Chapter 1

Introduction

The term combustion instability refers to a wide variety of oscillatory phenomena observed in combustion systems. Unstable combustion is not desirable and it can reduce efficiency and increase pollution. Considering that the combustion process leads to a localized heat release with high energy, there is no surprise that instability occurs. Combustion takes place inside a chamber with inlet-outlet conditions, where the acoustic system can also introduce oscillatory effects, like a Helmholtz oscillator. The coupled heat-release process and acoustic dynamics can produce a large amplitude pressure oscillation, which is called a “thermoacoustic instability”. The growth of the pressure oscillation degrades the system performance of a combustion engine. There are a variety of other sources of combustion instabilities including thermodiffusive instabilities, chemical-kinetic instabilities, convective instabilities, and Kelvin-Helmholtz instability [Kat98]. This research will focus mainly on the modeling of thermoacoustic instabilities produced by the coupling between heat release rate and combustor acoustic dynamics.

To model the thermoacoustic instability for a tube combustor with premixed laminar flame, we considered a feedback structure of the combustion system. The unsteady heat release rate adds energy to the combustor-acoustic system, while acoustic pressure oscillations affect the incoming velocity of fuel-oxidizer mixture feeding into the flame front. Figure 1 shows the diagram of the feedback scheme.
Figure 1. A feedback structure of linear/nonlinear parametric model

When unsteady heat release rate within a combustion chamber is coupled to the pressure oscillation associated with the chamber acoustics, the system becomes unstable and the response grows into a limit cycle. The system is called “self-excited”, and the linear part of the system is unstable with right half-plane poles. With any given input or initial conditions, the linear system response will grow without bound. The nonlinearity of the system causes the limit cycle response. We are interested in the prediction of the limit cycle occurrences, frequencies and oscillatory amplitudes.

Unsteady rate of change of heat release, also referred as flame dynamics, is the most complicated part of the modeling process due to the complexity of chemical kinetics and physical transport properties. In addition, acoustic dynamics play an important role in the thermoacoustic instability of combustion system. For a tube combustor, usually the longitudinal acoustic modes are dominant and tangential and radial modes are considered less important. Experiments show that acoustic modes are excited with harmonics and sub-harmonics due to nonlinear flame effects. Some other frequency peaks and modulations can also be introduced by flame dynamics.

Combustion researchers and designers agree that accurate descriptions of the instantaneous unsteady heat release rate with acoustic pressure oscillations will provide increased understanding of combustion instabilities. This research involves building a nonlinear, reduced-order model of flame dynamics both theoretically and experimentally,
which are coupled by a linear combustor acoustic model to predict thermoacoustic limit cycle frequencies and amplitudes.

1.1 Literature Review

As early as the beginning of the nineteenth century, combustion-driven acoustic oscillations were recognized by Higgins [Hig02]. Sound was produced while setting a flame inside an open-ended or closed-ended tube. This phenomenon was called the “singing flame” [Rau93]. The Rikje tube was invented in 1859 by placing a hot gauze in the lower half of open-ended vertical tube, which is called a “Rikje tube”. It was observed that the “sounded” oscillations were stronger or weaker while placing the heat-source at different locations. Rayleigh proposed a criterion for the development of this type of heat-driven oscillations, which are called thermoacoustic oscillations nowadays. Rayleigh’s criterion states that oscillations are magnified when heat addition is at the moment of the gas compression, or heat extraction is at the moment of gas expansion. It shows the importance of the phase of heat addition and acoustic pressure oscillations. Mathematically, Rayleigh’s criteria on a local basis can be written as an integration of the product of the oscillatory pressure $p(x,t)$ and unsteady heat release energy $e(x,t)$ over the time period under consideration [Kai98].

$$R(x) = \frac{1}{T} \int_{T} e(x,t)p(x,t)dt$$

If the integral $R(x)$ is positive, then oscillation is amplified. If $R(x)$ is negative, damping occurs. In other words, the phase difference or time lag $\tau$ between the heat release rate and the pressure oscillation determine whether the instability grows or decays. This leads to the most popular combustion control strategy – a phase shift controller. It senses pressure from the combustion chamber and adds time delay (phase shift) to the signal before feedback into the acoustic system by a pre-installed loud speaker used to control the dynamic system. Many experiments have been done without
an accurate and predictable combustion model [Zin00]. Zinn et al have designed adaptive controllers for suppression of combustion instability without a priori knowledge of the dynamics of the system.

