSRM-Based Maglev System

Design is made using rotating machine considerations, since the design of rotating machines is well known [56]. These design considerations are extended to linear machines through proper transformation [19], as described in Chapter 2. This transformation can be seen as cutting longitudinally through both the stator and translator cores and rolling the respective motor flat. It should be seen that the flux paths of an LSRM are very similar to its RSRM counterpart.

Popular 6/4 and 8/6 RSRM pole configurations are investigated for the implementation of the proposed maglev system. The first design issue for consideration is the feasible pole arc triangles associated with 6/4 and 8/6 pole machines [7], [37], [41]. These triangle boundaries are generated from the following conditions. Stator pole arc, $\beta_s$, should be less than rotor pole arc, $\beta_r$, to ensure that there exists ample space for accommodating the phase windings. $\beta_s$ must be greater than the minimum conduction angle for a phase winding excitation. The minimum conduction angle is equal to $2\pi$ divided by the multiplication of the number of rotor poles and the number of phases. This is required to ensure that the RSRM can start from any position with enough starting torque. The summation of pole arcs $\beta_s$ and $\beta_r$ should be less than one excitation cycle, which is given by $2\pi$ divided by the number of rotor poles. This is to guarantee that there exist unaligned position regions for phase advancing at high speed operation range. The above conditions for describing the feasible pole arc zones can be summarized using the following equations.

\[ \beta_s < \beta_r \]

(6.1)
\[ \beta_s > \frac{2\pi}{N_r \cdot N_s/2} = \frac{4\pi}{N_r \cdot N_s} \]  \hspace{1cm} (6.2) \\
\[ \beta_s + \beta_r < \frac{2\pi}{N_r} \]  \hspace{1cm} (6.3)

where \( \beta_s \) is the stator pole arc, \( \beta_r \) is the rotor pole arc, \( N_r \) is the number of rotor poles, and \( N_s \) is the number of stator poles, respectively. Figure 6.5 shows usable boundaries for these machines. Triangle abc is the usable boundary characterized by the preceding three constraints for 6/4 and 8/6 machines. Pole arcs of both the stator and rotor are varied and their inductance profiles are generated. Desirable profiles would exhibit wide and flat peak inductance regions for adjacent phases. The fully aligned flat inductance regions would be used to exert levitation or guidance control. To obtain small changes in the phase winding current carrying out the levitation control, the peak flat inductance regions need to be overlapped so that abrupt levitation force changes are not incurred during the switch of the phase excitation. This phase switch for the levitation control corresponds to transferring the generation of levitation force to the next sequential phase. The widely flat aligned inductance region places a special requirement for the presented LSRM application in maglev system. The difference between \( \beta_r \) and \( \beta_s \) must be larger than the minimum conduction angle for the phase winding excitation to ensure that a wide and flat peak inductance region is obtained. This is reflected in Figure 6.5(b) as triangle bde in the SRM with 8/6 pole combination and this requirement is derived as,

\[ \beta_r - \beta_s > \frac{2\pi}{N_r \cdot N_s/2} = \frac{4\pi}{N_r \cdot N_s}. \]  \hspace{1cm} (6.4)

This requirement can also be met in a 6/4 pole machine. However, this causes the machine to lack the flat unaligned region. This is graphically described by the small circle around the point b in Figure 6.5(a). In order to implement using a 6/4 pole machine, a minimum of two sets of actuators are required to ensure fully unaligned flat regions for phase current advancing and desirable phase overlap.

Ideal phase inductance profiles neglecting the fringing effect and magnetic core saturation are generated for both machine geometries. Ideal profiles for a 6/4 pole machine are shown in Figure 6.6. Note that a desirable overlapping peak inductance region for utilizing the normal force as levitation purpose can be obtained for \( \beta_s = 30^\circ \) and \( \beta_r = 60^\circ \). However, when these pole arcs are applied to a 6/4 pole machine, the machine exhibits negligible flat inductance region in
Figure 6.5 Feasible pole arc zones of SRM for utilizing the normal force as levitation and guidance forces.

(a) 6/4 pole combination.

(b) 8/6 pole combination.

Figure 6.5 Feasible pole arc zones of SRM for utilizing the normal force as levitation and guidance forces.
Figure 6.6 Ideal stator phase winding inductance profile vs. rotor position angle of 6/4 RSRM for various pole arc combinations.
the fully unaligned position. Phase current advancing becomes impossible with this pole arc set. This property of a 6/4 pole machine makes it a poor candidate for use in the proposed maglev application. The ideal inductance profiles for an 8/6 pole combination are shown in Figure 6.7. The 8/6 machine exhibits desirable properties of a wide and flat fully aligned inductance region for levitation and guidance force generation. The phases of the 8/6 machine have also widely flat region in the fully unaligned regions. This ensures that the phases will be excited in advance at high speed operation region. A desirable overlapping peak inductance region can be obtained for either $\beta_s = 18^\circ$ and $\beta_r = 35^\circ$ or $\beta_s = 15^\circ$ and $\beta_r = 35^\circ$ from the Figure 6.7. The rotary and linear machine representations corresponding to $18^\circ/35^\circ$ pole arc combination are shown in Figure 6.1 and Figure 6.4, respectively.

