MODELING TRAFFIC DISPERSION

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Dissertation submitted to the faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Civil Engineering

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November 2005
Blacksburg, Virginia

Keywords: Platoon Dispersion, Signalized Intersections, Delay, Robertson’s Platoon Dispersion Model, Microscopic Traffic Simulation, Macroscopic Traffic Simulation, TRANSYT, INTEGRATION, Calibration

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ABSTRACT

The dissertation studies traffic dispersion modeling in four parts. In the first part, the dissertation focuses on the Robertson platoon dispersion model which is the most widely used platoon dispersion model. The dissertation demonstrates the importance of the Yu and Van Aerde calibration procedure for the commonly accepted Robertson platoon dispersion model, which is implemented in the TRANSYT software. It demonstrates that the formulation results in an estimated downstream cyclic profile with a margin of error that increases as the size of the time step increases. In an attempt to address this shortcoming, the thesis proposes the use of three enhanced geometric distribution formulations that explicitly account for the time-step size within the modeling process. The proposed models are validated against field and simulated data.

The second part focuses on implementation of the Robertson model inside the popular TRANSYT software. The dissertation first shows the importance of calibrating the recurrence platoon dispersion model. It is then demonstrated that the value of the travel time factor \( \beta \) is critical in estimating appropriate signal-timing plans. Alternatively, the dissertation demonstrates that the value of the platoon dispersion factor \( \alpha \) does not significantly affect the estimated downstream cyclic flow profile; therefore, a unique value of \( \alpha \) provides the necessary precision. Unfortunately, the TRANSYT software only allows the user to calibrate the platoon dispersion factor but does not allow the user to calibrate the travel time factor. In an attempt to address this shortcoming, the document proposes a formulation using the basic properties of the recurrence relationship to enable the user to control the travel time factor indirectly by altering the link average travel time.

In the third part of the dissertation, a more general study of platoon dispersion models is presented. The main objective of this part is to evaluate the effect of the underlying travel time distribution on the accuracy and efficiency of platoon dispersion models, through qualitative and quantitative analyses. Since the data used in this study are generated by the INTEGRATION microsimulator, the document first describes the ability of INTEGRATION in generating
realistic traffic dispersion effects. The dissertation then uses the microsimulator generated data to evaluate the prediction precision and performance of seven different platoon dispersion models, as well as the effect of different traffic control characteristics on the important efficiency measures used in traffic engineering. The results demonstrate that in terms of prediction accuracy the resulting flow profiles from all the models are very close, and only the geometric distribution of travel times gives higher fit error than others. It also indicates that for all the models the prediction accuracy declines as the travel distance increases, with the flow profiles approaching normality. In terms of efficiency, the travel time distribution has minimum effect on the offset selection and resulting delay. The study also demonstrates that the efficiency is affected more by the distance of travel than the travel time distribution.

Finally, in the fourth part of the dissertation, platoon dispersion is studied from a microscopic standpoint. From this perspective traffic dispersion is modeled as differences in desired speed selection, or speed variability. The dissertation first investigates the corresponding steady-state behavior of the car-following models used in popular commercially available traffic microsimulation software and classifies them based on their steady-state characteristics in the uncongested regime. It is illustrated that with one exception, INTEGRATION which uses the Van Aerde car-following model, all the software assume that the desired speed in the uncongested regime is insensitive to traffic conditions. The document then addresses the effect of speed variability on the steady-state characteristics of the car-following models. It is shown that speed variability has significant influence on the speed-at-capacity and alters the behavior of the model in the uncongested regime. A method is proposed to effectively consider the influence of speed variability in the calibration process in order to control the steady-state behavior of the model. Finally, the effectiveness and validity of the proposed method is demonstrated through an example application.
ACKNOWLEDGEMENTS

I would like to give my special thanks to my advisor, Dr. Hesham Rakha, for all of his support and guidance throughout my doctoral studies. I have been fortunate in benefiting from his friendship, enlightening advice, and exemplary work ethic and commitment to quality of research work.

I would also like to express my appreciation to Dr. Kyoungho Ahn for his support and advices throughout my research work at VTTI. I would also like to thank my committee members Dr. Dusan Teodorovic, Dr. Antonio Trani, Dr. Pushkin Kachroo, and Dr. Montasir Abbas for their participation, encouragement and instructive suggestions.

Sincere gratitude goes to my father, Hosein Farzaneh, for his constant support and friendship.

Finally, this work is dedicated to my beloved wife, Behnoush Yeganeh, for all of her love, encouragement and support.
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CHAPTER 1
INTRODUCTION

The invention of the automobile has brought a new dimension to mobility and has had a prominent impact on the quality of life in urban areas. Specifically, it has contributed to the sprawl of cities and thus, to the development of the suburbs that inevitably intensified the transportation problem as a result of trips carried to and from these suburbs. This issue plus the existing traffic in urban areas has made a complicated situation for transportation engineers. Various solutions have been proposed and tested through the years such as flexible work scheduling and encouraging carpooling and the use of public transit. Despite these efforts, the lifestyle in North America does not lend itself to these solutions because of the high dependency on private vehicles.

Confronted with these problems, transportation engineers and planners have had two possible choices to manage and control the increasingly growing traffic in the urban networks. They can consider an increase of the network capacity by means of constructing new high capacity by-passes, bridges and roads. Unfortunately, this solution is very costly and often requires the reconstruction of downtown areas. An alternative solution is to try to enhance the capacity of existing networks through the use of traffic control techniques. In contrast to the first alternative, this solution is less costly and easily supported by local authorities in urban areas, and therefore transportation engineers tend to use this approach in addressing transportation problems. In fact, traffic engineering and control, nowadays, is the primary solution for growing traffic problem and plays a vital role in managing the increasing traffic demand on current roads.

1.1 PROBLEM DEFINITION

Among the techniques of urban traffic control, traffic signals are the most used and visible traffic control devices. The first three-color traffic signal was installed in 1920 in Detroit, Michigan. The invention of traffic lights has generated considerable benefits by increasing the capacity, reducing the delays, and consumption of energy and improving the safety by reducing the number of conflicts at roadway intersections. However, it must be remembered that the key
element of enjoying these benefits is the efficient signal timing. Specifically, an inefficient operating signal produces excessive delay, frustration, and wasted fuel.

Isolated intersections and arterials are the most important application of traffic signals. Traffic signals prevent chaos at busy intersections, but in the other hand, frequent stops that often occur on driving down a street with many signals, mainly arterials, do not sound pleasing at all. The number of stops on an arterial can be reduced by considering the effect of the interdependence of the signals and proper coordination of them. In fact, the coordination of traffic signals is a standard practice in traffic engineering with the objective of optimizing the use of the existing infrastructure by ensuring an adequate traffic flow through the network.

Many methods ranging from the use of simple time-distance diagrams to computer-oriented methods of Morgan and Little (1964) have been used to assist traffic engineers in achieving the desired coordination. During the last four decades, a considerable number of researches were carried out on the automated methods of traffic signal coordination, and have resulted in a number of tools including TRANSYT (Robertson, 1969) and SCOOT (Hunt et al., 1989). In spite of the complex algorithms that these systems use, their effectiveness to optimize traffic signals depends largely on the models they are using to describe the movement of the groups of the vehicles traveling between the signal-controlled intersections. Indeed, either in the offline signal models such as TRANSYT, or in the real-time control systems such as SCOOT, the modules of predicting the progression of vehicles along the arterials are incorporated, and based on the result of these modules the cycle length, the distribution of the green time and specifically the offset time between the signals are calculated. It thus appears that before any attempt to establish a plan aiming at improving the traffic flow through the network, a detailed understanding of traffic progression and the dynamics of the traffic flow is necessary.

The effect of a traffic signal, as a traffic control device, is to divide the traffic flow into a regular series of platoons of vehicles with initial time length not greater than the green phase of the signal. As the platoons move along the road they disperse and their time lengths increase. This mechanism is known as platoon dispersion. The platoon dispersion is caused, in part, by differences in drivers’ desired speeds and, mostly, as a result of vehicle interaction with other vehicles entering and exiting the roadway, which is commonly known as roadway side friction. Platoon dispersion models attempt to simulate the dispersion of traffic as it travels along a
roadway by attempting to estimate vehicle arrivals at downstream locations based on an upstream vehicle departure profile and a desired traffic-stream speed.

Observation of the diffusion of traffic platoons have been reported by a number of researchers. Some of the authors offered models which predict the length of the platoon in time or the time length of the platoon for various percentiles. Lighthill and Witham (1955) used a kinematic wave theory approach to describe the traffic platoon behavior as it travels along a roadway, but Pacey (1956) was the first person to introduce a model for predicting the downstream arrival flow rate considering the dispersion of traffic platoons. However, the most widely used platoon dispersion model is Robertson’s (1969) platoon dispersion model. This model has become a virtual universal standard platoon dispersion model and has been implemented in various traffic-simulation softwares, including TRANSYT (Robertson, 1969), SCOOT (Hunt et al., 1989), SATURN (Hall et al., 1980), and TRAFLO (Lieberman et al., 1980). Figure 1.1 depicts the Robertson platoon dispersion mechanism and the idea of optimizing the signal timing parameters. Consequently, the effectiveness of network signal models, such as TRANSYT and SCOOT, depends on the precision with which they predict the dispersion of moving platoons from an intersection to another.

Figure 1.1: Platoon dispersion on an arterial and optimization of signal timing.
A successful application of Robertson’s platoon dispersion model requires an appropriate calibration of the model parameters. Specifically, Guebert and Sparks (1989) showed that the accurate calibration of the Robertson platoon dispersion model parameters is critical in developing effective and efficient traffic signal timing plans. Furthermore, Manar (1994) examined the effect of the use of inappropriate platoon dispersion parameters in the TRANSYT software for a road section with three intersections in Montreal, Canada. He found that the use of the TRANSYT-7F’s manual recommended value of 0.25\(^1\) for the platoon dispersion factor \(\alpha\) will cause an extra total cost of 65,250 CND per year for the users of that section. Despite the significant impact the platoon dispersion parameters have on the signal timings that are estimated by the TRANSYT-7F software, the software manual does not provide an analytical framework for the calibration of the platoon dispersion model parameters. The state-of-practice has been the use of a goodness-of-fit approach to calibrate the model parameters.

Alternatively, Yu and Van Aerde (1995) developed an analytical framework for calibrating the platoon dispersion model parameters using a statistical analysis of the link travel-time distribution. Specifically, Yu and Van Aerde (1995) proposed a set of formulas to calibrate the parameters of Robertson’s platoon dispersion model based on the average travel time and the standard deviation of the travel time. However, as will be demonstrated later in the thesis, this approach is only valid for step sizes of 1-second duration.

In the other hand, the traffic dispersion can be translated to differences in driving behaviors in terms of desired speed selection, which is called speed variability in this document. Traffic microsimulation models use this phenomenon to produce more realistic driving behavior modeling. The usual way of modeling this phenomenon is to define drivers’ desired speed as a random variable and explain it with a probability distribution. It is observed that this distribution has significant effect on the resulting steady-state behavior of the traffic simulated by these models, but no one has studied and quantified this effect. The importance of this factor relies in the fact that steady-state behavior of each model determines the dynamics of the simulated traffic and therefore any factor that affects this behavior should be carefully considered in the calibration process.

\(^1\) For low friction condition: no parking, divided, turning provision, 12-ft lane width; suburban high-type arterial.
1.2 Research Objective

The main objective of this research is to investigate the shortcomings of the existing platoon dispersion models, and to improve these models and their calibration procedures. Since Roberson’s platoon dispersion model is the most widely used model, more attention will be given to investigate possible enhancements to this model.

In detail, four major objectives are considered for this research;

1. Perform a comprehensive study of macroscopic platoon dispersion models and investigate the effect of some important parameters, such as the modeling time step size, which has not been studied, on the performance of these dispersion models.

2. Develop enhancements to the platoon dispersion models based on the results of the previous part, and provide practical recommendations for users of these models.

3. Identify critical parameters within traffic dispersion models and develop calibration procedures for platoon dispersion models.

4. Investigate the effect of speed variability (microscopic platoon dispersion) on the steady-state behavior of microsimulation models and develop calibration procedures that account for the effect of speed variability on steady-state behavior.

1.3 Research Contributions

This Research develops enhancements to Yu and Van Aerde’s calibration procedure to overcome the limitations of Yu and Van Aerde formulation by explicitly considering the modeling time step in the analytical formulation. Furthermore, some analytical procedure is introduced to enable the TRANSYT’s users to overcome the limitation of the software regarding the use of appropriate parameters in order to get more reliable results. It is anticipated that the proposed procedures and enhancements in this research will have many practical and methodological implications to the traffic engineers who will be able to use them to enhance the performance of signal timing plans for an area. Furthermore, the research tries to quantify the effect of speed variability factor on the steady-state behavior of microsimulation models and propose a method to consider this factor in the calibration step. More specifically, this research effort makes the following contributions;

- Develops a generalized calibration procedure for the recurrence platoon dispersion model.
- Develops an analytical procedure to use the parameters derived from observation in the TRANSYT software.

- Develops a procedure to consider the microscopic traffic dispersion effect on the macroscopic characteristics of the traffic flow.

- Develops calibration procedures for microscopic traffic simulation software that accounts for traffic stream dispersion effects.

1.4 DISSERTATION LAYOUT

This dissertation is organized into 8 chapters. Chapter 1 contains a brief introduction to the topic and the problem overview. The second chapter provides a review of traffic signal coordination methods and platoon dispersion models. The literature review first discusses the importance of platoon dispersion modeling, then describes the state-of-the-art and state-of-the-practice macroscopic platoon dispersion models and their calibration methods, and finally explains how traffic dispersion is modeled within traffic microsimulation softwares. The third chapter provides an overview of the research methodology in terms of the problems and shortcomings of current calibration methods of dispersion models, proposed solutions, and consideration of traffic dispersion in microsimulation modeling.

Chapter 4 first shows the current calibration method for the popular Robertson model. The chapter demonstrates that the current procedures ignore the effect of the time interval size which causes error in flow prediction, and then proposes three methods to incorporate this factor in the model. Chapter 5 continues this effort by concentrating on the TRANSYT software structure. It first demonstrates the importance of the calibration of the travel time factor $\beta$ for the signal coordination task and then proposes an indirect calibration method to overcome the current limiting structure of the model.

In chapter 6, the dissertation investigates the effect of the underlying travel time distribution on the accuracy and performance of platoon dispersion models. The effect of different traffic characteristics on the platoon dispersion behavior is also studied in this chapter.

Chapter 7 studies microscopic traffic dispersion modeling using speed variability and investigates the effect of this factor on the steady-state behavior of microsimulation models. Subsequently, the chapter proposes a method to consider the effect of speed variability in the
calibration process. Finally, chapter 8 provides a summary of the findings and the conclusions of the research effort.
CHAPTER 2

LITERATURE REVIEW

This chapter provides a review of related research and identifies areas in which the literature should be expanded. This chapter is divided into five main subsections which are: definition, signal coordination techniques, platoon dispersion models, calibration of platoon dispersion models, and microscopic traffic dispersion modeling.

2.1 DEFINITION

Vehicles departing from a queue at a traffic signal typically travel in a platoon that disperses as vehicles travel further downstream. El-Reedy (1978) defined a platoon as a bunch of vehicles crossing the reference line with a time headway less than or equal to 4 seconds. The platoon dispersion happens mostly as the result of vehicle interaction with other vehicles entering and exiting the roadway which is commonly known as the roadway side friction. Beside the side friction, the difference in drivers’ desired speed also plays an important role in characterizing platoon dispersion, especially when the side friction is low.

Formation of platoons means that improved traffic flow can be achieved if the green phase at the downstream traffic signal is applied to coincide with the arrival of the platoon. To achieve this goal, traffic signals must be coordinated, or linked. Signal coordination improves the level of service on a road network where the spacing of traffic signals is such that isolated operation causes excessive delays. MUTCD: Millennium Edition, recommends that, “… signals within 0.5 mile of one another along a major route or in a network of intersecting major routes should be operated in coordination, preferably within interconnected controllers.” Furthermore, the Traffic Control System Handbook (FHWA, 1996) suggests interconnecting “adjacent traffic signals when the distance is less than approximately 70 times the desired average speed in ft/sec (m/s).”

2.2 SIGNAL COORDINATION TECHNIQUES

The timing of traffic signals to produce a coordinated progressive system has been the subject of considerable attention by traffic professionals. The simplest signal coordination method is the time-space diagram technique. Figure 2.1 illustrates the concept of the time-space diagram. This
method is a technique that attempts to maximize the bandwidth of uninterrupted passing through a set of successive signals given a progression speed. The main application of the time-space diagram is on linear arterials and cannot be used when the arterials are interconnected and form a network, especially as the turning movements make the process more complex. Moreover, this technique does not take into account the dispersion of the flow profile of released traffic from a signal, i.e. the variation of travel times of the vehicle in the platoon to cross the section between two adjacent crossroads.

Morgan and Little (1964) proposed a computer-oriented method for coordinating traffic signals. Their method uses the bandwidth as the objective function and tries to find the offsets that maximize the platoon bandwidth. This method also is applicable only for linear arterials and doesn’t consider the flow profiles, and some researchers (Seddon, 1971) criticized the philosophy of maximizing bandwidth.

![Figure 2.1: Space-time diagram for three intersections.](image)

Because of the difficulties and disadvantages of using the above techniques of signal coordination on a network, the Road Research Laboratory (RRL), in England, developed a method that calculates the timing plan and the optimal offsets for a network. The method is
known as the *RRL Combination Method* which was first described by Hillier (1965/66), which is based on four major assumptions:

1. Signal timing does not affect the amount of traffic flow and the route choice behavior of drivers.
2. All signals have a common cycle length or a sub-multiple of this cycle length.
3. For each signal, the distribution of green phases is known.
4. The delay experienced by the drivers in a direction along a link depends only on the timing of the traffic lights located at the two ends of the link; and is not affected by any other adjacent signals in the network.

The RRL Combination Method requires knowledge of the common cycle length of the network, which normally corresponds to the cycle length of the intersection that has heaviest traffic. The green phase duration is then calculated based on the ratio of flow of the approach to the saturation flow rate. The method also requires knowledge of the delay/difference-of-offset in each direction for each link. To obtain the delays/difference-of-offset relation, the combinative method needs to predict the flow profile at the end of each link during each interval or increment of the cycle, which is generally 2 or 3 seconds in duration. It thus can be seen that knowledge of the pattern in which a platoon of vehicles moves along a link is required to use the Combination Method. The method then calculates the delay on each link for each offset, and finally it determines the optimal offsets for all the signals in the network which minimize the total delay.

The fourth assumption of the Combination Method indicates since delay on a link is independent of the setting of any signals on other links, it is unnecessary to make trial combination of the delay/difference-of-offset table of each link with that of every other link in the network. This matter simplifies the calculations and allows the number of combinations to be reduced.

Hillier (1965/66) stated that the RRL combination method is applicable for a particular network type, namely those that can be reduced to a single link by successively combining links in series and in parallel. Allsop (1968) showed that the combination method can be adapted so as to apply to a wider range of networks. He proposed such a procedure that can be applied to many networks that cannot be reduced.
Robertson (1969) developed the TRANSYT model based on an improved version of the RRL Combination Method. The improvement that Robertson made in TRANSYT is that instead of using delay as the performance index which the program minimizes, he considered a new performance index which takes into account not only the delay but also the number of stops.

\[
\text{Performance Index} = \sum_{i=1}^{k} (d_i + K C_i) \tag{2.1}
\]

Where

- \(d_i\): the average delay on link \(i\) (veh-h/h),
- \(C_i\): the number of vehicles stopped on link \(i\), and
- \(K\): stop penalty factor (sec/stop), normally 4 (sec/stop).

Minimizing the number of stops of vehicles is actually the green-wave philosophy which is common in North America and Europe. If \(K\) is set equal to zero then the value of the performance index is equal to the total delay on the network. A non-zero value of \(K\) adds a penalty for each stop to the performance index and represents the delay experienced by vehicles in the queue during their deceleration and acceleration. Huddart and Turner (1969) stated that the use of stop penalties of 4 sec/stop very slightly increases the delay to traffic, but takes better account of the total economics, including accident risks.

### 2.3 Platoon Dispersion Models

It has been shown that to derive a link’s delay/offset relation, it is necessary to have a method to predict the traffic flow profile at the downstream end of the link. Observation of the diffusion of traffic platoons has been reported by a number of researchers such as Lighthill and Witham (1955), Pacey (1956), Lewis (1958), Graham and Chenu (1962), Herman, Potts and Rothery (1964), Dokerty (1967) and Hillier and Rothery (1967). Some of these authors offered models which predict the length of the platoon in time or the time length of the platoon for various percentiles. Lighthill and Witham (1955) used a kinematic wave theory approach to describe the platoon traffic behavior as it travels along a link, but Pacey (1956) was the first person to introduce a model for predicting the downstream arrival flow rate considering the dispersion of traffic platoons. Robertson (1969) used a recurrence relationship to describe the platoon dispersion phenomena. Because of the simplicity of applying this model, Robertson’s platoon
dispersion model became a virtual universal standard platoon dispersion model and has been implemented in various traffic simulation softwares. Seddon (1972a/b) showed that both Pacey’s and Robertson’s models are probability-based models with different probability density functions. Qiao et al. (2001) developed a traffic dispersion model based on a three layer Back-Propagation neural network. They used a field dataset to train the network, and they found that the trained network predicted the flow pattern for that specific link accurately.

2.3.1 The Kinematic Wave Theory
Lighthill and Whitham (1955) presented their theory in two papers; the first one giving a mathematical treatment of flood movement in long rivers, and the second paper giving a descriptive treatment of the traffic flow on long crowded roads. Their theory in both cases is exactly the same and is based on the fluid mechanics’ fundamental principle of mass conservation.

Considering \( t \) for time and \( x \) as the position of the vehicles along the road, and using the fluid mechanics concepts, the characteristics of the traffic flow can be described by the following quantities as a function of time and space; where \( k(x,t) \) is the density of traffic (veh/km), \( v(x,t) \) is the average space-mean speed of the traffic stream (km/h), and \( q(x,t) \) is the traffic flow rate (veh/h).

The mass conservation principle can be applied to traffic flow on a roadway similar to the motion of a fluid in a pipe. For an uninterrupted segment of roadway, if the entering flow is greater than the output flow, the mass conservation principle requires an equivalent increase in mass on this section since the vehicles do not disappear on the road. This means any difference between the number of vehicles entering and leaving a section of a roadway, can be explained as the variation of density.

For a finite section of a roadway ranging between \( x \) and \( x+dx \), the variation in the number of vehicles during time \( dt \) can be expressed as the difference between the number of vehicles arriving at \( x \), and number of vehicles leaving \( x+dx \);

\[
q \, dt - (q + \frac{\partial q}{\partial x} \, dx) \, dt
\]  

[2.2]
In addition, the same quantity can be expressed as the variation of the density between time \( t \) and \( t+dt \);

\[
(k + \frac{\partial k}{\partial t})\,dx - k\,dx
\]  

[2.3]

By equalizing these two expressions, one can obtain the vehicles conservation law;

\[
\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0
\]  

[2.4]

This equation, which assumes that there is no entrance or exit along the section, is known as the continuity equation of compressible fluids.

The wave theory follows from the equation of continuity, assuming that a functional form of flow-density relation is known. The speed of wave \( \omega \) then is the slope of the tangent to the \( q-k \) curve which is smaller than the mean speed of traffic \( u \) and indicates that the mean speed decreases with increase of concentration.

Kinematic waves do not disperse as the other waves do, but they experience a change in form because of the dependence of the wave speed \( \omega \) on the flow \( q \) carried by the waves. Accordingly, wave forms may suffer discontinuities due to the overtaking. Lighthill and Witham described this as shock waves.

The law of the motion of shock waves is derived from the conservation of vehicles which was described above. If the flow and concentration have the values \( q_1 \) and \( k_1 \) at one side, and \( q_2 \) and \( k_2 \) on the other side of a shock wave, which moves with speed \( \omega \), then the number of vehicles crossing it in the unit of time can be expressed as:

\[
q_1 - \omega k_1 = q_2 - \omega k_2
\]  

[2.5]

From which, the speed of the shock wave can be derived as:

\[
\omega = \frac{q_2 - q_1}{k_2 - k_1}
\]  

[2.6]

This is the slope of the vector joining the two points on the flow-density curve corresponding to the traffic states ahead of and behind the shock wave. Thus, the derivation of shock waves and
resulting traffic flow pattern depends on the knowledge of the q-k relation and the assumption that this relationship remains constant over a given length of road at a given time.

Figure 2.2: Example of wave paths.

Figure 2.3: Observed flow patterns with those predicted by Lighthill and Whitham (Seddon, 1971).
To use the shock waves to describe the change in a platoon’s form, each platoon is divided into small time steps, which means we divide the flow profile to separate waves. The flows and times at these intervals are then read from the smoothed starting flow profile and the wave speeds derived for each value of flow, and then a complete diagram of wave paths can be drawn as is shown in Figure 2.2. It is then a simple matter to derive the flow-time diagram at any point on the road as each wave carries a given flow. Figure 2.3 shows the results of Lighthill and Whitham kinematic wave theory for analyzing the platoon movement on Crescent Road, Manchester, England (Seddon 1971).