Putnam gave the same kind of description as Rayleigh’s criteria for the explanation of pressure oscillations within a combustion chamber [Put71]. Later, Culick gave a recent review of Rayleigh’s criteria and an elegant proof is also derived [Cul87]. Practically, if the heat release rate responds instantaneously to incoming velocity fluctuations, then the heat release rate changes will be exactly 90° out of phase with the acoustic pressure. This phase will not drive nor damp the oscillations. However, the heat transfer and the combustion chemical reaction (assuming it is not infinitely fast) introduce time delays, which vary the phase relationships between the heat source and acoustic oscillations.

Some important work was done earlier for liquid propellant rockets. It was based on the linear conservation equations for premixed combustion. This linear analysis results in the \( \tau-n \) model, where the \( \tau \) is the time delay from acoustic disturbance to the heat release rate changes due to combustion and \( n \) is the gain that controls the magnitude of the system response [Bau99]. Due to the complicated chemical process embedded in the actual combustion, it is a very hard task to predict the time lag globally. An alternative explanation is “hysteresis”, which was proposed as the nonlinear time-delay unit associated with a myriad of physical process in the combustion system’s response to pressure and velocity oscillations by Knoop and Culick etc. The characteristics of this sort of hysteresis in the combustion system can be measured experimentally to define the stability boundary. Also, it is implemented for combustion control with secondary fuel injection to minimize the pressure oscillation by way of nonlinear control theory [Kno97].

During the middle of the nineteen seventies, Culick studied the stability of one-dimension motions in a rocket motor, where he introduced the nonlinear behavior of acoustic waves in the combustion chamber [Cul76]. This work was concerned with the general problem of the nonlinear growth and limiting amplitude of acoustic waves in a
combustion chamber. The combustion response was assumed to be linear and could be characterized using an exponential growth for each mode. The limit amplitude was determined by a set of coupled nonlinear wave equations which can be solved numerically. Subject to the acoustic nonlinearity, the model was demonstrated on the analysis of rocket engine and other combustion chambers.

Some recent modeling work on combustion instability has been done by Annaswamy and Ghoniem, etc [Ann97]. From one-dimensional flow dynamics and conservation equations, they developed a linear unstable model for combustion control purposes. The linear acoustic modes are governed by the acoustic wave equation. A novel idea was building a linear flame model based on flame area theory. For a wrinkled, premixed laminar flame, the flame speed is almost a constant. Thus the heat release rate was considered proportional to the flame surface area. Their work results in a feedback structure with a first-order flame model coupling with a linear acoustic model. The model was linear and unstable. Furthermore, a set of different linear controllers and an adaptive controller were designed based on the above modeling work and simulation results were also presented.

In sharp contrast to Culick’s model, Sterling proposed that the only significant nonlinearities were originated from the combustion process and the acoustic dynamics could be considered purely linear [Ste93]. This modeling concept was “motivated by experimental results from a laboratory combustor that demonstrated the heat-addition and fluctuations are dominated by the Rayleigh mechanism.” A nonlinear expression for heat release rate is needed to describe the flame dynamics. Two simple ideas are suggested: 1) a time-delayed combustion response, which could be considered a variation of the $\tau$-n model with nonlinearity added; 2) instantaneous “kicks” occur equally spaced in time for heat release rate. Nonlinear dynamics were discussed for bifurcation parameters and limit cycle behavior. More specifically, Fannin et al presented a first-order heat-release model with fictitious quadratic and cubic nonlinearity attached [Fan99]. Bifurcation parameters were studied and stability analysis was performed on the nonlinear model with some numerical simulations. For lean premixed combustors, Peracchio and Proscia developed a nonlinear parametric model to capture the combustion instabilities. The model
represented the coupling between acoustics and heat-release by accounting for the effect of acoustic pressure oscillations on the equivalence ratio being delivered to the flame. If the phase between these two dynamic parts is proper, instability occurs and it grows into a limit cycle [Per98].

