Chapter 5

Free Motions of a Rigid-Body Breakwater

5.1 Introduction

In order to make the analyses of this problem more realistic, an in-depth rigid body analysis will be conducted. The point-mass case is a special case of the more general rigid-body case where the dimensions of the rigid body are infinitesimally small. The point-mass case needed to be performed before the rigid body investigation to make sure the formulation was correct. Now that a point-mass analysis has been performed and checked, the point-mass breakwater (PMBW) is now given dimensions and becomes a rigid-body breakwater (RBBW). To simplify the first RBBW analysis, free motions of the breakwater will be investigated as in the first problem.

5.2 Rigid-Body Model and Configuration

In this formulation, the generalized shape of the RBBW is a rectangle with a length dimension of A and a depth dimension of B as seen in Fig. 5.1. Also seen in Fig. 5.1 are labeled points of reference, which will be used later in the formulation of this problem. Points J and K are the left and right supports, respectively. Similarly, points V and W are the left and right mooring cable to

Fig. 5.1 Dimensional Parameters
breakwater connections, respectively. The point C is the center of mass of the RBBW. The coordinates X and Y define the position of this point in their positive directions, and the rotation, $\theta$, is measured counterclockwise about this point from the horizontal axis in its positive direction. The origin of the global X-Y axes is located at the center of the breakwater when it is at its lowest point, its equilibrium state with both cables taut.

Similar to the previous problems, the geometric configuration of the breakwater and its mooring system is arranged in such a manner that the components are symmetric. The two cables are of equal length and are suspended from the same height. Figure 5.1 shows the separation length of the supports as 2S, and the “taut” (natural) length of each cable as R. “Taut” in this sense means the cable has been stretched out and has reached its natural length. In order for the breakwater to float, the following condition must be satisfied:

$$R > S-A \quad (5.1)$$

With these dimensions, the distance H from the supports J and K to the connection points V and W when the RBBW is at its lowest point (the equilibrium state) may be defined by

$$(S-A)^2 + H^2 = R^2 \quad (5.2)$$

or

$$H = \sqrt{R^2 - (S-A)^2} \quad (5.3)$$

Motions of the RBBW must remain below the height of the supports (i.e., the breakwater not hitting the sea floor). Therefore the following restriction on Y must be met:

$$Y \leq H+B \quad (5.4)$$

Section 5.2.2.1 discusses this in more detail after some more parameters have been defined. There are three degrees of freedom in this system: $X(T)$, $Y(T)$, and $\theta(T)$, where $T = \text{time}$. 

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5.2.1 Rigid-Body Breakwater Shape

The RBBW is assumed to be symmetric vertically and horizontally about its center, C. Three shapes will be investigated. The first is a thin ring or circle of radius A with the cables connected at the level of the center of mass, as shown in Fig. 5.2.a. Thus B=0 for this case and the shape may be thought of as a bar. The second shape is a solid square, with B=A and the cables connected at the upper corners (Fig. 5.2.b). In the third case the RBBW is solid and rectangular in cross-section (Fig. 5.2.c), with B≠A. The shape of the RBBW does not matter, just the locations of the points C, V, and W. The mass moment of inertia, Iₖ, takes into account the shape of the RBBW. Thus, the mass moments of inertia, about the center of mass, are as follows:

for the circular shape,

\[ I_c = mA^2 \]  \hspace{1cm} (5.5)

and for the square or rectangular shapes,

\[ I_c = \frac{1}{3} m(A^2 + B^2) \]  \hspace{1cm} (5.6)

5.2.2 Boundary Equation Formulation

In order to develop equations for the boundaries of the RBBW case, the positions of the connection points, V and W, must be determined. The following positions of V and W were developed with respect to the respective support points, J or K. To determine the position equations, the breakwater is moved slightly with either one, both, or no cables taut in the positive X, Y, and θ directions. Thus, the general solution of the case with both cables slack is shown in Fig. 5.3.
Remembering that positive X is to the right, positive Y is up, and positive $\theta$ is counterclockwise, the following equations describe the positions of V and W with respect to J and K vertically and horizontally:

\[ \text{JV}_x = S + X - A \cos \theta - B \sin \theta \]  
(horizontal distance between J and V) \hspace{1cm} (5.7)  
\[ \text{JV}_y = H + B - Y + A \sin \theta - B \cos \theta \]  
(vertical distance between J and V) \hspace{1cm} (5.8)  
\[ \text{KW}_x = S - X - A \cos \theta + B \sin \theta \]  
(horizontal distance between K and W) \hspace{1cm} (5.9)  
\[ \text{KW}_y = H + B - Y - A \sin \theta - B \cos \theta \]  
(vertical distance between K and W) \hspace{1cm} (5.10)

Now that these vertical and horizontal positions are known, the distances $D_L$ and $D_R$ may be calculated from the Pythagorean Theorem.