An 8/6 pole LSRM with 40 watt power is designed by using the design procedure described in Chapter 2 and [19]. Finite element analysis is used to obtain the phase winding inductance, propulsion and normal force profiles including nonlinear saturation effect. The specifications of the machine design are given in Appendix C.4. The LSRM translator is moved from an unaligned position with respect to the stator to an aligned position for rated excitation currents and values of inductance, propulsion force, and normal force are obtained. Figures 6.8(a) and (b) show the flux distributions for rated phase current in the aligned position and unaligned position, respectively. Finite element analysis is used to determine the phase inductances and electromagnetic force components of two LSRM machines of $18^\circ/35^\circ$ and $15^\circ/35^\circ$ pole arc combinations. The pole arc combinations corresponding to RSRM machine parameters are translated to LSRM parameters using methods described in [19]. These inductance plots, shown in Figures 6.9 and 6.10, are used to determine propulsion and levitation forces, respectively. The force plots can be examined to determine levitation and propulsion phase algorithms for control. The finite element analysis results clearly show that the normal force overlap is good enough for smooth levitation and guidance control. These results also show that propulsion can be carried out by phases not associated with levitation or guidance control. The machine is capable of accomplishing both tasks simultaneously.

An example of a phase switching algorithm could be taken from the 13 to 23 mm translation region in Figure 6.9. In this region, phase B is generating the propulsion force. During phase B’s propulsion cycle, the levitation force is generated by phases D and A. Note that phases D and A can share the generation of the levitation force in this example. Phase A can also provide enough
Figure 6.7 Ideal stator phase winding inductance profile vs. rotor position angle of 8/6 RSRM for various pole arc combinations.

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Figure 6.7 Ideal stator phase winding inductance profile vs. rotor position angle of 8/6 RSRM for various pole arc combinations.
Figure 6.8 Flux distribution of LSRM with 18°/35° pole combination for rated phase current.

(a) Fully aligned position.

(b) Fully unaligned position.
Figure 6.9 Finite element analysis results of 8/6 LSRM corresponding to 18°/35° pole arc combination of 8/6 RSRM.
Figure 6.10 Finite element analysis results of 8/6 LSRM corresponding to 15°/35° pole arc combination of 8/6 RSRM.
levitation force for the entire span of phase B’s propulsive effort, if the required normal force is lower than that of the previous illustration of 325 N. This result demonstrates that control can be simplified if machine design is matched to vehicle dynamics and weight. The finite element analysis results imply that there should be smooth transition between a phase’s propulsion operation into the same phase’s normal force operation. This is true for all phases due to symmetry of the machine design. The propulsion phase of the machine is always producing both a propulsion and a normal force component. This normal force component is added to the levitation force generated by the phase winding in levitation mode. In this manner, integral LSRM-based maglev system can provide enough levitation force with reduced levitation phase current. Figure 6.11 shows the variation of the levitation phase winding current for generating the required lift force. As in previous example, phase A is generating the levitation force of 300 N and phase B is generating the propulsion force of 20 N. Notice that the normal force produced
by the propulsion phase reduces the current demand in the levitation phase. This reduces the average power and loss in the respective phases.

6.4 Merits and Demerits of the Proposed Maglev System

Integral LSRM-based maglev system has the following advantages:

1) Only two sets of LSRMs are required for the implementation of propulsion, levitation, and guidance system.
2) All LSRMs are used for acceleration and deceleration.
3) High utilization of normal forces for the levitation and guidance purposes is possible. This gives an overall high force density package for the entire system while meeting the requirements of propulsion, levitation, and guidance forces.
4) The mechanical hardware requirement decreases by going from three actuation systems to two actuation systems resulting in considerable savings in cost.
5) Simpler construction is achieved with the help of simple mechanical geometry of LSRM.

The disadvantages are as follows:

1) Control complexity increases.
2) Coordinated control of propulsion and levitation forces, as well as, propulsion and guidance forces is quite new and requires development for the proper operation of this proposed maglev system.
3) More power electronics subsystems are required to facilitate increased individual phase excitations. This cost is minimal and is offset by overwhelming mechanical cost savings.

6.5 Conclusions

This chapter describes the integral LSRM-based propulsion, levitation, and guidance system. The following contributions are made in this study.
1) A novel approach is proposed to an LSRM-based maglev system using only two sets of actuation systems for propulsion/levitation and propulsion/guidance control. This approach reduces mechanical complexity, thereby, greatly reducing the cost of the entire maglev system.

2) This approach was founded on RSRM machine designs and verified with finite element analysis results using the designed LSRM. The analysis proves that the proposed control technique can be implemented in practice.

3) Propulsion phases provide normal force components aiding levitation/guidance phases in this proposed maglev system. This reduces average power requirements to corresponding levitation/guidance phases.

4) The proposed system offers high utilization of inherent normal forces for levitation and guidance.