Seddon (1971) counted three major criticisms of the application of the Lighthill and Whitham theory to platoon dispersion. The first is the dependence of the method on having an accurate mathematical form of q-k relationship. The second criticism is that the predicted downstream flow profile does not increase in length, which means this method does not really take into account the dispersion of platoons. The reason for this is that the first and last waves, giving the front and rear of the platoon, are parallel and indeed representing zero flow with the constant speed of free flow speed $u_f$. The last criticism is that the method is a manual/graphical process and is very time-consuming, which deems it unsuitable for practical purposes, and thus this theory has not received application beyond the evaluation level.

### 2.3.2 Diffusion Theory

Pacey (1956) presented a purely kinematic theory to model the diffusion of a platoon of vehicles moving along a roadway. He developed his theory based on four basic assumptions:

- **Platoon dispersion arises only from the differences in speed between vehicles in the platoon,**
- **Passing is free,**
- **Vehicles travel at a constant speed,** and
- **The distribution of speeds of vehicles is normal.** This means that the probability that the speed of a vehicle lies between $v$ and $v+dv$ is given by:

$$ f(v)dv = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(v-v_0)^2}{2\sigma^2}} dv \quad [2.7] $$

Where
\( v_m \): the average speed of the vehicles in the platoon, and

\( \sigma^2 \): the variance of the speeds.

In the absence of any interference between the vehicles and by knowing the distribution of speeds of the vehicles, one can then determine the distribution of vehicles’ travel times between two points on a link. Assume that the distribution of speeds is defined by \( f(v).dv \) and the travel time for a distance of \( x \) is given by \( T=x/v \). Suppose that this travel time follows a certain distribution defined by \( g(T).dT \), which represents the probability that the travel time lies between \( T \) and \( T+dT \). Considering the relationship between speeds and travel times, this probability is the same as the probability that speed lies between \( v \) and \( v+dv \), where \( v=x/T \). Therefore we obtain;

\[
\frac{dv}{dT} g(T).dT = f(v).dv = f(v).\left(\frac{dv}{dT}\right) dT
\]

Substituting \( v \) by \( x/T \) and \( dv/dT \) by \( (-x/T^2) \) in Equation 2.8, we will have;

\[
\frac{dv}{dT} \frac{g(T).dT}{f(v).dv} = \frac{x}{T^2} \frac{g(T).dT}{f(v).dv} = \frac{x}{T^2} \frac{g(T).dT}{f(v).dv} = \frac{g(T).dT}{f(v).dv} = \frac{x}{T^2} \frac{g(T).dT}{f(v).dv}
\]

Minus sign is eliminated since the probability cannot be negative. Knowing that \( f(x/T) \) follows a normal distribution, Equation 2.9 can be rewritten as;

\[
g(T).dT = \frac{x}{T^2 \times \sigma \sqrt{2\pi}} e^{-\frac{(x^2-\alpha \sigma)^2}{2\sigma^2}} dT
\]

And finally by substituting \( \alpha = \sigma/v_m \) and \( T_m = x/v_m \), the final form of the distribution of vehicles travel times will be derived as:

\[
g(T).dT = \frac{T_m}{\alpha \times T \times \sqrt{2\pi}} e^{-\frac{(T_m-T)^2}{2\alpha^2 T^2}} dT
\]

Where

\( T \): the travel time,

\( T_m \): the travel time corresponding to the average speed, and

\( \alpha \): the diffusion constant.
This distribution, derived from the speeds’ distribution, is known as the transformed or reversed normal distribution. Having defined these two distributions, we can then predict the flow passing a point in downstream at a certain time period. If the flow past the upstream point in time between \( t \) and \( t+dt \) is \( q_1(t) \), the flow which will pass a certain downstream point at the time \( t+T \) will be \( q_1(t)g(T-t)dtdT \). Therefore total flow which will pass the downstream point during the time interval \((T, T+dT)\) will be:

\[
q_2(T)dt = \int q_1(t)g(T-t)dtdT 
\]

The integration is over all the values of \( T \) for which \( q_1(t) \) is non-zero. Since the flow profile is normally represented by a histogram, and since the discrete form of this equation would be computationally simple, it is useful to write Equation 2.12 in discrete form;

\[
q_2(j) = \sum_i q_1(i)g(j-i) 
\]

Where indices \( i \) and \( j \) refer to the discrete intervals of time at the first and second observation points respectively.

According to this formula, the flow in the \( j^{th} \) interval at the downstream observation is the summation of the flows in the \( i^{th} \) interval, for all values of \( i \), at the upstream observation point multiplied by the probability of the travel time is \((j-i)\).

Grace and Potts (1964) gave a more thorough mathematical treatment to Pacey’s model and showed that the model is equal to a one-dimensional diffusion equation. They then derived equations to predict the flow profile at a downstream point for a number of geometrical upstream flow profiles. They also made some calculations of the number of stopped vehicles at a downstream signal for different values of extension and offset time.

Denney (1989) used the general concept of Pacey’s theory and introduced a new dispersion model. He stated that “… the mechanism can be isolated from the assumption of normally distributed speed” and thus instead of using a transformed normal distribution for travel time, he used the actual observed distribution which was derived from the field study.

### 2.3.3 Robertson’s Recurrence Relationship

An alternative method for predicting platoon dispersion was developed by Robertson (1969), who used field data to derive an empirical method of predicting the platoon behavior. The method is
very simple to apply and makes use of a discrete recursive relationship. Robertson’s platoon dispersion model forms the core of the widely used TRANSYT program and has become virtually the universal standard and has been incorporated in a number of traffic simulation softwares, including the Split Cycle Offset Optimization Tool (SCOOT) (Hunt et al., 1989), SATURN (Hall et al., 1980), and TRAFLO (Lieberman et al., 1980).

The basic Robertson recursive platoon dispersion model takes the following mathematical form:

\[ q_i' = F \times q_{i-T} + (1 - F) \times q_{i-\Delta t} \quad [2.14] \]

\[ F = \frac{1}{1 + \alpha \beta T_a} \quad [2.15] \]

Where:

- \( q_{i-T} \): discharging flow over a time step \( \Delta t \) observed at the upstream signal at time \( t-T \);
- \( q_i' \): flow rate over a time step \( \Delta t \) arriving at the downstream signal at time \( t \);
- \( \Delta t \): time step duration, measured in the time intervals used for \( q_i' \) and \( q_i \);
- \( T \): minimum travel time on the roadway in units of time steps, equal to \( \beta T_a \);
- \( \alpha \): dimensionless platoon dispersion factor, express the degree of the dispersion of the platoon;
- \( \beta \): dimensionless travel time factor, equals to ratio of the average travel time of the first vehicle to the average travel time of all the vehicles in the platoon;
- \( F \): smoothing factor, and
- \( T_a \): mean roadway travel time, measured in units of time steps

The empirical values of \( \beta \) and \( \alpha \) can vary between 0 and 1. \( \beta = 1 \) and \( \alpha = 0 \) indicates the situation that a platoon remains compact and dispersion is minimum. Robertson suggested a value of 0.8 for \( \beta \) and to derive a value for \( \alpha \), he plotted the values of \( F \) which gave the best fit for the data from four sites in London against the average travel times and found \( \alpha = 0.5 \) gives the best fit for that case.

Equation 2.14 is applied by dividing the departure profile from an upstream traffic signal into a number of time steps. For example, the TRANSYT-7F model divides the cyclic profile into a
total of 60 time steps that typically range in duration between 1 to 3 seconds. Equation 2.14 expresses the arrival profile at the downstream signalized intersection at instant $t$ as a linear combination between the downstream flow one time step earlier ($q_{t-\Delta t}$) and the upstream departure flow $T$ steps earlier ($q_{t-T}$).

Seddon (1972) showed that Equation 2.14 can be rewritten in the following form;

$$q'_t = \sum_{i=T}^{\infty} F(1-F)^{t-i} \times q_{t-i}$$

[2.16]

This is a special case of the general form of Equation 2.13 which was proposed by Pacey (1956). Specifically, Equation 2.16 demonstrates that the downstream traffic flow that is computed using the Robertson platoon dispersion model follows a shifted geometric distribution. The geometric distribution gives the probability that a vehicle passing the upstream point in the $(t-i)$th interval is observed downstream in the $t$th interval. Seddon concluded since the Robertson’s method allows the existence of positive probabilities for unreasonably long travel times, an upper limit should be considered for the travel time to avoid having very big travel times.

### 2.3.4 Calibration

To apply the kinematic wave theory to platoon dispersion theory, one needs to have an accurate representation of the equilibrium flow-density relationship. Consequently, the calibration of a kinematic wave platoon dispersion model requires calibrating the $q$-$k$ relation.

Pacey’s diffusion theory approach of platoon dispersion modeling uses a normal distribution of vehicles’ speed, or in fact a transformed normal distribution of vehicles’ travel times on a link. Thus, to calibrate Pacey’s model, one needs to have the values of two parameters, namely the average speed of vehicles and the standard deviation of speed, or more simply average travel time and standard deviation of travel time. Pacey demonstrated that the mean speed was the critical of the two input variables. Once these parameters are quantified, they can be directly plugged into the model and the downstream flow can be estimated.

If the values of these parameters are unknown but the flow profile for upstream and downstream points are available, one can use a best fit approach as was described by Seddon (1972a). He varied the average travel time and standard deviation of travel time in a systematic manner and for each pair of them calculated the predicted flow. Then, the predicted flow profiles were
compared to the observed flow profiles and the sum of squared difference in each interval was calculated, and the best fit was found by minimizing the sum of squared error. For such a case, Pacey (1956) derived the average travel time from the centers of area of the flow patterns at the observation points, and chose the speed which gave the best fit to all the points combined. It was found that the average travel time derived from the flow histograms is quite close to the actual observed travel times.

Robertson’s platoon dispersion model uses two parameters; a platoon dispersion factor $\alpha$, and a travel time factor $\beta$. Robertson (1969) analyzed the traffic flow patterns observed by Hillier and Rothery (1965/66) at four sites in West London and determined that the values of $\alpha$ and $\beta$ from the best fit between the observed and calculated traffic flow patterns were equal to 0.5 and 0.8 respectively. Robertson (1969) also cautioned the users of TRANSYT that appropriate values of $\alpha$ and $\beta$ might be a function of site factors such as roadway width, gradient, parking, and others. Despite this matter, a fixed value of $\beta$ equal to 0.8 has been used in TRANSYT and users cannot change it.

Since the development of the Robertson’s platoon dispersion model, a number of studies have been conducted to evaluate its parameters. Most of these studies used a best fit approach to find the appropriate values of $\alpha$ and $\beta$. A summary of these studies is listed in Table 2.1.

Seddon (1972) carried out a theoretical investigation into the Robertson’s model and showed that the dispersion factor $\alpha$ should be a function of the travel time factor $\beta$. He found that the relationship between $\alpha$ and $\beta$ is in the following form:

$$\beta = \frac{1}{1 + \alpha}$$  \hspace{1cm} [2.17]

Seddon (1972) also found that when $\beta$ is equal to 0.8 the corresponding value of $\alpha$ is 0.25 which does not agree with the given values by Robertson (1969) that was confirmed with Manchester data. He stated that this inconsistency is due to taking the summation above to infinity when considering the shifted geometrical distribution, while for practical use the long tail is curtailed.

Yu and Van Aerde (1995) developed a method for calibrating the Robertson platoon dispersion factors ($\alpha$ and $\beta$) directly from the statistical properties of the travel-time experiences of individual vehicles. Specifically, the authors used the basic properties of the geometric
distribution of Equation 2.17 to derive the following three equations for calibrating the parameters of the Robertson platoon dispersion model.

\[ \alpha = \frac{1 - \beta}{\beta} \quad \text{[2.18]} \]

\[ \beta = \frac{2T_a + 1 - \sqrt{1 + 4\sigma^2}}{2T_a} \quad \text{[2.19]} \]

\[ F = \frac{\sqrt{1 + 4\sigma^2} - 1}{2\sigma^2} \quad \text{[2.20]} \]

Where:

\( \sigma \): standard deviation of link travel times (s), and

\( T_a \): mean roadway travel time (s).

### 2.4 Microscopic Traffic Dispersion Modeling

In the field of microscopic traffic simulation, platoon dispersion can be modeled as differences in driving behavior for different drivers. The concept of platoon dispersion in microscopic traffic modeling is capturing variability (randomness) in deterministic car-following models.

Unfortunately, this topic has not received the appropriate attention until recently. Most of the researches on microscopic traffic simulation have been on car-following models and the differences among them. However, recent studies (Ossen and Hoogendoorn, 2005, Brockfeld et al., 2005, Brockfeld et al.,2004 , and Punzo and Simonelli, 2005) have demonstrated that (a) the results of all the important car-following models are in the acceptable range (12% to 17% error) and no model can be denoted to be the best, and (b) considerable differences between the car-following behavior of individual drivers could be identified, and in fact the differences between individual drivers are larger than the differences between different models.

The conventional way that traffic dispersion phenomenon is modeled in microsimulators is to consider randomness around the results of the deterministic car following model, i.e. the deterministic desired speed or acceleration rate. There are several traffic microsimulation software available for microscopic analysis. Each of these models uses its own car-following
model and its specific dispersion modeling module. This section reviews some of these microscopic dispersion modules used in popular software packages.

Table 2.1: Summary of platoon dispersion studies conducted around the globe.

<table>
<thead>
<tr>
<th>Best-Fit Parameter</th>
<th>Condition</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\beta$</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.80</td>
<td>Three-lane dual carriageway; suburban high-type arterial</td>
</tr>
<tr>
<td>0.24</td>
<td>0.80</td>
<td>Typical suburban arterial roadway with two lanes in each direction; turn lanes provided</td>
</tr>
<tr>
<td>0.40</td>
<td>0.80</td>
<td>Three-lane carriageway with 10-15 percent commercial vehicles; reasonable freedom for overtaking</td>
</tr>
<tr>
<td>0.63</td>
<td>0.80</td>
<td>Two-way road 35 ft wide with two narrow lanes in the direction studied; 2-3 percent commercial vehicles; severely restricted overtaking</td>
</tr>
<tr>
<td>0.60</td>
<td>0.63</td>
<td>Single carriageway 33 ft wide on 5 percent downgrade; subject to 30-mph speed limit and clearway regulation during peak periods; bus volume of 12 veh/h in direction studied; 1378 ft downstream</td>
</tr>
<tr>
<td>0.70</td>
<td>0.59</td>
<td>Single carriageway 33 ft wide on 5 percent downgrade; subject to 30-mph speed limit and clearway regulation during peak periods; bus volume of 12 veh/h in direction studied; 1837 ft downstream</td>
</tr>
<tr>
<td>0.50</td>
<td>0.80</td>
<td>Characteristics ranging from single-lane flow with heavy parking and very restricted overtaking to multilane flow with no parking and relatively free overtaking</td>
</tr>
<tr>
<td>0.50</td>
<td>0.80</td>
<td>Heavy friction$^2$</td>
</tr>
<tr>
<td>0.50</td>
<td>0.80</td>
<td>Heavy friction</td>
</tr>
<tr>
<td>0.37</td>
<td>0.80</td>
<td>Moderate friction$^3$</td>
</tr>
<tr>
<td>0.35</td>
<td>0.80</td>
<td>Moderate friction</td>
</tr>
<tr>
<td>0.24</td>
<td>0.80</td>
<td>Low friction$^4$</td>
</tr>
<tr>
<td>0.25</td>
<td>0.80</td>
<td>Low friction</td>
</tr>
<tr>
<td>0.21</td>
<td>0.97</td>
<td>Low friction</td>
</tr>
<tr>
<td>0.15</td>
<td>0.97</td>
<td>Low friction</td>
</tr>
</tbody>
</table>

$^2$ Combination of parking, moderate to heavy turns, moderate to heavy pedestrian traffic, narrow lane width, traffic flow typical of CBD

$^3$ Light turning traffic, light pedestrian traffic, 11 to 12 ft lanes, possibly divided; typical of well-designed CBD arterial

$^4$ No parking, divided, turning provisions, 12-ft lane width; suburban high-type arterial
CORSIM was developed by the FHWA and combines two traffic simulation models: NETSIM for surface streets and FRESIM for freeway operations. NETSIM uses a unique sensitivity factor of 1/3600 while FRESIM uses the car-following sensitivity factor to account for different driving behaviors. NETSIM assumes 10 driver types and assigns each of them a sensitivity factor between 0.35 and 1.25 second using a uniform distribution (CORSIM User’s Manual. 1998). However users can change these default values in order to achieve a desired distribution; for example Schultz and Rilett (2004) used normal and lognormal distributions to improve the simulation results. In addition, NETSIM uses a user-defined distribution of desired speeds to represent driving behavior differences.

The car-following model used in VISSIM is a modified version of Wiedemann’s model (1974) and belongs to a family of models known as psychophysical or action-point models. This family of models uses thresholds or action-points where the driver changes his/her driving behavior. Drivers react to changes in spacing or relative speed only when these thresholds are crossed. The thresholds and the regimes are usually presented in relative speed/spacing diagrams for a pair of leader and follower vehicles.

VISSIM uses the following five random parameters to model different driving behavior between drivers and different times for a certain driver.

- **RND1**: (driver dependent) represents the difference between drivers in terms of distance. It is normally distributed with a mean of 0.5 and a standard deviation of 0.15.
- **RND2**: (driver dependent) accounts for individual drivers’ estimation ability. It has the same distribution as RND1.
- **NRND**: (driver independent) represents the variation in estimation ability for different moments. It has the same distribution as RND1.
- **RND4**: (driver dependent) accounts for drivers’ ability to control acceleration. It has the same distribution as RND1.
- **Desired speed**: (driver dependent) user can define the cumulative distribution of the desired speed.

Among these parameters only the desired speed can be controlled by users.

AIMSUN2 is designed and developed at the Universitat Politecnica de catalunya, Spain. AIMSUN2 uses the Gipps’ car-following model (1981) which consists of two components:
acceleration and deceleration sub-models. These two sub-models are explained as empirical formulations (SI units). The model accounts for behavior differences by using three random parameters:

- Maximum acceleration which a specific driver wishes to undertake,
- Most severe braking that a specific driver wishes to undertake, and
- The speed at which the driver wishes to travel (desired speed for a specific driver).

The INTEGRATION model uses a steady-state car-following model that was proposed by Van Aerde (1995) and Van Aerde and Rakha (1995). INTEGRATION accounts for drivers’ differences by using a speed variability factor and then randomly distributing desired speed among the drivers. The desired speed is considered to have either a normal or lognormal distribution.

It is observed that the most common way to account for driving behavior differences in traffic microsimulation software packages is the desired speed distribution or speed variability factor. Gipps (1981) noticed that the mean and standard deviation of the distribution of desired speeds affects the position and shape of the upper arm of the resulting steady-state speed-flow curve. This is very important, since the characteristics of the resulting speed-flow curve directly affects the dynamic behavior of the traffic on a roadway. The reason that the effect of speed variability is of special importance is the fact that any factor that alters the shape of the speed-flow relationship, directly affects the dynamic behavior of the simulated roadway. It is surprising that despite the importance of this issue, no one has studied the effect of this factor on the steady-state behavior of car-following models.

2.5 CONCLUSION

The literature review presented in this chapter provided some basic background information for the topic of signal coordination, traffic dispersion modeling and microscopic traffic dispersion modeling.

Most of the researches on the platoon dispersion modeling concentrate on the calibration of the TRANSYT platoon dispersion model. This is logical since the TRANSYT platoon dispersion model is the most widely used traffic dispersion model and the calibration heavily affects the outputs of the dispersion model and signal coordination system, and in fact improper calibration
results in excessive delay on the network. Almost all the efforts on this subject focused on finding good set of model parameters using best fit approach in order to minimize the difference between observed downstream flow profiles and the predicted one. This approach suffers from a very important shortcoming: it uses a limited dataset usually obtained from a specific site, and generalizes it for all the other places and traffic conditions. In the other hand, the results are usually described as the values of the parameters for an explanation of the site conditions. In the other word, the method is not an analytical method, and thus can not provide a general procedure for different situations and places.

In an attempt to overcome these drawbacks, Yu and Van Aerde (1995) developed a series of equations for calibrating the recurrence model based on the travel time statistics. Their approach overcomes the weaknesses of the goodness-of-fit approach, but suffers from a new problem. They did not realize that they used inconsistent units of time for different parts of the model, so their approach is correct only for one-second time steps and the error of the model increases as the time step size increases. In the other hand, however they provided a good procedure to calibrate the model, but they did not provide a way to use it in the popular TRANSYT software. These two issues have limited the application of their calibration procedure for traffic engineers. If these problems can be solved, users will be able to enjoy the Yu and Van Aerde’s calibration procedure to enhance the signal coordination plans.

Finally, beside the signal coordination systems, traffic microsimulators also use traffic dispersion models to increase the realism of the simulation process. Proper utilization of platoon dispersion phenomenon is a vital part of each traffic microsimulator. The conventional way to consider the traffic dispersion is to consider randomness around the deterministic car-following model. The most popular factor that has been used to account for drivers differences is the desired speed distribution or speed variability factor. It has been observed that the distribution of desired speed changes the steady-state behavior of a model, but unfortunately, no studies exist on this important issue. A quantitative analysis of this subject directly benefits the users of traffic simulation packages by increasing the accuracy of the calibration process.
CHAPTER 3

RESEARCH METHODOLOGY

The previous chapters identified the need for a comprehensive study of the limitations of the state-of-the-practice platoon dispersion models. This chapter introduces the proposed research approach in developing solutions to the identified limitations.

3.1 INTRODUCTION
This chapter describes the research methodology that is proposed to achieve the desired objectives. There are different methods utilized to predict platoon dispersion accurately. The most widely used platoon dispersion model is Robertson’s recursive model. A successful application of Robertson’s platoon dispersion model requires an appropriate calibration of the model parameters. The current research intends to develop better calibration procedures in order to enable the users to calibrate the model correctly. In addition, the research effort investigates alternative microscopic procedures for modeling the dispersion of traffic. Furthermore the possible links between platoon dispersion models and fuel consumption and emissions models is investigated. In the following sections, simple proposed methodologies to enhance the calibration methods and assess the environmental impacts (if applicable) are presented.

3.2 RESEARCH APPROACH
The research approaches include four basic tasks, as follows:

1. Identify the shortcomings and enhance the current recursive macroscopic platoon dispersion model,

2. Develop new calibration procedures for the recursive macroscopic platoon dispersion model,

3. Identify the critical variables that impact platoon dispersion behavior, and

4. Develop a calibration procedure to calibrate steady-state car-following models that accounts for the effect of speed variability on steady-state behavior.
3.2.1 IDENTIFY THE SHORTCOMINGS AND ENHANCE THE CURRENT RECURSIVE MACROSCOPIC PLATOON DISPERSION MODEL

Despite the significant impact the platoon dispersion parameters have on the signal timings that are estimated by the TRANSYT-7F software, the software manual does not provide an analytical framework for the calibration of the platoon dispersion model parameters. Usually the users use the default value or the values provided in the manual, which are derived from a limited number of studies using a best fit approach. Alternatively, Yu and Van Aerde (1995) developed an analytical framework for calibrating the platoon dispersion model parameters using a statistical analysis of link travel-time distribution. Specifically, Yu and Van Aerde proposed a set of formulas to calibrate the parameters of Robertson’s platoon dispersion model based on the average travel time and the standard deviation of the travel time. Yu and Van Aerde’s calibration procedure (1995) has a shortcoming; since Yu and Van Aerde considered travel times in units of seconds in the derivation of their calibration procedure, the procedure is only valid when a 1-second time step is considered. Consequently, the cyclic flow profile prediction error increases as the duration of the modeling time step increases.

Furthermore, using Yu and Van Aerde’s calibration procedure gives a travel time factor based on travel time statistics, and in many cases not equal to 0.8 which is assumed and fixed in TRANSYT software. This means a user can get a good set of parameters, but cannot use them, because of the software’s limitation.

To address the above mentioned problems, a series of simulations must be conducted to show the effect of step size on the results. To do this step, a set of traffic flow profiles on an arterial was used. The main advantages of using a microsimulator to produce such data are flexibility in generating data for different conditions, and the fact that microsimulator gives the individual travel time information which usually is not available for field data. However, it must be noticed that before using a microsimulator it must be validated in term of consistency with the real platoon dispersion pattern on streets.

3.2.2 DEVELOP NEW CALIBRATION PROCEDURES FOR THE RECURSIVE MACROSCOPIC PLATOON DISPERSION MODEL

After recognizing the problems and the factors that cause them, the next step is to develop a series of solutions to overcome the shortcomings. To accomplish these mission two main steps must be conducted:
1. A set of analytical generalizations of the Yu and Van Aerde calibration procedure is developed considering the effect of the aggregated flow pattern. This step needs a mathematical analysis of the recurrence model and Yu and Van Aerde’s calibration process.