Therefore

\[ D_L^2 = \text{JV}_x^2 + \text{JV}_y^2 \]  
(direct distance between J and V) \hspace{1cm} (5.11)  
\[ D_R^2 = \text{KW}_x^2 + \text{KW}_y^2 \]  
(direct distance between W and K) \hspace{1cm} (5.12)

Using Equations 5.7-5.10 in Equations 5.11 and 5.12 gives

\[ D_L^2 = (S + X - A \cos \theta - B \sin \theta)^2 + (H + B - Y + A \sin \theta - B \cos \theta)^2 \]  
(5.13)  
\[ D_R^2 = (S - X - A \cos \theta + B \sin \theta)^2 + (H + B - Y - A \sin \theta - B \cos \theta)^2 \]  
(5.14)

where the following conditions must be satisfied:
Equations 5.13 and 5.14 give the direct distances between the supports and the RBBW mooring cable connections at any time when the cables are slack. Thus, when a cable is taut and the conditions 5.15 and 5.16 are met, the equations describing the position of the RBBW between the boundaries may be developed similar to Section 2.2.2, using

\[ G_1 = D_R^2 - R^2 \]  
\[ G_2 = D_L^2 - R^2 \]

Thus the motion of the breakwater must remain in the region where

\[ G_1 \leq 0 \]  \hspace{1cm} (G_1 \text{ equation}) \hspace{1cm} (5.19)
\[ G_2 \leq 0 \]  \hspace{1cm} (G_2 \text{ equation}) \hspace{1cm} (5.20)

Recalling from Chapters 3 and 4, the boundaries for these problems are definite (i.e., fixed in space) in the point-mass cases and are drawn on the trajectory plots. However, the boundaries are more like regions in the rigid-body cases because \( x \) and \( y \) give the position of the center of gravity, which is not at the attachment points of the cables, and hence the boundaries are not plotted on the trajectory plots for the rigid-body problems.

5.2.2.1 Upper Boundary Restriction

The upper boundary restriction discussed in Section 5.2 becomes complicated when the RBBW is not horizontal (i.e., RBBW is rotated by some angle, \( \theta \)). Therefore, points V and W must remain below the upper boundary and must follow the following restrictions:

If the RBBW is circular,

\[ Y < H - A \]  

If the RBBW is rectangular,

\[ J V_y > 0 \]  
\[ K W_y > 0 \]

These conditions insure that the RBBW is below the upper boundary (i.e., the sea floor).
5.2.2.2 Rotation Restriction

In the rigid-body cases, a restriction was placed on the amount of rotation allowed. With the RBBW now having dimensions and the possibility to rotate, it is possible for the RBBW to rotate excessively. This motion is not desirable, since in reality the cables may wrap around the breakwater or become entangled. Therefore, the restriction of limiting the RBBW to rotations of ±π/2 or less was imposed. In the solution procedure, if this limit was reached, then the solution would stop, indicating that the initial conditions and parameters were not suitable to keep the rotations within the specified range.

5.3 Nondimensionalization

The terms used in the formulation of the problem have been nondimensionalized, so that the units will not be involved during this investigation. Length parameters were nondimensionalized by the cable spacing, $S$, and time by

$$\sqrt{\frac{S}{g}}$$

(5.24)

Mass is divided out of the equations of free motion. Uppercase letter symbols are used when terms have dimensions and lowercase letters are used to represent the nondimensionalized values. Thus, the variables become

$$x = \frac{X}{S}$$  \hspace{1cm} (5.25)

$$y = \frac{Y}{S}$$  \hspace{1cm} (5.26)

$$r = \frac{R}{S} > 1-a$$  \hspace{1cm} (5.27)

$$a = \frac{A}{S}$$  \hspace{1cm} (5.28)

$$b = \frac{B}{S}$$  \hspace{1cm} (5.29)

$$h = \frac{H}{S} = \sqrt{r^2 - (1-a)^2}$$  \hspace{1cm} (5.30)

$$t = T \sqrt{\frac{g}{S}}$$  \hspace{1cm} (5.31)

$$d_L = \frac{D_L}{S}$$  \hspace{1cm} (5.32)

$$d_R = \frac{D_R}{S}$$  \hspace{1cm} (5.33)

