CHAPTER V.

DYNAMIC RESPONSE CONTROL OF CANTILEVERS TO BLAST AND SONIC BOOM LOADINGS

5.1 Overview

During the evolution through the atmosphere and space the structure of flight vehicles may be exposed to a variety of time-dependent loads induced by nuclear blast, gust, sonic-boom, shock wave, fuel explosion, etc. Having in view the damaging effects played by blast loadings on structural integrity and operational life of flight vehicle, adequate methods to predict and control their structural dynamic response have to be devised. This is more imperative, as the next generations of aeronautical and space vehicle are likely to feature increasing structural flexibility and operate in severe environmental conditions.

With the advent of high performance composite material structures and their increased use in the aerospace industry and other fields of advanced technology, there is a need for further studies of the problem of structural response. This is due to the fact that the new composite material structures exhibit distinguishing features as compared to their metallic counterparts. For this reason, in order to accurately predict the response behavior of structures made of advanced composite materials, refined structural models have to be used. In particular, transverse shear as well as other non-classical effects must be included in their modeling. The recent investigations of the response to sonic booms and blasts have been concerned mainly with flat and curved panels [32, 56, 57]. In spite of the frequent use of thin-walled cantilevers beams in structures such as airplane wings, helicopter blades, robotic manipulator arms and space booms, very few studies have considered the behavior of thin-walled anisotropic cantilevers subjected to time-dependent external excitations.

The development of adequate approaches to the prediction of the response of composite thin-walled beams to time-dependent external pulses is of practical importance as far as safe design is concerned. In spite of its practical importance, studies of the control of the dynamic response of thin-walled beams to transient loadings via implementation of above mentioned dual technology are virtually absent in the specialized literature. In this sense, the present section is aiming to fill the existing gap in this field and is concerned with two related issues, namely: i) the development of a powerful mathematical methodology for determining the response of thin-walled cantilevers to time-dependent external excitations, and ii) the highlight of the influence of the various effects, including those related with the character of the considered pressure pulses on the dynamic response of thin-walled beam cantilevers.

Using structural tailoring and the induced strain actuation technology, the dynamic response of cantilevered thin-walled beams simulating an aircraft wing have been considered. In this context, both the dynamic response to harmonically time dependent excitations and the dynamic response to arbitrarily time-dependent excitations have considered.

The synergistic combination of the two control methodologies is demonstrated and pertinent conclusions about the influence of non-classical effects are outlined.

Although not restricted to this case, the analysis of the dynamic response to be developed in this work concerns mainly the case of an aircraft wing. If the airplane is in flight, the effect of the unsteady aerodynamic loads should be superposed to that due to the
time-dependent pressure pulses considered in this study. It is obvious that it would be extremely difficult to determine the response of the wing structure when exposed to such complex loadings. Having in view the linearity of the structural system as considered in the present study, the customary procedure (see e.g. Refs. 61, 65, 66) of breaking up the total response in structural responses belonging to each constituent load event will be adopted. In this context, herein, only the response to blast and sonic boom pressure pulses will be investigated.

5.2 Frequency Response Analysis to Harmonic Excitations

As for the dynamic response control of the structure to time-dependent external excitations, a powerful control scheme referred to as boundary moment control, using a combined feedback control law will be implemented.

For the numerical simulation, one considers that the structure is excited by a concentrated harmonic load located at \( z = z_o \), represented as:

\[
p_y(z,t) = F_o \delta(z-z_o)e^{i\omega t}
\]  

(5.1)

where \( F_o \) is its amplitude, \( \delta(z-z_o) \) a spatial Dirac delta function and \( \omega \) the excitation frequency. When \( p_y \) is located off the longitudinal axis of symmetry by an amount \( x_o \), then it gives rise to the sectional twist moment

\[
m_z(z,t) = F_o x_o \delta(z-z_o)e^{i\omega t}
\]  

(5.2)

Such a load and moment can be generated, among others, by the operation of a power system located on the wing at \((x_o, z_o)\).

