Chapter 6

Calculation Methodology

6.1 INTRODUCTION

6.2 DYNAMIC VIEW OF CALCULATION FLOW FOR A VR EXCAVATOR SIMULATOR SYSTEM
   6.2.1 Calculation Route 1 (Excavator Digging)
   6.2.2 Calculation Route 0 (Digging Operation Idle)
   6.2.3 Calculation Route 2 (Free Air Movement)

6.3 DIGGING MODE SELECTION
   6.3.1 Challenges in Digging Mode Selection
   6.3.2 Digging Mode Selection Algorithm
   6.3.3 Algorithm of Imaginary Joint Calculation of Multi-Joint Circular Trajectory

6.4 APPROXIMATION OF SOIL FAILURE ANGLE
   6.4.1 Summary of the Generalized Separation Model
   6.4.2 Sensitivity Analysis

6.5 APPROXIMATION OF BUCKET TRAVEL DISTANCE BETWEEN SEPARATION FAILURES
   6.5.1 Characteristics of Separation and Penetration Soil Failures
   6.5.2 Bucket Travel Distance between Separation Failures

6.6 CONCLUSION
Chapter 6

Calculation Methodology

VR Excavator Simulator Architecture and Pipeline

- Physics-Based Components
  - Physics-Based Excavator Model (Excavator Computational Model) ([Chapter 4])
  - Physics-Based Soil Model of Excavator Digging (Mathematical Model of Excavator Digging) ([Chapter 5])
    - Soil Resistance Models
    - Excavator Digging Modes
  - Simulation Engine ([Chapter 6])
    - Digging Mode Selection Algorithms
    - Computation Approximation
    - Internal Data Repository (Soil Configuration & Bucket Spatial Information)
    - Force Comparison
  - Soil Resisting Forces
- Graphical Components
  - Rendering Engine
  - Geometric Models
  - Graphical Terrain Model
  - Real-Time Graphics Output
- Interactive User Input Interface (Control Sticks)
- Interactive User Output Interface (Display Device)

Real-Time Input by Control Stick Movement

Chapter 6
6.1 Introduction

The preceding two chapters on the excavator computational model and the mathematical model of excavating digging provide two physics-based models for the calculations of bucket forces of an excavator and resisting forces of soil in order to account for the soil-tool (bucket) interaction in an excavator digging process. This interaction consideration is one of the requirements for a VR excavator simulator to be an engineering process tool. In this chapter, however, the focus is shifted to describe how these models are used for the development of a VR excavator simulator by providing a systematic and effective calculation scheme. In this regard, the main concerns of this chapter are 1) to provide detailed description on how calculations are performed from the perspective of the application of the models (proposed in Chapter 4 and Chapter 5) in VR simulator development; 2) to address issues that are encountered when the models are utilized in a VR simulator development; and 3) to enhance the calculation speed to achieve real-time interactivity of a VR simulator system.

Firstly, the component-based description of the calculation flow in a VR excavator simulation system presented in Section 3.3 is detailed in Section 6.2 to justify a bucket movement in soil media by presenting the data-and-function-based description of the computation flow in a particular time segment of digging operation.

In Section 6.3, algorithms to determine a digging mode for a segmental digging action are presented for the proposed excavator digging model to be applied in the calculation of soil resisting forces.

An expression to approximate a soil failure angle is proposed in Section 6.4 to lessen the calculation burden of the generalized separation model in determining separation resistance. Another approximation is made in Section 6.5 to quantify a bucket travel distance between separation failures since separation-based soil failing occurs discontinuously.
6.2 Dynamic View of Calculation Flow for a VR Excavator Simulator System

As described in Section 3.2, any digging process is composed of a series of digging action segments, each of which represents a certain digging mode during a short time period. As far as simulator development is concerned, the length of a segment (\( \Delta t \)) is dictated by a system speed. However, for a simulator to provide real-time graphical output to users, the number of segments needs to be 24 or more per second (Foley, vanDam, Feiner and Hughes 1996). Conversely, this implies that whole calculation cycle for each segmental digging action needs to be done in 1/24 second or less in a VR simulator system. In this section, with this segment as an unit in time scale, data and calculation flow in a VR simulator system is described in detail from the perspective of simulator development.

Figure 6.1 presents the computational process in a VR excavator simulator with an excavator operation time flow in a horizontal direction and a calculation time flow in a vertical direction. The segment in the middle represents a current operational time segment, the ends of which are marked by \( t_n \) (current time) and \( t_{n+1} \) (current time + segmental time increase). The current segment follows the previous segment and precedes the next operation segment. Therefore the time \( t_n \) also indicates the end of the previous segment and the time \( t_{n+1} \) indicates the start of the next segment.