5) Higher reliability is made possible in the proposed system with multiple sets of lower power LSRMs.
CHAPTER 7
CONCLUSIONS

7.1 Conclusions

Linear switched reluctance machine drives with electromagnetic levitation and guidance systems were investigated. Low and high speed transits are the potential applications and the same integrated system can also be applied for the general purpose industrial linear motion applications. Designs of the linear switched reluctance machine and dc electromagnet, analytical aspects of modeling and dynamics of the vehicle, and closed loop control have been discussed with comprehensive simulations and experimental results. The major contributions of this study are summarized as follows:

1) A novel design procedure for linear switched reluctance machine has been proposed using the current knowledge and design procedure of rotating switched reluctance machine.

2) Analytical predictions of fully aligned, two intermediate, and fully unaligned inductances using lumped-parameter magnetic equivalent circuit analysis have been developed and verified with finite element analysis.

3) A 4.8 m long, three-phase longitudinal LSRM prototype with active stator and passive translator configuration has been built based on the developed design procedure with 120 stator poles spread along its length and tested with a converter and controller.

4) Experimental correlation of inductances, propulsion and normal forces has confirmed the validity of the proposed design procedure. Even though the modeling of end effects is considered in the analytical calculation procedure, small discrepancies in the unaligned inductance prediction are presented. This may need an accurate three-dimensional finite element analysis.
5) A design procedure for dc electromagnet has been presented using analytic relations between the design variables, and the two-dimensional finite element analysis has been used to fine-tune the final design. The dc electromagnet prototype and track have been built based on the final dimensions, and tested with a converter and controller. To provide a linear propulsion force for the vehicle, a 6 m long, four-phase longitudinal LSRM prototype with active translator and passive stator configuration has also been built and tested.

6) For the stable operation of the electromagnetic levitation system, the air gap position and force/current control loops have been designed using PID and PI controllers, respectively. Modeling of each control loop has been derived using single-input single-output system under the assumption that two sets of four dc electromagnets were used for the independent control of levitation and guidance systems, respectively. The design procedures for the PID position and PI force/current controllers were systematically derived. Each controller was implemented and tested using the analog control circuitry.

7) A feedforward compensation strategy for the levitation air gap control has been proposed to reject the external force disturbance mainly caused by the normal force component generated in the LSRM propulsion drive system. The feedforward control command is calculated in the LSRM propulsion controller and is used in the levitation controller. The enhancement of ride quality and system efficiency for the LSRM-propelled electromagnetic levitation system is achieved.

8) Extensive dynamic simulations and experimental results for the levitation air gap position and force/current control loops have been presented with the prototype system. Experimental correlation has proved the validity of the controller design procedure based on the single-input single-output model, and showed the feasibility of the LSRM-propelled electromagnetic levitation system.

9) For accomplishing the independent control of the lateral vehicle motion, a set of guidance structure containing four dc electromagnets, four gap sensors, and track has been built and tested with corresponding converter and controller.
10) To overcome the guideway irregularities and coupled motion dynamics between the two front magnets or two rear magnets, the air gap position command of the front or rear electromagnets was generated as a half of the sum of two front or rear air gap position feedbacks, respectively. This has been implemented and tested using analog control circuitry.

11) For the stable operation of the electromagnetic guidance system, the air gap position and force/current control loops have been designed using PD and PI controllers, respectively, based on the single-input single-output model. Each control loop was implemented and tested using the analog control circuitry. The design procedure for the PD position controller was systematically derived.

12) Dynamic simulations and experimental results for the guidance air gap position and force/current control loops are presented using the 6 m long prototype system. Experimental results correlate the simulation results, and it confirms the feasibility of the independent electromagnetic guidance system. Experimental results also show the validity of the air gap position command generation.

13) A novel approach has been proposed to an LSRM-based maglev system using only two sets of actuation systems for propulsion and levitation/guidance control. This approach reduces mechanical complexity, thereby, greatly reducing the cost of the entire maglev system. This approach was founded on RSRM machine designs and verified with linear machine finite element analysis results. The analysis proves that the proposed control technique can be implemented in practice. Propulsion phases provide normal force components aiding levitation/guidance phases in this proposed maglev system. This reduces average power requirements to corresponding levitation/guidance phases. The proposed system offers high utilization of inherent normal forces for levitation and guidance. Higher reliability is made possible in the proposed system with multiple sets of lower power LSRMs.
7.2 Recommendations for Future Work

The following avenues of research are recommended for possible investigation in future studies:

1) Three-dimensional finite element analysis considering the end effects.

2) Analysis and design of a multivariable controller for an LSRM-propelled, and magnetically levitated and guided actuation systems.

3) Experimental investigation of a novel topology in which only two sets of linear switched reluctance machines are utilized to control individually propulsion, levitation, and guidance forces.
APPENDIX A:
CALCULATION OF AIR GAP PERMEANCE

The method for analytical estimation of the permeance of prefixed flux paths is originally developed in [12] for use in the electromagnetic devices and is introduced in Appendix A. In general, the lumped parameter magnetic circuit analysis requires reluctances and they are computed from the permeances for various flux paths. The accuracy of the magnetic equivalent circuits is dependent on the reluctances and hence on the permeances. However, three-dimensional flux distribution makes the task of the designer difficult to estimate the permeance accurately. For this reason, the analytical air gap permeance calculations use predetermined magnetic flux paths in between the poles, consisting of straight lines and concentric circular arc segments.