To derive frequency response solution, we first insert Eq. (5.1) and Eq. (5.2) into Eq. (4.17) and obtain the generalized force vector

\[
Q(t) = \int_0^L \left[ F_o \delta(z-z_o) \phi_1^T(z) \quad 0^T \quad F_o x_o \delta(z-z_o) \phi_3^T(z) \right] dz e^{i\omega t} = Q e^{i\omega t}
\]  

(5.3)

where

\[
Q = \left[ F_o \phi_1^T(z_o) \quad 0^T \quad F_o x_o \phi_3^T(z_o) \right]^T
\]  

(5.4)

is a constant vector. Then, expressing the particular solution of Eq. (4.19) in the form

\[
X(t) = x_p e^{i\omega t}
\]  

(5.5)

we conclude that

\[
x_p = \left[ i\omega I - A \right]^{-1} BQ
\]  

(5.6)

in which \( I \) is the identity matrix. Equation (5.6) can be used in conjunction with Eq. (4.8) to obtain the frequency response corresponding to \( v_o \) and \( \phi \).

A number of plots display the frequency response corresponding to a harmonic load located at the beam tip. Figures 5.1 and 5.2 present frequency response plots of the normalized deflection for shearable and unshearable beams exhibiting free and restraint warping, for \( \theta = 60^\circ \). As the displayed results reveal, transverse shear flexibility is responsible for a shift in resonance towards lower frequencies as well as for an increase in the deflection amplitude. One also concludes that discard of transverse shear yields an overestimation of the resonance frequency and underestimation of the deflection response.
Moreover, comparing Fig. 5.1 \( (AR = 6) \) with Fig. 5.2 \( (AR = 16) \), it is evident that resonance frequencies increase dramatically with a decrease in the beam aspect ratio.

Figure 5.3 displays the frequency response of TWBs featuring free warping and the ply-angle \( \theta = 90^\circ \), at which the bending stiffness reaches a maximum (see Fig. 3.3). As the results reveal, transverse shear exerts a small influence on the deflection amplitude. It also becomes apparent that in the absence of transverse shear, a shift in the resonance frequencies towards higher frequencies occurs. From Figures 5.2 and 5.3, it may be concluded that, for the same beam aspect ratio, a variation of the ply-angle from \( \theta = 90^\circ \) to \( \theta = 60^\circ \) yields a decrease of the resonance frequencies. Figures 5.4 and 5.5 depict the frequency response corresponding to the twist amplitude for shearable and unshearable beam of ratio \( AR = 6 \) and ply-angle \( \theta = 60^\circ \), for the case of activated and unactivated beams. The results reveal that in the free warping case (see Fig. 5.4) the influence of the transverse shear is more prominent than for the beams with warping restraint (see Fig. 5.5). The results also reveal that the inadvertent neglect of the transverse shear can yield an underestimation of the twist and an overestimation of the resonance frequency.

All previously displayed results are based on the assumption that the concentrated load is located on the \( z \)-axis. However, if there is a load eccentricity, then both the deflection and the twist are affected. Figure 5.6 depicts the normalized steady-state deflection as a function of the feedback gain with the normalized eccentricity \( \bar{x}_o = x_o/c \) acting as a parameter. The plots reveal the obvious trends that \( \bar{V} \) increase with \( c \) and decrease with an increase in the control gain. Figure 5.7 displays the normalized bending moment at the wing root versus the control gain with the ply-angle as a parameter. The figure shows that the bending moment has a maximum for the value \( \theta = 0^\circ \) of the ply-angle, for which the bending stiffness \( a_{33} \) has a minimum, and a minimum for \( \theta = 90^\circ \), where \( a_{33} \) has a maximum (see Fig. 3.3). This implies that tailoring and feedback control can be used simultaneously to reduce the bending moment, thus avoiding overdesign at the root of the beam. The figure also reveals that for relatively large values of the feedback gain, tailoring becomes less efficient than for low feedback gains. This suggests the necessity of a trade-off between the two techniques.

Finally, it should be pointed out that the mass and stiffness of the actuators were included in all numerical results obtained here. It is common practice, to ignore them, so that a comparison of results obtained by including and ignoring the mass and stiffness of actuators should prove of interest. Figure 5.8 shows such a study in the form of two sets of frequency response plots, the set in solid lines corresponding to the case in which the mass and stiffness of the actuators are considered and the dashed lines corresponding to the case in which they are not. As can be concluded from the Fig. 5.8, the effect of ignoring the mass and stiffness of the actuators is to predict a resonance frequency about 10\% smaller and higher response amplitudes than the actual ones. The case depicted in Fig. 5.8 seems to present a worse than average picture. In the other frequency response plots presented in this paper, the resonance frequency obtained by ignoring the mass and stiffness of the actuators (results not shown) was about 4\% - 6\% smaller.