From these nondimensionalizations, Equations 5.13, 5.14, 5.17, and 5.18 become

$$d_L^2 = (1 + x - a \cos \theta - b \sin \theta)^2 + (h + b - y - a \sin \theta - b \cos \theta)^2$$  \hspace{1cm} (5.34)
\[ d_R^2 = (1-x-\cos \theta + b \sin \theta)^2 + (h+b-y-\sin \theta - b \cos \theta)^2 \]  
\[ g_1 = d_R^2 - r^2 \]  
\[ g_2 = d_L^2 - r^2 \]  

5.4 Equations of Motion

Inside the bounded region, both cables are slack, and the only force acting on the RBBW during free vibration is gravity. Therefore the equations of motion (EOM’s) are

\[ m \frac{d^2X}{dT^2} = 0 \]  
\[ m \frac{d^2Y}{dT^2} = -mg \]  
\[ I \frac{d^2\theta}{dT^2} = 0 \]

After the nondimensionalization, the basic EOM’s become

\[ \frac{d^2x}{dt^2} = 0 \]  
\[ \frac{d^2y}{dt^2} = -1 \]  
\[ \frac{d^2\theta}{dt^2} = 0 \]

The solutions for this formulation are

\[ x = c_1 + c_2 t \quad y = c_3 + c_4 t - \frac{1}{2} t^2 \quad \theta = c_5 + c_6 t \]  
\[ \dot{x} = c_2 \quad \dot{y} = c_4 - t \quad \dot{\theta} = c_6 \]  
\[ \ddot{x} = 0 \quad \ddot{y} = -1 \quad \ddot{\theta} = 0 \]

where \( \dot{} = \frac{d}{dt} \) and

where the constants for motion following \( t=t_i \) (initial time or impact time) are

\[ c_1 = x_i - c_2 t_i \quad c_3 = y_i - c_4 t_i + \frac{1}{2} t_i^2 \quad c_5 = \theta_i - c_6 t_i \]
5.5 Rigid-Body Impact

Now a detailed derivation of impact response equations will be formulated to determine the new initial conditions after impact for the RBBW problems.

5.5.1 Definition of Rigid-Body Impact

Similar to the definition of “impact” discussed in Chapter 2, impact is defined when a cable becomes taut. “Taut” in this sense means that the cable reaches its natural length which in this problem is from a support to an attachment point on the RBBW, as seen in Figs. 5.4 and 5.5, and not to the center of mass as in the PMBW problems. When a cable becomes taut, it is said that the breakwater is hitting a fictitious boundary which is defined by the mathematical equations for \( g_1 = 0 \) or \( g_2 = 0 \) discussed in Section 5.2.2. Once one of the cables has become taut, an impact is felt and the breakwater rebounds in the opposite direction. However, it can be seen that now the boundaries depend on the size of the RBBW and the angle at which it is rotated before impact, and this complicates the impact response of the RBBW.

5.5.2 Formulation of Impact Response

The impact response of the RBBW is not the same as in the PMBW problems. Before, the impact conditions were

\[
v^n_+ = -ev^-_n
\]

(5.59)

for the normal velocity, and no change in tangential velocity. However, with the rigid-body case, three impact conditions are needed and several modifications to the previous impact equations were required. It is no longer assumed that the tangential velocity, \( v_t \), remains unchanged at the time of impact, because of the rotations involved. The rotation of the RBBW at the time of impact has a significant effect on the impact velocities. This is because if the center of mass of the RBBW is not in line with the normal velocity
vector of the taut cable, then a spinning will occur at the time of impact. This causes the RBBW to rotate in the opposite direction to the direction it was rotating before impact. This may be compared to the motions of a yo-yo, which unrolls down a string and then, acting like a pendulum, rotates around the end of the string and rolls back up (Bürger 1984). The mathematical models, which describe this behavior with respect to a RBBW, will now be described.

Equation 5.59 will continue to be used as the first impact condition. The second impact condition is that the resultant of the impulsive force during impact acts longitudinally along the taut cable. The third impact response is that the sudden change in the angular momentum of the RBBW relative to its center of mass is equal to the sum of the moments induced by the impulsive forces about the center of mass (Synge and Griffith 1959).