The stator and translator poles at a given position determine the air gap geometry. The poles are rectangular in shape and separated from each other by an air gap, as shown in Figure A.1. The total flux linkages can be estimated using five basic forms of flux paths, namely, parallelepiped, semicircular cylinder, half annulus, spherical quadrant, and spherical shell quadrant, and they are shown in Figure A.2.
Figure A.2 Magnetic flux paths and their plane surfaces of the five basic forms.
A.1 Parallelepiped

This is the most basic geometry of the magnetic flux paths and the flux lines are perpendicular to the material or plane surface. Calculation of the cross-section area and flux path length is simple and the permeance is given by,

$$
P_1 = \mu_0 \cdot \frac{\text{Mean Cross-Section Area}}{\text{Mean Flux Path Length}} = \frac{\mu_0 \cdot hd}{g} \quad \text{(A.1)}$$

where \(\mu_0\) is the permeability of free space, \(h\) is the height of the flux path, \(d\) is the depth of the flux path, and \(g\) is the air gap length.

A.2 Semicircular Cylinder

The mean length of the flux path is equal to the length of a line drawn midway between the diameter and the semi-circumference, and this is equal to \(1.211 \cdot g\). The mean cross-section area of the flux path is estimated by dividing the volume of the flux path by its mean length. The permeance is then derived as,

$$
P_2 = \mu_0 \cdot \frac{\text{Mean Cross-Section Area}}{\text{Mean Flux Path Length}} = \mu_0 \cdot \frac{\pi gd^2}{8 \cdot 1.211g} = 0.268 \cdot \mu_0d. \quad \text{(A.2)}$$

A.3 Half Annulus

There are two different formulas for calculating the permeance of half annulus determined by the ratio between the air gap length and annulus thickness. When \(g \geq 3t\) where \(t\) is the width of the fringing flux path, the permeance is given by,

$$
P_3 = \mu_0 \cdot \frac{\text{Mean Cross-Section Area}}{\text{Mean Flux Path Length}} = \mu_0 \cdot \frac{td}{\pi \left(\frac{g + t}{2}\right)} = 0.637 \cdot \frac{\mu_0td}{g + t}. \quad \text{(A.3)}$$

When \(g < 3t\), the permeance is given by,

$$
P_3 = \int_{g/2}^{(g+t)/2} \frac{\mu_0d}{\pi r} dr = \frac{\mu_0d}{\pi} \ln \left(1 + \frac{2t}{g}\right) = 0.318 \cdot \mu_0d \cdot \ln \left(1 + \frac{2t}{g}\right). \quad \text{(A.4)}$$
A.4 Spherical Quadrant

The mean length of the flux path is equal to the length of a line drawn 0.65 times the distance between the center of the sphere and the circumference, and this is equal to 1.311\( \cdot \)g. The mean cross-section area of the flux path is also estimated by dividing the volume of the flux path by its mean length. Therefore, the permeance is given by,

\[
P_4 = \mu \cdot \frac{\text{Mean Cross-Section Area}}{\text{Mean Flux Path Length}} = \mu_0 \cdot \frac{\frac{\pi g^3}{24} \cdot \frac{1}{1.31g}}{1.31g} = 0.076 \cdot \mu_0 g.
\]  \hspace{1cm} (A.5)

A.5 Spherical Shell Quadrant

If the mean cross-section area is assumed to be half of the maximum cross-section area of the magnetic flux path, the permeance is given by,

\[
P_5 = \mu \cdot \frac{\text{Mean Cross-Section Area}}{\text{Mean Flux Path Length}} = \mu_0 \cdot \frac{\frac{\pi t(g + t)}{8}}{\frac{\pi (g + t)}{2}} = 0.25 \cdot \mu_0 t.
\]  \hspace{1cm} (A.6)

A.6 Total Permeance Value of the Air Gap Flux Paths

By using (A.1) to (A.6), the total permeance of the air gap flux paths shown in Figure A.1 can be calculated as,

\[
P_{\text{Total}} = \mu_0 \cdot \frac{hd}{g} + 0.536 \cdot \mu_0 (h + d) + 0.636 \cdot \mu_0 (h + d) \cdot \ln\left(1 + \frac{2t}{g}\right) + 0.304 \cdot \mu_0 g + \mu_0 t. \quad (A.7)
\]
Detailed equations for the calculation of permeance in the poles and in the back iron of the stator and translator are derived in Appendix B. Figure B.1 shows the typical magnetic flux paths of the LSRM for 4 different translator position regions. With reference to Figure 2.9 and Figure B.1, the equations for the mmf, mean length of the flux path and its cross-section area are derived for all the four regions. Half-machine symmetry is assumed.

Figure B.1 Typical magnetic flux paths of the LSRM for 4 different translator position regions.