2. Validate the proposed solutions using field and microsimulation data.

3.2.3 **DEVELOP A METHOD TO INCORPORATE THE EFFECT OF SPEED VARIABILITY IN THE CALIBRATION OF MICROSIMULATION MODELS**

In the field of microscopic traffic simulation, platoon dispersion can be modeled as differences in desired speeds for different drivers. In other words, the concept of platoon dispersion in microscopic traffic modeling is the way that randomness of a deterministic car-following model is captured. The conventional way that traffic dispersion phenomenon is modeled in microsimulators is to consider randomness around the deterministic car-following model, i.e. the deterministic desired speed or acceleration rate. It was observed that this randomness causes a change in steady-state behavior of the corresponding car-following model.

This research attempts to analyze the microscopic platoon dispersion modeling concept based on speed variability and develop a method to include the impact of speed variability in the calibration process. This task needs a study of the steady-state behavior of the available car-following models. This research assists microsimulation users by providing the basic tool to control the shape of the steady-state model, and increasing the accuracy of the calibration process.
MACROSCOPIC MODELING OF TRAFFIC DISPERSION: ISSUES AND PROPOSED SOLUTIONS

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Submitted to ASCE Journal of Transportation Engineering, 2004

ABSTRACT
The paper improves upon the Yu and Van Aerde calibration procedure of the TRANSYT-7F platoon dispersion model particularly for time steps that are greater than 1 s in duration and develops three generalized platoon dispersion models that explicitly account for the effect of the time step duration on traffic dispersion. The paper validates the proposed models utilizing two datasets. The first dataset includes field data that were gathered in Montréal, Canada, while the second dataset was generated using the INTEGRATION microscopic traffic-simulation software. The results demonstrate that the predicted flow profile using the proposed platoon dispersion models provides a good fit to field-observed and simulated profiles, regardless of the modeling time step that is considered, while the results also demonstrate the deficiencies of Yu and Van Aerde formulation, especially for time steps that are greater than 1 s in duration.

INTRODUCTION
Vehicles departing from a queue at a traffic signal typically travel in a platoon that disperses as vehicles travel further downstream. In part, the platoon dispersion is caused by differences in drivers’ desired speeds and, mostly, as a result of vehicle interaction with other vehicles entering and exiting the roadway, which is commonly known as the roadway side friction. Platoon dispersion models attempt to simulate the dispersion of a traffic stream as it travels along a roadway by attempting to estimate vehicle arrivals at downstream locations based on an upstream vehicle departure profile and a desired traffic-stream speed.

The most widely used platoon dispersion model is Robertson’s (1969) platoon dispersion model. This model, which is also known as the TRANSYT platoon dispersion model, has become a virtual universal standard platoon dispersion model and has been implemented in various traffic
simulation softwares. A successful application of Robertson’s platoon dispersion model requires an appropriate calibration of the model’s parameters. Specifically, Guebert and Sparks (1989) showed that the accurate calibration of the Robertson platoon dispersion model parameters was critical in developing effective and efficient traffic signal timing plans. Despite the significant impact the platoon dispersion parameters have on the signal timings that are estimated by the TRANSYT software, the software manual does not provide an analytical framework for the calibration of the platoon dispersion model parameters. The state-of-practice has been the use of a goodness-of-fit approach to calibrate the model parameters. Alternatively, Yu and Van Aerde (1995 and 2000) developed an analytical framework for calibrating the platoon dispersion model parameters using a statistical analysis of the link travel-time distribution. Specifically, Yu and Van Aerde proposed a set of formulae to calibrate the parameters of Robertson’s platoon dispersion model based on the average travel time and the standard deviation of the travel time, as will be described later in further detail.

This paper demonstrates that the accuracy of the predicted downstream flow profile using the parameters derived by Yu and Van Aerde is highly dependent on the duration of the modeling time step because the smoothing factor \((F)\) has units of time. Specifically, as the length of the time step increases, the prediction accuracy decreases. The paper highlights the deficiencies of the Yu and Van Aerde calibration procedure and develops a modified formulation that overcomes the identified deficiencies. The proposed models are derived using a generalized parametric second-by-second platoon dispersion analysis of the basic TRANSYT platoon dispersion model. The proposed models are then validated using two data sets: namely, a field and simulation dataset.

**Robertson’s Recursive Formulation**

The most commonly used macroscopic approach to the mathematical modeling of the platoon dispersion process is the Robertson platoon dispersion model, which was developed for the TRANSYT software (Robertson, 1969).

The basic Robertson recursive platoon dispersion model takes the following mathematical form:

\[
q_i^t = F \times q_{i-1} + (1 - F) \times q_{i-\Delta t} \tag{4.1}
\]
\[
F = \frac{1}{1 + \alpha \beta T_a}
\]  

[4.2]

Where:

\(q_{t-T}\): discharging flow over a time step \(\Delta t\) observed at the upstream signal at time \(t-T\);

\(q'_i\): flow rate over a time step \(\Delta t\) arriving at the downstream signal at time \(t\);

\(\Delta t\): modeling time step duration, measured in units of time steps;

\(T_a\): mean roadway travel time, measured in units of time steps;

\(T\): minimum travel time on the roadway, measured in units of time steps (\(T = \beta T_a\));

\(\alpha\): platoon dispersion factor (unitless);

\(\beta\): travel time factor (unitless); and

\(F\): smoothing factor (time steps\(^{-1}\)).

Seddon (1972) rewrote Equation 4.1 in the form

\[
q'_i = \sum_{i=T}^{+\infty} F(1-F)^{i-T} \times q_{t-i}
\]  

[4.3]

Where:

\(i\): the interval number for which an upstream flow is observed downstream. This integer variable ranges from \(T\) (minimum travel time in units of time steps) to infinity;

Equation 3 demonstrates that the downstream traffic flow that is computed using the Robertson platoon dispersion model follows a shifted geometric series. The geometric series estimates the contribution of an upstream flow in the \((t-i)\)th interval to the downstream flow in the \(t\)th interval.

A successful application of Robertson’s platoon dispersion model relies on the appropriate calibration of the model parameters. Robertson (1969) assumed the travel-time factor \(\beta\) to be fixed at a value of 0.8, and it has since been fixed at 0.8, while the platoon dispersion factor \(\alpha\) was allowed to vary between 0.0 and 1.0, depending on the level of friction along the roadway. The TRANSYT-7F User’s Guide (Wallace et al. 1984) recommends that the platoon dispersion factor \(\alpha\) vary depending on the site specific geometric and traffic conditions.
Several studies have demonstrated that the use of the TRANSYT-7F default platoon dispersion parameters and the use of a travel-time factor of 0.8 result in significant errors in the modeling of platoon movement along roadways and thus result in inefficient traffic signal timings. Yu and Van Aerde (1995 and 2000) not only demonstrated that the travel-time factor ($\beta$) depends on the platoon dispersion factor ($\alpha$) but also used the basic properties of the geometric distribution of Equation 4.3 to derive the following three equations for calibrating the parameters of the Robertson platoon dispersion model. The interested reader may refer to the literature (Yu and Van Aerde, 1995 and 2000) for a more detailed description of how the formulations were derived.

$$\beta = \frac{1}{1 + \alpha} \text{ or } \alpha = \frac{1 - \beta}{\beta}$$ \hfill [4.4]$$

$$\beta = \frac{2T_a \pm 1 - \sqrt{1 + 4 \sigma'^2}}{2T_a}$$ \hfill [4.5]$$

$$F = \frac{\sqrt{1 + 4 \sigma'^2} - 1}{2 \sigma'^2}$$ \hfill [4.6]$$

Where:

$\sigma'$ : standard deviation of link travel times (s), and

$T'_a$ : mean roadway travel time (s).

It should be emphasized that the $T'_a$ and $\sigma'$ parameters are in units of seconds as opposed to units of time steps, as was the case for the variable $T_a$ in Equations 4.1 through 4.3.

Equation 4.4 demonstrates that the value of the travel time factor ($\beta$) is dependent on the value of the platoon dispersion factor ($\alpha$), and thus a $\beta$ of 0.8, which is currently implemented in the TRANSYT software, results in inconsistencies in the formulations. Furthermore, Equation 4.6 demonstrates that the Robertson platoon dispersion model requires the calibration of a single parameter ($\alpha$), given that the travel-time factor ($\beta$) is dependent on the platoon dispersion factor, and can be calibrated from the expected roadway travel time ($T'_a$) and the travel-time variance ($\sigma'^2$). Incorporating Equation 4.4 into Equation 4.2, $F$ can be rewritten as

$$F = \frac{1}{1 + (1 - \beta)T_a} = \frac{1 + \alpha}{1 + \alpha(1 + T_a)}$$ \hfill [4.7]$$
Equation 7 demonstrates that the smoothing factor \((F)\) can be expressed as a function of either \(\beta\) or \(\alpha\). Using \(\alpha\) and \(\beta\) values that do not satisfy Equation 4.4 results in an average travel time that is inconsistent with the desired input average travel time.

**EXAMPLE APPLICATION OF STATE-OF-PRACTICE FORMULATIONS**

Seddon (1972) demonstrated that the Robertson platoon dispersion model assumes that the traffic-stream travel time distribution is a shifted geometric distribution. It is demonstrated in this paper that because Yu and Van Aerde considered travel times in units of seconds in deriving their calibration procedure, the procedure is only valid for a 1 s time step. Specifically, the cyclic flow profile prediction error is demonstrated to increase as the modeling time step increases because the smoothing factor \((F)\) is not dimensionless, which results in travel time probabilities that are inconsistent with the desired travel times.

For illustration purposes the impact of the modeling time step on the downstream flow profile prediction error is analyzed for a simple upstream flow profile, as demonstrated in Figure 4.1. The downstream flow profile, for different time step durations, was compared to the predicted flow profiles considering a 1 s step size. The figure distinguishes between prediction and data aggregation errors, in order to demonstrate the need to address the prediction error problem. The figure clearly demonstrates that the predicted downstream flow profile changes as the modeling step size increases and that the prediction error increases with an increase in the modeling step size. Specifically, as the modeling step size increases, the model erroneously estimates a higher level of dispersion.

**PROPOSED TRAFFIC DISPERSION MODEL ENHANCEMENTS**

The shortcomings of the Yu and Van Aerde calibration procedure arises from two factors; first, the smoothing factor \((F)\) is not unitless, and second, lack of consistency in units of several parameters within the formulation. Specifically, the unit of time used in calculating the \(\alpha, \beta\) and \(F\) parameters are in seconds while the units of time for the remainder parameters are in units of time interval durations. The following sections describe three proposed formulations that overcome the identified shortcomings.
A simple approach to overcome the shortcoming of Yu and Van Aerde method for calibrating the Robertson platoon dispersion model is to disaggregate the upstream flow profile to a 1 s time step assuming that the flow rate is constant in each time step (i.e. ignoring the variability within the time step). Subsequently, the dispersion of the disaggregated upstream flow profile can be performed utilizing Equation 4.3 to predict a disaggregated downstream flow profile using the parameters derived from Equations 4.4 through 4.6. Finally, the downstream disaggregated flow profile can then be aggregated to the desired time step to estimate the aggregated downstream flow profile. The proposed approach addresses the prediction accuracy but does not address the aggregation accuracy, which was demonstrated in Figure 4.1 to be minor compared to the prediction accuracy.

**FIRST APPROACH: SECOND-BY-SECOND PARAMETRIC ANALYSIS**

Figure 4.1: Upstream and predicted downstream flow profiles.
The proposed approach is initially described and derived for a step size of 3 s for illustration purposes and is then generalized for any step size. Figure 4.2 illustrates the second-by-second parametric analysis for a 3 s time-step example.

Figure 4.2: Parametric second-by-second platoon dispersion derivation.

Considering a single 3 s flow rate \( (q) \) departing from an upstream traffic signal, the flow can be disaggregated into three equal 1 s flow profiles-each of flow rate \( q \). Subsequently, the downstream flow profile for each of the three 1 s flow pulses can be estimated using Equation 4.3, as illustrated in Figure 4.2. Subsequently, the disaggregated flow profile can be aggregated to generate the desired 3 s flow profile. The first 1 s upstream flow rate of \( q \) results in flows \( q'_1 \), \( q'_2 \), and \( q'_3 \) during the first 3 s time interval of the downstream profile. Similarly, the second 1 s upstream flow pulse, which is temporally shifted by 1 s, produces flows \( q'_{1} \) and \( q'_{2} \) during the first 3 s time interval at the downstream location. Finally, the third 1 s flow of \( q \) results in a
single flow rate of $q'_{ij}$ at the downstream location during the first 3 s interval. Aggregating the downstream flow profile considering a 3 s time step produces a flow rate of $q'_{t1}$, as

$$q'_{t1} = \frac{1}{3} \left[ (q'_{r1} + q'_{r2} + q'_{r3}) + (q'_{s1} + q'_{s2} + q'_{s3}) \right] = q'_{r1} + \frac{2}{3} q'_{r2} + \frac{1}{3} q'_{r3} \tag{4.8}$$

Substituting the downstream flows that are derived using Equation 4.3 into Equation 4.8, the aggregated first 3 s flow rate can be computed as

$$q'_{t1} = \left[ (1 - F)^0 + \frac{2}{3} (1 - F)^1 + \frac{1}{3} (1 - F)^2 \right] \times F \times q \tag{4.9}$$

For the second and third time steps the same approach can be applied as follows

$$q'_{t1} = \frac{1}{3} \left[ (q'_{r1} + q'_{r2} + q'_{r3}) + (q'_{s1} + q'_{s2} + q'_{s3}) \right] = \frac{1}{3} q'_{s2} + \frac{2}{3} q'_{s3} + q'_{s4} + \frac{2}{3} q'_{s5} + \frac{1}{3} q'_{s6}
= \left[ \frac{1}{3} (1 - F)^1 + \frac{2}{3} (1 - F)^2 + \frac{1}{3} (1 - F)^3 \right] \times F \times q \tag{4.10}$$

and

$$q'_{t1} = \frac{1}{3} \left[ (q'_{r1} + q'_{r2} + q'_{r3}) + (q'_{s1} + q'_{s2} + q'_{s3}) \right] = \frac{1}{3} q'_{s5} + \frac{2}{3} q'_{s6} + q'_{s7} + \frac{2}{3} q'_{s8} + \frac{1}{3} q'_{s9}
= \left[ \frac{1}{3} (1 - F)^2 + \frac{2}{3} (1 - F)^3 + \frac{1}{3} (1 - F)^4 + \frac{2}{3} (1 - F)^5 \right] \times F \times q \tag{4.11}$$

Generalizing Equations 4.9, 4.10, and 4.11 for all time intervals, the aggregated 3 s downstream flow profile can be computed, as follows.

$$q'_{t} = \sum_{i=0}^{n} \left[ F_{q_{t-i}} \times \sum_{k=\max[3i-2,0]}^{3i+2} \left( (1 - F)^k \times \frac{3-i-k}{3} \right) \right] \tag{4.12}$$

Where:

$q'_{t}$ : aggregated 3 s downstream flow rate at time interval $t$ (veh/h) (where $t$ represents the mid-point of the time interval);

$q_{t}$ : aggregated 3 s upstream flow rate at time interval $t$ (veh/h) ; and

$F$ : smoothing factor calculated using Equation 4.7.
Generalizing Equation 4.12 for any bin size, the final formulation is derived as

\[
q_i' = \sum_{i=0}^{\infty} \left( F \times q_{i-(T'+in)} \times \sum_{k=\max(i,n-(n-1),0)}^{n+(n-1)} \left( 1-F \right)^{k} \times \frac{n-i-n-k}{n} \right)
\]

[4.13]

Where:

- \( q_i' \): aggregated n-second downstream flow rate at time interval \( t \) (veh/h);
- \( q_t \): aggregated n-second upstream flow rate at time interval \( t \) (veh/h); and
- \( n \): time step duration (s).

\( T' \) is equal \( \beta \cdot T' \), while the parameters \( \alpha \), \( \beta \), and \( F \) are calibrated using Equations 4.4, 4.5 and 4.6, respectively. Equation 4.13 demonstrates that the aggregated downstream traffic flow depends on the size of the time interval. In fact, Equation 4.13 is a generalized form of the geometric distribution that ensures consistency across different time interval sizes, since it ensures consistency in the time units across the various model parameters.

SECOND APPROACH: SECOND-BY-SECOND PARAMETRIC ANALYSIS IGNORING DIFFERENCES IN DISPERSION WITHIN A TIME INTERVAL

While the approach that was described earlier generalizes the Yu and Van Aerde calibration procedure for time intervals greater than 1 s in duration, it is computationally intensive and complex. The model complexity arises from the fact that the model disaggregates a flow profile to its lowest temporal resolution (time interval of 1 s) prior to dispersing the flow profile and subsequently re-aggregates the downstream flow profile. A simpler approach can be derived by performing a second-by-second parametric analysis, however in this case the dispersion of each 1 s flow pulse within the modeling time interval is assumed to be identical.

In this approach, it is assumed that every 1 s upstream flow pulse within the time interval \((i-T)\) produces the same flow in the \(i^{th}\) time interval of the downstream profile. In other words, all 1 s upstream flows in interval \((i-T)\) have the same downstream profile as the downstream flow shown in Figure 4.2a. Performing the same analysis that was done in the previous section \(q_{i+1}'\) can be calculated as follows:

\[
q_i' = \frac{1}{3} \left[ (q_i' + q_{i+2} + q_{i+3}) + (q_i' + q_{i+2} + q_{i+3}) + (q_i' + q_{i+2} + q_{i+3}) \right] = q_i' + q_{i+2} + q_{i+3}
\]

[4.14]
Repeating the same derivation that was explained in the previous section the Final formulation can be written as

\[
q_i' = \sum_{i=0}^{\infty} \left( F \times q_{i-(T+i.n)} \times \sum_{k=i}^{(n-1)} (1-F)^k \right)
\]  \[4.15\]

Comparing Equation 4.15 to Equation 4.3, it is evident that Equation 4.15 is identical to Equation 4.3 and therefore can be recast as

\[
\sum_{k=i.n}^{i+n(n-1)} F_i (1-F)^k = g(i-T) = GCDF(i.n+n) - GCDF(i.n)
\]  \[4.16\]

Where:

\(GCDF(\cdot)\) : Cumulative probability of the shifted geometric distribution.

Equation 4.15 demonstrates that the aggregated downstream traffic flow profile can be estimated using the corresponding geometric distribution while ensuring consistency in the scaling between the geometric distribution and cyclic flow profile. Like Equation 4.13, Equation 4.15 ensures consistency between the statistical dispersion distribution and the temporal time steps of the upstream cyclic profile, however the accuracy of the dispersion model is less than the earlier formulation (Equation 4.13) because traffic dispersion is assumed to be identical for all 1 s sub-intervals within the modeling time interval.

**Third Approach: Equivalent Dispersion Distribution**

Both of the above approaches consider a geometric distribution for a 1 s step size and generalize the formulation to consider non-1 s time intervals. Another approach to achieve consistency across the various step sizes is to find a set of \(\beta, F\) and \(\alpha\) parameters for the desired step size. To obtain this new set of parameters, Equations 4.4, 4.5, and 4.6 can be rewritten as

\[
\alpha_n = \frac{1-\beta_n}{\beta_n}
\]  \[4.17\]

\[
\beta_n = \frac{2T_s + 1 - \sqrt{1 + 4\sigma^2}}{2T_s}, \text{ and}
\]  \[4.18\]

\[
F_n = \frac{\sqrt{1 + 4\sigma^2} - 1}{2\sigma^2}
\]  \[4.19\]
Where:

\( \beta_n, F_n \) and \( \alpha_n \): model parameters for step size of \( n \) seconds,

\( \sigma \): standard deviation of link travel times (in units of time steps) equals to \( \sigma'/n \), and

\( T_a \): mean roadway travel time (in units of time steps) equals to \( T'_a/n \).

Substituting \( \sigma \) and \( T_a \) for \( \sigma'/n \) and \( T'_a/n \) in Equations 4.18 and 4.19 gives

\[
\beta_n = \frac{2T'_a + n - \sqrt{n^2 + 4\sigma'^2}}{2T'_a}, \quad \text{and} \\
F_n = n \times \frac{\sqrt{n^2 + 4\sigma'^2} - n}{2\sigma'^2} \quad \text{[4.20]}
\]

Equations 4.20 and 4.21 demonstrate that the parameters \( \beta_n, F_n \) and \( \alpha_n \) are dependent on the size of the modeling time interval. In addition, it can be observed that the relationship between \( \beta_n, F_n \), and \( \alpha_n \) is as

\[
F_n = \frac{1}{1 + \alpha_n \beta_n T_a} = \frac{n}{n + (1 - \beta_n)T'_a} \quad \text{[4.22]}
\]

**MODEL VALIDATION**

This section describes the validation effort of the proposed models using two datasets. These datasets include a field dataset that was gathered in Montréal, Canada, by Manar (1994) and a dataset that was generated as part of this study using the INTEGRATION microscopic traffic simulation software.

**MONTRÉAL FIELD DATA**

The field dataset that is utilized for validation purposes was gathered by Manar (1994) in Montréal, Canada. The test site is a section of Papineau Ave. between Rue De Louvein and Emile Journault Ave., Montréal, Canada. Papineau Ave. is a 6-lane arterial roadway (3-lanes per direction of travel). Three video cameras were installed in the field to observe and record the progression of platoons along the roadway. The video cameras were set up at locations A, B, and C, as illustrated in Figure 4.3. A single bus line passed through the section with a frequency of 10 minutes, and thus had minimum impacts on the study. Unfortunately, the volume-to-capacity (v/c) ratio along the study section was not provided in the text and thus cannot be reported.
Using the video data, the number of vehicles within 2 s intervals was recorded over multiple cycles at the three locations, however only the average flow profile was provided in the literature. Because the travel time mean and variance were not available, the platoon dispersion parameters were estimated by minimizing the sum of the squared error between the observed and estimated 1 s flow profiles by varying the $T'_a$ and $\sigma'$ parameters and calculating the $\beta$, $F$ and $\alpha$ parameters using Equations 4.17, 4.18 and 4.19.

![Figure 4.3: Montreal field test site and simulated network configuration.](image)

The upstream flow profile (at point A) was then aggregated to reflect time steps of 4 and 6 s in addition to the base case of 2 s. The downstream flow profile was estimated at locations B and C using the proposed dispersion models by calibrating the $\alpha$ and $\beta$ parameters using the Yu and Van Aerde procedures, as summarized in Table 4.1.

<table>
<thead>
<tr>
<th>Dist. (m)</th>
<th>Step Size (s)</th>
<th>Travel Time (s)</th>
<th>Platoon Dispersion Factor - $\alpha$</th>
<th>Travel Time Factor - $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>2</td>
<td>19.00 7.60</td>
<td>0.59 0.59 0.59 0.59 0.59 0.59</td>
<td>0.63 0.63 0.63 0.63 0.63 0.63</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>0.45 0.45 0.45 0.45 0.45 0.45</td>
<td>0.37 0.37 0.37 0.37 0.37 0.37</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td>0.37 0.37 0.37 0.37 0.37 0.37</td>
<td>0.73 0.73 0.73 0.73 0.73 0.73</td>
</tr>
<tr>
<td>300</td>
<td>2</td>
<td>30.50 11.3</td>
<td>0.54 0.54 0.54 0.54 0.54 0.54</td>
<td>0.65 0.65 0.65 0.65 0.65 0.65</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>0.45 0.45 0.45 0.45 0.45 0.45</td>
<td>0.39 0.39 0.39 0.39 0.39 0.39</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td>0.39 0.39 0.39 0.39 0.39 0.39</td>
<td>0.72 0.72 0.72 0.72 0.72 0.72</td>
</tr>
</tbody>
</table>

Figure 4.4 demonstrates how the estimated downstream flow profiles compared to the field-observed profiles for a 6 s time-step configuration. The figure clearly demonstrates a deterioration in the accuracy of the estimated downstream flow profile when the Yu and Van Aerde formulation is applied. Alternatively, the proposed models are able to estimate the downstream profile with a level of accuracy that does not deteriorate as the modeling time step
increases, as demonstrated in Figure 4.5. Figure 4.5 clearly depicts an increase in the Yu and Van Aerde model overall prediction error as the analysis step-size increases. The overall prediction error is composed of two components: a prediction and a white noise random error. As the modeling time step increases, the random error decreases, given that more vehicles are considered within each time step. Consequently, in the case of Yu and Van Aerde formulation, although the random error decreases as the time step increases, the prediction error increases substantially, resulting in an increase in the overall error. In the case of the proposed models, however, the total error decreases slightly as the time step increases because the prediction error remains virtually constant, while the random error decreases as the time step increases.

![Location C: 6 s Time Step](image)

**Figure 4.4: Observed and predicted downstream flow profiles (Montréal Data).**

Amongst all the models that were examined, excluding the Yu and Van Aerde formulation, the proposed model-1 produces the least error compared to all other cases, which is expected since it utilizes a higher level of resolution (1 s analysis). The proposed model-3 produces the second highest accuracy followed by model-2. The results demonstrate the level of consistency that each model is able to maintain for different time step values.