### 5.3 Time-Dependent Loads Associated with Blast and Sonic-Boom Pulses

The response of elastic structures to time-dependent external excitations, such as sonic boom and blast loadings, is a subject of much interest in the design of aeronautical and space vehicles as well as of marine and terrestrial ones (see e.g. Crocker (1969, 1970), Crocker and Hudson (1969) and Houlston \textit{et al.} (1985)). For the case of blast loadings, various analytical expressions have been proposed and discussed [58-63]. As it was clearly established, the blast wave reaches the peak value in such a short time that the structure can be assumed to be loaded instantly. Due to the relative small dimensions of the structure when compared to the blast and sonic boom wave front, it may also be assumed that the pressure is uniformly distributed over the structure [57]. In accordance with above
mentioned references the overpressure associated with the blast pulses can be described in terms of the modified Friedlander exponential decay equation as:

\[ p_y(s, z, t) = P_m \left( 1 - \frac{t}{t_p} \right) e^{-\frac{d t}{t_p}} \]  

(5.7)

where the negative phase of the blast is included. In Eq. (5.7), \( P_m \) denotes the peak reflected pressure in excess of the ambient one; \( t_p \) denotes the positive phase duration of the pulse measured from the time of impact of the structure and \( a' \) denotes a decay parameter which has to be adjusted to approximate the overpressure signature from the blast tests. A depiction of the ratio \( p_y/P_m \) vs. time for various values of the ratio \( d'/t_p \) and a fixed value of \( t_p \) is displayed in Fig. 5.9(a). As it could be inferred, the triangular load may be viewed as a limiting case of Eq. (5.7) occurring for \( d'/t_p \to 0 \).

As concerns the sonic-boom loading, this can be modeled as an N-shaped pressure pulse arriving at a normal incidence. Such a pulse corresponds to an idealized far field overpressure produced by an aircraft flying supersonically in the earth's atmosphere or by any supersonic projectile rocket or missile[58, 59, 64]. The overpressure signature of the N-wave shock pulse can be described by

\[ p_y(s, z, t) = \begin{cases} 
P_m \left( 1 - \frac{t}{t_p} \right) & \text{for } 0 < t < rt_p \\
0 & \text{for } t < 0 \text{ and } t > rt_p 
\end{cases} \]  

(5.8)

where \( r \) denotes the shock pulse length factor and \( P_m \) and \( t_p \) maintain the same meaning as in the case of blast pulses. It may easily be seen that: i) for \( r = 1 \) the N-shaped pulse degenerates into a triangular pulse; ii) for \( r = 2 \) a symmetric N-shaped pressure pulse is obtained; while iii) for \( 1 < r < 2 \), as shown in Fig. 5.9(b), the N-shaped pulse becomes an asymmetric one.

Another special case emerging from blast and sonic-boom pulses corresponds to a step pulse. The case is obtained from Eq. (5.7), when \( t_p \to \infty \) and from Eq. (5.8) when \( r = 1 \) and \( t_p \to \infty \).

In addition, the cases of the sine and rectangular pressure pulses described as:

\[ p_y(s, z, t)(\equiv p_y(t)) = \begin{cases} 
P_m \sin \pi t / t_p & 0 \leq t \leq t_p \\
0 & t > t_p 
\end{cases} \]  \hspace{1cm} \text{sine pulse}  

\[ p_y(s, z, t)(\equiv p_y(t)) = \begin{cases} 
P_m & 0 \leq t \leq t_p \\
0 & t > t_p 
\end{cases} \]  \hspace{1cm} \text{rectangular pulse}  

(5.9)
will considered in the study of the dynamic response.

For the problem at hand, only distributed forces will be considered, implying that in the forthcoming developments \( m_c(z,t) = 0 \).