(a) Aligned position region. (b) Intermediate position region I. (c) Intermediate position region II. (d) Unaligned position region.
B.1 Aligned Position Region

B.1.1 MMF Per Path

\[ F_{i,2,4,6} = \frac{T_{ph}}{2} \cdot i \]  
\[ F_{3,5,7} = \frac{(h_s - t/2)}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \]  

where the subscript denotes the chosen flux path.

B.1.2 Mean Path Length and Mean Cross-Section Area

**Stator pole:**
\[ l_{sp1,2,4,6} = h_s + C_{sy}/2 \]  
\[ l_{sp3,5,7} = h_s - t/2 + C_{sy}/2 \]  
\[ A_{sp1-7} = L_w w_{sp} \]  

**Stator yoke:**
\[ l_{sy1-7} = 1.5 \cdot (w_{sp} + w_{ss}) \]  
\[ A_{sy1-7} = L_w C_{sy} \]  

**Translator pole:**
\[ l_{tp1,2,4,6} = h_r + C_{ry}/2 \]  
\[ l_{tp3,5,7} = h_r - t/2 + C_{ry}/2 \]  
\[ A_{tp1-7} = L_w w_{tp} \]  

**Translator yoke:**
\[ l_{ty1-7} = w_{tp} + w_{ts} \]  
\[ A_{ty1-7} = L_w C_{ry} \]  

B.2 Intermediate Position Region I

B.2.1 MMF Per Path

\[ F_{i,2,3,7} = \frac{T_{ph}}{2} \cdot i \]  
\[ F_4 = \frac{(h_s - t_2/2)}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \]
\[
F_5 = \frac{(h_s - d_2)}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \quad \text{(B.15)}
\]
\[
F_6 = \frac{(h_s - t_2)}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \quad \text{(B.16)}
\]
\[
F_8 = \frac{(h_s - (t_1 + t_2)/4)}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \quad \text{(B.17)}
\]
\[
F_9 = \frac{(h_s - d_2/2)}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \quad \text{(B.18)}
\]

B.2.2 Mean Path Length and Mean Cross-Section Area

**Stator pole:**
\[
l_{sp1,2,3,7} = h_s + C_{sy}/2 \quad \text{(B.19)}
\]
\[
l_{sp4,10} = h_s - t_2/2 + C_{sy}/2 \quad \text{(B.20)}
\]
\[
l_{sp5} = 2 \cdot ((h_s - d_2)/2 + C_{sy}/2) \quad \text{(B.21)}
\]
\[
l_{sp6} = 2 \cdot ((h_s - t_2)/2 + C_{sy}/2) \quad \text{(B.22)}
\]
\[
l_{sp8} = h_s - (t_1 + t_2)/4 + C_{sy}/2 \quad \text{(B.23)}
\]
\[
l_{sp9} = h_s - d_2 + C_{sy}/2 \quad \text{(B.24)}
\]
\[
A_{sp1-10} = L_w w_{sp} \quad \text{(B.25)}
\]

**Stator yoke:**
\[
l_{sy1-4,7-10} = 1.5 \cdot (w_{sp} + w_{ss}) \quad \text{(B.26)}
\]
\[
l_{sy5,6} = w_{sp} + w_{ss} \quad \text{(B.27)}
\]
\[
A_{sy1-10} = L_w C_{sy} \quad \text{(B.28)}
\]

**Translator pole:**
\[
l_{tp1,2,4,7} = h_r + C_{ry}/2 \quad \text{(B.29)}
\]
\[
l_{tp3} = h_r - t_1/2 + C_{ry}/2 \quad \text{(B.30)}
\]
\[
l_{tp8} = h_r - (t_1 + t_2)/4 + C_{ry}/2 \quad \text{(B.31)}
\]
\[
l_{tp9} = h_r - t_1/2 + C_{ry}/2 \quad \text{(B.32)}
\]
\[
l_{tp10} = h_r - d_3/2 + C_{ry}/2 \quad \text{(B.33)}
\]
\[
A_{tp1-4,7-10} = L_w w_{tp} \quad \text{(B.34)}
\]

**Translator yoke:**
\[
l_{ty1-4,7-10} = w_{tp} + w_{ts} \quad \text{(B.35)}
\]
\[
A_{ty1-4,7-10} = L_w C_{ry} \quad \text{(B.36)}
\]

**Pole tip:**
\[
l_{pt1,2,7} = t_1/2 \quad \text{(B.37)}
\]
\[
A_{pt1,2,7} = L_w d_1 \quad \text{(B.38)}
\]
B.3 Intermediate Position Region II

B.3.1 MMF Per Path

\[ F_{1,2,3,7,8} = \frac{T_{ph}}{2} \cdot i \]  
\[ F_{4,11} = \left(\frac{h_s - t_4}{2}\right) \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_5 = \frac{h_s - d_5}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_6 = \frac{h_s - t_4}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_9 = \frac{h_s - (t_3 + t_4)/4}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_{10} = \frac{h_s - d_5/2}{h_s} \cdot \frac{T_{ph}}{2} \cdot i \]