This may be stated mathematically by

\[ I_c \Delta \dot{\theta} = \hat{M} \]

(5.60)

where

- \( I_c \) is the moment of inertia about the center of mass
- \( \Delta \dot{\theta} \) is the change in angular velocity
- \( \hat{M} \) is the summation of the impulsive force moments
- (\( \wedge \)) means that the term is an impulsive action

The specific terms associated with both boundaries for these conditions will be expanded upon during more detailed formulations in subsequent sections.

### 5.5.3 Impact Response when \( g_1 = 0 \)

Impact parameters will now be developed for \( g_1 = 0 \). Solving Equation 5.36 for \( y \) where \( g_1 = 0 \) gives

\[ y = h + b - a \sin \theta - b \cos \theta - \sqrt{r^2 - (1 - x - a \cos \theta + b \sin \theta)^2} \]

(5.65)
Thus, to get the slope of the boundary at a given point, the derivative is taken:

\[ \frac{dy}{dx} = \frac{(1 - x - a \cos \theta + b \sin \theta)}{\sqrt{1^2 - (1 - x - a \cos \theta + b \sin \theta)^2}} \]  

(5.66)

The angle, \( \phi \), from Section 2.5.2.1 is the same for this case and is used in the following derivations. Seen in Fig 5.4, this angle is formed when the right cable is taut.

### 5.5.3.1 Normal and Tangential Velocities at \( g_1 = 0 \)

By including the RBBW dimensions and its potential to rotate, and from the simple geometry in Section 2.5.2.2, the normal and tangential velocities may be obtained. In the RBBW problems, the point W is actually the point hitting the boundary, and not the center, C. The equations in this section are derived with respect to displacements from the center, but are actually with respect to the attachment points with some geometric transformations. Thus, the following equations were developed based on the position and velocity of point W when it strikes the boundary \( g_1 = 0 \):

\[ \dot{v}_x^- = x^- - a \theta^- \sin \theta - b \theta^- \cos \theta \]  

(5.67)

\[ \dot{v}_y^- = y^- + a \theta^- \cos \theta - b \theta^- \sin \theta \]  

(5.68)

\[ v_n^- = -v_x^- \sin \phi - v_y^- \cos \phi \]  

(5.69)

\[ v_i^- = -v_x^- \cos \phi + v_y^- \sin \phi \]  

(5.70)

### 5.5.3.2 Impact Solution for \( g_1 = 0 \)

Now the impact condition equations and their solutions will be derived. By using the first condition, Equation 5.59, and Fig. 5.4, the impact equation becomes

\[ -\dot{x}^+ \sin \phi - y^+ \cos \phi + (CW_y \sin \phi - CW_x \cos \phi) \theta^+ = -ev_n^- \]  

(5.71)

where

\[ CW_x = a \cos \theta - b \sin \theta \]  

(5.72)

\[ CW_y = a \sin \theta + b \cos \theta \]  

(5.73)
Next, by applying the second condition to Fig. 5.4, the second impact equation is developed:

$$\dot{KW}_x x^+ - \dot{KW}_x y^+ = \dot{KW}_x x^- - \dot{KW}_x y^-$$  \hspace{1cm} (5.74)

Finally, by again using Fig. 5.4, the following impulsive forces and moments may be derived:

$$\hat{F}_x = m\Delta x = m(x^+ - x^-)$$  \hspace{1cm} (5.75)

$$\hat{F}_y = m\Delta y = m(y^+ - y^-)$$  \hspace{1cm} (5.76)

$$\hat{M} = I_c \Delta \theta = I_c (\dot{\theta}^+ - \dot{\theta}^-)$$  \hspace{1cm} (5.77)

Summing the moments about the center, C, gives

$$-\hat{F}_x CW_y + \hat{F}_y CW_x - \hat{M} = 0$$  \hspace{1cm} (5.78)

Plugging Equations 5.75–5.77 into Equation 5.78 gives the third impact equation

$$-CW_y x^+ + CW_x y^+ - I_c \dot{\theta}^+ = -CW_y x^- + CW_x y^- - I_c \dot{\theta}^-$$  \hspace{1cm} (5.79)

After grouping the terms of Equations 5.71, 5.74, and 5.79 with respect to the velocities after impact, the equations may be put into matrix form, giving the following impact velocity matrix equation:

**Fig. 5.4. Right Cable Taut with RBBW Moved in Positive Directions**
\[
\begin{bmatrix}
 a_{11} & a_{12} & a_{13} \\
 a_{21} & a_{22} & a_{23} \\
 a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
 x^+ \\
y^+ \\
 \theta^+
\end{bmatrix}
= 
\begin{bmatrix}
 b_1 \\
b_2 \\
b_3
\end{bmatrix}
\] (5.80)

where
\[
a_{11} = -\sin \phi 
\] (5.81)
\[
a_{12} = -\cos \phi 
\] (5.82)
\[
a_{13} = \sin \theta \sin \phi + b \sin \theta \cos \phi - a \cos \theta \cos \phi + b \cos \theta \sin \phi 
\] (5.83)
\[
a_{21} = K W_y 
\] (5.84)
\[
a_{22} = -K W_x 
\] (5.85)
\[
a_{23} = 0 
\] (5.86)
\[
a_{31} = -C W_y 
\] (5.87)
\[
a_{32} = C W_x 
\] (5.88)
\[
a_{33} = -I_c 
\] (5.89)
\[
b_1 = -e v_n 
\] (5.90)
\[
b_2 = K W_y x^--K W_x y^- 
\] (5.91)
\[
b_3 = -C W_y x^--C W_x y^- - I_c \theta^- 
\] (5.92)

Then, by using Cramer’s Rule, the velocities after impact may be determined.

The analytical solution, along with the Newton’s Method for convergence, will be used to analyze the cases in this problem. The solution procedure is similar to the previous problems with exceptions brought about by the inclusion of the rotational degree of freedom.

**5.5.4 Impact Response when g_2=0**

Similar to the derivation of the g_1=0 impact response, the impact parameters will now be developed for g_2=0. Solving Equation 5.37 for y where g_2=0 gives
y = h + b + a \sin \theta - b \cos \theta - \sqrt{r^2 - (1 + x - a \cos \theta - b \sin \theta)^2} \tag{5.93}

Thus, to get the slope of the boundary at a given point, the derivative is taken:

\[
\frac{dy}{dx} = \frac{1 + x - a \cos \theta - b \sin \theta}{\sqrt{r^2 - (1 + x - a \cos \theta - b \sin \theta)^2}} \tag{5.94}
\]

The angle, $\psi$, from Section 2.5.3.1 is the same for this case and is used in the following derivations. Seen in Fig 5.5, this angle is formed when the left cable is taut.

### 5.5.4.1 Normal and Tangential Velocities at $g_2 = 0$

By including the RBBW dimensions and its potential to rotate, and from the simple geometry in Section 2.5.3.2, the normal and tangential velocities may be obtained. In the RBBW problems, the point V is actually the point hitting the boundary and not the center, C. The equations in this section are derived with respect to displacements from the center, but are actually with respect to the attachment points with some geometric transformations. Thus, the following equations were developed based on the position and velocity of point V when it strikes the boundary $g_2 = 0$:

\[
v_x^- = x^- + a \theta^- \sin \theta - b \theta^- \cos \theta \tag{5.95}
\]

\[
v_y^- = y^- - a \theta^- \cos \theta - b \theta^- \sin \theta \tag{5.96}
\]

\[
v_n^- = v_x^- \sin \psi - v_y^- \cos \psi \tag{5.97}
\]

\[
v_t^- = v_x^- \cos \psi + v_y^- \sin \psi \tag{5.98}
\]

### 5.5.4.2 Impact Solution for $g_2 = 0$

Now the impact condition equations and their solutions will be derived. By using the first condition, Equation 5.59, and Fig. 5.5, the impact equation becomes

\[
x^+ \sin \psi - y^+ \cos \psi + (-CV_x \sin \psi + CV_x \cos \psi) \theta^+ = -ev_n^- \tag{5.99}
\]

where

\[
CV_x = a \cos \theta + b \sin \theta \tag{5.100}
\]
CV_y = -a \sin \theta + b \cos \theta \quad (5.101)

Next, by applying the second condition to Fig. 5.5, the second impact equation is developed:

\[ J V_y x^+ + J V_x y^+ = J V_y x^- + J V_x y^- \quad (5.102) \]

Finally, by again using Fig. 5.5, the following impulsive forces and moments may be derived:

\[ \ddot{F}_x = m \Delta x = m(x^+ - x^-) \quad (5.103) \]
\[ \ddot{F}_y = m \Delta y = m(y^+ - y^-) \quad (5.104) \]
\[ \ddot{M} = I_c \Delta \dot{\theta} = I_c (\dot{\theta}^+ - \dot{\theta}^-) \quad (5.105) \]