The calculation of the current segment is mostly based on various databases (depicted as rounded rectangles) at the current time \( t_n \). These databases are maintained in their corresponding components (angled brackets inside the databases) in the simulator system as the end products of the calculation from the previous segment. The databases include 1) excavator geometrical data in a physics-based excavator model (or excavator computational model), 2) terrain geometrical data in a simulation engine, 3) excavator geometrical data in a graphical excavator model, and 4) terrain geometrical data in a graphical terrain model. The data in each database is persistent and available throughout the current segment calculation until they are updated and replaced at the end of the current segment calculation.
The calculation process of the current segment begins when the operation time gets to $t_{n+1}$, which defines the moment when a particular digging operation is realized after the segmental time increase ($\Delta t$) from $t_n$. From this moment on until the process gets to the next operational time segment, calculations are performed at $t_{n+1}$ vertically downward as shown in Figure 6.1, interacting with the various data persistent from the time $t_n$. In the figure, a calculation task is presented as a dotted rectangle block. The calculation process, however, follows various calculation routes depending upon a certain routing decision in one of the calculation blocks as described in the following.
Figure 6.1.1 Data-and-Function-Based Calculation Flow in a VR Excavator Simulator
Figure 6.1.2 Data-and-Function-Based Calculation Flow in a VR Excavator Simulator
Figure 6.1.3 Data-and-Function-Based Calculation Flow in a VR Excavator Simulator
Figure 6.1.4 Data-and-Function-Based Calculation Flow in a VR Excavator Simulator
6.2.1 Calculation Route 1 (Excavator Digging)

At $t_{n+1}$, the physics-based excavator model (or excavator computational model) reads in digitized input signals from the operator controls to determine the extent of valve openings in hydraulic circuits for a bucket, a stick, a boom and the revolving upper body. The non-zero valve values read at $t_{n+1}$ mean that hydraulic oil has flowed in a circuit(s) with that amount of valve opening(s) during the current time segment, implying the movements of corresponding mechanical parts. (The case of zero valve values is discussed as Route 0 in Section 6.2.2). Therefore, it is necessary at this calculation time to check whether an excavating tool (bucket) was embedded in soil at $t_n$ in order to determine the necessity of soil resistance calculation. The simulation engine retrieves the excavator geometrical data from the physics-based excavator model (or excavator computational model) and its terrain geometrical data from the simulation engine at $t_n$ to evaluate whether the bucket is in contact with the soil. If this is the case, the calculation follows the Route 1 as shown in the figure 6.1. Otherwise, it follows the Route 2.

Calculation Route 1-1 (Bucket Force Determination)

The next calculation step in the Route 1 branches out in two ways. One branch (Route 1-1) leads to the calculation of bucket forces and the other branch (Route 1-2) leads to soil resistance calculation. In the first block following the Route 1-1, the physics-based excavator model calculates a ram force for each cylinder by multiplying a valve pressure by an inside cross-sectional area of the cylinder and by considering the valve opening. The ram force(s) is (are) immediately used in the next block to determine bucket forces (Detailed description of calculating bucket forces are presented in Chapter 4). It is, however, essential to understand that at this moment the bucket has not advanced at all but is fixed on its boundary with soil media. This situation justifies a condition in which the surrounding soil media functions as reacting body for the bucket force(s) to be generated. Therefore, the physics-based excavator model refers to its geometrical data
saved at \( t_n \) to calculate the bucket forces. These bucket forces are kept in the simulation engine for later comparison with soil resisting forces.

**Calculation Route 1-2 (Soil Resistance Calculation)**

While the calculation process follows the Route 1-1 for the bucket forces, it also branches out to the Route 1-2 for soil resistance calculation. The physics-based excavator model in the first block on the Route 1-2 calculates the hydraulic oil volume passing through each valve by multiplying each valve value, a flow rate of the hydraulic oil (volume/time) and the segmental time \( \Delta t \). The oil volume for each cylinder is, in turn, converted to the piston travel distance by dividing it by the cylinder area of the corresponding valve. By combining each travel distance from all active cylinders, the bucket’s end effect and spatial location is determined and saved as tentative excavator geometrical data in the physics-based excavator model. It is important to understand the reason this information is tentative. In order to justify the bucket’s advancement in soil media, it needs to be checked if the bucket forces are enough to overcome the soil resisting forces. Since the bucket advancement at this moment was calculated without considering the soil resistance, the bucket spatial information is only imaginary, therefore tentative. This imaginary bucket geometrical data is used to determine soil resisting forces in the next calculation block, where a digging mode is selected for the current digging operational segment. The imaginary bucket spatial information represents a situation that the bucket has advanced through the soil media, during which soil resistance forces must have been generated. Differently put, the imaginary bucket spatial information defines the end point of soil resistance generation, which was initiated from the moment the bucket moved at \( t_n \). Therefore, the simulation engine retrieves this information from the excavator geometrical data at \( t_n \) and the tentative excavator geometrical data at \( t_{n+1} \) along with its terrain geometrical data at \( t_n \) to determine a proper digging mode for the current segment. The description on digging mode selection using this information (the excavator geometrical data and the terrain geometrical data at \( t_n \) and the tentative excavator geometrical data at \( t_{n+1} \)) is detailed in Section 6.3. Once the
digging mode is identified for the current segment, the simulation engine checks if the bucket has traveled enough to justify the generation of penetration resistance, separation resistance, or both. This issue is discussed in Section 6.5.

**Calculation Route 1-2-1 (Resistance) and Route 1-2-2 (No Resistance)**

Once the resistance generation is justified (Route 1-2-1), the simulation engine refers to the excavator geometrical data and the terrain geometrical data to prepare input data for the physics-based soil model of excavator digging (or mathematical model of excavator digging) so that the model determines the corresponding resistance forces. The calculated resistance forces are compared with the bucket forces (calculated previously following the Route 1-1). If the resistance generation is not justified (Route 1-2-2), the resistance is assumed to be non-existent.