5.4 Dynamic Response Control to Blast and Sonic Boom Loadings

Herein, a number of successive steps aiming to derive a solution to the dynamic response problem already developed in Chapter 4 will be presented. The virtual work is expressed as

\[
\delta W = \int_0^L \rho \delta v_o \, dz
\]  

(5.10)

The system of equations of motion expressed in matrix form (Eq. (4.18)) are rewritten here

\[
M' \ddot{q}(t) + H \dot{q}(t) + K q(t) = \dot{Q}(t)
\]  

(5.11)

where

\( M' = M + E \)

In state-space form, Eq. (5.11) is written as:

\[
\dot{X}(t) = AX(t) + BQ(t)
\]  

(5.12a)

where

\[
A = \begin{bmatrix}
0 & I \\
-M^{-1}K - M^{-1}H & M^{-1}
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
\end{bmatrix}
\]  

(5.12b,c)

Applying Laplace transform to each side of Eq. (5.12a) yields

\[
\bar{X}(s) = (sI - A)^{-1} X(0) + (sI - A^{-1}) \bar{BQ}(s)
\]  

(5.13)

where the overbars denote the Laplace transforms of the counterpart quantities without the overbar, \( X(0) \) denotes the initial state vector while \( s \) denotes Laplace transform variable.

The state \( X(t) \) is obtained as the inverse Laplace transform of Eq. (5.13):

\[
X(t) = \mathcal{L}^{-1}[\Phi(s)X(0)] + \mathcal{L}^{-1}[\Phi(s)\bar{BQ}(s)]
\]  

(5.14a)

where

\[
\Phi(s) \equiv (sI - A)^{-1}
\]  

(5.14b)

and \( \mathcal{L}^{-1} \) denotes the inverse Laplace transform operation.

The top half of the state vector, Eq. (5.14a) can be used in conjunction with Eqs. (4.8) to obtain system responses corresponding to \( v_o, \theta, \) and \( \phi \), which are expressed, respectively, as

\[
v_o(z,t) = \sum_{i=1}^{N} \varphi_i(z)q_i(t), \quad \theta(z,t) = \sum_{i=1}^{2N} \varphi_i(z)q_i(t), \quad \phi(z,t) = \sum_{i=2N+1}^{3N} \varphi_i(z)q_i(t)
\]  

(5.15)

where \( \varphi_i \) are space-dependent trial functions and \( q_i \) are time-dependent generalized coordinates.
The following numerical illustrations concern the dynamic response of a cantilevered thin-walled beam incorporating bending-twist coupling, (Eq. (3.6)), as well as of beams featuring transverse-isotropy, (Eq. (3.9)), subjected to time-dependent external excitations. The displayed results correspond to the case of zero initial conditions and of $P_m = 500 \frac{\text{lb}}{L}$. In Figs. 5.10-5.14 the dimensionless deflection $\tilde{V}(=v_o/L)$ responses of the beam tip to a sonic boom pressure signature are displayed. Figure 5.10 reveals the efficiency of the tailoring technique to confine the increase of the transverse deflection. From this plot it appears that for $\theta = 90^\circ$, at which the maximum bending stiffness is reached (see Fig. 3.3), a minimum deflection throughout the positive and negative phases of the pulse and even in the free motion range (i.e. for $t > t_p$ when the wave has left the structure) is reached.

Figures 5.11 and 5.12 reveal the quantitative and qualitative differences in the response deflection due to a symmetric ($r=2$), asymmetric ($r=1.5$) and a degenerated ($r=1$) sonic boom pulse and due to the positive phase duration of the pulse, $t_p$, respectively. The trends as appearing from these graphs are similar to the ones displayed in Refs. 6 and 7 in the case of a flat panel.

In Fig. 5.13 the effects of the transverse shear flexibility of the material of the structure, measured in terms of $E\,G'$ is highlighted. Herein $E$ and $G'$ denote the Young’s modulus in the plane of isotropy and transverse shear modulus, respectively, where $E\,G'=0$ corresponds to the non-shearable (Bernoulli-Euler) beam model. Whereas during the positive and negative phases of the blast, transverse shear has an almost negligible effect on the deflection, in the free motion range its effect is rather strong and it becomes evident that the classical theory inadvertently underestimates the deflection. The same trend emerges also from Fig. 5.14.