Summing the moments about the center, C, gives

\[ \ddot{F}_x CV_y + \ddot{F}_y CV_x + \ddot{M} = 0 \quad (5.106) \]

Plugging Equations 5.103–5.105 into Equation 5.106 gives the third impact equation

\[ CV_y x^+ + CV_x y^+ + I_c \dot{\theta}^+ = CV_y x^- + CV_x y^- + I_c \dot{\theta}^- \quad (5.107) \]
After grouping the terms of Equations 5.99, 5.102, and 5.107 with respect to the velocities after impact, the equation may be put into matrix form, giving Equation 5.80 with

\[ a_{11} = \sin \psi \]  
\[ a_{12} = -\cos \psi \]  
\[ a_{13} = \sin \theta \sin \psi - b \cos \theta \sin \psi + a \cos \theta \cos \psi + b \sin \theta \cos \psi \]  
\[ a_{21} = J V_y \]  
\[ a_{22} = J V_x \]  
\[ a_{23} = 0 \]  
\[ a_{31} = C V_y \]  
\[ a_{32} = C V_x \]  
\[ a_{33} = I_c \]  
\[ b_1 = -e v_n \]  
\[ b_2 = J V_y \dot{x} + J V_x \dot{y} \]  
\[ b_3 = C V_y \dot{x} + C V_x \dot{y} + I_c \dot{\theta} \]  

The analytical solution, along with the Newton’s Method for convergence, will be used to analyze the cases in this problem. The solution procedure is similar to the previous problems with exceptions brought about by the inclusion of the rotational degree of freedom.

### 5.6 Convergence to a Boundary

Similar to the PMBW problems, Newton’s Method will be employed to converge the RBBW to a boundary. The development of Newton’s Method for this investigation is discussed in Section 2.6. In order to use Newton’s Method, the derivatives of \( g_1 \) and \( g_2 \) are required. Using Equations 5.36 and 5.37, the derivatives with respect to time are found to be
\[ g_1 = 2(1 - x - a \cos \theta + b \sin \theta)(-\dot{x} + a \dot{\theta} \sin \theta + b \dot{\theta} \cos \theta) \]
\[ + 2(h + b - y - a \sin \theta - b \cos \theta)(-\dot{y} - a \dot{\theta} \cos \theta + b \dot{\theta} \sin \theta) \]  
(5.120)

and

\[ g_2 = 2(1 + x - a \cos \theta - b \sin \theta)(\dot{x} + a \dot{\theta} \sin \theta - b \dot{\theta} \cos \theta) \]
\[ + 2(h + b - y + a \sin \theta - b \cos \theta)(\dot{y} + a \dot{\theta} \cos \theta + b \dot{\theta} \sin \theta) \]  
(5.121)

where

\[ \dot{x}(t), \dot{x}(t), y(t), \dot{y}(t), \theta(t), \text{ and } \dot{\theta}(t) \] are seen in Equations 5.44-5.49.

Again, the value \( g \) was converged to a tolerance of \( 10^{-6} \), and the corresponding converged time becomes the impact time.

With the more general rigid-body solution now formulated, the FORTRAN program previously used may be modified to account for the changes in the solution and response at impact. This program may be used because the solution procedure discussed in Section 2.7 does not change for the RBBW problems.

**5.7 Analyzed Cases**

Several different cases were analyzed to investigate how the variation of different parameters affected the motions of the breakwater. With this problem, several more parameters and degrees of freedom (DOF’s) are involved. These parameters and initial conditions are summarized in Table 5.1 where \( v_x = \dot{x}(0), v_y = \dot{y}(0), \text{ and } v_\theta = \dot{\theta}(0) \).

These include the size of the RBBW and the initial rotation and angular velocity of the RBBW. This table shows the case number, which was used to identify each case; this number is a series of numbers taken from the parameters and initial conditions. In this problem, the case number uses \( r, e, a, b, x(0), \dot{x}(0), y(0), \dot{y}(0), \theta(0), \text{ and } \dot{\theta}(0) \) in this order to identify it. For example, the standard case has a case number of 1910121100.