B.3.2 Mean Path Length and Mean Cross-Section Area

**Stator pole:**

\[ l_{sp1,2,3,7,8} = h_s + C_{sy}/2 \]  
\[ l_{sp4,11} = h_s - t_4/2 + C_{sy}/2 \]  
\[ l_{sp5} = 2 \cdot ((h_s - d_5)/2 + C_{sy}/2) \]  
\[ l_{sp6} = 2 \cdot ((h_s - t_4)/2 + C_{sy}/2) \]  
\[ l_{sp9} = h_s - (t_3 + t_4)/4 + C_{sy}/2 \]  
\[ l_{sp10} = h_s - d_5 + C_{sy}/2 \]  
\[ A_{sp1-11} = L_{w} w_{sp} \]

**Stator yoke:**

\[ l_{sy5,6} = w_{sp} + w_{ss} \]  
\[ A_{sy1-11} = L_w C_{sy} \]

**Translator pole:**

\[ l_{tp1,2,4,8} = h_r + C_{ry}/2 \]  
\[ l_{tp3} = h_r - t_3/2 + C_{ry}/2 \]  
\[ l_{tp7} = C_{ry}/2 \]  
\[ l_{tp9} = h_r - (t_3 + t_4)/4 + C_{ry}/2 \]
\[ I_{tp10} = h_r - t_3/2 + C_{ry}/2 \]  
\[ I_{tp11} = h_r - d_6/2 + C_{ry}/2 \]  
\[ A_{tp1-4,7-11} = L_w w_{tp} \]  

**Translator yoke:**  
\[ I_{ty1-4,8-11} = w_{tp} + w_{ts} \]  
\[ I_{ty7} = 1.5 \cdot (w_{tp} + w_{ts}) \]  
\[ A_{ty1-4,7-11} = L_w C_{ry} \]  

**Pole tip:**  
\[ I_{pt1,2,8} = d_4 \]  
\[ A_{pt1,2,8} = L_w d_5 \]  

**B.4 Unaligned Position Region**

**B.4.1 MMF Per Path**

\[ F_1,7,8 = \left(\frac{h_s - t_5/2}{h_s}\right) \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_2 = \left(\frac{h_s - t_5}{h_s}\right) \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_3 = \left(\frac{h_s - t_5 - d_8/2}{h_s}\right) \cdot \frac{T_{ph}}{2} \cdot i \]  
\[ F_{4,6,9,10} = \frac{T_{ph}}{2} \cdot i \]  
\[ F_5 = \left(\frac{h_s - d_7}{h_s}\right) \cdot \frac{T_{ph}}{2} \cdot i \]  

**B.4.2 Mean Path Length and Mean Cross-Section Area**

**Stator pole:**  
\[ I_{sp1,7,8} = h_s - t_5/2 + C_{sy}/2 \]  
\[ I_{sp2} = h_s - t_5 + C_{sy}/2 \]  
\[ I_{sp3} = h_s - t_5 - d_8/2 + C_{sy}/2 \]  
\[ I_{sp4,6,9} = h_s + C_{sy}/2 \]  
\[ I_{sp5} = 2 \cdot ((h_s - t_5 - d_8)/2 + C_{sy}/2) \]  
\[ I_{sp10} = h_s - d_9/2 + C_{sy}/2 \]
\[ A_{sp1-10} = L_w w_{sp} \]  \hspace{1cm} (B.78)

**Stator yoke:**
\[ l_{sy1-4,6-10} = 1.5 \cdot (w_{sp} + w_{ss}) \]  \hspace{1cm} (B.79)
\[ l_{sy5} = w_{sp} + w_{ss} \]  \hspace{1cm} (B.80)
\[ A_{sy1-11} = L_w C_{sy} \]  \hspace{1cm} (B.81)

**Translator pole:**
\[ l_{tp1,7,8} = h_r - t_5/2 + C_{ry}/2 \]  \hspace{1cm} (B.82)
\[ l_{tp2,3} = h_r + C_{ry}/2 \]  \hspace{1cm} (B.83)
\[ l_{tp4} = h_r - t_5 - d_9/2 + C_{ry}/2 \]  \hspace{1cm} (B.84)
\[ l_{tp6,9} = C_{ry}/2 \]  \hspace{1cm} (B.85)
\[ l_{tp10} = C_{ry}/2 - d_9/2 \]  \hspace{1cm} (B.86)
\[ A_{tp1-4,6-10} = L_w w_{tp} \]  \hspace{1cm} (B.87)

**Translator yoke:**
\[ l_{ty1-4,7,8} = 1.5 \cdot (w_{tp} + w_{ts}) \]  \hspace{1cm} (B.88)
\[ l_{ty6,9,10} = w_{tp} + w_{ts} \]  \hspace{1cm} (B.89)
\[ A_{ty1-4,6-10} = L_w C_{ry} \]  \hspace{1cm} (B.90)
### APPENDIX C: MACHINE PARAMETERS