In Figs. 5.15-5.16 the effects of a blast load on the response behavior are highlighted. Figure 5.15 which includes the negative phase duration of the blast shows that with the increase of the parameter $a'/t_p$ lower deflection amplitudes are emerging in the positive phase of the blast. Figure 5.16 highlights the effect of the ply-angle and of the free and constrained warping models on the dynamic response behavior. The results reveal that while the ply-angle plays a significant role in confining the deflection response, for high aspect ratio beams, the warping inhibition plays a negligible role. In Fig. 5.17, the time-history of the twist angle of the beam tip is recorded. This graph highlights the strong effect played by the tailoring technique towards reducing the twist. Figures 5.18 and 5.19 record the deflection response of the beam to a rectangular pulse. While in Fig. 5.18 the effect of the ply-angles is highlighted, in Fig. 5.19 the effect of transverse shear is displayed. The results in these graphs reveal again the great influence played by the ply-orientation and transverse shear on the transverse deflection response amplitude. The strong influence of transverse shear on deflection amplitude in the free motion range becomes evident also from Fig. 5.19. Figure 5.20 displays the transverse deflection response to a step pulse. In addition to the effect played by the ply-angle, that of the warping restraint emerges clearly from this plot. The results reveal that warping inhibition plays a stronger role in confining the increase of the deflection amplitudes at ply-angles resulting in lower deflection amplitudes.

Next, the following obtained results reveal the powerful role played by the dual control methodology mentioned earlier towards enhancing the dynamic response of thin-walled beam cantilevers to various overpressure signatures.

In Figs. 5.21 and 5.22 the effects of the dual control methodology on the dynamic response of the beam when subjected to a blast loading are highlighted. In these figures the case of a non-shearable beam free to warp is considered. The results in these plots reveal the strong beneficial influence played by the combined tailoring and adaptive technology resulting in a reduction of the deflections in both the forced and free motion ranges. They also reveal that implementation of a ply-angle yielding an increase of the bending stiffness
results not only in a dramatic reduction of the deflection in the free motion range, but also in a shift of the deflection peaks towards larger times.

A similar conclusion applies when the acceleration feedback control strategy is implemented (see Fig. 5.22). One should remark that within the velocity feedback control methodology, the closed-loop eigenvalues are complex-valued quantities, and as a result, damping is generated. This explains why in the free motion range, for the activated structure and in contrast to the non-activated one, the response amplitudes die out as time unfolds. Incorporation of both velocity and acceleration feedback control strategies results in both a reduction of response amplitude and the shift of their maxima towards larger times.

In Figs. 5.23 and 5.24 the open and closed-loop dynamic response to a rectangular pressure pulse are considered. The results reveal that with the increase of the velocity feedback gain, a dramatic reduction of the deflection and twist throughout the forced and free motion ranges, and specially within the latter one, can be achieved.

The effect of the warping restraint upon the open and closed-loop dynamic response of the beam subjected to a rectangular pressure pulse is addressed in Fig. 5.25. The results reveal that in the forced motion range, although the warping restraint yields a reduction of the deflection amplitudes as compared with their free warping counterpart, the reduction is not so large as that experienced in the free motion regime. Furthermore, when the beam is activated, for the same feedback gain, the difference of the deflection amplitudes as experienced in the free and constrained warping beam models, are more modest in the free motion regime than in the forced motion regime.

In Figs. 5.26 and 5.27 the open and closed-loop dynamic response of cantilevers exposed to a sonic-boom signature are displayed. The results reveal again the powerful effect played by the velocity feedback control in reducing the dynamic deflection amplitude in the forced and free motion regimes.

Finally, Figure 5.28 depicts the dynamic response to a sinusoidal pressure pulse. The results from this plot reveal that implementation of the dual control methodology can play a powerful role in controlling the response in both the forced motion range where the tailoring technique plays a more stronger role, and in the free motion range, where the strain actuation technology becomes more efficient.

For the sake of implementation of LQR scheme incorporating boundary moment, the non-shearable beam model (Bernoulli-Euler model), Eq. (3.9m), (3.9n,o) and Eq. (3.10c) are used and rewritten here,

\[ a_{33} v_o^{'''} + b_1 \ddot{v}_o - (b_4 + b_{44}) \dddot{v}_o = p, \]  

(5.16)

with the associated BCs at \( z = 0 \)

\[ v_o = v_o' = 0 \]  

(5.17a,b)

and at \( z = L \)

\[ a_{33} v_o''' = -M_x^a, \quad v_o''' = 0 \]  

(5.18a,b)

The corresponding discretized equations of motion becomes

\[ [M^h + M^a] \ddot{q} + [K^h + K^a] q = [Q] - [F] u \]  