The standard case will be explained in detail later and is denoted by the shaded cells in
Table 5.1. Further, the initial conditions shown in this table are put into a series of groups. These groups were used to see how varying only one parameter would affect the breakwater’s response. The value of $h$ is given Table 5.1 because, as seen in Equation 5.30, the value of $h$ is based upon the size of the RBBW; thus, the size of the region is dependent on the RBBW size as well. This value will be shown for reference when looking at the trajectory plots.
Table 5.1. Parameters and Initial Conditions for Free Motions of a Rigid-Body Breakwater

<table>
<thead>
<tr>
<th>Case #</th>
<th>reabxy,vy,theta</th>
<th>r</th>
<th>h</th>
<th>e</th>
<th>a</th>
<th>b</th>
<th>x</th>
<th>vx</th>
<th>y</th>
<th>vy</th>
<th>theta</th>
<th>vtheta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1910121100</td>
<td>1.5</td>
<td>1.200</td>
<td>0.9</td>
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5.7.1 Standard Case

A standard case was developed for this problem as a starting point for the variation of parameters and initial conditions. The standard parameters and initial conditions for this RBBW problem are:

\[
\begin{align*}
  r & = 1.5 \\
  a & = 0.1 \\
  \theta(0) & = 0.0 \\
  x(0) & = 0.2 \\
  y(0) & = -0.1 \\
  \dot{x}(0) & = 0.0 \\
  \dot{y}(0) & = 0.0 \\
\end{align*}
\]

As in the point mass free motion case, the breakwater is started above its equilibrium height and off to one side. However, the RBBW, in this case, is not started from a large height as in the previous case. This is because it is felt that the modeling should be as realistic as possible and the RBBW would actually be near the bottom most of the time, as discussed in Section 4.3.1. Further, after running a few cases, it was found that the restriction of \( \theta \leq \pi/2 \), discussed in Section 5.2.2.2, was violated with \( y(0) = 1.0 \), as in the first problem. The RBBW is given some \( x \) and \( y \) velocities to give it some initial push, since there are no external forces other than gravity acting on the RBBW. Further, it is felt that the rotational ability of the RBBW will add to its motions, and thus no initial rotation or rotational velocity is specified.

As seen in Table 5.1 and in the standard case parameters \( a \) and \( b \) (Equation 5.122) the RBBW will be modeled, most of the time, with a horizontal but not a vertical dimension. This condition may be thought of as a bar supported by two strings, and is meant to model the circular RBBW, as seen in Fig. 5.2.a, where the mooring lines are attached to the breakwater at the sides. Also seen in Table 5.1, other shapes including squares and rectangles, will be analyzed. The circular RBBW will be modeled as a thin ring with a nondimensional mass moment of inertia of
\[ I_c = ma^2 \]  
with \( m = 1.0 \).

This is because the inflatable structures proposed may be modeled as a thin ring in cross section. A circular shape was chosen for the standard case because this is the shape most likely to be used in practice. The solid square and rectangular shapes of the RBBW will have a nondimensional mass moment of inertia of

\[ I_c = \frac{1}{3} m(a^2 + b^2) \]

with \( m = 1.0 \).

### 5.8 Analysis of Data

Cases were analyzed and data was collected to see what types of behavior the RBBW exhibited under free motions. Again, several types of graphs were used to evaluate the responses of the RBBW, but now the time history plot of rotation vs. time and the phase plane plot of angular velocity vs. rotation were also generated. Special attention must be paid to the fact that the RBBW has the ability to rotate. There might be a position of the RBBW where \( g_1 \) and \( g_2 \) are close to zero but the RBBW is not at the origin. This is due to the fact that the RBBW is free to rotate, thus giving it the possibility of tensioning both cables by rotating its center while not being at the origin. This situation occurred when the RBBW was near the origin and was treated as though the RBBW would settle to the equilibrium state because gravity is the only force acting and would tend to pull the RBBW down. It is not possible for the RBBW to rise out of this state because gravity is the only force acting. This situation will be discussed in more detail in Section 6.5.2 when wave forcing is added to the system. However, most of the solutions settled down towards the origin (i.e., the equilibrium state).

#### 5.8.1 Observations

The radius was the first parameter varied to see how the RBBW would respond. There do not appear to be any significant differences between the motions of a case that has a radius of 1.5 and a case that has a radius of 2.5. The only notable difference between the
two radii is that the boundaries for the 1.5 radius have a sharper point at the bottom of the region than for the 2.5 radius case, which is shallower, as discussed previously. Figures 5.6 and 5.7 illustrate the difference in size and range of motions of the RBBW, dependent upon the mooring cable length (radius). Note that the Figs. 5.6 and 5.7 are plotted to a similar scale.