Modeling combustion instability is a very complicated task. An alternative approach is modern computational fluid dynamics (CFD). Taking advantage of the advancement of computational power, CFD has been applied to simulate the combustion process and interactions between acoustics and reacting flows. Notable works have been done by Wang and Yang for ramjet simulations [Wan97]. However, they still could not solve all scales and geometries for the large dimensions of the combustors. Recently, Quinn presented a simple one-dimensional simulation tool by solving the Navier-Stokes equations [Qui98]. With assumptions of simplified chemical kinetics, it simulated the system by solving conservation equations with prescribed initial conditions. Compared to CFD, it took less computational power due to relatively heavy assumptions, but it was still restricted by the given conditions and the result is only valid for each run of simulation.
1.2 Research Objectives

At the present time, no physically based nonlinear model exists for even such simple combustion systems as a Rijke tube. Although limit cycle frequencies can be predicted based on linear acoustic analysis, limit cycle amplitudes have not been predicted. The goal of this research is to build an analytical model from the physical combustion process and the acoustic dynamics of a premixed laminar flame combustor. It will be a nonlinear, reduced-order flame model coupled by a linear acoustic model to exhibit thermoacoustic limit cycle oscillations. This model will predict the thermoacoustic instabilities in terms of frequencies as well as limit cycle amplitudes. To achieve this, the following work has been done in this research:

- **Develop a physically-based, theoretical, nonlinear, reduced-order, dynamic model for laminar premixed flame.**

  Combustion instability is generated by a highly nonlinear system. The most complete solution to the problem is to solve conservation equations for mass, momentum, and energy subject to appropriate boundary conditions. Numerical simulation has been performed in recent years, but it is hampered by the variety of scales. This research is starting from the system point of view to focus more on the physical properties of the combustion instabilities. To solve the conservation equations, finite element methods (FEM) or finite difference methods (FDM) are applicable to obtain simulation results. But the order of FEM/FDM model is as high as the number of mesh points. Our approach is to develop a physically based reduced-order model from those partial differential equations to provide a simple structure that describes the flame dynamics accurately. Starting from the governing conservation equations, the heat release dynamics are described by partial differential equations that are simulated by a finite-difference method. Using proper orthogonal decomposition (POD) and a generalized Galerkin procedure, the infinite-dimensional PDE model can be reduced to a set of low-order nonlinear ordinary differential equations. The issues of model order versus accuracy and the selection of mode shapes to be used in the reduction are discussed. The challenges of reduced-order modeling for nonlinear systems are also discussed in the context of
modeling of flame dynamics. A reduced-order nonlinear heat release model is developed and simulated to reproduce the describing function of the full-order finite difference model.

- **Model linear acoustics of combustor and construct a closed-loop model to predict thermoacoustic limit cycle frequencies and amplitudes.**

The thermoacoustic closed-loop model is a feedback structure where the flame dynamics module is coupled by combustor acoustics as stated in Figure 1. With the assumption of an intensively localized combustion within a tube combustor, a combustor acoustic model is developed from modified wave equations driven by heat sources and given boundary conditions. Integrated with the reduced-order nonlinear heat release model developed above, this feedback structure reproduces thermoacoustic pressure oscillations and predicts limit cycle frequencies as well as amplitudes. The spectral character matches up with the experimental measurement of a laboratory tube combustor at Virginia Tech in terms of oscillation frequency peaks and modulations. Linear stability analysis can show the unstable pole locations and oscillation frequencies. But nonlinear analysis determines the limit cycle amplitudes. Our goal is to model a closed-loop nonlinear system to predict accurately the oscillation frequencies and limit cycle amplitudes.

- **Explore input-output modeling and nonlinear system identification techniques to generate an empirical nonlinear flame model from experimental data.**

An investigation of nonlinear system identification techniques is conducted for the purpose of constructing an empirical flame model from experiments. A comparison of different model structures, input-output differential equations and state space forms, is presented. Furthermore, a frequency domain, nonparametric identification technique based on principles of harmonic balancing is proposed and applied to empirical nonlinear heat release dynamic modeling. This technique will model the nonlinearity of the combustion system from experimental data. The idea of describing functions is used to investigate the nonlinearity of the flame model. With a small sinusoidal perturbation
input sweeping all frequencies, the linear frequency response function for heat release rate is obtained. Increasing the input amplitude to higher levels allows the nonlinear effects of flame dynamics to be captured. An empirical nonlinear flame model is identified with the measured experimental data.