**Microscopic Simulation Analysis**

An additional validation effort was conducted using the INTEGRATION (Van Aerde and Rakha, 1995 and 2003; Rakha and Crowther, 2003) traffic-simulation software. The INTEGRATION software, which was developed over the past two decades, has not only been validated against
standard traffic flow theory (Rakha and Van Aerde, 1996; Rakha and Crowther, 2002), but has also been utilized for the evaluation of real-life applications (Rakha et al., 1998; Rakha et al., 2000). The INTEGRATION software utilizes a system of car-following models to capture both steady state and non-steady state longitudinal vehicle motion along a roadway section. The steady state behavior is characterized by vehicles traveling at identical cruising speeds \((du/dt=0)\). Alternatively, the non-steady state behavior characterizes how vehicles move from one steady state to another, which involves either vehicle decelerations or accelerations. In addition, the software models vehicle lane-changing behavior to capture the lateral movement of vehicles along a roadway segment and has been validated against field data (Rakha and Zhang, 2005).

![Location B: 200 m Downstream](image1)

![Location C: 300 m Downstream](image2)

**Figure 4.5: Error in predicted downstream flow profile.**

The steady-state longitudinal motion is based on a link-specific microscopic car-following relationship that is calibrated macroscopically to yield the appropriate target aggregate speed-flow attributes for that particular link. The steady state car-following model, which was proposed by Van Aerde (1995) and Van Aerde and Rakha (1995), combines the Pipes and Greenshields models into a single-regime model (Rakha and Crowther, 2002), as
Specifically, the first two terms constitute the Pipes steady state model (Pipes, 1953), and the third term constitutes the Greenshields steady state model (Greenshields, 1935). This combination provides a functional form that includes four parameters that require calibration using field data (constants $c_1$, $c_2$, $c_3$ and the roadway free-speed $u_f$). The first two terms of the relationship provide the linear increase in vehicle speed as a function of the distance headway, and the third term introduces curvature to the model and ensures that the vehicle speed does not exceed the free-speed. Equations 4.24 through 4.27 are utilized to compute the $c_1$, $c_2$, and $c_3$ constants based on four parameters; the roadway free-speed, speed-at-capacity, capacity, and jam density (Rakha and Crowther, 2002). These parameters can be calibrated to loop detector data (Van Aerde and Rakha, 1995).

\[
h = c_1 + c_2 u + \frac{c_2}{u_r - u} [4.23]\]

Once the vehicle’s speed is computed, the vehicle’s position is updated every 0.1 seconds to reflect the distance that it travels during each previous 0.1 seconds. The vehicle’s headway and speed is then re-computed. The modeling of traffic dispersion is achieved by modeling stochastic vehicle speeds about the steady-state speed-flow-density relationship. Specifically, the user specifies a speed coefficient of variation (CV), and the software models a normally distributed white noise about the steady-state desired speed. A full validation of traffic dispersion modeling within the INTEGRATION software is beyond the scope of this paper; however a brief discussion is presented. A future publication will focus on the microscopic aspects of modeling traffic dispersion.
Several studies (Dion et al. 2004; Hellinga et al. 2004; Rakha and Zhang, 2004) showed that INTEGRATION’s outputs are consistent with both observed field data and fundamental traffic flow theory. However, prior to utilizing the INTEGRATION software in this study, a basic validation of the traffic dispersion module is presented using two sample field datasets. The first dataset was gathered by Denney (1989) in Houston, Texas and the second dataset was gathered by Castle and Bonniville (1985) in Kuwait City, Kuwait. The Houston data contained the observed average flow profiles at an upstream signal and at a checkpoint 300 m (990 ft) downstream on a 3-lane arterial. The observed average speed and standard deviation of speeds were reported as 48.3 km/h (44 ft/s) and 5.9 km/h (5.4 ft/s), respectively which results in an observed speed coefficient of variation (CV_{obs}) of 12.3%. Given that vehicle interactions may reduce vehicle speed variability a slightly higher value of CV was input to the simulation software (CV_{in} = 15%). The network was modeled as a 300 m 3-lane link, with a free flow speed of 50 km/h, a speed-at-capacity of 40 km/h, a saturation flow of 1800 veh/h/lane, and jam density of 100 veh/km/lane. Figure 4.6a and b illustrate the resulting average simulated flow
profile from 10 random simulations at downstream locations for both datasets superimposed on
the observed flow profiles. In the case of the Houston dataset, the simulated average speed and
standard deviation were 48.2 km/h and 6.1 km/h (CV_{out}=12.7%), which is very similar to the
field observed parameters. Consequently, the results demonstrate a high degree of consistency
between simulated and field observed data in terms of aggregate trip measures (trip mean and
variance) and in the progression of vehicles within platoons.

The Kuwait City data contained the observed profiles at an upstream traffic signal and two
checkpoints 430 and 953 m downstream for a 4-lane arterial (Riyadh street). The reported
average speeds to these check points were 67.3 and 71.1 km/h, respectively. The standard
deviation of the speed was not provided in the literature, but using Equations 4.2 and 4.21
assuming reported average journey times of 23 and 48.2 s the speed CV_{obs} was estimated to be
25%. Again, the network was coded in the INTEGRATION software as a 953 m 4-lane link,
with a free flow speed of 73 km/h, a speed-at-capacity of 68 km/h, a saturation flow rate of 1800
veh/h/lane, a jam density of 100 veh/km/lane, and CV_{in} of 27.5%. The resulting simulated
average speeds and coefficient of variations were 67.9 km/h and 18.7% and 69.7 km/h and
17.7%, at the 430 for 953 m checkpoints, respectively. These results that are illustrated in Figure
4.6 demonstrate that INTEGRATION provides a pattern of traffic progression and dispersion
that is consistent with field observed traffic behavior both for short and long distances, and
therefore can be used for the modeling of platoon dispersion behavior.

Subsequently, a synthetic dataset similar to the Montréal field dataset was constructed for
validation purposes. Vehicles departed from an upstream traffic signal and were monitored as
they traveled downstream along a three-lane roadway. Three loop detectors were placed within
the simulation environment. The first loop detector was located immediately upstream the
signalized intersection, while the other two detectors were located 200 and 300 m downstream of
the traffic signal, respectively. The loop detectors recorded data at 2 s intervals as was done in
the Montréal case study. Figure 4.3 depicts the network layout, while Table 4.2 summarizes the
roadway and network characteristics that were simulated.
Table 4.2: Characteristics of simulated roadway.

<table>
<thead>
<tr>
<th>Link Characteristic</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roadway length (m)</td>
<td>300</td>
</tr>
<tr>
<td>Free-flow speed (km/h)</td>
<td>50</td>
</tr>
<tr>
<td>Speed-at-capacity (km/h)</td>
<td>35</td>
</tr>
<tr>
<td>Capacity (veh/h/lane)</td>
<td>1800</td>
</tr>
<tr>
<td>Jam density (veh/km/lane)</td>
<td>100</td>
</tr>
<tr>
<td>Number of lanes</td>
<td>3</td>
</tr>
<tr>
<td>Number of loop detectors</td>
<td>3</td>
</tr>
<tr>
<td>Speed coefficient of variation</td>
<td>10%</td>
</tr>
<tr>
<td>Entering headway distribution</td>
<td>100% Random (Exponential Distribution)</td>
</tr>
<tr>
<td>Total Simulation Time (s)</td>
<td>600</td>
</tr>
</tbody>
</table>

The simulation run continued for 600 s and consisted of six distinct platoons of vehicles that departed from the upstream traffic signal. Travel-time variability was captured by modeling randomness in vehicle speeds as a random variable that followed a normal distribution with a CV=10% about the mean steady-state desired speed. All simulated vehicles were passenger cars and set as probes to record their individual travel time experiences. These travel times were utilized to compute the expected travel time and travel time variance, which in turn were utilized to calibrate the $\alpha$ and $\beta$ parameters, as was described earlier. Additionally, no mid-block flow were considered and all vehicles are allowed to perform lane changing based on INTEGRATION’s lane changing rules (Rakha and Zhang, 2005). The demand to capacity (v/c) ratio of the approach was 1.12, which resulted in an over-saturated approach. A full sensitivity analysis of v/c ratio on traffic dispersion was discussed by Manar (1994). A summary of the calibrated parameters is provided in Table 4.3. Two time steps were considered in the analysis: namely, a 2 and 6 s time step. The simulated and estimated downstream flow profiles were computed using the proposed models and the Yu and Van Aerde formulation. Figure 4.7 demonstrates the downstream flow profile for the proposed model-1 (which produced the least error among all the proposed models) and the Yu and Van Aerde formulation for a 6 s time step. As was the case for the field data analysis, the accuracy of Yu and Van Aerde formulation deteriorates as the time step increases, while the accuracy of the proposed models increases slightly, as demonstrated in Figure 4.8.
Table 4.3: Characteristics of simulated dataset.

<table>
<thead>
<tr>
<th>Dist. (m)</th>
<th>Step Size (s)</th>
<th>Travel Time (s)</th>
<th>Platoon Dispersion Factor - $\alpha$</th>
<th>Travel Time Factor - $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>2</td>
<td>17.38</td>
<td>1.59</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>2</td>
<td>25.44</td>
<td>2.29</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.7: Simulated and predicted flow profiles.

Figure 4.8: Error in predicted downstream flow profiles.
DISCUSSION OF RESULTS
Farzaneh and Rakha (2005) studied the effect of travel distance and various platoon dispersion parameters on the efficiency of the Robertson recursive model. A full description of the results is beyond the scope of this paper, however it suffices to mention that the study concluded that the approach delay is more sensitive to the $\beta$ parameter than it is to the $\alpha$ parameter, and that the effect of $\beta$ is more significant for larger signal spacing distances. Alternatively, other studies have shown that using a unique value of $\alpha$ provides a reasonable accuracy (Retzko and Schenk, 1993), which together with the results of Farzaneh and Rakha study implies that using a fixed value of $\beta$ and calibrating $\alpha$, as is implemented in TRANSYT, is only appropriate for short distances (less than 1 km).

In comparing the different calibration methods as part of this study, two factors were considered, namely the accuracy and the simplicity of each method for practical use. The methods developed in this paper provide adequate accuracy and require minimum data for calibration purposes, which makes the proposed methods a better choice for practical use. Furthermore the third proposed method offers a very simple approach to modeling traffic dispersion.

Finally, it should be noted that the Robertson recursive dispersion model assumes that vehicles travel at their desired speed and are not constrained by the surrounding traffic. In other words, this model does not account for the interaction of vehicles in a platoon. This assumption results in a modeling of larger dispersion than is typically observed in the field or within a simulation environment. Based on the results of this study it is demonstrated that the recursive platoon dispersion models are adequate for the modeling of traffic dispersion over short travel distances (less than 1 km), however further studies are required to validate traffic dispersion models for longer travel distances.

STUDY CONCLUSIONS
The paper demonstrates the importance of Yu and Van Aerde calibration procedure for the commonly accepted Robertson platoon dispersion model, which is implemented in the TRANSYT software. The paper demonstrates that the formulation results in an estimated downstream cyclic profile with a margin of error that increases as the size of the time step increases. In an attempt to address this shortcoming, the paper proposes the use of three enhanced geometric distribution formulations that explicitly account for the time-step size within
the modeling process. The proposed models were validated against field and simulated data. The results clearly demonstrate that the proposed model prediction error is not affected by the size of the modeling step size.

It is anticipated that the implementation of the proposed formulations can enhance the accuracy of traffic dispersion modeling that is key to the design of off-line and real-time traffic-signal control systems. Furthermore, the proposed models can be integrated within an Advanced Traveler Information System (ATIS) to enhance dynamic roadway travel time predictions.

ACKNOWLEDGEMENT
The authors acknowledge the financial support of the Mid-Atlantic University Transportation Center (MAUTC) in conducting this research effort. The authors also acknowledge the anonymous reviewers for enhancing the quality of the paper.

REFERENCES
Manar, A. (1994). Modelisation de la Dispersion du Trafic Entre les Carrefours, PhD Dissertation at the Civil Engineering Department, Universite de Montréal, Montréal, Canada.


CALIBRATION OF TRANSYT TRAFFIC DISPERSION MODEL: ISSUES AND PROPOSED SOLUTIONS

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Submitted to ASCE Journal of Transportation Engineering, 2005

ABSTRACT
The paper demonstrates some inherent limitations of the TRANSYT software with regards to the calibration of the recurrence platoon dispersion model and more specifically, the modification of the travel time factor. Subsequently, the paper develops a procedure that overcomes this limitation by adjusting the average travel time in the model in order to control the value of travel time factor indirectly. Furthermore, the paper presents numerical examples in order to provide a preliminary investigation of different calibration procedures of the recurrence relationship. Although the dataset used for this purpose was generated using the INTEGRATION microscopic traffic-simulation software, the procedures are general and intended for use with field data. The calibration procedure that is developed in this paper is demonstrated to produce the best results in terms of simplicity and accuracy.

INTRODUCTION
Interdependence of the neighboring signals in a traffic signalized network and proper coordination of these signals has been the subject of many studies. The interest in the subject arises from the fact that traffic signals are the most influential traffic control devices in urban and arterial networks. A well-designed traffic signal system ensures adequate traffic flow through the network, while an inefficient traffic signal system produces excessive delay, frustration, and wasted fuel.

Among the different signal coordination methods, the Road Research Laboratory (RRL) Combination Method (Hillier, 1966-1965) is the most widely used traffic signal coordination procedure. The combinational technique is a computer-based method that computes the set of optimum traffic signal offsets that minimizes the total delay within a network. The method
utilizes the departure flow profile at each intersection to estimate the arrival platoon at the downstream signalized intersection. In modeling the movement of platoons along roadways, platoon dispersion models attempt to capture the dispersion of a platoon as it travels downstream. These models estimate vehicle arrivals at downstream locations based on an upstream vehicle departure profile and an average traffic-stream space-mean speed.

The most widely used platoon dispersion model is Robertson’s (1969) platoon dispersion model. This model has become a virtual universal standard platoon dispersion model and has been implemented in various traffic-simulation software, including TRANSYT (Robertson, 1969), SCOOT (Hunt et al., 1981), SATURN (Hall et al., 1980), and TRAFLO (Lieberman and Andrews, 1980). A successful application of Robertson’s platoon dispersion model requires an appropriate calibration of the model’s parameters, which include the platoon dispersion factor ($\alpha$) and the travel time factor ($\beta$). Specifically, Guebert and Sparks (1989) showed that the accurate calibration of the Robertson platoon dispersion model parameters was critical in developing effective and efficient traffic signal timing plans. Despite the significant impact that platoon dispersion parameters have on the effective modeling of traffic dispersion and their subsequent impact on the selected optimum signal timings, the software’s structure only allows the modeler to modify one of the two parameters that characterize traffic dispersion, namely the platoon dispersion factor. Alternatively, the software assumes that the travel time factor is fixed at 0.8.

The objectives of this paper are three-fold. First, the paper demonstrates the limitations of the TRANSYT software with regards to calibrating the platoon dispersion model. Second, the paper proposes a methodology that enables the users to calibrate the TRANSYT dispersion model effectively by providing an approach for controlling the travel time factor indirectly using the basic properties of Robertson’s recurrence relationship. Third, the paper compares different calibration procedures and demonstrates the effectiveness of these calibration procedures using some example applications.

**TRANSYT Traffic Dispersion Model**

This section describes the state-of-practice TRANSYT platoon dispersion model. The calibration procedures and enhancements of the Robertson’s platoon dispersion model are also described.
ROBERTSON’S RECURSIVE FORMULATION

Robertson (1969) developed an empirical recursive relationship to describe the dispersion of traffic, which forms the core of the popular TRANSYT software, commonly known as TRANSYT-7F in North America. Because of the simplicity of applying the recursive formulation, Robertson’s model has become the standard platoon dispersion model and has been incorporated in a number of softwares.

The basic Robertson’s recursive platoon dispersion model takes the following mathematical form:

\[ q'_i = F(q_{i-T} + (1-F)q'_{i-M}) \]  \[ F = \frac{1}{1 + \alpha\beta T_a} \]

Where:

- \( q_{i-T} \): discharging flow over a time step \( \Delta t \) observed at the upstream signal at time \( t-T \);
- \( q'_i \): flow rate over a time step \( \Delta t \) arriving at the downstream signal at time \( t \);
- \( \Delta t \): modeling time step duration, measured in units of time steps;
- \( T_a \): mean roadway travel time, measured in units of time steps;
- \( T \): minimum travel time on the roadway, measured in units of time steps (\( T = \beta T_a \));
- \( \alpha \): platoon dispersion factor (unitless);
- \( \beta \): travel time factor (unitless); and
- \( F \): smoothing factor (time steps\(^{-1}\)).

Seddon (1972) rewrote Equation 5.1 in the form

\[ q'_i = \sum_{i=T}^{\infty} F_i (1-F)^{i-T} q_{t-i} \]

Where:

- \( i \): the interval number for which an upstream flow is observed downstream. This integer variable ranges from \( T \) (minimum travel time) to infinity;
- \( q_{t-i} \): discharging flow over a time step \( \Delta t \) observed at the upstream signal at time \( t-i \);
Equation 5.3 demonstrates that the downstream traffic flow that is computed using the Robertson platoon dispersion model follows a shifted geometric series. The geometric series estimates the contribution on an upstream flow in the \((t-i)^{th}\) interval to the downstream flow in the \(t^{th}\) interval. Robertson (1969) assumed the travel-time factor \(\beta\) to be fixed at a value of 0.8, and it has since been fixed at 0.8 in the TRANSYT software, while the platoon dispersion factor \(\alpha\) was allowed to vary between 0.2 and 0.5, depending on the level of friction along the roadway. The TRANSYT-7F User’s Guide (Wallace et al, 1984) recommends that the platoon dispersion factor \(\alpha\) vary depending on the site specific geometric and traffic conditions and provides three recommended values for three roadway conditions, namely low friction, moderate friction, and high friction (Table 5.1). The typical procedure for calibrating the platoon dispersion factor is to select the platoon dispersion factor that minimizes the sum-of-squared error between field-observed and estimated downstream flow profiles for a given upstream flow profile.

<table>
<thead>
<tr>
<th>Roadway Characteristic</th>
<th>Definition of Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy friction</td>
<td>Combination of parking, moderate to heavy turns, moderate to heavy pedestrian traffic, narrow lane width, traffic flow typical of CBD</td>
</tr>
<tr>
<td>Moderate friction</td>
<td>Light-turning traffic, light-pedestrian traffic, 11 to 12 ft lanes, possibly divided; typical of well-designed CBD arterial</td>
</tr>
<tr>
<td>Low friction</td>
<td>No parking, divided, turning provision, 12-ft lane width; suburban high-type arterial</td>
</tr>
</tbody>
</table>

**CALIBRATION OF TRANSYT’S DISPERSION MODEL**

Since the development of the Robertson platoon dispersion model, a number of studies have been conducted to evaluate the model parameters. Most of these studies used a best fit approach to find the appropriate values of \(\alpha\) and \(\beta\) as summarized by McCoy et al. (1983). These studies also demonstrated that the use of the TRANSYT-7F default platoon dispersion parameters results in significant errors in the modeling of platoon movement along roadways and thus results in inefficient traffic signal timings; however, these studies did recommend an alternative calibration procedure.

Yu and Van Aerde (1995 and 2000) not only demonstrated that the travel-time factor \(\beta\) is dependent on the platoon dispersion factor \(\alpha\) but also developed a method for calibrating the
Robertson platoon dispersion factors \((\alpha \text{ and } \beta)\) directly from the statistical properties of the travel-time experiences of individual vehicles. Specifically, the authors used the basic properties of the geometric distribution of Equation 5.3 to derive the values of the travel time factor and platoon dispersion factor from the expected \((T'_a)\) roadway travel time and the travel-time variance \((\sigma'^2)\).

Rakha and Farzaneh (2005) showed that because Yu and Van Aerde (1995 and 2000) considered travel times in units of seconds in the derivation of their calibration procedure, the procedure is only valid when a 1-second time step is considered. Consequently, the cyclic flow profile prediction error increases as the duration of the modeling time step increases. Rakha and Farzaneh (2005) also provided three enhanced formulations to overcome the shortcomings of Yu and Van Aerde’s calibration procedure. In this study, we use the third method since it is the simple and provides adequate accuracy. The following three equations show Rakha and Farzaneh’s (2005) third formulation:

\[
\beta_n = \frac{1}{1 + \alpha_n} \quad \text{or} \quad \alpha_n = \frac{1 - \beta_n}{\beta_n} \tag{5.4}
\]

\[
\beta_n = \frac{2T'_a + n - \sqrt{n^2 + 4\sigma'^2}}{2T'_a} \tag{5.5}
\]

\[
F_n = n - \frac{\sqrt{n^2 + 4\sigma'^2} - n}{2\sigma'^2} \tag{5.6}
\]

Where:

- \(\beta_n, F_n\) and \(\alpha_n\): model parameters for step size of \(n\) seconds,
- \(\sigma'\): standard deviation of link travel times (s), and
- \(T'_a\): mean roadway travel time (s).

Equations 5.4, 5.5, and 5.6 demonstrate that the values of \(\alpha, \beta,\) and \(F\) are dependent on the size of the time interval.

**Problem Description**

As was mentioned earlier, the successful application of Robertson’s platoon dispersion model relies on the appropriate calibration of the model parameters. However, all versions of the
TRANSYT software only allow for the calibration of the platoon dispersion factor and do not allow for the calibration of the travel time factor. A number of studies have attempted to quantify the impact of the platoon dispersion model parameters on the optimized signal timings. These studies have produced differing and in some instances contradicting results.

For example, McCoy et al. (1983) studied two cases in the United States and found that the optimum values for $\alpha$ and $\beta$ were different from the values provided in the TRANSYT manual. Consequently, the authors concluded that the software should be modified to enable users to specify both the $\alpha$ and $\beta$ parameters. Similarly, Guebert and Sparks (1989) conducted a parametric sensitivity analysis to study the effect of the calibrated platoon dispersion factors on the final optimized signal timing plan. The authors showed that the accurate calibration of the Robertson platoon dispersion model parameters is critical in developing effective and efficient traffic signal timing plans. Alternatively, Retzko and Schenk (1993) used the TRANSYT-8 (Vincent et al., 1980) to study the effect of the deviation of the correct value of $\alpha$ on the resulting optimized signal timings for three networks. The authors found that despite the changes in the platoon dispersion factor $\alpha$, the optimized signal timings were not significantly affected. Consequently, the authors suggested that the use of a unique value of $\alpha$ provides sufficient accuracy. Contrary to the previous studies, Manar (1994) examined the effect of the use of inappropriate platoon dispersion parameters using the TRANSYT-7F software for a road section composed of three intersections in Montreal, Canada. Manar found that the use of the recommended platoon dispersion factor of 0.25 incurred 65,250 CND per year in additional user costs as a result of the resulting inefficient signal timings.

Consequently, as part of this study, an attempt was made to quantify the effect of the recursive platoon dispersion model parameter values on the traffic performance at traffic signals by conducting a sensitivity analysis using data generated by the INTEGRATION software (Van Aerde, 2003). The INTEGRATION model represents the movement of individual vehicles at a 1 hertz resolution, based on a steady-state car-following relationship for each link and driver/vehicle specific acceleration and deceleration constraints. A detailed description of the model calibration procedures is beyond the scope of this paper but is described in detail in the literature (Van Aerde and Rakha, 1995, Rakha and Crowther, 2003). In terms of platoon dispersion behavior, Rakha and Farzaneh (2005) showed that INTEGRATION’s traffic
dispersion modeling is consistent with the field observed data demonstrating the validity of the software for modeling traffic dispersion.

The configuration that is used in this study consists of a three-lane arterial of 1-km length with a pre-timed traffic signal on the entrance link. Vehicles departing from the upstream traffic signal were monitored as they traveled downstream along the roadway. Specifically, six loop detectors were placed on the roadway. The first loop detector was located immediately upstream of the signalized intersection, while the other five detectors were located downstream of the signalized intersection at a spacing of 200 meters. The loop detectors gathered data at 3-second intervals. Figure 5.1 depicts the network layout, while Table 5.2 summarizes the roadway and network characteristics of the three cases that were simulated.

<table>
<thead>
<tr>
<th>Link Characteristic</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roadway length (m)</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Free-flow speed (km/h)</td>
<td>50</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>Speed-at-capacity (km/h)</td>
<td>35</td>
<td>45</td>
<td>32</td>
</tr>
<tr>
<td>Capacity (veh/h/lane)</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
</tr>
<tr>
<td>Jam density (veh/km/lane)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Number of lanes</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Number of loop detectors</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Speed coefficient of variation (percent)</td>
<td>15</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Entering headway distribution (% Random)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Total Simulation Time (s)</td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>Cycle Length (s)</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Effective Green Time (s)</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

For each case, the simulation run continued for 1200 seconds and consisted of seventeen distinct platoons of vehicles that departed from the upstream traffic signal. All the vehicles were passenger cars. Travel-time variability was captured through the use of a normally distributed function about the steady-state car-following model. The user has control over the level of randomness by specifying a Coefficient of Variation (CV) for the desired level of randomness.
All simulated vehicles were set as probes to record their individual travel times in computing the travel time mean and variance for the calibration of the $\alpha_n$ and $\beta_n$ parameters using Equations 5.4 through 5.6. All the required procedures (data retrieving, platoon dispersion modeling, optimal offset search, and delay calculation) were implemented within MATLAB 6.0.

