Appendix A

Procedure for Calculating Cavity Expansion Pressure
(Yu and Houlsby’s Model)
This procedure is abstracted from the paper by Yu and Houlsby.

(i) Calculate initial stress $p_0$ that is the pressure corresponding to the initial cavity with radius $a_0$.

(ii) For a given cavity pressure $p$, greater than initial yielding cavity pressure $p_1$, calculate $R$ from the following equation.

$$R = \frac{\frac{m + \alpha}{\alpha(1 + m)\left[Y + (\alpha - 1) \cdot p_0\right]}}{\frac{m(\alpha - 1)}{\alpha(1 + m)\left[Y + (\alpha - 1) \cdot p_0\right]}}$$

where, $m = 1$ (cylindrical cavity) & 2 (spherical cavity), $\alpha = \frac{1 + \sin \phi}{1 - \sin \phi}, \quad Y = \frac{2 \cdot c \cdot \cos \phi}{1 - \sin \phi}$

(iii) Evaluate $\Lambda_1$ from the equation – only a few terms are sufficient.

$$\Lambda_1(x, y) = \sum_{n=0}^{\infty} A_n^1$$

where, $A_n^1 = \frac{y^n}{n!} \ln x, \quad if \quad n = \gamma \quad (\gamma = \frac{\alpha(\beta + m)}{m(\alpha - 1)\beta}, \quad \beta = \frac{1 + \sin \psi}{1 - \sin \psi})$

$$A_n^1 = \frac{y^n}{n!(n-\gamma)} [x^{n-\gamma} - 1], \quad otherwise$$

(iv) Evaluate $a/a_0$ from the following and from that the cavity pressure can be determined.

$$\frac{a}{a_0} = \left\{ \frac{R^{-\gamma}}{(1 - \delta)^{\beta + m} / \beta - (\gamma / \eta)\Lambda_1(R, \xi)} \right\}^{\beta + m}$$

where, $\delta = \frac{Y + (\alpha - 1) p_0}{2(1 + \alpha)G}, \quad (G = \frac{E}{2(1+\nu)})$

$$\eta = \exp\left\{ \frac{(\beta + m) \cdot (1 - 2\nu) \cdot [Y + (\alpha - 1) \cdot p_0] \cdot [1 + (2 - m) \cdot \nu]}{E \cdot (\alpha - 1) \cdot \beta} \right\}$$

$$\xi = \frac{[1 - \nu^2 \cdot (2 - m)] \cdot (1 + m) \cdot \delta \cdot \left[\alpha \cdot (\beta + m \cdot (1 - 2\nu) + 2\nu - \frac{m \cdot \nu \cdot (\alpha + \beta)}{1 - \nu \cdot (2 - m)} \right]}{(1 + \nu) \cdot (\alpha - 1) \cdot \beta}$$