- **Perform a nonlinear describing function analysis using nonlinear heat release dynamics data from an experimental flat flame burner. Thermoacoustic limit cycle frequencies and amplitudes are predicted.**

The describing functions of the flame dynamics near the 2nd acoustic modal frequency are measured through experiments. This is the key for understanding the nonlinearity of the heat release model within the frequency range of the limit cycles. With a closed loop structure, the describing function can be used to predict the limit cycle behavior of the combustion instabilities, and feedback controllers can be designed using information from the describing functions. The final result of this nonlinear modeling work will be to predict the limit cycle frequency and amplitude of a model-scale combustor. To simulate the dynamic response of the nonlinear flame model and capture transients of the system, a low-order nonlinear heat release model is identified to reproduce the measured describing functions. This model can be applied to simulate thermoacoustic instabilities, to predict limit cycles, and to assist in control system design.

In conclusion, there is a major need for physically-based, nonlinear, reduced-order models of dynamic heat release. This research is to meet this need and build nonlinear models for a laminar premixed flame both from first-principles and from experiments, while striving to maintain a simplicity that will make the models suitable for the analysis and control design.
1.3 Organization of the Dissertation

The following chapters of this dissertation are used to describe the details of the flame dynamics modeling for thermoacoustic instabilities. Immediately following this chapter, we start to develop the physically-based heat release model from chemical kinetics for a one-dimensional burner-stabilized flame by examining the basic governing equations. Steady-state temperature and species mass fraction profiles are obtained using a shooting method. Furthermore, the time-dependent solutions are simulated with the finite difference method. These full-order simulations of heat release dynamics from PDEs are considered as the basic reference in the next stage of reduced-order flame modeling.

In the third chapter, we discuss model order reduction by principle orthogonal decomposition (POD) and a generalized Galerkin procedure. From the collection of finite difference simulation data, linear and nonlinear flame models were obtained, where the input is velocity perturbation and output is the heat release rate. A weakly nonlinear flame model was identified using pseudo random binary sequence (PRBS) inputs, and the model limitation for large inputs is discussed. To build a model containing all necessary nonlinearities of the flame dynamics, a selection of simulation data with an input near 180Hz with various amplitudes was used to extract basis functions. A thorough discussion is presented to relate the order of the reduced-order modeling to the completeness and accuracy of reproducing describing functions near critical unstable frequencies. The challenges of reduced-order modeling were also pointed out for a real system.

The fourth chapter explains input-output modeling and nonlinear system identification techniques for a real system, such as nonlinear flame dynamics. The discussion explores the relevance and difference between two forms of nonlinear model structure, the input-output differential equation and the state-space nonlinear description. A nonparametric system identification technique in the frequency domain, based on the principle of harmonic balancing, is used. A subsystem modeling technique is proposed and demonstrated together with discussion of the difficulty of system identification of high-order, complicated systems.
The describing function technique is very useful for nonlinear system limit cycle analysis. In the fifth chapter, we present the measurement of describing functions from experiments and build an empirical nonlinear model fitting the data. To capture the describing function of a flame, a flat flame burner is used to measure OH* fluctuations, which are assumed proportional to the heat release rate changes, resulting from sinusoidal velocity perturbations at various amplitudes within the frequency range of 160-200Hz. An empirical nonlinear model with cubic nonlinearities is identified to fit the measured describing functions. Numerical simulation is performed to validate the identified model and reproduce the measured describing functions.

One of the important goals of building a reduced-order flame model is to predict the occurrence, frequency and amplitude of the thermoacoustic limit cycle. For this reason, a two-mode linear acoustic model is developed for the Rijke tube combustor in Chapter 6. Subsequently, linear stability analysis was performed with a linear flame model coupled by the acoustics. Furthermore, nonlinear analysis and simulation were presented to demonstrate the occurrence of thermoacoustic instabilities with the first-principles nonlinear flame model. Compared to experiments, this model predicts thermoacoustic limit cycle frequency as well as amplitudes with reasonable accuracy.

The last major element of this research work is describing function analysis of the experimental thermoacoustic instability. With the flame data captured from experiments, describing function methods were applied to estimate closed-loop system limit cycles. With the two-mode linear acoustic model developed earlier, the thermoacoustic limit cycle can be estimated from Nyquist diagram intersections of the unsteady heat release rate describing functions and the linear acoustics frequency response function (FRF). Compared to experiments, the describing function analysis predicts the limit cycle amplitude accurately.

The final chapter concludes this research work on reduced-order modeling of nonlinear flame dynamics. The contributions of this research are summarized and suggestions for future work are discussed.