This response may also be seen in the varying of the dimensions a and b. As the dimensions increase, it may be seen that the bottom of the region becomes flatter. This flatness is further exaggerated by the increase of the radius (Figs. 5.8–5.11). Note that Figs. 5.6-5.11 are all plotted to the same scale for comparison purposes. This flatness can be seen in Fig. 5.1 where the size of the shape would cause the connection points, V and W, to move apart or together. Further, the smaller the dimensions, the more rotations occur. This is because the moment of inertia is less, and thus the resistance to rotation is less and the RBBW rotates more. Figures 5.12 through 5.14 show that when the dimension a is increased for a rectangular section from 0.1 to 0.2 to 0.3 with b=0.1, the magnitude and amount of rotation are decreased. Thus, larger dimensions give a larger moment of inertia and the more the rotation is being resisted. Numerically, the dimensional values with the associated nondimensional mass moments of inertia for the various shapes may be seen in Table 5.2.
Fig. 5.6. $y$ vs. $x$, Case 1910121100  
($r=1.5$, $a=0.1$, $b=0.0$)

Fig. 5.7. $y$ vs. $x$, Case 2910121100  
($r=2.5$, $a=0.1$, $b=0.0$)

Fig. 5.8. $y$ vs. $x$, Case 1911121100  
($r=1.5$, $a=0.1$, $b=0.1$)

Fig. 5.9. $y$ vs. $x$, Case 2911121100  
($r=2.5$, $a=0.1$, $b=0.1$)

Fig. 5.10. $y$ vs. $x$, Case 1921121100  
($r=1.5$, $a=0.2$, $b=0.1$)

Fig. 5.11. $y$ vs. $x$, Case 2921121100  
($r=2.5$, $a=0.2$, $b=0.1$)
Fig. 5.12. \( \theta \) vs. \( t \), Case 1911121100 (\( a=0.1, b=0.1 \))

Fig. 5.13. \( \theta \) vs. \( t \), Case 1921121100 (\( a=0.2, b=0.1 \))

Fig. 5.14. \( \theta \) vs. \( t \), Case 1931121100 (\( a=0.3, b=0.1 \))
Table 5.2 Moments of Inertia for Different Shapes

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The variation of the coefficient of restitution, \( e \), in this problem does not produce any noticeable characteristic differences from the phenomena previously discussed. Figures 5.15 through 5.18 show the normal velocity before impact vs. time plots for \( e \) ranging from 1.0 to 0.9 to 0.7 and to 0.5. The time scale in Fig. 5.15 is different than in Figs. 5.16-5.18. One point which should be noted is that during the solution of the \( e=1.0 \) case, the rotation of the RBBW surpassed the \( \pi/2 \) restriction. This is because no energy is lost, thus the motions are not damped out and excessive rotations become likely.

Fig. 5.15. \( v_n^- \) vs. \( t \), Case 1110121100 (\( e=1.0 \))

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Fig. 5.16. $v_n$ vs. $t$, Case 1910121100 ($e=0.9$)

Fig. 5.17. $v_n$ vs. $t$, Case 1710121100 ($e=0.7$)

Fig. 5.18. $v_n$ vs. $t$, Case 1510121100 ($e=0.5$)
There was no significant change in behavior when the initial values of $\theta$ and $\dot{\theta}$ were varied with the values seen in Table 5.1. However, intuitively it makes sense that if the initial rotation or rotational velocity were increased enough, the RBBW might rotate past its rotation restriction of $\pi/2$ before it would settle to the bottom.

5.8.2 Point-Mass Case

When the size of the RBBW is set to zero, the solution for the PMBW is obtained. This was done as a check on the general RBBW solution. A modification was necessary in the impact equations because of problems in the mathematical solution of the matrix in Equation 5.80. The modification consisted of letting $\dot{\theta}^+ = \dot{\theta}^-$ which is acceptable since $\dot{\theta}$ does not appear in the special PMBW case of the RBBW case. A comparison of the trajectories may be seen in Figs. 5.19 and 5.20, where the standard case from the PMBW problem was analyzed. Some data points were selected randomly from the two solutions and have been compiled in Table 5.3 to show the equivalence. This comparison indicates that the PMBW case is just a special case of the more general RBBW solution.

Fig. 5.19. $y$ vs. $x$, PMBW-Case 19461111
Fig. 5.20. $y$ vs. $x$, RBBW-Case 1900461100
Table 5.3 Comparison of Point-Mass Breakwater Solution with the Rigid-Body Breakwater Solution

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