**Calculation Route 1-a (Bucket forces greater than Resistance Forces)**

If the bucket forces are greater than or equal to the resistance forces (, which is the condition for the bucket move in soil), the calculation process follows the Route 1-a. Following the route, the simulation engine updates the excavator data and the terrain data by deleting the excavator data at and finalizing the tentative excavator data and by changing the terrain data at according to the bucket movement. The simulation engine at the same time sends signals downstream to the graphical models to update their data for the excavator and the terrain.

**Calculation Route 1-b (Resistance Forces greater than Bucket Forces)**

If the resistance forces are greater than the bucket forces (the Route 1-b), the imaginary (tentative) bucket movement through the soil media can not be justified. This is essentially a situation that the bucket is stuck in the middle of the digging operation. Therefore, the simulation engine keeps the excavator and terrain data of physics-based
and graphical components at $t_n$ as the data at $t_{n+1}$ and nullifies the tentative excavator data.

### 6.2.2 Calculation Route 0 (Digging Operation Idle)

The Route 0 is the calculation path of the situation that the valve values are all equal to none, which signifies that a digging operation is idle in the current time segment. This route leads to the next block of the simulation engine, which commands to keep the previous excavator and terrain geometrical data of physics-based components as well as of graphical components. This route is exactly the same as the Route 1-b in the sense that the excavator and terrain data are not changed.

### 6.2.3 Calculation Route 2 (Free Air Movement)

The Route 2 represents the situation that the bucket moves in air without experiencing any resistance from the soil. Therefore, following this calculation route, there is no need to calculate resistance forces and perform force comparison. The tentative and imaginary excavator spatial movement of the Route 1-2 is now considered to be completely possible since it moves in air. Therefore, on this route, the simulation engine only updates the excavator geometrical data for physics-based and graphical parts in the simulator and maintaining the same terrain geometrical data at $t_{n+1}$.

Whatever route the calculation may follow for the current segment, the databases (the excavator geometrical data in the physics-based excavator model, the terrain geometrical data in the simulation engine, the excavator geometrical data in the graphical excavator model, and the terrain geometrical data in the graphical terrain model) at time $t_{n+1}$ are available for the next digging operation segment and the same process repeats until the simulation operator quits the simulation.
6.3 Digging Mode Selection

Digging mode selection for a digging action segment is an essential step toward the calculation of soil resisting forces since a digging mode itself reveals the types of underlying soil resisting forces induced by the segmental digging action. The preceding chapter on the excavator digging modes (Section 5.5) is essential to understanding that each digging mode represents an unique digging situation in terms of its digging mechanism(s) or a type(s) of soil resistance, which is in turn determined by the factors such as a specific bucket part(s) used, the relationship between bucket motion and movement direction, and the surrounding soil. However, in order for a proper digging mode to be selected, this information needs to be complemented with additional criteria, refined and systematically organized in a finite list of unambiguous instructions that can be used in a simulator system. This method is proposed in this section as a digging mode selection algorithm.

6.3.1 Challenges in Digging Mode Selection

One of the issues in developing a digging mode selection algorithm is that the algorithm should not only capable of determining a proper digging modes for a digging action but also capable of identifying unreasonable digging actions, which can not be applied in normal excavator digging processes. Simulator users are not necessarily expected to always generate reasonable digging actions but they frequently make unacceptable digging actions. This is truer in the case of novice operators than in the case of skilled operators. Therefore the digging mode selection algorithm should be equipped with a capability that identifies abnormal digging actions so that the simulator system can react properly.

The other issue in selecting a digging mode for a digging action segment is that the simulation system, specifically the digging mode selection algorithm, only deals with very small period of time, which in turn implies that a bucket movement during this short
segment is very small. This presents some difficulties because it is difficult to determine which digging mode is applied in the time segment as the portion of bucket trajectory calculated is very small. This problem is addressed in the next section.

6.3.2 Digging Mode Selection Algorithm

Figure 6.2 presents a proposed digging mode selection algorithm that describes systematic procedures to identify a digging mode for a particular digging action segment. A digging action segment is defined as the action between the bucket spatial information data at the start of a segment \((t_n)\) and the tentative bucket spatial information data at the end of the segment \((t_{n+1})\). With the digging action segment as an input, the digging mode selection process goes through various criteria and, as a result, the digging mode selection algorithm either matches a segmental digging action to a digging mode, or it identifies a segmental digging action that is operationally not applicable (O/N/A) or unacceptable (N/A) in an excavator digging. The digging mode selection algorithm classifies all possible digging actions, either normal or abnormal.
Figure 6.2 Digging Mode Selection Algorithm
The digging mode selection algorithm starts from the assumption that bucket tip movement corresponding to a digging action segment follows a circular trajectory no matter how small the segment might be and no matter how many excavator joints (such as a bucket joint, a stick joint, and a boom joint) are involved in moving the bucket. Often times a bucket moves through movement of only one joint (a stick joint, for example), in which case the bucket tip obviously goes through a circular trajectory around one joint as shown in Figure 6.3. However, even in a short digging segment, multiple joints can be simultaneously utilized to meet various motional needs in a digging process. A linear motion of a bucket is an example of this multi-joint-based bucket manipulation. Even this linear trajectory, however, is considered as a circular trajectory of a large radius. Therefore, even for multiple-joint-based digging action segments, circular trajectories can be constructed.