The value of the platoon dispersion factor $\alpha_n$ was varied between 0.25 and 0.50 at increments of 0.05. Similarly, the travel time factor $\beta_n$ was varied between 0.70 and 0.95 at increments of 0.05. Using Equations 5.1 and 5.2, the traffic flow profile at each downstream check point was calculated for each pair of $\alpha_n$ and $\beta_n$ combination. To study the effect of the analysis step size on the results, three step sizes were selected for the prediction phase: 1, 3, and 6 seconds. It must be noted that these step sizes were only used for flow prediction purposes, while the delay estimation was conducted using a step size of 1 second.

In computing the optimum offset, all downstream virtual signals (signals that are considered at different locations for delay and offset calculation purposes only) were assumed to operate at a common cycle length and the signal timing plan of the upstream traffic signal. The optimum offset for each virtual downstream traffic signal was computed using the projected downstream flow profile using a simple hill-climbing search algorithm. The search algorithm minimized a performance index (PI) function, which was a weighted combination of vehicle delay and stops, as follows:

$$d_i^\prime = d_i + KC_i \quad [5.7]$$

Where

- $d_i^\prime$ : the total delay for $i$-th intersection (veh-s/lane),
- $d_i$ : the average delay for $i$-th intersection (veh-s/lane),
- $C$ : the number of vehicles stopped behind $i$-th intersection, and
- $K$ : stop penalty factor (s/stop), normally 4 (s/stop).

A 4-second/stop equivalency was selected in order to be consistent with the TRANSYT-7F manual. The optimum offset for each $\alpha_n$ and $\beta_n$ combination was then applied to the arrival flow profile to compute the total delay and number of vehicle stops using deterministic queuing theory.
Since the final results are qualitatively the same for all investigated cases, only the results for the first case are presented in Figures 5.2 through 5.5.

![Diagram of Variation of PI as Function of Travel Time Factor](image)

Figure 5.2: Variation of PI as function of travel time factor using 6-second step size (first case).

Figures 5.2 and 5.4 demonstrate the variation in the PI associated with different values of the travel time factor $\beta_n$ for step sizes of 1 and 6 seconds, respectively. Alternatively, Figures 5.3 and 5.5 illustrate the variation in the PI as a function of the platoon dispersion factor $\alpha_n$. A comparison of the two sets of figures clearly demonstrates that the variation in the PI is significantly higher in the case of $\beta_n$ than for $\alpha_n$ values. Furthermore, the effect of $\beta_n$ increases as the distance of travel increases (600, 800, and 1000 m), while $\alpha_n$ has a minimum impact on the PI. Alternatively, as the travel distance decreases the impact of $\alpha_n$ on the PI increases while the impact of $\beta_n$ on the PI decreases; however, the impact of $\beta_n$ remains higher for most cases. This phenomenon is attributed to the fact that for shorter distances the dispersion is minimal and vehicle platoons typically remain intact; therefore, with a sub-optimal offset most of the vehicles
can discharge during the green phase. On the other hand, as vehicles travel farther downstream, vehicle platoons disperse significantly and thus the start time of the green phase becomes critical.

Furthermore, a comparison of the results was conducted for different temporal step sizes including step sizes of 1, 3, and 6 seconds. The results demonstrate that the PI is more sensitive to the variation in $\alpha_n$ and $\beta_n$ values than to the modeling step size; however, the overall trends appear to be similar. In our case, since the original data were collected at 3-second intervals and then disaggregated to 1-second data, the difference between the results for 1-second and 3-second time step sizes is not significant; however, the results for a 6-second step size shows more variation in comparison to the 1 and 3 second step sizes. Overall, the results indicate that the PI is more sensitive to $\beta_n$ than $\alpha_n$.

In conclusion, the findings of this sensitivity analysis can be summarized as follows:

- Proper calibration of the recursive platoon dispersion model is important to achieve and maintain a good signal timing plan.
The PI is more sensitive to the value of the travel time factor $\beta_n$ than the platoon dispersion factor $\alpha_n$ and thus the calibration of $\beta_n$ is more critical than the calibration of $\alpha_n$.

The importance of calibrating $\beta_n$ is more significant for larger signal spacing distances.

Using a unique value of $\alpha_n$ provides a reasonable accuracy as was suggested by Retzko and Schenk (1993).

Considering these conclusions and recognizing that the current versions of the TRANSYT software do not allow the user to vary the travel time factor $\beta_n$ from its set value of 0.8, it becomes a challenge to calibrate the TRANSYT software. Although a number of researchers (McCoy et al., 1983, Manar, 1994) have suggested that the TRANSYT software should be revised to allow users to control the value of the travel time factor, these recommendations have not been addressed. Consequently, we are offering a solution that does not require modifications to the code, as is described in the following section.
**PROPOSED SOLUTION**

The problem with calibration of the TRANSYT software arises from the fact that the software uses a fixed value for the travel time factor and only provides the user with control over the platoon dispersion factor which was demonstrated earlier in the paper to have a smaller impact on estimating the optimum signal timing plan.

The first step in addressing this problem is to analyze Robertson’s formulation and its elements. The objective is to maintain the level of prediction error as produced by the optimum $\alpha$ and $\beta$ parameters. Equation 5.3 demonstrates that the model has two main factors, namely the minimum travel time $(T=\beta.T_a)$ and the smoothing factor $F$ which is equal to $1/(1+\alpha.\beta.T_a)$ (Equation 5.2). Consequently, based on Equation 5.3, if the values of $T$ and $F$ are held constant, the model will produce identical dispersion behavior.

Assume that for a certain link $\alpha^O$ and $\beta^O$ are the optimum dispersion and travel time factors that result in a good timing plan and that $\alpha$ and $\beta$ are the corresponding TRANSYT input parameters.
The $\alpha^O$ and $\beta^O$ parameters can be calibrated using the Rakha and Farzaneh (2005) calibration procedure (Equations 5.4 through 6) or through the use of a best fit approach. Recognizing that the TRANSYT’s travel time parameter is equal to 0.8 and utilizing Equations 5.2 and 5.3 with $\alpha^O$ and $\beta^O$, we predict the correct downstream flow profile, which is a predicted traffic-flow profile that gives a signal timing close enough to the optimum timing plan obtained from the observed traffic flow. In order to produce identical downstream profiles using TRANSYT’s parameters ($\alpha$ and $\beta$), the following equalities must be satisfied:

$$\beta T_a = 0.8 T_a = \beta^O T_a$$  \[5.8\]

$$\alpha \beta T_a = 0.8 \alpha T_a = \alpha^O \beta^O T_a$$  \[5.9\]

Where

$T_a$: user coded average travel time in TRANSYT (s), and

$T^O_a$: observed average travel time (s).

If we ensure that $\alpha = \alpha^O$ then maintaining $0.8T_a$ to equal $\beta^O T_a$ the model provides an estimate of the average travel time that is coded in the TRANSYT software in order to produce an identical downstream profile as produced by the $\alpha^O$ and $\beta^O$ parameters. The value of the average travel time $T_a$ can be calculated as follows

$$T_a = \frac{\beta^O T_a}{0.8} = 1.25 \beta^O T_a$$  \[5.10\]

Equation 5.10 demonstrates that by altering the average travel time that is input into the TRANSYT software, the model users can indirectly control the value of travel time factor. It should be noted that the link specific platoon dispersion factor can be modified using the link specific platoon dispersion card, as described in the TRANSYT-7F manual (Wallace et al, 1983).

A legitimate concern about the use of Equation 5.10 may be that by altering the average link travel time the results of the software may be adversely affected. In addressing this concern it should be noted that Equation 5.10 guarantees that the TRANSYT software produces the desired downstream flow profile. Consequently, the vehicle delay and stop estimates would be correct given that all computations are based on the arrival cyclic profile. However, it should be noted that by applying Equation 5.10 the total travel time estimates would be altered since TRANSYT
uses the user-defined average link travel time to estimate the total network travel time. Consequently, this parameter should be used with caution.

**NUMERICAL EXAMPLE OF CALIBRATION METHODS**

This section attempts to provide a preliminary investigation of different calibration procedures. This effort serves two purposes: first, it explains different choices that users have to calibrate TRANSYT’s platoon dispersion model and second, it provides a preliminary validity analysis for each of the methods.

The data used for this purpose is the dataset that was generated and used in the problem description section. Seven different calibration methods were considered and applied to the data, as summarized in Table 5.3. Five of the seven calibration approaches use the state-of-practice best-fit technique to calibrate the $\alpha$ and $\beta$ parameters while the sixth approach uses the Rakha and Farzaneh approach to calibrate the model parameters (Equation 5.4 through 5.6), and the final approach considers the TRANSYT-7F default parameters. The average travel time for the Rakha and Farzaneh formulation was estimated based on simulated probe travel time experiences generated by the INTEGRATION software. Alternatively, the average travel time for the remainder scenarios was calculated as the distance between the centers of gravity of the area under the upstream and downstream flow profiles, as is commonly done in practice.

<table>
<thead>
<tr>
<th>Method</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Best fit, varying average travel time and standard deviation of travel times, separately for each downstream intersection</td>
</tr>
<tr>
<td>M2</td>
<td>Rakha and Farzaneh method, separately for each downstream intersection</td>
</tr>
<tr>
<td>M3</td>
<td>Best fit, varying $\alpha$ and $\beta$, separately for each downstream intersection</td>
</tr>
<tr>
<td>M4</td>
<td>Best fit, considering $\beta = 0.8$ and varying $\alpha$ separately for each downstream intersection</td>
</tr>
<tr>
<td>M5</td>
<td>TRANSYT’s default values, $a = 0.35$ and $\beta = 0.8$</td>
</tr>
<tr>
<td>M6</td>
<td>Best fit, varying $a$ and $\beta$, for all downstream intersections collectively</td>
</tr>
<tr>
<td>M7</td>
<td>Best fit, considering $\beta = 0.8$ and varying $a$, all downstream intersections collectively</td>
</tr>
</tbody>
</table>

The vehicle delay and stop estimates were made using deterministic queuing theory. As was the case earlier, three time-step sizes were considered, namely 1, 3, and 6 seconds. These time steps were used to predict the downstream flow profile in searching for the optimum platoon dispersion parameters using the best-fit technique. Two different flow profiles were used for the 1-second analysis. The first profile was generated by disaggregating the 3-second flow profile.
and the second profile was generated by disaggregating the 6-second flow profile. Furthermore, for the best-fit approaches, two methods were utilized to estimate the optimum platoon dispersion parameters. The first method minimized the error for all seventeen platoons simultaneously while the second approach minimized the error considering a single randomly selected cyclic profile.

Results demonstrated that none of the calibration methods guarantees that the derived calibrated parameters result in minimum delay (This is the delay calculated by applying the offset derived from the predicted downstream flow profile) for all traffic and roadway instances. Furthermore, Table 5.4 demonstrates that methods that use a fixed value of the travel time parameter ($\beta = 0.8$) (methods M4, M5, and M7) tend to produce greater delay estimates on average compared to the other methods. Second, methods that use the best-fit approach (methods M1, M3, and M6) tend to produce the least delay estimates of all three methods. Finally, on average, the Rakha and Farzaneh calibration method (method M2) provided better timing plans than the fixed travel time parameter methods ($\beta = 0.8$).

<table>
<thead>
<tr>
<th>Factor</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>Percent sample with least extra delay</td>
<td>45.0</td>
</tr>
<tr>
<td>Percent sample with largest extra delay</td>
<td>13.3</td>
</tr>
<tr>
<td>Maximum extra delay (percent)</td>
<td>17.3</td>
</tr>
</tbody>
</table>

In comparing different calibration methods two factors play key roles: the accuracy and efficiency of the method and the simplicity and applicability of the method to different roadway and traffic conditions. As described above, the best fit methods tend to yield better results in terms of precision and efficiency; however the use of such methods requires extensive data collection that deemed them impractical. In contrast, the formulation proposed by Rakha and Farzaneh (2005) provides adequate precision and efficiency, and at the same time is easy to apply, which makes it the best candidate for practical use. The ease of application arises from the fact that it only requires tracking a sample of vehicles to estimate the travel time mean and variance.
Another finding is that for most of the cases the resulting delay estimates are similar whether a single or multiple platoons are considered. These results are encouraging because it indicates that for most situations observation of a single platoon provides adequate accuracy. Furthermore, it was also observed that in general a 6-second step size results in the highest delay. This issue implies that better resolution (smaller step sizes) provides better efficiency in terms of delay.

The literatures suggest that calibrating the platoon dispersion parameters by minimizing the deviation between the estimated and observed downstream profiles would result in estimating the optimum signal-timing plan. In contrast, we found that approximately 30 percent of our investigated cases (217 out of 712 cases) did not result in the optimum signal timings. The reason of this finding is the fact that the vehicle travel time distribution is not necessarily a shifted geometric distribution as is assumed in the platoon dispersion model. In contrast, studies have shown that the distribution of vehicle travel times is more consistent with a normal, lognormal or a gamma distribution rather than a geometric distribution (Tracz, 1975, Polus, 1979).

**CONCLUSION**

The paper demonstrates the importance of calibrating the recurrence platoon dispersion model. The paper clearly demonstrates that the value of the travel time factor $\beta$ is critical in estimating appropriate signal-timing plans. Alternatively, the paper demonstrates that the value of the platoon dispersion factor $\alpha$ does not significantly affect the estimated downstream cyclic flow profile; therefore, a unique value of $\alpha$ provides the necessary precision. Unfortunately, the TRANSYT software allows the user to calibrate the platoon dispersion factor but does not allow the user to calibrate the travel time factor. In an attempt to address this shortcoming, the paper proposes a formulation (Equation 5.10) using the basic properties of the recurrence relationship to enable the user to control the travel time factor indirectly by altering the link average travel time.

Finally, the paper presents some numerical examples to demonstrate the effectiveness of different calibration methods of the recurrence platoon dispersion model. Although the dataset used for this purpose was generated using the INTEGRATION microscopic traffic-simulation software the procedures are general and intended for use with field data. It is anticipated that the
implementation of the proposed formulations can enhance the accuracy of the traffic dispersion model within the TRANSYT software and thus produce better signal timings.

ACKNOWLEDGEMENTS
The authors acknowledge the financial support of the Mid-Atlantic University Transportation Center (MAUTC) and the Virginia Department of Transportation (VDOT) in conducting this research effort.

REFERENCES


PLATOON DISPERSION MODELS: EFFECT OF UNDERLYING TRAVEL TIME DISTRIBUTION

Mohamadreza Farzaneh and Hesham Rakha

ABSTRACT
The main objective of the paper is to evaluate the effect of the underlying travel time/speed distribution on the accuracy and efficiency of platoon dispersion models using data generated by the INTEGRATION software. Consequently, the paper first validates the INTEGRATION traffic dispersion modeling behavior. Subsequently, the paper utilizes the simulation output to evaluate the prediction precision and performance of seven different platoon dispersion models. The results demonstrate that in terms of prediction accuracy the resulting flow profiles from all models are very similar for short lengths (less than 800m). The model prediction error increases as the travel distance increases because the models fail to capture the interaction of vehicles as they travel. In terms of efficiency, the study demonstrates that the type of model has minimum effect on the optimum offset; instead the signal spacing has a larger impact on the prediction error. Furthermore, the paper demonstrates that the explicit modeling of differences in driver behavior is critical in obtaining realistic results.

INTRODUCTION
The interest in platoon dispersion arises from the fact that traffic signal coordination systems require the prediction of downstream flow profiles in order to estimate appropriate signal timing plans. Platoon dispersion models generally estimate vehicle arrivals at downstream locations based on an upstream vehicle departure profile and an average traffic-stream space-mean speed. Current state-of-practice platoon dispersion models assume that vehicles in a platoon do not interact with each other and thus can travel at a constant speed, and therefore the dispersion of platoons can be modeled using an appropriate travel time/speed distribution.

Seddon (1971, 1972a and 1972b) in his series of papers on platoon dispersion studied the different models of platoon dispersion. He used a data set that was collected in England to compare the different models. He examined the recurrence model (Robertson, 1969) and
diffusion model (Pacey, 1956) and based on his observation and simulation results he concluded that “there appears to be little to choose between the Pacey and Robertson method on accuracy or efficiency”.

Hartley and Powner (1971) found that a rectangular distribution of travel times gives a similar arrival flow profile to a transformed normal distribution. Furthermore, Tracz (1975) expanded Hartley and Powner’s work and used a trapezoid distribution of travel time. She investigated the effect of the time distribution for a link with a maximum length of 400 m, and found the results of this model to be similar to Robertson’s model predictions.

Yu and Van Aerde (1995) studied the effect of the underlying travel time distribution on the predicted downstream cyclic flow profile. They considered a normal speed distribution, geometric and normal travel time distribution. Through an example for a short link, the authors demonstrated that although the fundamental probability distribution was significantly different considering a single upstream flow pulse, the dispersion of a cyclic flow profile produced minimum differences in the estimated downstream flow. Consequently, it was concluded that the particular shape of the statistical distribution that is used to represent the dispersion modeling has a marginal effect on the predicted downstream flow profile. Rakha and Farzaneh (2004) investigated this assumption using the same approach used by Yu and Van Aerde (1995) by adopting lognormal travel time and speed probability distribution functions in addition to those pdf’s that were used by Yu and Van Aerde (1995). They also found that the differences in the predicted downstream flow profiles from different pdf’s are marginal.

Although all of the above research efforts found that the effect of the underlying travel time/speed distribution is not important in the modeling of traffic dispersion, these studies only considered short distances of travel because it is commonly believed that after 800m arrivals are random. Smelt (1984) investigated this hypothesis for a 1200 m roadway in Australia and found that downstream flow from the signalized intersection had not reached random flow and in contrast vehicles still traveled in distinct platoons. Castle and Bonneville (1985) studied the potential benefit of signal coordination over distances of 1500 m. They concluded that although the reductions of delay as a result of signal coordination reduced as the distance between signals increased, however reductions in delay were still significant for road lengths between 500 and 2000 m.
Wang et al. (2003) performed a study on platoon dispersion models using six travel time/speed distributions using field data gathered on two roadways of 1.35 and 0.93 km long. The study concluded that the recursive model (Robertson, 1969) gives good results for short distances, while for longer distances the lognormal and normal distributions are better (0.93 and 1.35 km).

These results demonstrate the need for a more in-depth and comprehensive study of the effect of underlying travel time/speed distribution on the accuracy and efficiency of the platoon dispersion models. The objective of this study is three-fold. First, the paper demonstrates INTEGRATION’s ability in modeling platoon dispersion adequately. Second, the paper evaluates the effect of the statistical distribution on the prediction accuracy and the efficiency of traffic signal coordination systems. Third, the paper investigates that which roadway and characteristics have the highest impact on the dispersion behavior and travel time distribution.

**Platoon Dispersion Models**

In this study, we considered seven platoon dispersion models in order to characterize the effect of the model type on the prediction accuracy and performance. All seven platoon dispersion models can be described using the general form:

\[
q'_{t_i} = \sum_{i=T}^{\infty} g(i-T)q_{t_{i-1}}
\]  \[6.1\]

Where:

- \(q'_{t_i}\): Arrival flow at the downstream intersection at time \(t\) (veh/h);
- \(q_{t_i}\): Departure flow at the upstream intersection at time \(t\) (veh/h);
- \(T\): Travel time between two observation points (units of time steps); and
- \(g(i-T)\): Probability of a travel time of \((i-T)\) time steps.

Different platoon dispersion models are produced by adopting different probability distribution functions for travel time. These pdf’s express either the distribution of individual travel times or distribution of individual space-mean-speeds which can be converted to a travel time distribution.

In the selection of travel time/speed pdf’s we considered pdf’s that are reported to yield relatively successful models in literature. There are other options like Gamma, Poisson and binomial distributions of travel times, which either are too complicated for dispersion modeling...
or are found not appropriate for platoon dispersion modeling (Wang et al., 2003), and therefore were not considered in this study. The formulation of selected models are described in this section.

**NORMAL DISTRIBUTION OF SPEED: PACEY’S PLATOON DISPERSION MODEL**

In an unpublished research note at the Road Research Laboratory, Pacey (1956) presented a purely kinematic platoon dispersion model that is remarkably simple. Specifically, Pacey claimed that the only changes in the shape of a platoon of vehicles released from a signalized approach arise from differences in vehicle speeds within the platoon assuming that any vehicle proceeds with the same speed irrespective of the number or distribution of vehicles on the road, and that vehicles are able to pass slow moving vehicles in order to maintain their desired speed.

In his derivation, Pacey adopted a normal distribution as the distribution of vehicle speeds within a platoon. He showed that using the distribution of vehicle velocities \( f(v).dv \), it is possible to obtain the distribution of vehicle travel times \( g(T).dT \) between any two observation points. Using the distribution of travel times, Pacey demonstrated that the downstream flow within a time interval can be estimated using the discrete form described in Equation 6.1. To calibrate Pacey’s model, one needs two parameters; namely, the average speed \( V \), and the speed standard deviation, \( \sigma_v \).

**GEOMETRIC DISTRIBUTION OF TRAVEL TIMES: ROBERTSON’S RECURSIVE MODEL**

Robertson (1969) developed an empirical recursive relationship to describe the dispersion of traffic, which forms the core of the popular TRANSYT software, commonly known as TRANSYT-7F in North America. Because of the simplicity of applying the recursive formulation, Robertson’s model has become the standard platoon dispersion model and has been incorporated in a number of software, including SCOOT (Hunt et al., 1989), SATURN (Hall et al., 1980), and TRAFLO (Lieberman et al., 1980).

The basic Robertson’s recursive platoon dispersion model takes the following mathematical form:

\[
q'_i = Fq_{i-1} + (1 - F)q'_{i-\Delta t}
\]  \[6.2\]

\[
F = \frac{1}{1 + \alpha \beta T_a}
\]  \[6.3\]

Where:
$\Delta t$: time step duration, measured in the time intervals used for $q'_t$ and $q_t$;

$T$: minimum travel time on the roadway in units of time steps, equal to $\beta T_a$;

$\alpha$: dimensionless platoon dispersion factor;

$\beta$: dimensionless travel time factor;

$F$: smoothing factor, and

$T_a$: mean roadway travel time, measured in units of time steps.

Seddon (1972b) showed that although Equation 6.3 seems different from the general form of Equation 6.1, it is equivalent to a shifted geometric distribution for travel times, and therefore it can be rewritten in the following form:

$$q'_t = \sum_{i=1}^{\infty} F_i (1 - F)^{i-1} q_{t-i}$$ \[6.4\]

To calibrate the Robertson’s platoon dispersion model, one needs to find appropriate values for $\alpha$ and $\beta$. Yu and Van Aerde (1995) provided a set of equations to calculate $\alpha$ and $\beta$ values based on the observed average and standard deviation of travel times. Rakha and Farzaneh (2004) expanded Yu and Van Aerde’s work to consider the effect of analysis step size and developed a set of formulations to consider this factor. The following Equations show Rakha and Farzaneh’s (2004) third formulation which is used in this study to calibrate Robertson’s dispersion model.

$$\alpha_n = \frac{1 - \beta_n}{\beta_n}$$ \[6.5\]

$$\beta_n = \frac{2T_a + n - \sqrt{n^2 + 4\sigma^2}}{2T_a}$$ \[6.6\]

$$F_n = \frac{\sqrt{n^2 + 4\sigma^2} - n}{2\sigma^2}$$ \[6.7\]

Where:

$\beta_n$, $F_n$ and $\alpha_n$: model parameters for step size of $n$ seconds,

$\sigma$: standard deviation of link travel times (s), and

$T_a$: mean roadway travel time (s).
NORMAL, LOGNORMAL, AND UNIFORM DISTRIBUTION OF TRAVEL TIMES

The distribution of travel time in Equation 6.1, g(i-T), can be substitute by any desired probability function. Using Pacey’s approach, we can substitute g(i-T) with Normal, Lognormal and Uniform probability distribution functions. Figure 6.1 shows the travel time distributions collected on Interstate 35 near San Antonio, Texas. The data were collected for a 10-mile section. It is found that Gamma, normal, and lognormal probability distributions are close to the observed distribution of travel times for different segments. The results of a chi-square goodness-of-fit test is also provided in the figure and demonstrates that the distribution of travel times is similar to a gamma, normal and lognormal distribution, and therefore in this study we also consider normal and lognormal distributions.

Hartley and Powner (1971) tried a uniform distribution of travel times. Tracz (1975) also used another form of rectangular travel time to model the traffic dispersion. In both cases, they stated that this selection was not based on any theoretical basis; however these distributions were selected simply because these distributions can be generated relatively easily in practice. In this study we only took at the uniform distribution of travel times. To calibrate the platoon dispersion models using lognormal and uniform distributions of travel times, we need the average travel time and standard deviation of travel times.

LOGNORMAL AND UNIFORM DISTRIBUTION OF SPEED

Using the same methodology of Pacey’s model, we can substitute a probability distribution function of speeds, f(v), with lognormal and uniform pdf’s. Wang et al. (2003) reported a successful use of lognormal speed distribution to model platoon dispersion on longer distances. Although no literature was found that used uniform distribution of speed for dispersion modeling, it was also considered it in the study.