#### C.1 Parameters of the 6/4 LSRM in Chapter 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power capacity</td>
<td>67.5 W</td>
</tr>
<tr>
<td>Rated linear velocity</td>
<td>1.5 m/s</td>
</tr>
<tr>
<td>Rated propulsion force</td>
<td>45 N</td>
</tr>
<tr>
<td>Rated normal force at aligned position</td>
<td>944.6 N</td>
</tr>
<tr>
<td>DC link input voltage</td>
<td>160 V</td>
</tr>
<tr>
<td>Rated phase winding current</td>
<td>8.5 A</td>
</tr>
<tr>
<td>Number of turns per phase</td>
<td>AWG #17, 210 Turns/phase</td>
</tr>
<tr>
<td>Phase winding current density</td>
<td>4.73 A/mm²</td>
</tr>
<tr>
<td>Aligned phase winding inductance</td>
<td>31.96 mH</td>
</tr>
<tr>
<td>Unaligned phase winding inductance</td>
<td>6.38 mH</td>
</tr>
<tr>
<td>Phase winding resistance</td>
<td>0.8 Ω/phase</td>
</tr>
<tr>
<td>Stator pole arc of the equivalent RSRM</td>
<td>30°</td>
</tr>
<tr>
<td>Rotor pole arc of the equivalent RSRM</td>
<td>36°</td>
</tr>
<tr>
<td>Air gap length</td>
<td>1 mm</td>
</tr>
<tr>
<td>Translator pole width</td>
<td>24 mm</td>
</tr>
<tr>
<td>Translator slot width</td>
<td>36 mm</td>
</tr>
<tr>
<td>Translator pole height</td>
<td>15 mm</td>
</tr>
<tr>
<td>Translator yoke width</td>
<td>24 mm</td>
</tr>
<tr>
<td>Translator core stack width</td>
<td>56 mm</td>
</tr>
<tr>
<td>Stator pole width</td>
<td>20 mm</td>
</tr>
<tr>
<td>Stator slot width</td>
<td>20 mm</td>
</tr>
<tr>
<td>Stator pole height</td>
<td>37 mm</td>
</tr>
<tr>
<td>Stator yoke width</td>
<td>20 mm</td>
</tr>
<tr>
<td>Stator core stack width</td>
<td>50 mm</td>
</tr>
</tbody>
</table>
### C.2 Parameters of the 8/6 LSRM in Chapter 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power capacity</td>
<td>40 W</td>
</tr>
<tr>
<td>Rated linear velocity</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Rated propulsion force</td>
<td>40 N</td>
</tr>
<tr>
<td>Rated normal force at aligned position</td>
<td>266.3 N</td>
</tr>
<tr>
<td>DC link input voltage</td>
<td>160 V</td>
</tr>
<tr>
<td>Rated phase winding current</td>
<td>8.5 A</td>
</tr>
<tr>
<td>Number of turns per phase</td>
<td>AWG #19, 360 Turns/phase</td>
</tr>
<tr>
<td>Phase winding current density</td>
<td>6.51 A/mm²</td>
</tr>
<tr>
<td>Aligned phase winding inductance</td>
<td>35.3 mH</td>
</tr>
<tr>
<td>Unaligned phase winding inductance</td>
<td>20.4 mH</td>
</tr>
<tr>
<td>Phase winding inductance</td>
<td>2.2 Ω/phase</td>
</tr>
<tr>
<td>Stator pole arc of the equivalent RSRM</td>
<td>18°</td>
</tr>
<tr>
<td>Rotor pole arc of the equivalent RSRM</td>
<td>22°</td>
</tr>
<tr>
<td>Air gap length</td>
<td>3 mm</td>
</tr>
<tr>
<td>Translator pole width</td>
<td>12 mm</td>
</tr>
<tr>
<td>Translator slot width</td>
<td>18 mm</td>
</tr>
<tr>
<td>Translator pole height</td>
<td>44 mm</td>
</tr>
<tr>
<td>Translator yoke width</td>
<td>12 mm</td>
</tr>
<tr>
<td>Translator core stack width</td>
<td>60 mm</td>
</tr>
<tr>
<td>Stator pole width</td>
<td>15 mm</td>
</tr>
<tr>
<td>Stator slot width</td>
<td>25 mm</td>
</tr>
<tr>
<td>Stator pole height</td>
<td>15 mm</td>
</tr>
<tr>
<td>Stator yoke width</td>
<td>15 mm</td>
</tr>
<tr>
<td>Stator core stack width</td>
<td>68 mm</td>
</tr>
</tbody>
</table>
### C.3 Parameters of the DC Electromagnet