![Diagram](image.png)

Figure 6.3 Circular Trajectory by a Single-Joint-Based Rotation (ex. by Stick Joint)
The idea of a bucket tip moving on a circular trajectory for any digging action segment is critical information in that it allows the simulation system to overcome the problem of dealing with an infinitesimal bucket movement for the digging segment. Specifically, instead of the simulation engine dealing with an infinitesimal change in location between bucket geometrical data at the start and the end of a digging segment, the simulation engine, by identifying moving parts (or corresponding joints) and determining their changed angles, can construct a circular trajectory that encompasses the bucket tip at both ends of the segment. Then this circular trajectory is used as a base line, to which the selection algorithm applies most of its criteria. The construction of the circular trajectory for a multi-joint-based bucket movement is detailed in the next section (Section 6.3.3).

The digging mode selection algorithm includes the following selection criteria:

- 1st criterion - a bucket movement on a circular trajectory;
- 2nd criterion - relation of a separation blade with a tangent line at a bucket tip on a circular trajectory;
- 3rd criterion - an embedding condition of a secondary separation blade in soil;
- 4th criterion - a direction of a separation blade’s inside-face normal vector relative to the tangent of a bucket tip on a trajectory circle; and
- 5th criterion - a direction of a separation blade’s inside-face normal vector relative to a circular bucket movement.

The application of these criteria in the digging mode selection algorithm is described as follows with an example of a digging action segment in Figure 6.4.
Figure 6.4 Example of Digging Mode Selection Algorithm Application

Figure 6.4 presents a typical digging action (an example of the curl case <C10> in the digging mode selection algorithm chart in Figure 6.2) in which a stick rotates around a stick joint, therefore forming a circular trajectory. The current digging action segment is defined by the two bucket tips (gray-colored and black-colored), which is a part of the circular trajectory. Note that $c$ is a vector representing the circular trajectory, $n$ is a vector denoting the separation blade’s inside-face normal vector, and $t$ is a tangent line on the circular trajectory at the tip of the bucket.

Given this information, the simulation engine follows the procedures in the digging mode selection algorithm charts (Figure 6.2). It first checks if the bucket curls in toward the machine or uncurls away from the machine. In this specific example shown in Figure 6.4, the bucket curls in toward the machine. The simulation engine then interrogates if the angle of the separation blade with the tangent line $t$ at the bucket tip on the circular trajectory is either tangential, normal, or angled. The example shows that the separation
blade forms an acute angle with the tangent line \( t \). The algorithm continues to determine the embedding condition of the secondary separation blade. The embedding condition is classified as either deep or shallow. In the example, the secondary separation plate is not even in soil, so the determination is made for ‘shallow’. The next criterion determines whether the normal vector \( n \) (to the separation blade’s inside-face) acts in a direction toward inside the circle or outside the circle. In the example the normal vector \( n \) acts toward inside the circle. Lastly, the simulation engine determines if the normal vector \( n \) and the circular trajectory vector \( c \) act in the same direction. In the example they act in the same direction as shown in Figure 6.4. The whole selection process has led to the digging mode III (separation and the penetration resistance) as represented in Figure 6.5.

To demonstrate the digging mode selection algorithm’s capability in identifying an operationally not applicable (O/N/A) digging situation, imagine another digging action segment. Assume that, for this digging action segment, the simulation engine follows the same selection route as in the example of Figure 6.4 until the 5\(^{th}\) criterion (a direction of a separation blade’s inside-face normal vector relative to a circular bucket movement), which means that a normal vector \( n \) and a circular trajectory vector \( c \) act in opposite directions. The whole selection process determines that this digging action segment does not match with a normal digging mode and represents an operationally-not-applicable (O/N/A) case as shown in Figure 6.5. An example case of this digging action segment is presented in Figure 6.6. The reason that the digging action segment is O/N/A is that the outside face of the separation blade is used to separate soil.
Figure 6.5 Selection Process Examples of Two Digging Action Segments
Figure 6.6 Example of Operationally Non-Applicable Digging Action Segment

Figure 6.7 shows all the cases identified by the digging selection algorithm with specific examples to help visualize digging action segments corresponding to different cases. Due to the inclusion of many selection criteria, some cases in the digging selection algorithm are not even possible to achieve in reality. These cases are indicated as N/A in Figure 6.2.
Figure 6.7.1 All Digging Cases in Digging Selection Algorithm (Curling Actions)
Figure 6.7.2 All Digging Cases in Digging Selection Algorithm (Curling Actions)
Figure 6.7.3 All Digging Cases in Digging Selection Algorithm (Curling Actions)
Figure 6.7.4 All Digging Cases in Digging Selection Algorithm (Curling Actions)
Figure 6.7.5 All Digging Cases in Digging Selection Algorithm (Curling Actions)
6.3.3 Algorithm of Imaginary Joint Calculation of Multi-Joint Circular Trajectory

As demonstrated in the last section, the first thing that the simulation engine has to do before it starts a digging mode selection process is to identify a circular trajectory matching to a current digging action segment. The challenge is that unless a bucket moves against only one joint (either a bucket joint, a stick joint, or a boom joint), the construction of the circular trajectory is not a trivial matter because it relies on two bucket tip points with infinitesimal distance between them. In this section, therefore, an algorithm is proposed that accurately and swiftly (from a computational perspective) constructs a circular trajectory for a multi-joint-based digging action segment.