MODEL COMPARISON

This section describes the effort of evaluating the effect of underlying travel time/speed distribution on the accuracy and efficiency of platoon dispersion models. In this study distances of up to 2 km are considered in order to cover a wide range of distances. Additionally, in order to provide realistic traffic conditions, the effect of other factors such as flow level, level of speed variability, side flow level, and number of lanes is also considered. Two types of comparisons...
are performed: qualitative, and quantitative. The details of the process and the results of each part are discussed in the following sections.

Figure 6.1: Observed distributions of travel times on I-35 South.
INTEGRATION MODELING OF TRAFFIC DISPERSION

The INTEGRATION model represents the movement of individual vehicles in a time-stepping fashion, based on a steady-state car-following relationship for each link. It should be mentioned that INTEGRATION is a fully microscopic simulation model; however, the microscopic rules used in it have been carefully calibrated in order to capture the most important macroscopic traffic characteristics. A detailed description of the model calibration procedures is beyond the scope of this paper but is described in detail in the literature (Van Aerde and Rakha, 1995; Rakha and Crowther, 2003).

INTEGRATION models traffic dispersion through the use of a speed variability factor. Unlike macroscopic platoon dispersion models, the dispersion modeling within the INTEGRATION software captures differences in vehicle desired speeds in addition to vehicle interactions and their impacts on a vehicle’s desired speed. Model users can control the level of speed variability by selecting a link specific speed coefficient of variation (standard deviation divided by mean). Various aspects of the INTEGRATION software have been validated against field data and basic traffic flow theory (Dion et al. 2004, Helinga et al. 2004, and Rakha and Zhang 2004). Also Rakha and Farzaneh (2004) showed that data generated by INTEGRATION is consistent with the traffic dispersion modeled using Robertson’s model (1969) for short distances (less than 800m). Since the Robertson platoon dispersion model has been extensively validated against field data, consistency for short distances in another indication that the INTEGRATION software is suitable for the modeling of traffic dispersion.

To further demonstrate the validity of the INTEGRATION software for the modeling of traffic dispersion, two validation efforts are performed here. If we assume that platoon dispersion results from differences in vehicle travel times, then a comparison between simulated and field observed travel times would be a good indication of the modeling validity. Figure 6.2 shows the travel time distribution derived from the simulation output for the network presented in Figure 6.5, along with the corresponding normal and lognormal distributions, and Kolmogorov-Smirnov (K-S) goodness of fit test results. The detail of the simulation procedure is described in later in the paper. A comparison of Figure 6.2 to the field observed distributions that are presented in Figure 6.1 demonstrates that the INTEGRATION software is qualitatively consistent with field observed travel times.
In addition, the INTEGRATION software was used to model sample field traffic dispersion data. The data used in the analysis were gathered by Denney (1989) in Houston, Texas. The data contain observed average flow profiles at an upstream signal and at a checkpoint 300 m (990 ft) downstream for a 3-lane arterial. The field observed average speed and speed standard deviation were reported as 48.3 km/h (44 ft/s) and 5.9 km/h (5.4 ft/s), respectively. Consequently, the speed coefficient of variation (COV$_{obs}$) is 12.3%. Figure 6.3a depicts the average flow profile at the upstream traffic signal.

The roadway was coded in the INTEGRATION software as a 300 m 3-lane link, with a free flow speed of 50 km/h, a speed-at-capacity of 40 km/h, a saturation flow of 1800 veh/h/lane, and a jam density of 100 veh/km/lane.

An important factor in the modeling of traffic dispersion is estimating the desired speed coefficient of variation (COV$_{in}$), which characterizes differences in driver speed selection. It must be noted that differences in desired speeds will be curbed by the vehicle’s ability to attain its desired speed. This ability to achieve one’s desired speed is clearly dependent on the level of congestion along a roadway. Consequently the output speed coefficient of variation (COV$_{out}$) will differ from COV$_{in}$. Therefore, a higher value of COV was coded as the input to the software.
Two simulation batches were performed; the first batch had a $\text{COV}_{\text{in}}=10\%$ while the second had a $\text{COV}_{\text{in}}=15\%$ and each batch consisted of 10 simulations with different random seeds.

a. Observed average upstream flow profile

b. Observed and simulated average flow profile at 300 m downstream ($\text{COV}_{\text{in}} = 10\%$)

c. Observed and simulated average flow profile at 300 m downstream ($\text{COV}_{\text{in}} = 15\%$)

Figure 6.3: Observed and simulated flow profiles for Houston data.
Figures 6.3b and c illustrate the average simulated flow profiles (average of 10 runs) 300 m downstream superimposed on the observed flow profile. The resulting average speeds were 48.3 and 48.2 km/h for a COV\textsubscript{in} of 10\% and 15\%, respectively, which is consistent with the field observed average speed of 48.3\%. The resulting speed standard deviations were 4.2 (COV\textsubscript{out}=8.7\%) and 6.1 km/h (COV\textsubscript{out}=12.7\%) for a COV\textsubscript{in} of 10\% and 15\%, respectively, which are consistent with the field observed COV. Both figures demonstrate a good level of consistency between the resulting flow profiles generated by the INTEGRATION software and the field observed flow profiles, though the profile for a COV\textsubscript{in} of 15\% appears to provide a better fit given that the COV\textsubscript{out} is closer to the field observed COV.

**QUALITATIVE ANALYSIS**

In an attempt to demonstrate the effect of the underlying travel time distribution on the performance of accuracy of traffic dispersion models, the dispersion of an upstream flow profile was modeled considering various dispersion distributions for distances of 200, 500, 1000, 1500, and 2000 m, as demonstrated in Figure 6.4. An analysis step size of 2 seconds was considered for an 80-second cycle length with 50 seconds of effective green time. The figure demonstrates that the predicted downstream flow profiles 200 and 500 m downstream of the traffic signal are very similar for all distributions. The differences in flow profiles become more noticeable at a location 1000 m downstream, and this difference becomes more significant as the distance of travel increases to 1500 and 2000 m. The figure demonstrates that the geometric distribution predicts downstream profiles that are significantly different from the other dispersion models.

In comparing the predicted flow profiles the normal travel time distribution is considered as the base case and all R\textsuperscript{2} are computed relative to the base case, as summarized in Table 6.1. The results of Table 6.1 confirm the conclusions that were made based on qualitative observation. Except for 200 m where the R\textsuperscript{2} associated with some of the models is slightly less than the 500 m location, the results demonstrate that in general the greater the distance of travel, the smaller the value of R\textsuperscript{2}. The predicted flow profile using the geometric distribution is least similar to the profiles predicted by other models.
Figure 6.4: Upstream and predicted downstream flow profiles.
Table 6.1: $R^2$ between predicted downstream flow profile from normal time distribution and other models.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Geometric Time</th>
<th>Lognormal Time</th>
<th>Uniform Time</th>
<th>Normal Speed</th>
<th>Lognormal Speed</th>
<th>Uniform Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>99.2%</td>
<td>100.0%</td>
<td>99.9%</td>
<td>99.4%</td>
<td>99.3%</td>
<td>99.3%</td>
</tr>
<tr>
<td>500</td>
<td>99.0%</td>
<td>99.9%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.6%</td>
</tr>
<tr>
<td>1000</td>
<td>98.1%</td>
<td>99.8%</td>
<td>99.4%</td>
<td>99.7%</td>
<td>99.8%</td>
<td>99.5%</td>
</tr>
<tr>
<td>1500</td>
<td>95.9%</td>
<td>99.7%</td>
<td>98.8%</td>
<td>99.2%</td>
<td>99.7%</td>
<td>98.9%</td>
</tr>
<tr>
<td>2000</td>
<td>94.5%</td>
<td>99.6%</td>
<td>97.8%</td>
<td>98.8%</td>
<td>99.6%</td>
<td>98.7%</td>
</tr>
</tbody>
</table>

**QUANTITATIVE ANALYSIS**

This section is dedicated to the quantitative analysis of the effect of the underlying travel time/speed distribution on the accuracy and efficiency of the traffic dispersion models. The main objective of this section is to quantify the conclusions that were made earlier. The analysis consists of two steps: first, data generation, and then, statistical data analysis regarding the effect of the considered parameters on the accuracy and efficiency of the platoon dispersion models.

Previous studies comparing the different travel time/speed distributions compared the error between predicted and observed profile flow profiles (Seddon, 1972, Tracz, 1975, Denney, 1986). Alternatively, other studies quantified the impact of different dispersion approaches on the computed optimum offsets (Hartley and Powner, 1971). Finally, some studies quantified the impact of various dispersion models on the efficiency of an optimum timing plan (e.g. delay, queue length, and number of stops) (Wang et al., 2003). In this study, we combine all three aspects in the evaluation of alternative platoon dispersion models, as follows:

- Percent mean square error between the observed flow profile and predicted profile,
- Deviation of predicted optimal offset\(^5\) from observed optimal offset\(^6\),
- Percent of extra of additional Performance Index (PI) associated with a specific traffic dispersion model. The PI was computed as

\[
PI = d + KC, \quad [6.8]
\]

where $PI$ is the total delay for the intersection (veh-s/lane), $d$ is the average delay for i-th intersection (veh-s/lane), $C$ is the number of vehicles stopped behind i-th intersection, and $K$ is the stop penalty factor (sec/stop), normally 4 (sec/stop). A 4-second/stop equivalency was

\(^5\) The offset that minimizes P.I. for predicted flow profile.

\(^6\) The offset that minimizes P.I. for observed flow profile.
selected in order to be consistent with the TRANSYT-7F default value. The total delay and number of stops were computed using deterministic queuing theory that was coded in MATLAB.

The data were generated using the INTEGRATION software (Van Aerde, 1990). The network used in this study was composed of a unidirectional arterial connecting nodes 1 and 12, as illustrated in Figure 6.5. An upstream traffic signal, located at node 2, served as the master signal to which the offset of downstream virtual signals were referenced. The upstream signal was considered to have a 2-phase signal timing plan operating at a 60 s cycle length with 35 seconds of effective green time for the main arterial and 10 (2x5) seconds of lost time. Minor streets intersected the main street at 200 m intervals and they have stop signs. The presence of stop sign is to force the drivers coming to main stream to stop and look for appropriate gaps, similar to what is observed in the field.

The main arterial (node 1 to node 12) was assigned a free-flow speed of 60 km/h, a speed-at-capacity of 48 km/h, and a saturation flow rate of 1800 veh/h/lane. All minor streets were 1-lane links with a free-flow speed of 40 km/h, a speed-at-capacity of 32 km/h, and a saturation flow rate of 1200 veh/h/lane. The jam density on all links was 100 veh/km/lane and all vehicles were passenger cars.

The origin-destination (O-D) demand was composed of a major demand (from node 1 to node 12) and several side-street demands (from/to side streets). Side-street demands produced turning and

---

7 Signals that in reality don’t exist, but they are considered at a section for delay and offset calculation purpose
weaving movements on the main road and provided more realistic traffic conditions. Using side-street demands also introduces errors in the dispersion modeling because of changes in flow rates along the major arterial. In order to minimize these errors the total side-street demand was kept below 10% of the saturation flow rate. Furthermore, the number of vehicles entering and exiting at different side streets was maintained equal in order to ensure that the total demand remained the same. Eleven loop detectors were placed on the roadway. The first loop detector was located immediately upstream of the signalized intersection, while the other five detectors were located downstream of the signalized intersection at spacing of 200 meters. The loop detectors gathered data every 2 seconds.

Four related traffic characteristics were considered in the simulation in order to study the accuracy and efficiency of platoon dispersion models under different traffic conditions. Table 6.2 lists these four parameters as well the various levels that were considered in the simulation. Overall these factors provided 24 different scenarios (traffic conditions). Thirty replications were simulated for each scenario to provide enough power for statistical inference. For each replication, the simulation run continued for 1800 seconds and consisted of 26 distinct platoons of vehicles that departed from the upstream traffic signal. All simulated vehicles were set as probes to record their individual travel times in computing the expected and travel-time variance for the calibration of the platoon dispersion models. No left or right turns were modeled at the upstream intersection (node 2). Furthermore, no traffic was considered from/to the minor road at this intersection.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Definition</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>n\text{\textsubscript{l}}</td>
<td>Number of lanes\textsuperscript{8}</td>
<td>2 and 3 Lanes</td>
</tr>
<tr>
<td>q\text{\textsubscript{m}}</td>
<td>Main flow from Upstream</td>
<td>High and Normal</td>
</tr>
<tr>
<td>var\text{\textsubscript{u}} (CV\textsubscript{\text{\text{\textsubscript{m}}}})</td>
<td>Speed variability in INTEGRATION, represents the difference between drivers in speed selection</td>
<td>10% and 20%</td>
</tr>
<tr>
<td>q\text{\textsubscript{s}}</td>
<td>Incoming/outgoing flow from/to minor roads</td>
<td>High, Medium, and Zero</td>
</tr>
<tr>
<td>d</td>
<td>Distance to downstream section</td>
<td>200, 400, …, 2000 m</td>
</tr>
</tbody>
</table>

\textsuperscript{8} the demand level per lane is the same for both cases, total demand for 2-lane scenarios is two-third of total demand for corresponding 3-lane cases
Figure 6.6 illustrates the procedure used to analyze the data. The step size used for flow profile prediction and offset optimization was 2 seconds. The following sections describe the results of the analysis for each of the MOEs.

**Percent Fit Error**

An ANOVA analysis was performed to evaluate the impact of underlying travel time/speed distribution on the fit error between predicted and observed flow profiles. The results ($F=189499$) shows that statistically there is enough evidence that the pdf type has an effect on the fit error. Furthermore, linear regression between the type of pdf and fit error is used to inspect how big this effect is. $R^2$ between the type of pdf and fit error is 0.5% which implies that the strength of this effect is very weak.

In further analyzing the data, the normal travel time model was arbitrarily taken as the base case and a correlation analysis was performed between the fit error for this base case and other models over all scenarios for all distances. Table 6.3 shows the results of this analysis. The geometric distribution has the least $R^2$ for all distances while all other models have $R^2$ greater than 99.18%. The pattern presented in Table 6.3 confirms the conclusion we made in the previous section that the geometric distribution yields a flow profile that has the highest difference with the outcome of the other models and this difference increases as the distance of travel (travel time) increases. It is also observed that the resulting $R^2$ for the geometric
distribution in Table 6.3 are higher than the corresponding values in Table 6.1. This is because the upstream flow demand used for Table 6.1 had minimum variability while the upstream demands used for Table 6.2 had a higher level of vehicle speed variability which weakens the effect of pdf differences.

Table 6.3: R² between predicted downstream flow profile from normal time distribution and other models.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Geometric Time</th>
<th>Lognormal Time</th>
<th>Uniform Time</th>
<th>Normal Speed</th>
<th>Lognormal Speed</th>
<th>Uniform Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>96.70%</td>
<td>99.98%</td>
<td>99.60%</td>
<td>99.83%</td>
<td>99.88%</td>
<td>99.45%</td>
</tr>
<tr>
<td>400</td>
<td>99.29%</td>
<td>99.99%</td>
<td>99.93%</td>
<td>99.97%</td>
<td>99.98%</td>
<td>99.89%</td>
</tr>
<tr>
<td>600</td>
<td>99.49%</td>
<td>99.99%</td>
<td>99.96%</td>
<td>99.94%</td>
<td>99.96%</td>
<td>99.86%</td>
</tr>
<tr>
<td>800</td>
<td>99.32%</td>
<td>99.98%</td>
<td>99.96%</td>
<td>99.92%</td>
<td>99.96%</td>
<td>99.86%</td>
</tr>
<tr>
<td>1000</td>
<td>98.98%</td>
<td>99.97%</td>
<td>99.95%</td>
<td>99.85%</td>
<td>99.96%</td>
<td>99.82%</td>
</tr>
<tr>
<td>1200</td>
<td>98.53%</td>
<td>99.95%</td>
<td>99.91%</td>
<td>99.76%</td>
<td>99.94%</td>
<td>99.75%</td>
</tr>
<tr>
<td>1400</td>
<td>97.68%</td>
<td>99.94%</td>
<td>99.81%</td>
<td>99.69%</td>
<td>99.92%</td>
<td>99.65%</td>
</tr>
<tr>
<td>1600</td>
<td>97.11%</td>
<td>99.94%</td>
<td>99.67%</td>
<td>99.68%</td>
<td>99.91%</td>
<td>99.53%</td>
</tr>
<tr>
<td>1800</td>
<td>97.28%</td>
<td>99.95%</td>
<td>99.55%</td>
<td>99.72%</td>
<td>99.91%</td>
<td>99.39%</td>
</tr>
<tr>
<td>2000</td>
<td>97.40%</td>
<td>99.96%</td>
<td>99.46%</td>
<td>99.77%</td>
<td>99.92%</td>
<td>99.18%</td>
</tr>
</tbody>
</table>

Figure 6.7 compares the 95% confidence limit of the relative fit error for the geometric distribution and all other models combined. It can be seen that on average for all distances the geometric distribution provides a greater fit error when compared to other models. This difference is caused by the fact that the geometric distribution assumes that the majority of vehicles travel at higher speeds, while all other models consider an equal dispersion at the front and rear of the platoon.
Deviation from Optimal Offset (Offset Deviation)
The results of linear regression between the type of model and deviation from the observed optimal offset ($R^2 = 0.02\%$, Regression’s $F = 14.97$, Coefficient’s p-value = 0.0011) shows that although statistically the type of model has an effect on the offset deviation, this effect is very small.

Furthermore, Figure 6.8 depicts the percentage of cases that fall in different deviation intervals as a function of the distance of travel for Robertson’s dispersion model (geometric distribution of time). The figure demonstrates that for distances up to 600 m the model yields very good offsets,
i.e. almost all the cases results in less than a 5-second deviation from the optimum offset, but for longer distance the efficiency declines. For example, for 800 m only 1.7% of the cases have an offset deviation greater than 10 seconds. This percentage is 31.8% for a 1.2 km spacing and 53.3% for 2 km spacing. These results emphasize the poor prediction quality of the model for longer distances. Similar findings are observed for all macroscopic platoon dispersion models because the models ignore the interaction of vehicles.

**Change in Performance Index**

A similar analysis was applied to compute the percent of increase in the PI associated with modeling using a specific traffic dispersion model. The linear regression yields $R^2 = 0.06\%$, $F = 29.15$, and p-value = 6.71E-08. These results are obviously consistent with the results for offset deviation and indicate that although the type of model affects the percent of extra delay, this effect is very small.

Figure 6.9 depicts the percentage of cases that produce different PI values relative to the optimum PI using the Robertson model. Similar trends were observed for other models. Figure 6.9 demonstrates that for distances up to 600 m the percent extra delay associated with the use of the Robertson model is relatively small, i.e. greater than 70% of the cases results in less than 10% percent extra delay. But again for longer distances the efficiency of the model declines, (e.g. the percentage of cases with an increase in PI in excess of 10% is 45.4% for 800 m, 48.1% for 1.2 km, and 63.2% for 2 km). The general trend is that as the distance of travel increases the efficiency of the macroscopic platoon dispersion models decreases. It is important to note that the rate of decline in efficiency for longer distances is less than that for offset deviation. This is because as the distance increases the sensitivity of PI to offset deviation decreases, as is observed in Figure 6.10, and therefore it is less sensitive to the distance of travel.
Figure 6.9: Percent of cases having different percent of extra P.I. for Robertson’s model.

**DISCUSSION OF THE RESULTS**

The above analyses show that the underlying travel time/speed distribution has a minimum effect on both prediction precision and efficiency. In addition to this, the results also indicate that the accuracy and efficiency of the models in the form of Equation 6.1 declines as the travel time/distance increases. The main reason for this is the underlying assumption that the vehicles in this family of models travel at a constant speed, thus ignoring the vehicle’s interaction, which in reality restricts the speed selection. This assumption results in flatter and more dispersed flow profiles than observed in reality, though this error is not significant for short distances (less than 800m) it is critical for long signal spacing. Figure 6.4 clearly shows that as the distance increases the predicted flows by the platoon dispersion models become flatter and platoons become less distinct. This reveals the inherent problem of this family of models in the prediction the flow profiles for distances greater than 800 m.
Figure 6.10: Offset-delay relationship for virtual signals at different distances.

Figure 6.11: Average reduction in P.I. moving from random offsets to full coordination (%).

A simple analysis is performed to investigate the potential average benefit of using a 100%-accurate signal coordination (zero offset deviation) versus not-coordinated situation (random offset distribution). In conducting the analysis, five random generated offsets were considered for each downstream checkpoint in each simulation run, and then using the same procedures that were discussed earlier the percent extra PI was computed. This provided 3600 (720*5) values for each virtual signal. The average percent additional PI caused by non-optimal offsets is depicted in Figure 6.11. Although the figure clearly demonstrates that the benefit from signal coordination
declines drastically as the signal spacing increases, the benefits of signal coordination are still significant (27.8% PI reduction). Consequently, there is a need to develop new platoon dispersion models which are able to provide the same prediction accuracy for all travel time/distances. The use of a microscopic approach is one potential method of achieving this objective.

**Complementary Analysis**

The data that were utilized in the quantitative analysis that was presented earlier provided the opportunity to perform a complimentary analysis to investigate the effect of the described parameters on some of the important traffic flow factors. These factors are listed in Table 6.4. The main objective of this analysis was to identify the traffic characteristics of a roadway (Table 6.2) that affect traffic dispersion.

| Table 6.4: Traffic related factors considered in complimentary study. |
|----------------|---------------------------------|
| Factor | Definition |
| $T_a$ | Average Travel Time |
| $CV_T$ | Travel Times Coefficient of Variation ($\sigma_T/T_a$) |
| $V_a$ | Average Speed (space-mean-speed) |
| $CV_V$ | Speed Coefficient of Variation ($\sigma_V/V_a$) |
| $\alpha$ | Recurrence Model’s Platoon Dispersion Factor (Equation 6.5) |
| $\beta$ | Recurrence Model’s Travel Time Factor (Equation 6.6) |

The method used in this study was a stepwise linear regression. For each of the dependent variables (traffic related factors from table 6.4) a forward stepwise linear regression was performed in order to identify the important predictor variables amongst the variables listed in Table 6.2. From the results in Table 6.5 the following conclusion can be drawn:

- For all the considered factors the effect of the number of lanes is negligible.

- The average travel time in the non-congested regime is linearly related to the signal spacing. The effects of the other roadway characteristics are not significant.
### Table 6.5: Complimentary analysis results.