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal air gap</td>
<td>3 mm</td>
</tr>
<tr>
<td>Maximum air gap</td>
<td>6 mm</td>
</tr>
<tr>
<td>Nominal air gap flux density</td>
<td>0.528 T</td>
</tr>
<tr>
<td>Maximum load per electromagnet</td>
<td>15 Kg</td>
</tr>
<tr>
<td>Viscous damping coefficient</td>
<td>≈ 0 N/(m/s)</td>
</tr>
<tr>
<td>Rated electromagnet winding current</td>
<td>3.5 A</td>
</tr>
<tr>
<td>DC link input voltage</td>
<td>160 V</td>
</tr>
<tr>
<td>Attraction force at 3 mm, 3.5 A</td>
<td>177.5 N</td>
</tr>
<tr>
<td>Attraction force at 6 mm, 7 A</td>
<td>189.4 N</td>
</tr>
<tr>
<td>Number of turns per electromagnet</td>
<td>AWG #19, 720 Turns/magnet</td>
</tr>
<tr>
<td>Electromagnet winding current density</td>
<td>5.36 A/mm²</td>
</tr>
<tr>
<td>Electromagnet winding resistance</td>
<td>3.25 Ω/electromagnet</td>
</tr>
<tr>
<td>Rated copper loss</td>
<td>39.8 W</td>
</tr>
<tr>
<td>Winding inductance at 0.2 mm, 3.5 A</td>
<td>245.2 mH</td>
</tr>
<tr>
<td>Winding inductance at 3.0 mm, 3.5 A</td>
<td>108.0 mH</td>
</tr>
<tr>
<td>Winding inductance at 6.0 mm, 3.5 A</td>
<td>62.80 mH</td>
</tr>
<tr>
<td>Magnet pole width (Longitudinal width)</td>
<td>30 mm</td>
</tr>
<tr>
<td>Magnet pole depth (Core stack width)</td>
<td>25 mm</td>
</tr>
<tr>
<td>Magnet pole height</td>
<td>35 mm</td>
</tr>
<tr>
<td>Winding window width</td>
<td>40 mm</td>
</tr>
<tr>
<td>Magnet yoke thickness</td>
<td>25 mm</td>
</tr>
<tr>
<td>Track yoke depth</td>
<td>33 mm</td>
</tr>
<tr>
<td>Track yoke thickness</td>
<td>25 mm</td>
</tr>
</tbody>
</table>
C.4 Parameters of the 8/6 LSRM in Chapter 6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power capacity</td>
<td>40 W</td>
</tr>
<tr>
<td>Rated linear velocity</td>
<td>1 m/s</td>
</tr>
<tr>
<td>Rated propulsion force</td>
<td>40 N</td>
</tr>
<tr>
<td>Rated normal force at aligned position</td>
<td>401.9 N</td>
</tr>
<tr>
<td>DC link input voltage</td>
<td>160 V</td>
</tr>
<tr>
<td>Rated phase winding current</td>
<td>8.5 A</td>
</tr>
<tr>
<td>Number of turns per phase</td>
<td>AWG #17, 380 Turns/phase</td>
</tr>
<tr>
<td>Phase winding current density</td>
<td>4.09 A/mm²</td>
</tr>
<tr>
<td>Aligned phase winding inductance</td>
<td>47.02 mH</td>
</tr>
<tr>
<td>Unaligned phase winding inductance</td>
<td>31.33 mH</td>
</tr>
<tr>
<td>Phase winding resistance</td>
<td>1.41 Ω/phase</td>
</tr>
<tr>
<td>Stator pole arc of the equivalent RSRM</td>
<td>18°</td>
</tr>
<tr>
<td>Rotor pole arc of the equivalent RSRM</td>
<td>35°</td>
</tr>
<tr>
<td>Air gap length</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>Translator pole width</td>
<td>12 mm</td>
</tr>
<tr>
<td>Translator slot width</td>
<td>18 mm</td>
</tr>
<tr>
<td>Translator pole height</td>
<td>56 mm</td>
</tr>
<tr>
<td>Translator yoke width</td>
<td>12 mm</td>
</tr>
<tr>
<td>Translator core stack width</td>
<td>60 mm</td>
</tr>
<tr>
<td>Stator pole width</td>
<td>23 mm</td>
</tr>
<tr>
<td>Stator slot width</td>
<td>17 mm</td>
</tr>
<tr>
<td>Stator pole height</td>
<td>23 mm</td>
</tr>
<tr>
<td>Stator yoke width</td>
<td>23 mm</td>
</tr>
<tr>
<td>Stator core stack width</td>
<td>68 mm</td>
</tr>
</tbody>
</table>
APPENDIX D: EXPERIMENTAL SETUP

D.1 Experimental Setup with the 6/4 LSRM in Chapter 2

Figure D.1 Experimental setup with the 6/4 LSRM in Chapter 2.
D.2 Experimental Setup with the LSRM-Propelled Electromagnetic Levitation and Guidance Systems

Figure D.2 Experimental setup with vehicle and track.

Figure D.3 Propulsion, levitation, and guidance subsystems.
Figure D.4 Converters and control boards on the vehicle.
BIBLIOGRAPHY


Byeong-Seok Lee was born in Buan, Korea, in 1964. He received the B.S. and M.S. degrees in Electrical Engineering from Seoul National University, Seoul, Korea, in 1987 and 1989, respectively. From 1989 to 1995, he was with the Research & Development Center, DAEWOO Heavy Industries & Machinery Ltd., Incheon, Korea, where he worked on research and development of permanent magnet electric machines and their power-electronic control for mechatronics system and machine tool applications. In the Fall of 1995, he joined the Ph.D. program in the Bradley Department of Electrical and Computer Engineering at Virginia Polytechnic Institute and State University, Blacksburg, VA. His research interests include electric machine design and power-semiconductor-controlled motion control systems. He is a member of the IEEE, and the IEEE Industrial Applications, Industrial Electronics, and Power Electronics Societies.

Byeong-Seok Lee