Figure 6.8 shows the circular trajectory construction of a stick-boom-joint-activated digging action segment. In this specific example, during the current time segment defined by the gray-colored bucket and the black-colored bucket in space, the stick rotates curling against its own joint, \( P_{st,j} (x_{st,j}, y_{st,j}) \) by an angle, \( \Delta_{st,j} \) and the boom rotates curling against its own joint, \( P_{bm,j} (x_{bm,j}, y_{bm,j}) \) by an angle, \( \Delta_{bm,j} \). Note that the angle changes at the stick joint \( \Delta_{st,j} \) and the boom joint \( \Delta_{bm,j} \) are exaggerated for clear presentation.

If \( \Delta_{bm,j} = 0 \), a circular trajectory will be formed with the stick joint, \( P_{st,j} (x_{st,j}, y_{st,j}) \) as its center. If \( \Delta_{st,j} = 0 \), a circular trajectory will be formed and its center coincides with the stick joint, \( P_{bm,j} (x_{bm,j}, y_{bm,j}) \). If these angle changes are non-zero, the center of a resultant circular trajectory will be on a line that connects two joints, \( P_{st,j} (x_{st,j}, y_{st,j}) \) and \( P_{bm,j} (x_{bm,j}, y_{bm,j}) \). The center point of this trajectory is called an imaginary joint, \( P_{im,j} (x_{im,j}, y_{im,j}) \) since it does not coincide with any other joints. The imaginary joint, \( P_{im,j} (x_{im,j}, y_{im,j}) \) for a stick-boom-joint-activated digging action segment, therefore, is expressed as in the expressions (6-1), (6-2), and (6-3).
Figure 6.8 Stick-Boom-Joint-Activated Circular Trajectory
(Example 1)
\[ P_{im \_j}(x_{im \_j}, y_{im \_j}) = P_{st \_j} + (P_{bm \_j} - P_{st \_j}) \times \left( \frac{\Delta_{bm \_j}}{\Delta_{bm \_j} + \Delta_{st \_j}} \right) \]  \hfill (6-1)

\[
\begin{align*}
x_{im \_j} &= x_{st \_j} + (x_{bm \_j} - x_{st \_j}) \times \left( \frac{\Delta_{bm \_j}}{\Delta_{bm \_j} + \Delta_{st \_j}} \right) \\
y_{im \_j} &= y_{st \_j} + (y_{bm \_j} - y_{st \_j}) \times \left( \frac{\Delta_{bm \_j}}{\Delta_{bm \_j} + \Delta_{st \_j}} \right)
\end{align*}
\hfill (6-2)
\]

\[
\begin{align*}
x_{im \_j} &= x_{st \_j} + (x_{bm \_j} - x_{st \_j}) \times \left( \frac{\Delta_{bm \_j}}{\Delta_{bm \_j} + \Delta_{st \_j}} \right) \\
y_{im \_j} &= y_{st \_j} + (y_{bm \_j} - y_{st \_j}) \times \left( \frac{\Delta_{bm \_j}}{\Delta_{bm \_j} + \Delta_{st \_j}} \right)
\end{align*}
\hfill (6-3)
\]

where,
- \( P_{im \_j}(x_{im \_j}, y_{im \_j}) \), \( P_{st \_j}(x_{st \_j}, y_{st \_j}) \), and \( P_{bm \_j}(x_{bm \_j}, y_{bm \_j}) \) are the locations of an imaginary joint, a stick joint, and a boom joint, respectively.
- \( \Delta_{st \_j} \) and \( \Delta_{bm \_j} \) are the rotated angles of a stick and a boom at its own joint, respectively.
- Positive when curling and negative when uncurling.

Note that an angle change, \( \Delta \) at a joint is a positive value when a curling rotation takes place and it is a negative value when an uncurling rotation takes place.

In a case that \( \Delta_{bm \_j} = 0 \) (that is, only stick rotation), \( P_{im \_j}(x_{im \_j}, y_{im \_j}) = P_{st \_j}(x_{st \_j}, y_{st \_j}) \), and in a case that \( \Delta_{st \_j} = 0 \) (only boom rotation), \( P_{im \_j}(x_{im \_j}, y_{im \_j}) \) matches with \( P_{bm \_j}(x_{bm \_j}, y_{bm \_j}) \) as expected. Also, as shown in Figure 6.8, in a case that \( \Delta_{bm \_j} = +18 \) and \( \Delta_{st \_j} = +22 \), \( P_{im \_j} \) lies on a line between the stick joint and the boom joint. As a result, the circular trajectory connecting two bucket tips (solid gray line and black line) is constructed as shown in the figure.

As another example, Figure 6.9 shows a digging action segment, in which \( \Delta_{bm \_j} = -11 \) and \( \Delta_{st \_j} = +22 \). The imaginary joint of this digging segment is on outside the line segment defined by the stick joint and the boom joint.
Figure 6.9 Stick-Boom-Joint-Activated Circular Trajectory
(Example 2)
Through the same analytical approach, an imaginary joint calculation of a circular trajectory for a stick-boom-joint-activated digging action segment can be extended for a three-joint-activated digging action segment as follows.