<table>
<thead>
<tr>
<th>Model</th>
<th>$R^2$</th>
<th>Coefficients’ Sign</th>
<th>p-values of Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a = f(d)$</td>
<td>0.9923</td>
<td>+</td>
<td>3.69E-71</td>
</tr>
<tr>
<td>$T_a = f(var_u)$</td>
<td>0.0045</td>
<td>+</td>
<td>5.76E-09</td>
</tr>
<tr>
<td>$T_a = f(q_m)$</td>
<td>0.0008</td>
<td>+</td>
<td>0.0165</td>
</tr>
<tr>
<td>$T_a = f(q_s)$</td>
<td>0.0002</td>
<td>+</td>
<td>0.1066</td>
</tr>
<tr>
<td>$T_a = f(nl)$</td>
<td>&lt; 0.0001</td>
<td>-</td>
<td>0.6440</td>
</tr>
<tr>
<td>CV $T_a = f(var_u)$</td>
<td>0.8120</td>
<td>+</td>
<td>0.00</td>
</tr>
<tr>
<td>CV $T_a = f(d)$</td>
<td>0.0629</td>
<td>-</td>
<td>9.55E-05</td>
</tr>
<tr>
<td>CV $T_a = f(q_m)$</td>
<td>0.0346</td>
<td>-</td>
<td>3.00E-07</td>
</tr>
<tr>
<td>CV $T_a = f(q_s)$</td>
<td>0.0129</td>
<td>-</td>
<td>2.62E-22</td>
</tr>
<tr>
<td>CV $T_a = f(nl)$</td>
<td>0.0017</td>
<td>+</td>
<td>2.77E-04</td>
</tr>
<tr>
<td>CV $T_a = f(var_u,d)$</td>
<td>0.8749</td>
<td>+, -</td>
<td>0.00, 0.00</td>
</tr>
<tr>
<td>CV $T_a = f(var_u,d,q_m)$</td>
<td>0.9096</td>
<td>+, - , -</td>
<td>0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>CV $T_a = f(var_u,d,q_m,q_s)$</td>
<td>0.9226</td>
<td>+, - , - , -</td>
<td>0.00, 0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>$V_a = f(var_u)$</td>
<td>0.5329</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_a = f(d)$</td>
<td>0.1726</td>
<td>+</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_a = f(q_m)$</td>
<td>0.1375</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_a = f(q_s)$</td>
<td>0.0712</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_a = f(nl)$</td>
<td>&lt; 0.0001</td>
<td>-</td>
<td>0.8574</td>
</tr>
<tr>
<td>$V_a = f(var_u,d)$</td>
<td>0.7056</td>
<td>-, +</td>
<td>0.00, 0.00</td>
</tr>
<tr>
<td>$V_a = f(var_u,d,q_m)$</td>
<td>0.8493</td>
<td>- , + , -</td>
<td>0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>$V_a = f(var_u,d,q_m,q_s)$</td>
<td>0.9145</td>
<td>- , + , - , -</td>
<td>0.00, 0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>CV $V_a = f(var_u)$</td>
<td>0.7963</td>
<td>+</td>
<td>0.00</td>
</tr>
<tr>
<td>CV $V_a = f(d)$</td>
<td>0.0769</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>CV $V_a = f(q_m)$</td>
<td>0.0330</td>
<td>-</td>
<td>1.20E-54</td>
</tr>
<tr>
<td>CV $V_a = f(q_s)$</td>
<td>0.0179</td>
<td>-</td>
<td>2.53E-30</td>
</tr>
<tr>
<td>CV $V_a = f(nl)$</td>
<td>0.0020</td>
<td>+</td>
<td>8.27E-05</td>
</tr>
<tr>
<td>CV $V_a = f(var_u,d)$</td>
<td>0.8733</td>
<td>+, -</td>
<td>0.00, 0.00</td>
</tr>
<tr>
<td>CV $V_a = f(var_u,d,q_m)$</td>
<td>0.9064</td>
<td>+, -, -</td>
<td>0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>CV $V_a = f(var_u,d,q_m,q_s)$</td>
<td>0.9244</td>
<td>+, -, -, -</td>
<td>0.00, 0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>$\alpha = f(var_u)$</td>
<td>0.7486</td>
<td>+</td>
<td>0.00</td>
</tr>
<tr>
<td>$\alpha = f(d)$</td>
<td>0.0526</td>
<td>+</td>
<td>1.71E-86</td>
</tr>
<tr>
<td>$\alpha = f(q_m)$</td>
<td>0.0284</td>
<td>-</td>
<td>3.00E-47</td>
</tr>
<tr>
<td>$\alpha = f(q_s)$</td>
<td>0.0110</td>
<td>-</td>
<td>3.21E-19</td>
</tr>
<tr>
<td>$\alpha = f(nl)$</td>
<td>0.0014</td>
<td>+</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\alpha = f(var_u,d)$</td>
<td>0.8011</td>
<td>+, +</td>
<td>0.00, 0.00</td>
</tr>
<tr>
<td>$\alpha = f(var_u,d,q_m)$</td>
<td>0.8297</td>
<td>+, + , -</td>
<td>0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>$\alpha = f(var_u,d,q_m,q_s)$</td>
<td>0.8407</td>
<td>+, + , - , -</td>
<td>0.00, 0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>$\beta = f(var_u)$</td>
<td>0.7469</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$\beta = f(ln(d))$</td>
<td>0.0982</td>
<td>-</td>
<td>0.00</td>
</tr>
<tr>
<td>$\beta = f(q_m)$</td>
<td>0.0257</td>
<td>+</td>
<td>6.31E-43</td>
</tr>
<tr>
<td>$\beta = f(q_s)$</td>
<td>0.0094</td>
<td>+</td>
<td>9.77E-17</td>
</tr>
<tr>
<td>$\beta = f(nl)$</td>
<td>0.0015</td>
<td>-</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\beta = f(var_u,ln(d))$</td>
<td>0.8452</td>
<td>- , -</td>
<td>0.00, 0.00</td>
</tr>
<tr>
<td>$\beta = f(var_u,ln(d),q_m)$</td>
<td>0.8711</td>
<td>- , - , +</td>
<td>0.00, 0.00, 0.00</td>
</tr>
<tr>
<td>$\beta = f(var_u,ln(d),q_m,q_s)$</td>
<td>0.8806</td>
<td>- , - , + , +</td>
<td>0.00, 0.00, 0.00, 0.00</td>
</tr>
</tbody>
</table>
The parameters that have significant influence on the travel time variability are driving behavior differences, signal spacing, and the level of congestion. As is expected, more diversity in driving behavior results in more travel time variability. On the other hand, higher levels of congestion and longer signal spacing result in less travel time variability. These findings are attributed to the lower level of travel freedom as more vehicles are introduced on a roadway.

All roadway characteristics except for the number of lanes have a significant contribution to the average speed (space-mean-speed). More traffic, from upstream and side streets, reduces the average speed, which is logical considering the speed-flow relationship in the uncongested regime. As vehicles travel farther downstream they reach steady-state conditions with higher speed and therefore the average speed is higher for longer distances. Surprisingly, more differences amongst the drivers causes a decrease in the average speed as slower vehicles slow down faster vehicles.

Variability in speed is mostly a result of differences in driver characteristics; the more the difference between drivers, the more the variability in speed. In addition, the traveled distance and traffic conditions have noticeable effects on the variability in speeds.

The values of both $\alpha$ and $\beta$ highly depend on the distance of travel (average travel time) and differences in driver behavior. For example, $\alpha$ increases as the travel distance and variability increase while $\beta$ decreases as these factors decrease. Given that $\alpha$ is the platoon dispersion factor, which represents the amount of dispersion a platoon experiences, therefore the two factors that obviously increase traffic dispersion will also increase the $\alpha$ value. On the other hand, $\beta$ is the proportion of the arrival time of the first vehicle in the platoon to the average travel time of all the vehicles in the platoon. More dispersion means that the first vehicle earlier than the average vehicle. This finding clearly demonstrates the need for link specific $\alpha$ and $\beta$ factors.

CONCLUSIONS
The primary purpose of this paper was to evaluate the effect of the underlying travel time/speed distribution on the accuracy and efficiency of platoon dispersion models. Prior to conducting the analysis the INTEGRATION traffic dispersion modeling was validated against field data by
comparing travel time distributions and predicted downstream flow profiles. Consequently, the INTEGRATION software was utilized to generate the synthetic data used for this study.

Through two different analyses on seven platoon dispersion models, qualitative and quantitative, the paper investigated the effect of the underlying travel time/speed distribution on platoon dispersion modeling performance. The analysis considered both prediction accuracy and efficiency of signal timings.

The paper demonstrated that in terms of accuracy, the prediction precision of Robertson’s model (geometric travel time distribution), which assigns the majority of dispersion to the leading edge of a flow profile, is less accurate. The difference is not noticeable for short distances of travel (less than 800m), however the difference increases as the distance of travel increases. The predicted flow profiles for all models, except for the geometric travel time distribution, offer minimum differences. However, all models fail with travel distances in excess of 800m.

In terms of efficiency, the paper demonstrates that the underlying travel time/speed distribution has a minimum impact on the optimum offset selection and the resulting performance index. This is mainly because of the fact that for short distances all the investigated models predict almost identical downstream flow profiles, while for longer distances (greater than 1 km) the prediction accuracy is poor. The decline in the prediction precision as distance increases is the result of the fundamental assumption of this family of models that ignores the interaction among vehicles. It is anticipated that a model that can include traffic stream dynamics within platoon dispersion modeling, will highly enhance signal coordination performance especially for distances longer than 1 km.

The paper demonstrates that differences in driver behavior (in desired speeds) have the highest effect on the distribution of travel times and speeds, and therefore it is important to implement this factor accurately in any traffic simulation software.

ACKNOWLEDGEMENTS
The authors acknowledge the financial support of the Mid-Atlantic University Transportation Center (MAUTC) and the Virginia Department of Transportation (VDOT) in conducting this research effort.
REFERENCES
IMPACT OF SPEED VARIABILITY FACTOR ON STEADY-STATE CAR-FOLLOWING BEHAVIOR

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Accepted for Presentation at TRB Annual Meeting, January 2006

ABSTRACT

The paper analyzes the steady-state behavior of car-following models within state-of-the-practice commercial traffic simulation software. The car-following models are classified based on their uncongested regime steady-state behavior into two categories. Apart from the INTEGRATION software that employs the Van Aerde car-following model, the research demonstrates that all state-of-the-practice traffic simulation software assume a constant desired speed that is insensitive to the level of congestion within the uncongested regime. The paper then quantifies the effect of speed variability on the steady-state characteristics of car-following models. The paper not only demonstrates that the speed variability has a significant impact on the speed-at-capacity, but also develops procedures for the calibration of the steady-state relationship while accounting for speed variability. Finally, the effectiveness and validity of the proposed procedure is demonstrated through an example illustration.

INTRODUCTION

The rapid development of personal computers over the last few decades has provided the necessary computing power for advanced traffic micro-simulators. Today, microscopic traffic simulation software is widely accepted and applied in all branches of transportation engineering as an efficient, cost effective, and safe analysis tool. One of the main reasons for this popularity is the ability of microscopic traffic simulation software to reflect the dynamic nature of the transportation system in a stochastic fashion.

The core of microscopic traffic simulation software is a car-following model that characterizes the longitudinal motion of vehicles. The process of car-following consists of two levels, namely modeling steady-state and non-steady-state behavior (Rakha and Passumarthty, 2004). Ozaki
(1993) defined steady-state as conditions in which the vehicle acceleration and deceleration rate is within a range of ±0.05 g. Another definition of steady-state or stationary conditions is provided by Rakha (In Press) as the conditions when traffic states remain practically constant over a short time and roadway distance. Steady-state car-following is extremely critical to traffic stream modeling given that it influences the overall behavior of the traffic stream. Specifically, it determines the desirable speed of vehicles at different levels of congestion, the roadway capacity, and the spatial extent of queues. Alternatively, non-steady-state conditions govern the behavior of vehicles while moving from one steady state to another through the use of acceleration and deceleration models. The acceleration model is typically a function of the vehicle dynamics while the deceleration model ensures that vehicles maintain a safe relative distance to the preceding vehicle thus ensuring that the traffic stream is asymptotically stable.

Traffic stream models describe the motion of a traffic stream by approximating for the flow of a continuous compressible fluid. The traffic stream models relate three traffic stream measures, namely: flow rate (q), density (k), and space-mean-speed (u). Gazis et al. (1961) were the first to derive the bridge between microscopic car-following and macroscopic traffic stream models. Specifically, the flow rate can be expressed as the inverse of the average vehicle time headway. Similarly, the traffic stream density can be approximated for the inverse of the average distance headway for all vehicles within a section of roadway. Therefore every car-following model can be represented by its resulting steady-state traffic stream model. Different graphs relating each pair of the above parameters can be used to show the steady-state properties of a particular model, however for this study, the speed-distance headway (u-h) and speed-flow (q-u) relationships are used to demonstrate the steady-state behavior of different models. The latter curve is of more interest, since it is more sensitive to the calibration process and the shape and nose position of the curve determines the behavior of the resulting traffic stream.

A reliable use of micro-simulation software requires a rigorous calibration effort. Because traffic simulation software are commonly used to estimate macroscopic traffic stream measures such as average travel time, roadway capacity, and average speed, the state-of-the-practice is to systematically alter the model input parameters to achieve a reasonable match between desired macroscopic model output and field data (Dowling et al. 2004). Since the macroscopic flow characteristics are mostly related to steady-state conditions, this requires the user to calibrate the parameters of the steady-state relationship and therefore the knowledge of the steady-state
behavior of the car-following model is necessary in this process. It should be mentioned that under certain circumstances, the non-steady-state behavior can also influence steady-state behavior (Rakha, In Press), however since this is not the general case the focus of this paper will be on steady-state conditions.

Over the past decade, several car-following models have been proposed and described in the literature. Brackstone and McDonald (1999) categorized the car-following models based on their non-steady-state logic into five groups, namely: Gazis-Herman-Rothery (GHR) models, safety distance models, linear models, Psycho-physical or action point models, and fuzzy logic based models. However, as it was mentioned above the measures that are usually used by transportation engineers are those of macroscopic nature which are mostly affected by the steady-state behavior of the model. Therefore a new classification based on the steady-state behavior would be of more interest from a practical standpoint.

Despite the differences in modeling logic and steady-state behavior of different traffic simulation software, several studies have shown that the output from these software are similar (comparisons are typically done for the modeling of highways). This raises the question how different models with different behaviors can produce, to some extent, similar results. Gipps (1981) noticed that the mean and standard deviation of the distribution of desired speeds affects the position and shape of the upper arm of the resulting steady-state speed-flow curve. This is very important, since the characteristics of the resulting speed-flow curve directly affects the dynamic behavior of traffic on a roadway. Despite the importance of this issue, no one has studied the effect of this factor on the steady-state behavior of car-following models. Gipps’ finding suggests that it could be hypothesized that speed variability allows different models to produce comparable steady-state behaviors.

The goals of this paper are two-fold. First, to classify the state-of-practice car-following models based on their steady-state properties in the uncongested regime, and second, to study the effect of speed variability on the steady-state behavior of different models. The paper first discusses and classifies the steady-state characteristics of six car-following models that are used in commercial and academic traffic micro-simulation software (except Greenshields’ model). Subsequently, the effect of the speed variability on the steady-state behavior of different classes of models is analyzed and a method is provided to consider this effect in the calibration process.
of the car-following model. Finally, an example application of the proposed method is presented and discussed.

**Traffic Simulation Car-Following Model**

The modeling of car-following and traffic stream behavior requires a mathematical representation that captures the most important features of the actual behavior. In this treatment, the relationships obtained by observation, experimentation, and reasoning are given; the researcher attempts to express their steady-state behavior in a graphical form, and classify them based on their steady-state representation.

Typically, car-following models characterize the behavior of a following vehicle (vehicle $n+1$) that follows a lead vehicle (vehicle $n$). This can be presented by either characterizing the relationship between a vehicle’s desired speed and the distance headway (speed formulation), or alternatively by describing the relationship between the vehicle’s acceleration and the speed differential (acceleration formulation).

Over the last few decades, several car-following and traffic stream models have been proposed and utilized in micro-simulation software packages. This section describes the steady-state characteristics of six of the state-of-practice and state-of-art car-following models, including Pipes’ model (CORSIM), Greenshields’ model, Gipps’ model (AIMSUN2), Wiedemann’s model (VISSIM), Fritzsche’s model (PARAMICS), and Van Aerde’s model (INTEGRATION). Consequently, each model is characterized based on its steady-state behavior in the uncongested regime.

It should be noted again that this study only describes car-following behavior under steady-state conditions, when the lead vehicle is traveling at a constant speed and both the lead and follower vehicles have an identical car-following behavior, i.e. $h_{n+1} \approx h_{\text{desired}}, \Delta u_{n+1} \approx 0$, where $h_{n+1}$ is the distance headway between the lead vehicle (vehicle $n$) and following vehicle (vehicle $n+1$) and $\Delta u_{n+1}$ is the relative speed between the lead and following vehicles. In addition to these two conditions, it is assumed that no randomness (variability) is observed. Although unrealistic, this assumption is crucial to this analysis since, as it was mentioned earlier, the variability (randomness) affects the steady-state characteristics and therefore in order to be able to characterize this effect first we have to learn about the models in absence of it and then investigate any changes caused by this factor.
CORSIM

CORSIM is developed by the Federal Highway Administration (FHWA) and combines two traffic simulation models: NETSIM for surface streets and FRESIM for freeway roadways. Although, each of these models uses a different car-following formulation, Rakha and Crowther (2003) have shown that in steady state conditions both models revert to the form,

\[ u = \min \left( u_i, \frac{h_{i+1} - h_i}{c_3} \right) \]  \[ \text{[7.1]} \]

where \( h_j \) is the distance headway when vehicles are completely stopped in a queue (km), \( u_f \) is the roadway free-speed (km/h), and \( c_3 \) is a driver sensitivity factor (h). If we assume all vehicles are identical then the vehicle subscripts can be dropped from the formulation. The value of the driver sensitivity parameter \( c_3 \) is fixed and equal to 1/3600 in the case of NETSIM, and in the case of FRESIM can be computed as follows (Rakha and Crowther, 2003),
where \( q_c \) is the capacity of the link (veh/h/lane). This car-following behavior is identical to the Pipes car-following model. The Pipes car-following model and the traffic stream models that evolve from it are multi-regime in nature, with different models for the congested versus uncongested regimes, as illustrated in Figure 7.1. Specifically, the Pipes model assumes that the desired speed is insensitive to the traffic density in the uncongested regime; therefore its steady-state behavior in the uncongested regime is constrained and flat. Equation 7.1 and Figure 7.1 obviously illustrate that the Pipes model assumes that the speed-at-capacity equals the free-speed (\( u_c = u_f \)), and therefore it is an externally constrained flat-top model. The Pipes model has three degrees of freedom which means that the calibration of this model requires the determination of three parameters, namely: the roadway free-speed (\( u_f \)), the spacing of vehicles at jam density (\( h_j \)), and the roadway capacity (\( q_c \)).

**Greenshields’ Model**

Although the Greenshields model is not implemented in any commercial traffic micro-simulation software, it is presented because of the historical importance of this model and its common and simple use in textbooks and transportation planning macroscopic models.

Greenshields (1953) proposed the first and most famous single-regime traffic stream model. This model assumes that the relationship between speed (\( u \)) and density (\( k \)) is linear, as shown in Equation 7.3. The car-following that evolves from Greenshields’ traffic stream model is provided in Equation 7.4 (speed formulation). As was the case with the Pipes model, given that all vehicles are assumed identical the vehicle index in Equation 7.4 can be dropped.

\[
\begin{align*}
    \frac{c_3}{q_c} &= \frac{h_j}{u_f} \\
    u &= u_f \left( 1 - \frac{k}{k_j} \right) \quad [7.3] \\
    u &= u_f \left( 1 - \frac{h_j}{h} \right) \quad [7.4]
\end{align*}
\]

In contrast to Pipes’ model, the Greenshields car-following and traffic stream model is a single regime model and thus does not need to enforce the roadway free-speed as an external constraint.
This difference is illustrated in Figure 7.1 which illustrates that the Greenshields model, unlike the Pipes’ model, is an internally constrained curved-top model.

The Greenshields model assumes that \( u_f = 2u_c \), and therefore the capacity is determined by \( q_c = (u_f k_j)/4 \). Consequently, the Greenshields’ model has two degrees of freedom and can be calibrated by estimating the values of two parameters: \( u_f \) and either \( h_j \) or \( q_c \).

![Flow Speed Graph](image1)

![Distance Headway Graph](image2)

Figure 7.2: Steady-state behaviour of the Gipps model.

**AIMSUN2**

AIMSUN2 is designed and developed at the Universitat Politecnica de Catalunya, Spain. AIMSUN2 uses The Gipps car-following model (1981) which consists of two components: acceleration and deceleration sub-models. These two sub-models are explained using empirical formulations (SI units) illustrated by Equations 7.5 and 7.6, respectively;

\[
u^*_n(t + T) = u_n(t) + 2.5u_{n+1} \max T \left( 1 - \frac{u_n(t)}{u_{n+1} \max} \right) \sqrt{0.025 + \frac{u_{n+1}(t)}{u_{n+1} \max}} \tag{7.5}\]
\[ u_{n+1}^d(t + T) = a_{n+1}^{\text{max}} T + \sqrt{\left((d_{n+1}^{\text{max}} T)^2 - d_{n+1}^{\text{max}} \right)} \left( 2\{x_n(t) - L_n - x_{n+1}(t)\} - u_{n+1} T - \frac{u_n(t)^2}{d_{n}^{\text{des}}} \right) \]  \[7.6\]

where \( u_{n+1}(t) \) is the speed of vehicle \( n+1 \) at time \( t \), \( u_{n+1}^{\text{max}} \) is the desired speed of vehicle \( n+1 \), \( a_{n+1}^{\text{max}} \) is the maximum acceleration for vehicle \( n+1 \), \( T \) is the reaction time, \( d_{n+1}^{\text{max}} \) is the maximum deceleration desired by vehicle \( n+1 \) (\( d_{n+1}^{\text{max}} < 0 \)), \( x_n(t) \) is the position of vehicle \( n \) at time \( t \), \( L_n \) is the effective length of vehicle \( n \), and \( d_{n}^{\text{des}} \) is an estimation of the desired deceleration of vehicle \( n \). The final speed of vehicle \( n+1 \) during time interval \((t, t+T)\) is the minimum of \( u_{n+1}^a \) and \( u_{n+1}^d \). Equation 7.5 demonstrates that the Gipps model is constrained by the constant desired speed (which is equal to free-speed) of the vehicle. Wilson (2001) performed a mathematical analysis of the Gipps model and showed that this model is a flat-top model in steady-state conditions, as illustrated in Figure 7.2. Again, we ignore vehicle/driver differences the vehicle index can be dropped. Furthermore, if we assume instantaneous vehicle reactions the time index can also be dropped.

**VISSIM**

The car-following model used in VISSIM is a modified version of Wiedemann’s model (1974) and belongs to a family of models known as psychophysical or action-point models. This family of models uses thresholds or action-points where the driver changes his/her driving behavior. Drivers react to changes in spacing or relative speed only when these thresholds are crossed. The thresholds and the regimes they define are usually presented in the relative speed/spacing diagram for a pair of lead and follower vehicles. Figure 7.3a illustrates such a diagram for the VISSIM car-following model (Fellendorf and Vortisch, 2000).

For the purposes of this study only the area identified as steady-state is of interest. This area has the mentioned criteria for steady-state behavior (\( h_{n+1} = h_{\text{desired}}, \Delta u_{n+1} \approx 0 \)). The only issue is that the desired headway is an interval (\( ABX \leq h \leq SDX \)) instead of a single value as was the case for the previous models. Given that \( \Delta u_{n+1} \approx 0 \), only the boundaries of desired headway interval (\( ABX \) & \( SDX \)) determine the steady-state characteristics of the VISSIM car-following model. Ignoring any vehicle differences, the ABX and SDX parameters can be calculated using Equations 7 through 9:

\[ AX = h_j + 0.5 \]  \[7.7\]
Figure 7.3: VISSIM's car-following model. a. thresholds and regimes, b & c. steady-state behaviour.

\[ BX = BX_{add} + 0.5 \times BX_{mult} \quad \text{[7.8]} \]

\[ ABX = AX + BX \sqrt{u}, \quad u \leq u_{\text{desired}} \quad \text{[7.9]} \]
and the $SDX$ ranges between 1.5 and 2.5 times the $ABX$ parameter, where $BX_{add}$ and $BX_{mult}$ are user-defined calibration parameters.

Equation 7.9 demonstrates that the parameters $ABX$ and $SDX$ are not internally constrained and thus an external maximum speed constraint ($u \leq u_{desired}$) must be enforced. Given that the desired speed is insensitive to traffic conditions ($u_{desired} = u_c = u_f$), the uncongested steady-state behavior of VISSIM is similar to Pipes’ car-following model, and therefore the VISSIM car-following model is also an externally constrained flat-top model, as illustrated in Figure 7.3b and 3c.

The distribution of headways in the steady-state area determines the resulting roadway capacity. Through running some simulations, it was observed that for small $ABX$ values ($ABX < 30$ m), resulting vehicles’ steady-state headways can be assumed to be uniformly distributed between $ABX$ and 30 m. In contrast, for longer $ABX$ distances ($ABX > 30$ m) vehicles’ steady-state headways tend to converge to the $ABX$ value. This behavior is consistent with field driving behavior observed by Brackstone et al. (2002).

**PARAMICS**

The car-following model utilized in Paramics, like the VISSIM model, is a psychophysical car-following model that was developed by Fritzsche (1994). Fritzsche’s model uses the same modeling concept as VISSIM’s car-following model. The difference between these two models is the way thresholds are defined and calculated. Figure 7.4a depicts the Fritzsche model’s thresholds in the $\Delta u - \Delta x$ plane.

The area corresponding to steady-state conditions is almost identical to VISSIM’s car-following model. The headway for this regime lies between the desired headway ($AD$) and the risky distance ($AR$). These two boundaries are determined using Equations 7.10 and 7.11.

$$AR = A_0 + T_r \times u_v$$  \hspace{1cm} [7.10]  

$$AD = A_0 + T_D \times u_{v+1}$$  \hspace{1cm} [7.11]

where $A_0$ is gross standstill distance, $T_r$ is the risky time gap (usually 0.5 s), $T_D$ is the desired time gap (recommended value: 1.8 s).

Similar to VISSIM’s car-following model, the desired speed constraint must be enforced externally and since the desired speed is insensitive to traffic conditions ($u_{desired} = u_c = u_f$),
therefore the resulting steady-state behavior is an externally constrained flat-top relationship, as demonstrated in Figure 7.4b and c.

Figure 7.4: Fritzsche’s car-following model a. thresholds and regimes, b & c. steady-state behaviour.
INTEGRATION

The INTEGRATION model uses a steady-state car-following model that was proposed by Van Aerde (1995) and Van Aerde and Rakha (1995). The functional form of the Van Aerde model combines the Greenshields and Pipes models, as demonstrated in Equation 7.12.

\[ h = c_1 + c_2 u_{n+1} + \frac{c_2}{u_i - u_{n+1}} \]  \[ [7.12] \]

\[ c_1 = \frac{u_i}{k_j u_c^2} (2u_c - u_i) \]  \[ [7.13] \]

\[ c_2 = \frac{u_i}{k_j u_c^2} (u_i - u_c)^2 \]  \[ [7.14] \]

\[ c_3 = \frac{1}{q_c} - \frac{u_i}{k_j u_c^2} \]  \[ [7.15] \]

The calibration of the Van Aerde car-following model requires estimating four parameters, namely \( c_1, c_2, c_3 \) and \( k_j \) utilizing Equations 13 to 15. These four parameters are a function of the roadway free-speed \( (u_f) \), the speed-at-capacity \( (u_c) \), capacity \( (q_c) \), and jam density \( (k_j) \).