Figure 6.10 shows a digging action segment example, in which a bucket, a stick, and a boom rotate against their own joints. Firstly, the first two joints, the bucket joint, \( P_{bk,j} (x_{bk,j}, y_{bk,j}) \) and the stick joint, \( P_{st,j} (x_{st,j}, y_{st,j}) \), are focused to find the intermediate imaginary joint, \( P_{im_2j} (x_{im_2j}, y_{im_2j}) \) of the circular trajectory due to the rotations of the bucket and the stick. This point is calculated using the expression (6-4) (equivalently, (6-5) and (6-6) ) and lies on outside the line segment defined by the bucket joint and the stick joint. Then the calculation continues with this intermediate imaginary joint, \( P_{im_2j} (x_{im_2j}, y_{im_2j}) \) and \( P_{bm,j} (x_{bm,j}, y_{bm,j}) \) to find the final imaginary joint of the circular trajectory due to the rotations of the bucket, the stick, and the boom using the expression (6-7) (equivalently, (6-8) and (6-9) ). As a result, the final circular trajectory connecting two bucket tips (solid gray line and black line drawings) is constructed as shown in Figure 6.10.

\[
P_{im_{-2}j}(x_{im_{-2}j}, y_{im_{-2}j}) = P_{bk_{-j}} + (P_{st_{-j}} - P_{bk_{-j}}) \times \left( \frac{\Delta_{st_{-j}}}{\Delta_{st_{-j}} + \Delta_{bk_{-j}}} \right)
\]

\[
\begin{align*}
x_{im_{-2}j} &= x_{bk_{-j}} + (x_{st_{-j}} - x_{bk_{-j}}) \times \left( \frac{\Delta_{st_{-j}}}{\Delta_{st_{-j}} + \Delta_{bk_{-j}}} \right) \\
y_{im_{-2}j} &= y_{bk_{-j}} + (y_{st_{-j}} - y_{bk_{-j}}) \times \left( \frac{\Delta_{st_{-j}}}{\Delta_{st_{-j}} + \Delta_{bk_{-j}}} \right)
\end{align*}
\]

\[
P_{im_{-3}j}(x_{im_{-3}j}, y_{im_{-3}j}) = P_{im_{-2}j} + (P_{bm_{-j}} - P_{im_{-2}j}) \times \left( \frac{\Delta_{bm_{-j}}}{\Delta_{bm_{-j}} + (\Delta_{st_{-j}} + \Delta_{bk_{-j}})} \right)
\]

\[
\begin{align*}
x_{im_{-3}j} &= x_{im_{-2}j} + (x_{bm_{-j}} - x_{im_{-2}j}) \times \left( \frac{\Delta_{bm_{-j}}}{\Delta_{bm_{-j}} + (\Delta_{st_{-j}} + \Delta_{bk_{-j}})} \right) \\
y_{im_{-3}j} &= y_{im_{-2}j} + (y_{bm_{-j}} - y_{im_{-2}j}) \times \left( \frac{\Delta_{bm_{-j}}}{\Delta_{bm_{-j}} + (\Delta_{st_{-j}} + \Delta_{bk_{-j}})} \right)
\end{align*}
\]
where,
- $P_{im_2j}(x_{im_2j}, y_{im_2j})$ : location of an imaginary joint of an intermediate circular trajectory due to rotations of a bucket and a stick
- $P_{im_3j}(x_{im_3j}, y_{im_3j})$ : location of an imaginary joint of a final circular trajectory due to rotations of a bucket, a stick and a boom
- $P_{bk_j}(x_{bk_j}, y_{bk_j}), P_{st_j}(x_{st_j}, y_{st_j}),$ and $P_{bm_j}(x_{bm_j}, y_{bm_j})$ : location of a bucket joint, a stick joint, and a boom joint, respectively
- $\Delta_{bk_j}, \Delta_{st_j},$ and $\Delta_{bm_j}$ : rotated angle of a bucket, a stick and a boom at its own joint, respectively
- : positive when curling and negative when uncurling
Figure 6.10 Bucket-Stick-Boom-Joint-Activated Circular Trajectory
6.4 Approximation of Soil Failure Angle ($\rho$)

In this section, the generalized separation model’s inherent problem in being used in the simulator system is described and a solution to correct this problem is proposed.

6.4.1 Summary of the Generalized Separation Model

As proposed in Chapter 5, the generalized separation model is summarized as follows.

\[
R_s = \frac{-ADF \cdot \cos(\beta + \rho + \phi) + W \cdot \sin(\alpha + \rho + \phi) + CF_1 \cdot \cos \phi + 2 \cdot ACF \cdot \cos \phi + 2 \cdot SF_2 \cdot \cos \phi}{\sin(\beta + \rho + \delta + \phi)}
\]

(5-3)

where,

- $\alpha$: angle between an inclined terrain surface and a horizontal plane
- $\beta$: angle between a separation blade and a terrain surface
- $\gamma$: angle between a soil failure plane and a terrain surface (soil failure angle)
- $\delta$: soil-metal friction angle
- $\Phi$: soil internal friction angle
- $ADF$ (adhesional force) = $c_a \cdot A_3 = c_a \cdot B \cdot L$

\[c_a: \text{soil adhesion}
A_3: \text{area of bucket separation plate in contact with soil ('abed')}
B: \text{separation plate width}
L: \text{length of separation blade in contact with soil}
\]

- $W$ (soil wedge) = $\gamma \cdot B \cdot A_3 = \gamma \cdot B \cdot [0.5 \cdot (L \cdot \sin \beta) \cdot (L \cdot \cos \beta + L \cdot \sin \beta / \tan \rho)]$

\[= 0.5 \cdot \gamma \cdot B \cdot L^2 \cdot \sin \beta \cdot (\cos \beta + \sin \beta / \tan \rho)
\]

$\gamma$: soil unit weight

\[A_3: \text{area of triangular rupture surface ('abc' or 'def')}
\]

- $CF_1$ (cohesional force) = $c \cdot A_1 = c \cdot B \cdot L \cdot \sin \beta / \sin \rho$

\[c: \text{soil cohesion}
A_1: \text{area of rectangular failure surface ('bcfe')}
\]
\[ ACF(\text{adhesional-cohesional force}) = c^* \cdot A_3 = c^* \cdot \{0.5 \cdot (L \cdot \sin \beta) \cdot (L \cdot \cos \beta + L \cdot \sin \beta / \tan \rho)\} \]
\[ = 0.5 \cdot c^* \cdot L^2 \cdot \sin \beta \cdot (\cos \beta + \sin \beta / \tan \rho) \]
\[ c^*: \text{soil adhesion-cohesion} \] (5-7)

\[ SF_2(\text{friction force}) = K_0 \cdot \gamma \cdot z \cdot \tan \Phi^* \cdot A_2 \]
\[ = 0.5 \cdot K_0 \cdot \gamma \cdot z \cdot \tan \Phi^* \cdot L^2 \cdot \sin \beta \cdot (\cos \beta + \sin \beta / \tan \rho) \]
\[ K_0: \text{coefficient of lateral earth pressure at rest} \]
\[ z: \text{depth from the wedge top (point 'c' or 'f') to the wedge centroid} \]
\[ \Phi^*: \text{combined friction angle of } \delta (\text{soil-metal friction angle}) \text{ and } \Phi (\text{soil internal friction angle}) \] (5-8)

\[ c^* = \frac{c_a \cdot \text{Area}_{abx} + c \cdot \text{Area}_{bcx}}{\text{Area}_{abc}} \quad \phi^* = \frac{\delta \cdot \text{Area}_{abx} + \phi \cdot \text{Area}_{bcx}}{\text{Area}_{abc}} \] (5-9)

\[ \therefore z = L \cdot (\cos \beta + \frac{\sin \beta}{\tan \rho}) \cdot \sin \alpha - \frac{L}{3} \cdot \left[ \cos(\beta - \alpha) + \frac{\sin(\beta - \alpha)}{\tan(\alpha + \rho)} \right] \cdot \left[ \sin^2 \alpha \cdot (\cos \beta + \frac{\sin \beta}{\tan \rho})^2 - \sin^2(\beta - \alpha) \right] \]
\[ \sin \beta \cdot (\cos \beta + \frac{\sin \beta}{\tan \rho}) \] (5-10)

Therefore, the generalized separation model is a function of many variables as shown in below.

\[ R_s = f(\rho, \alpha, \beta, \phi, \delta, c, c_a, \gamma, K_0, B, L) \] (5-12)

All these factors are known, except for the soil failure angle (\( \rho \)), which means that the separation resistance cannot be determined directly. The soil failure angle and the resulting separation resistance force, however, can be determined indirectly by adopting the passive earth pressure theory that states that (passive) soil failure occurs when the resistance is minimum. Therefore, either through an iteration or by a trial-error method, the separation resistance (\( R_s \)) – or, the soil failure angle (\( \rho \)) can be determined by finding the minimum resistance value.
6.4.2 Sensitivity Analysis

An iterative method of determining separation resistance implies a heavy computational burden for the simulator system. Unless the model supports the idea of instant resistance calculation, direct model application slows the system speed significantly. Therefore, in this sub-section, through sensitivity analysis, factors affecting a soil failure angle are identified and these factors are correlated with a soil failure angle ($\rho$).

Among the factors in the expression (5-12), $\gamma$, $K_0$, $c$, $c_a$, $B$, and $L$ are identified as the factors affecting only the magnitude of the separation resistance. Figure 6.11 shows the effect of $L$(length of separation blade in contact with soil) on the separation resistance magnitude and the soil failure angle. Changing the values of L does not result in the change in soil failure angle, $\rho$ but does result in the change of the separation resistance.

![Diagram showing the effect of separation blade length on separation resistance and soil failure angle](image)

Figure 6.11 Effect of Separation Blade Length in Contact with Soil ($L$)
The following is a well-known expression proposed by Rankine that relates a soil internal friction angle ($\phi$) and a soil failure angle in a passive earth pressure of a retaining wall (Das 1991).

$$\rho = 45^\circ - \frac{\phi}{2}$$  \hspace{1cm} (6-10)

Also, it is suggested that soil friction ($\phi$ and $\delta$) approximately determines the shape of failure zone, that is, a soil failure angle (McKyes 1985).

Therefore, only factors that need to be examined through a sensitivity analysis are $\alpha$ (angle between an inclined terrain surface and a horizontal plane) and $\beta$ (angle between a separation blade and a terrain surface). And their relation with $\rho$ (soil failure angle) needs to be determined.

The sensitivity analysis is performed in the following order.

(Step 1)
With a fixed angle of $\alpha$ and varying angles of $\beta$, a series of $R_s$ (Separation Resistance)-$\rho$ (soil failure angle) diagrams are constructed for each $\beta$. For each $R_s$-$\rho$ diagram, a range of $\rho$ in which resistance becomes a minimum is obtained. An example of this $R_s$-$\rho$ diagram is presented in Figure 6.12. The complete analysis results are included in Appendix B.