The Van Aerde model is a single regime model which combines the Greenshields and Pipes models to address the main flaws of these models. Specifically, the model overcomes the shortcoming of the Pipes model in which it assumes that vehicles’ desired speeds are insensitive to traffic density in the uncongested regime, which has been demonstrated to be inconsistent with a variety of field data from different facility types (Rakha and Crowther, 2003). Alternatively, the model overcomes the main shortcoming of the Greenshields model, which assumes that \( u_c = 0.5u_f \) and the speed-flow relationship is parabolic, which again is inconsistent with field data from a variety of facility types as demonstrated by Rakha and Crowther (2003). The Van Aerde model is an internally constrained curved-top model, as clearly demonstrated in Figure 7.1.

SAMPLE TRAFFIC STREAM DATA

Figure 7.5 illustrates sample data from a variety of roadways that are provided in the literature (May, 1990) or obtained from the field. These data include a Dutch freeway, a German Autobahn, a tunnel (Holland Tunnel, NY), and an arterial street in the UK being monitored using the Split Cycle and Offset Optimization Tool (SCOOT) system.
The data provided in Figure 7.5a are obtained from a section of the Amsterdam Ring Road, Netherlands, which has a speed limit of 100 km/h. The shape of the upper portion of the speed-flow relationship appears to be close to linear (flat-top) as a result of the speed limit restriction. In contrast, Figure 7.5b, c, and d illustrate a more parabolic fit to the data when the roadway geometry and control is more restrictive than the roadway speed limit. It must be mentioned that there is a very high speed limit on the German Autobahn, and therefore the roadway geometry is a restricting factor. Super-imposed on these data are the fitted Van Aerde model. The parameters of the fitted traffic stream model are also provided in the figures. Since the curves provide very good fits, the parameters of these curves are taken as a reliable estimate of the observed traffic stream’s principal parameters ($u_f$, $u_c$, $q_c$, and $k_j$). The resulting $u_c$ to $u_f$ ratio ($u_c/u_f$) for the Dutch freeway data is equal to 0.87, while for the German Autobahn, tunnel data, and arterial street data the values are 0.64, 0.51, and 0.50, respectively. Matching these ratios with the corresponding data, it is obvious that as the $u_c/u_f$ ratio decreases the curvature of the upper portion of the speed-flow relationship increases, therefore this ratio is a reliable representation of the uncongested regime’s curvature.

The data illustrated above demonstrate that the uncongested regime of the speed-flow relationship is not flat; instead, it has curvature and $u_c \neq u_f$. Alternatively, as was demonstrated in
the previous section, all commercially available traffic micro-simulation software packages, except for INTEGRATION, assume a constant flat-top steady-state model for the uncongested regime (assume $u_c = u_f$). However, the simulation results from various studies (Gipps, 1981, and Fellendorf and Vortisch, 2000) which used these models showed $u_c < u_f$ and a curvature in the uncongested regime. This raises the question “why a curvature is observed while the models assume that there is no such a curvature, and how can we control this curvature to produce the desired steady-state behavior.” This question will be answered in the remainder of the paper.

Gipps (1981) found that the distribution of desired speeds affects the position and shape of the upper arm of the resulting steady-state speed-flow curve. Remembering that one of the main assumptions was made to derive the steady-state behavior of the models was the absence of any variability (behavior difference), Gipps’ finding suggests that the answer to the above question might be differences in drivers’ desired speed. This can be expressed as; the resulting steady-state behavior is a result of the interaction of the steady-state car-following model (ignoring behavioral differences) plus the effect of variability.

The reason that the effect of speed variability is of special importance is the fact that any factor that alters the shape of the speed-flow relationship, directly affects the dynamic behavior of the simulated roadway. It is surprising that despite the importance of this issue, it has not been studied in the literature. In the next section, this issue is studied and a method is proposed to effectively consider the influence of speed variability in the calibration process.

**IMPACT OF SPEED VARIABILITY ON TRAFFIC STREAM PARAMETERS**

This section characterizes the impact of the desired speed variability on the shape and parameters of steady-state traffic stream and car-following models. In conducting the study a number of facilities with different steady-state behaviors and different levels of speed variability were coded in the INTEGRATION micro-simulator and the results were analyzed to quantify the effect of speed variability on the steady-state parameters. The main reason of choosing the INTEGRATION software for this study is that it is the only model that allows the user to input a speed-at-capacity that differs from the free-speed and therefore control the shape of the steady-state relationship.

INTEGRATION considers the speed variability by using a user defined desired speed coefficient of variation ($CV^{in}$), which characterizes differences in driver in speed selection. The software
then determines the desired speed of each vehicle using a normal or lognormal distribution with a mean equal to the steady-state desired speed \( u_{desired}^m \) that is computed using Equation 7.12 and a standard deviation that is computed as \( \sigma = CV_{in}^m \times u_{desired}^m \). Although other software may have more flexibility in terms of defining the shape of the desired speed distribution, various studies have showed that a normal or lognormal distribution is a good approximation for the speed distribution. For this study the normal distribution of speeds is considered and implemented in the coding. Before going any further, it must be mentioned that the superscript “in” is used for parameters coded into the simulation software, and the superscript “out” refers to parameters that are derived as outputs from the simulation software.

The purpose of this study is to characterize the traffic stream parameters in absence of any conditions that alter the basic traffic stream characteristics, such as weaving sections and traffic signals. Therefore, the network configuration and O-D table used for this study were selected for a basic roadway condition. Figure 7.6 depicts the network layout used in this study. All the links in the network have 2 lanes. A loop detector was considered 800 m downstream of the network entrance to gather data when vehicles reach steady-state conditions. The on-ramp which is located 2 km downstream was introduced to produce congestion upstream. The O-D table was constructed in a way that all the traffic stream regimes (free-flow regime, capacity, and congested regime) were observable at the detector location. All the vehicles were allowed to perform lane changing maneuvers based on INTEGRATION’s lane changing logic and no bias was considered toward any lane.

![Figure 7.6: Network used in the simulation study.](image)

The combination of five free-flow speeds \( u_f \) and six saturation flow values \( q_c \) was considered to cover different roadway facilities. However, only 19 pairs were considered realistic (e.g. a low free-speed speed with a high lane capacity was considered unrealistic). For each set of combinations four values of the \( u_c^{in} / u_f^{in} \) ratio were considered (0.50, 0.67, 0.83, and 1.00) and
five levels of variability ($CV^{in} = 0\%, 5\%, 10\%, 15\%, \text{and} 20\%$) were assigned. For all the scenarios the jam density was considered to be 150 veh/km/lane. To achieve statistically significant estimates of the parameters with a 95\% confidence limit, the number of repetitions was computed considering the standard deviation of the first 10 replications. In total a minimum of 3800 (19 x 4 x 5 x 10) simulation runs are executed; each consists of 2 hrs of traffic simulation.

The loop detector gathered speed, flow, and occupancy data at 1 min intervals. The traffic stream parameters of each simulation run were estimated using the SPD_CAL software (Van Aerde and Rakha, 1995). SPD_CAL finds the optimum free-speed, speed-at-capacity, capacity, and jam density by minimizing the normalized orthogonal error between the observed data and the functional relationship. A detailed description of SPD_CAL logic is beyond the scope of this paper but can be referenced elsewhere (Rakha, In Press). The resulting steady-state parameters were then checked against the input parameters to identify any statistically significant differences. The results demonstrated that for all cases the required numbers of replications was less than 10 with a significance level of 5\%.

A stepwise linear regression is utilized to identify the critical variables that affect the four traffic stream parameters ($u_f$, $u_c$, $q_c$, and $k_j$). Of all the four traffic stream parameters, only $k_j$ remained unchanged between inputs and outputs for all the simulation runs. Among the remaining three parameters, $u_c$ and $q_c$ were found to have a significant correlation to the speed variability level $CV^{in}$ ($R^2 = 37\%$, and 55\% for $u_c$, and $q_c$ subsequently), $u_c^{in}$ ($R^2 = 19\%$, and 8\% for $u_c$, and $q_c$ subsequently) and $u_c^{in}/u_f^{in}$ ($R^2 = 49\%$, and 26\% for $u_c$, and $q_c$ subsequently), however the correlation of the these traffic stream parameters with the values of the $u_f^{in}$, and $q_c^{in}$ were weak ($p$-value<0.05 but $R^2<1\%$). $u_f$ was found to have significant correlation only to $CV^{in}$ ($R^2 = 94\%$). Since the $R^2$ corresponding to $u_c$, and $q_c$ for $u_c^{in}/u_f^{in}$ was at least twice as $R^2$ for $u_c^{in}$, it is decided that $u_c^{in}/u_f^{in}$ and $CV^{in}$ were sufficient to describe differences between simulated input and output parameters. It must be noted that $u_c^{in}/u_f^{in}$ is found to have no significant correlation with $u_f$ ($p$-value>0.05), however to be consistent with the other parameters it is decided to use $u_c^{in}/u_f^{in}$ in the analysis.

Figure 7.7 illustrates the relationship between the coded input parameters and the estimated parameters derived from the simulation software. The thick lines in the graphs represent the
mean parameter values, while the thin lines are obtained using linear interpolation of the simulation results. Figure 7.7a illustrates not only demonstrates that the shape of the steady-state traffic stream model changes as the level of speed variability \((CV^{in})\) varies, but also that this change is more significant for greater as the speed-at-capacity approaches the free-speed.

**Figure 7.7:** Effect of speed variability on the steady-state parameters.

Consequently, the results demonstrate that model users can, to a limited extent, control the curvature of the uncongested steady-state behavior using the \(CV^{in}\) input variable. Figure 7.7a also demonstrates the impact that \(CV^{in}\) has on the shape of the steady-state model for different car-following models (i.e. different values of \(u_{c}^{in}/u_{f}^{in}\)). For example, the flat-top models \((u_{c}^{in} = u_{f}^{in})\), on average, are capable to produce steady-state patterns with \(0.8u_{f}^{out} \leq u_{c}^{out} \leq 0.9u_{f}^{out}\) by varying the \(CV^{in}\) parameter, however they are incapable to produce steady-state behavior like the ones presented earlier in Figure 7.5b, c, and d. Figure 7.7b demonstrates that the output free-speed decreases linearly as the speed variability increases. The reduction remains fairly constant for all \(u_{c}^{in}/u_{f}^{in}\) values. Figure 7.7c demonstrates that flat-top models are more sensitive to speed
variability than curved uncongested regime models. Specifically, for the curved-top models \((u_c \neq u_f)\) the effect of speed variability is almost identical for all \(u_c^{in}/u_f^{in}\) ratios. The biggest effect of \(CV^{in}\) is on the speed-at-capacity \((u_c)\), as demonstrated by Figure 7.7d. The Figure 7.7d clearly demonstrates that the output speed-at-capacity declines as the \(CV^{in}\) increases, and the reduction is significant for higher \(u_c^{in}/u_f^{in}\) ratios.

PROPOSED CALIBRATION PROCEDURE AND EXAMPLE APPLICATION

The graphs in Figure 7.7 can be used to calibrate micro-simulation traffic models in order to replicate a desired steady-state behavior by altering the speed variability factor. The recipe of the proposed procedure is as follows:

**Step 1**-Determine the traffic stream parameters of the desired steady-state behavior \((u_f^{out}, u_c^{out}, q_c^{out}, \text{and } k_j^{out})\) for each link. This is usually done by fitting a curve to loop detector data obtained from the field.

**Step 2**-

a. If the model is flexible, i.e. can have \(u_c \neq u_f\), determine the desired level of speed variability \((CV^{in})\) using field data. Subsequently, find the closest line to point \((CV^{in}, u_c^{out}/u_f^{out})\) in Figure 7.2a to identify the desired input ratio of speed-at-capacity to free-speed \((u_c^{in}/u_f^{in})\).

b. If the model is a flat-top model, draw a horizontal line from the desired \(u_c^{out}/u_f^{out}\) value that intersects the line with \(u_c^{in}/u_f^{in} = 1.0\), then draw a vertical line down to the axis and read the desired \(CV^{in}\) value.

**Step 3**-Use the \(u_c^{in}/u_f^{in}\) and \(CV^{in}\) values of step 2 to compute the \(u_f^{in}, u_c^{in}, \text{and } q_c^{in}\) parameters using Figure 7.7b, c, and d. Consider \(k_j^{in}\) to equal \(k_j^{out}\).

**Step 4**-Code the computed \(u_f^{in}, u_c^{in}, q_c^{in}, \text{and } k_j^{in}\) input parameters to the micro-simulation software.

The proposed calibration procedure was validated using the INTEGRATION software. The network used in the validation effort had the same layout shown as was presented earlier Figure 7.6 except that all freeway links had three lanes instead of two lanes. The desired steady-state behavior was \(u_f^{out}, u_c^{out}, q_c^{out}, k_j^{out}\) of 80 (km/h), 66.4 (km/h), 2000 (veh/h/lane), 120
(veh/km/lane), respectively. Using Figure 7.7a with a $u_{c_{out}}/u_{f_{out}}$ ratio of 0.83 (66.4/80) can be produced using a flat-top model with a $CV_{in}$ of 10% (case-1) and 20% (case-2), or using the Van Aerde model with $u_{c_{in}}/u_{f_{in}}$ ratio of 0.83 and $CV_{in}$ of 5% (case-3). For each of these three cases, the input parameters are estimated using Figure 7.7b, c, and d to be (89.89, 89.89, 2174, 120), (100, 100, 2247, 120), and (85.1, 70.6, 2028, 120) for cases 1, 2, and 3, respectively. After finding the input parameters, the three cases were coded in the INTEGRATION software using an O-D demand matrix that was the same for all three cases to provide data points over the entire regimes. Figure 7.8 illustrates the results of the simulation runs for all the cases. Figure 7.8a illustrates the desired steady-state speed-flow relationship together with the input relationships for each of the cases. The figure clearly demonstrates significant differences in the speed-flow relationships, however all three cases resulted in almost identical static and dynamic steady-state behavior.

<table>
<thead>
<tr>
<th>Description</th>
<th>$u_{c}$ (km/h)</th>
<th>$u_{f}$ (km/h)</th>
<th>$q_{c}$ (veh/h)</th>
<th>$k_{j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1 Pipes + CV=10%</td>
<td>89.9</td>
<td>89.9</td>
<td>2173</td>
<td>120</td>
</tr>
<tr>
<td>Case2 Pipes + CV=20%</td>
<td>100</td>
<td>100</td>
<td>2247</td>
<td>120</td>
</tr>
<tr>
<td>Case3 Van Aerde + CV=5%</td>
<td>85.1</td>
<td>70.6</td>
<td>2020</td>
<td>120</td>
</tr>
<tr>
<td>Output Desired behavior</td>
<td>80</td>
<td>66.4</td>
<td>2000</td>
<td>120</td>
</tr>
</tbody>
</table>

Figure 7.8: Application example of the proposed method.

The example presented in this section illustrates how the proposed method can be used to control the steady-state behavior of micro-simulation software. It also demonstrates the validity of the proposed calibration procedure for different number of lanes and different values of $k_{j_{in}}$. 115
CONCLUSION
The paper describes the steady-state car-following behavior for the state-of-the-practice commercial traffic micro-simulation software. The models are then classified into two categories depending on the shape of the uncongested regime of the speed-flow curve as: flat-top models and curved-top models. The paper demonstrates that apart from the INTEGRATION software, VISSIM, Paramics, AIMSUN2, and CORSIM all consider a flat-top model. The paper then studies the impact of driver differences on the shape of the fundamental diagram and the key model parameters ($u_f$, $u_c$, $q_c$, and $k_j$). The paper demonstrates that driver differences have significant influence on the observed steady-state behavior of micro-simulation models. Specifically, these differences alter the speed-at-capacity and therefore change the shape of the fundamental speed-flow diagram. The paper also demonstrates that flat-top models are incapable of produce highly curved steady-states behaviors (i.e. $u_c^{\text{out}}/u_f^{\text{out}}$ ratios less than 0.8) through the modeling of differences in desired speeds. Finally, calibration procedures were developed to allow model users to achieve a desired steady-state behavior. The validity and effectiveness of the proposed procedures were demonstrated through an example illustration.

It should be noted, however, that some of the discussed software may allow modelers to calibrate the desired steady-state behavior through other variables. Consequently, it is recommended that the effect of these other forms of variability be investigated.

REFERENCES


CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

8.1 STUDY CONCLUSIONS
The research presented in this document analyzes traffic dispersion phenomenon and provides some enhancements for effective modeling and calibration of this phenomenon. The conclusion of this research work can be summarized in three categories; calibration of Robertson’s model, sensitivity analysis, and microscopic modeling.

8.1.1 CALIBRATION OF ROBERTSON’S MODELS
The dissertation identifies the shortcomings of the calibration procedure proposed by Yu and Van Aerde

- Proper calibration of the recursive platoon dispersion model is important to achieve and maintain a good signal timing plan. The proposed method by Yu and Van Aerde (1995) provides the necessary tool to calibrate the model based on observed travel time statistics.

- The Yu and Van Aerde calibration method suffers from a shortcoming in the formulation. The original formulation doesn’t take into account the time interval step size. This results in an estimated downstream cyclic profile with a margin of error that increases as the size of the time step increases.

- Three methods are proposed to address this shortcoming. The first method uses a second-by-second analysis and is at the same time the most accurate and most computationally extensive method. The second method simplifies the first method by ignoring the dispersion within each interval. The resulting accuracy is less than the first method; however, the difference is within acceptable range. The third method is the simplest method which uses a scaling factor and provides a level of accuracy close to the second method.

- The proposed models were validated against field and simulated data. The results clearly demonstrate that the proposed model prediction error is not affected by the size of the modeling step size. It is anticipated that the implementation of the proposed formulations can
enhance the accuracy of traffic dispersion modeling that is key to the design of off-line and real-time traffic-signal control systems.

- The resulting delay is more sensitive to the value of the travel time factor $\beta_n$ than the platoon dispersion factor $\alpha_n$ and thus the calibration of $\beta_n$ is more critical than the calibration of $\alpha_n$. The importance of calibrating $\beta_n$ is more significant for larger signal spacing distances. On the other hand, using a unique value of $\alpha_n$ provides a reasonable accuracy as was suggested by Retzko and Schenk (1993).

- The popular TRANSYT software assumes that the travel time factor is fixed at 0.8. The document demonstrates the effect of this limitation on the efficiency of the resulting signal coordination plan and develops a procedure that overcomes this limitation by adjusting the average travel time in the model in order to control the value of the travel time factor indirectly.

- The dissertation presents some numerical examples to demonstrate the effectiveness of different calibration methods of the recurrence platoon dispersion model. Results indicate that (a) none of the calibration methods guarantees that the derived calibrated parameters result in minimum delay and (b) the third proposed method provides better results than default values.

### 8.1.2 Sensitivity Analysis

The dissertation uses the INTEGRATION microsimulation software to generate data to evaluate the prediction precision and performance of seven different platoon dispersion models. The conclusions can be summarized as follows:

- Almost all of the previous studies used relatively short distances (less than 1 km) to investigate the platoon dispersion behavior. The investigation in this study uses long distances (up to 2 km) to analyze the behavior.

- Among the investigated travel time distributions, the geometric distribution (which is utilized in TRANSYT) produces the highest prediction error. Results suggest that symmetric distributions are better suited for dispersion modeling. Normal and lognormal distributions result in better flow profile prediction than others. However, it is observed that for short distances the results of all the models are within the acceptable range and for long distances
the predicted flow profile by all the models are not inaccurate because they ignore vehicle interaction effects. Therefore, it can be concluded that none of the models are superior.

- The lack of accuracy for long distances for the family of models studied in this research arises from the fact that this form of modeling assumes that all the vehicles in a platoon have the same distribution of travel time. This assumption is not consistent with actual behavior on the road, since the vehicles at the front of the platoon have more freedom to choose their speed, while the vehicles in the middle or back of the platoon are limited by vehicles ahead and therefore experience a more restricted driving environment.

- In terms of efficiency two performance indices are examined, namely percent extra delay and offset deviation. The study demonstrates that the type of model has a very weak effect on these measures. On the other hand, the distance of travel has the biggest impact on the efficiency of the models. All the models perform well for short distances (less than 800m), and as the distance increases the efficiency deteriorates.

- The simulation results demonstrate that even for long travel distances vehicles remain in platoons, and therefore signal coordination is still beneficial for long signal spacing (greater than 800m). This opposes the widely accepted assumption that for longer distances, arrivals of vehicles will be random; therefore considering platoon dispersion for longer distances only provides small benefit from coordination. In fact it was found that for a 2 km roadway appropriate signal coordination can decrease the delay up to 28% on average.

- The parameters that have significant influence on the travel time variability are driving behavior differences, distance from upstream traffic signal, and incoming flow from upstream signal. As would be expected, more diversity in driving behavior means more variability in travel times. On the other hand, higher flows and longer distances results in less travel time variability. This is because higher flows result in less freedom for drivers and thus more uniform driving patterns.

- All the link characteristics except for the number of lanes have significant impacts on average speed (space-mean speed). More traffic, from upstream and side streets, reduces the average speed, which is logical considering the speed-flow relationship in the uncongested regime. As vehicles travel farther downstream they reach a steady-state with higher speeds
and therefore the average speed will be higher for longer distances. Surprisingly, more difference amongst the drivers causes a decrease in the average speed.

- As is expected, the variability in speeds is mostly controlled by drivers’ differences; the higher the difference between drivers, the more the variability in speed.

- The values of both $\alpha$ and $\beta$ highly depend on the distance of travel (average travel time) and drivers’ differences. $\alpha$ is an increasing function of both of these parameters, and $\beta$ is a decreasing function of them. Considering the concept that $\alpha$ and $\beta$ are representing, the mentioned pattern is quite reasonable. $\alpha$ is the platoon dispersion factor and represents the amount of dispersion a platoon experiences and therefore the two factors that obviously increase the dispersion will also increase the $\alpha$ value. On the other hand, $\beta$ is the proportion of the arrival time of the first vehicle in the platoon to the average travel time of all the vehicles in the platoon. More dispersion means the first vehicle arrives in less proportion of average travel time, and therefore the parameters that increase the dispersion will decrease the $\beta$ value.

8.1.3 MICROSCOPIC MODELING

- The dissertation validates the dispersion module of the INTEGRATION microsimulation software. The current version of INTEGRATION uses a speed variability factor to simulate the differences among drivers in desired speed selection. Comparison to field data demonstrated that the model is able to capture the dispersion behavior accurately and in a realistic fashion.

- The dissertation derives the steady-state car-following behavior of some popular microsimulation models. It is illustrated that with one exception, INTEGRATION which uses the Van Aerde car-following model, all the software assume that the desired speed in the uncongested regime is insensitive to the level of congestion. This assumption is inconsistent with observed field behavior, especially on facilities with low geometric standards or very high speed limits.

- The document evaluates the effect of desired speed distribution on the shape and parameters of the steady-state behavior of microsimulation models. In this study a number of facilities with different steady-state behaviors and different levels of speed variability were considered.
The results demonstrate that speed variability affects the shape of the steady-state behavior curve to some extent.

- Plots are generated to quantify the effect of speed variability on different traffic stream parameters. The graphs show that by using the speed variability to control the resulting steady-state behavior of the models, the users of flat top speed-flow models are unable to produce all traffic conditions simply by varying the speed variability.

- A method is proposed to effectively consider the influence of speed variability in the calibration process in order to control the steady-state behavior of the model. Finally, the effectiveness and validity of the proposed method is demonstrated through an example application.

**8.2 Recommendations for Future Research**

The following areas of research should be pursued to expand the current research work on macroscopic and microscopic modeling of traffic dispersion:

- More field data on platoon dispersion are required to expand the model and cover wider range of facilities. Such a data should also include dispersion along longer distances.

- The current structure of models assumes the same travel time distribution for all vehicles in a platoon. This assumption causes higher dispersion for long distances than is observed in the field. A method that can handle differences inside a platoon can strongly improve the macroscopic dispersion modeling and provide accurate results for longer distances. Intelligent system modeling methods, such as fuzzy systems and neural networks, potentially can be used for this purpose.

- There are very few researches available on the direct links of traffic dispersion and environmental impacts of traffic. Specifically, there is a lack of knowledge about the relation between vehicles’ emissions and traffic dispersion pattern. A comprehensive study of this issue will help to understand the interaction of traffic dispersion and the environmental impacts, and also help to developing new energy and emissions models.

- In this document only the effect of the speed variability factor is investigated. However, some models consider differences in driving behavior by assuming some other variability factors. This additional randomness can also potentially affect the steady-state behavior;
therefore further research is required to quantify the effect of these factors on the steady-state behavior of a model.

- The conventional way to model traffic dispersion microscopically is to consider randomness around a deterministic car-following or traffic stream model. This approach usually assumes that the level of randomness is the same along the entire deterministic model. Consequently, the approach is a one degree of freedom process. Improvements could be achieved by varying the level of randomness depending on the level of congestion in the vicinity of the vehicle.
BIBLIOGRAPHY


[58]. Rakha, H. Validation of Van Aerde’s Simplified Steady-State Car-Following and Traffic Stream Model. Submitted to *Transportation Science*.


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- Applications of Intelligent Systems in Transportation Engineering (Neural Networks, Fuzzy Systems, and Genetic Algorithms)
- Travel Demand Modeling

PUBLICATIONS AND PRESENTATIONS