(Step 2)
With the information of the step 1, a $\rho$-$\beta$ diagram is constructed for each $\alpha$. An example of this $\rho$-$\beta$ diagram is presented in Figure 6.13. The complete analysis results are included in Appendix C.

(Step 3)
Step 1 and 2 are repeated for different angles of $\alpha$ and, therefore, a series of $\rho$-$\beta$ diagrams are developed. From these diagrams, a trend of associations of $\alpha$ and $\beta$ with $\rho$ is identified and formulated.
Figure 6.12 Example of $R_s$ (Separation Resistance)-$\rho$ (Soil Failure Angle) Diagram
Figure 6.13

Example of $\rho$ (Soil Failure Angle)- $\beta$ (Separation Blade Angle with respect to Terrain Surface Angle) Diagrams for Different Angles of $\alpha$ (Angle between an Inclined Terrain Surface and a Horizontal Plane)
The sensitivity analysis result is that all the $\rho - \beta$ diagrams (See Appendix C) are almost the same in terms of the range of $\rho$ (soil failure angle) for $\beta$ (angle of separation blade with a terrain) regardless of the angle of $\alpha$ (angle of terrain slope), which is indicative of the fact that $\alpha$ is not the factor affecting $\rho$. Therefore, using Figure 6.13 as a representative of all the $\rho - \beta$ diagrams and combining the suggestions of Rankine and McKyes, the following relationship is concluded.

$$\rho = (45 - \frac{\phi + \delta}{2}) + (45 - \frac{\beta}{2}) \quad (6-11)$$

Using this expression, the generalized separation model is capable of making resistance prediction without an iterative process. Therefore it can lessen the calculation burden of a simulator system.
6.5 Approximation of Bucket Travel Distance between Separation Failures

There is one more issue to be addressed for the generalized separation model to be complete and fully functional in the simulator system. In this section, the issue is discussed and a solution is proposed thereafter.

6.5.1 Characteristics of Separation and Penetration Soil Failures

In order to describe what consideration needs to be added to the generalized separation model in its use in a simulator system, a simple excavation process, made up of a digging mode I and II, is presented in Figure 6.14. The excavation starts from a bucket penetrating downward as shown in Figure 6.14 a. When it gets to a desired depth, the bucket moves horizontally and consequently densifying soil in front of the bucket (Figure 6.14b-1). Once the bucket reaches to a point of condition that soil can no longer resist, a separation failure instantly develops, which is manifested by soil failure line as shown in Figure 6.14b-2. The bucket moves again to continue separation failures (Figure 6.14c).

There is one distinct difference between failings induced by a penetration mechanism and a separation mechanism. When a bucket is in a penetrating motion such as that shown in Figure 6.14a, soil failing is continuous in progress as the bucket moves. When the bucket is engaged in a separational motion such as in Figure 6.14b, however, soil failing does not develop on a continuous basis but occurs discontinuously because soil needs to be in a passive earth pressure state before it develops failure. Therefore this discontinuous characteristic of soil separation failure requires the consideration of bucket travel distance between separational failures.
Figure 6.14 Simple Excavation Process by Penetration and Separation
6.5.2 Bucket Travel Distance between Separation Failures

The travel distance of an excavator bucket in soil between separational failures can be approximated using a geotechnical triaxial soil test result. Figure 6.15 presents a typical soil failure of a triaxial test, where an axial strain in a soil specimen takes place from the point of axial force loading until the specimen ultimately gets failed by the axial force. This situation is analogous to separation-blade-induced soil failure cases if loading caps are replaced with a bucket separation blade.

![Figure 6.15 Typical Triaxial Soil Test](image)

Given an axial strain-at-failure \( \varepsilon_f \), the compressed length of the specimen \( \Delta l \) is determined as follows. Note that the length of a soil specimen usually runs from \( 2d \sim 2.5d \) [ASTM, 1996 #51].

\[
\Delta l = (2.0 \ d \sim 2.5d) \ \varepsilon_f
\]  

(6-12)
Therefore, by replacing $d$ with $L$ (length of separation blade in contact with soil), a bucket travel distance between separation failures, $T_s$, is approximated as follows.

$$T_s = (2.0 \ L \sim 2.5 \ L) \ e_f$$

(6-13)

Even though this expression is only a rough approximation, the consideration on the bucket travel distance is important for the following reason. When the simulation engine matches a certain digging action segment with a separation-including digging mode (either mode I, III, IV, or V), it does not necessarily mean that a separation plate goes through a separation resisting force during that segmental time period unless the separation blade has traveled a separation failure bucket distance ($T_s$) from the previous separation failure. If the actual bucket travel is less than the separation failure bucket distance ($T_s$), then the separation resistance is considered to be non-existent.
6.6 Conclusion

In this chapter, systematic and effective calculation methods for the development of a VR excavator simulator system are developed. The main objectives of this chapter are achieved by

1) proposing complete calculation procedures for a VR simulator system to describe in detail how computations are performed from the perspective of simulator development;

2) developing a digging mode selection algorithm and a bucket travel distance approximation, which address the issues of the proposed models being utilized in VR simulator development; and

3) proposing an algorithm (imaginary joint calculation algorithm for multi-joint circular trajectory) and a separation soil failure angle approximation method, which increase simulation speed in general.