(5.19a)

where \( M^h, M^a, K^h \) and \( K^a \) are the mass and stiffness matrices of the host structure and actuator, respectively, \( Q \) is a generalized force vector, while \( F \) denotes piezoeactuator influence vector.
The corresponding state-space representation of Eq. (5.19a) is

\[ \dot{X}(t) = AX(t) + BU(t) + Wu \]  

where

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \quad \text{and} \quad W = \begin{bmatrix} 0 \\ -M^{-1}F \end{bmatrix}
\]

In addition, \( M = M^h + M^a \), \( K = K^h + K^a \)

### 5.5 Influence of Locations and Size of Piezo-Patch Actuators and Sensors

Until now, the control based on the piezoactuators spread over the entire beam span, polarized in the thickness direction under an applied electric field \( \xi_3 \) (out-of-phase voltage), was considered.

However, for evident reasons, piezo-patch actuators are becoming increasingly popular.

To illustrate the influence of location and size of a single piezoactuator patch, we consider here the equations of motion for the non-shearable beam model. For this case, the pertinent equation of motion is considered again here

\[ -a_{33}v_0'' + p_y - M^h_z'' = b_1\dot{v}_0 - (b_4 + b_{44})\ddot{v}_0 \]

with the associated BCs at \( z = 0 \)

\[ v_0 = v_0' = 0 \]  

and at \( z = L \)

\[ a_{33}v_0'' = 0 \quad , \quad v_0''' = 0 \]  

Symmetric piezoactuator pairs mounted on the thin-walled beam surface (Fig. 5.29) are considered. It should be remarked that the flexural stiffness term \( a_{33} \) includes contributions from both the host structure denoted as \( a^h_{33} \), and from piezoactuator patch, denoted as \( a^a_{33} \). The latter one is expressed as
\[ a_{i3} = \mathbf{f} \left[ \left( A_{i1} - \frac{A_{j3}}{A_{11}} \right) y^2 + D_{i1} \left( \frac{dx}{ds} \right)^2 \right] R(s,z) ds \]  

(5.26a)

Similarly, the mass terms can be cast in the form

\[ b_4 = b_4^h + b_4^a, \quad b_{14} = b_{14}^h + b_{14}^a \]  

(5.26b)

where

\[ (b_4^h, b_4^a) = \mathbf{f} m_o (1, y^2) R(s,z) ds, \quad b_{14}^a = \mathbf{f} m_2 \left( \frac{dx}{ds} \right)^2 R(s,z) ds \]  

(5.26c)

\[ m_o \text{ and } m_2 \text{ being associated with the piezoactuator patch, while } R(s,z) \text{ is a space function defining the distribution in the } s \text{ and } z \text{ directions of the piezoactuator.} \]

The spatial dependence of the piezoactuator patch for the problem at hand is included explicitly through the piezoelectrically induced moment term \( M_x^e \) in Eq. (2.5.5b), as

\[ M_x^e = \mathbf{f} \sum_{k=1}^{n} \zeta_3^{(k)} t^e \varepsilon_3^{(k)} R(s,z) \left[ y \left( 1 - \frac{A_{12}}{A_{11}} \right) + \frac{dx}{ds} \left( \frac{B_{12}}{A_{11}} - n_k^a \right) \right] ds \]

\[ = c R(z) V(t) \]

\[ = c V(t) \left[ H(z - z_1) - H(z - z_2) \right] \]  

(5.27a)

where the Heaviside distribution \( H(\cdot) \) locates the distributed moment between the points \( z_1 \) and \( z_2 \) along the beam span.

Using the identity

\[ \int_{z_1}^{z_2} \delta^j (z - z') h(z) dz = (-1)^j h^j (z') \]  

(5.27b)

which is valid when \( z' \in [z_1, z_2] \), and that \( H''(z) = \delta'(z) \) where \( \delta(\cdot) \) is the spatial Dirac’s distribution, the piezoelectrically induced moment term in the equation of motion Eq. (5.23) is expressed as

\[ M_x^{e'n} = c V(t) \left[ \delta'(z - z_2) - \delta'(z - z_1) \right] = u \left[ \delta'(z - z_2) - \delta'(z - z_1) \right] \]

(5.28)

Therefore, the corresponding piezoactuator influence vector \( F_i \) is expressed as

\[ F_i = \left[ \phi_i'(z_2) - \phi_i'(z_1) \right] \]

(5.29)

The corresponding discretized equations of motion are expressed as

\[ [M^h + M^e] \ddot{q} + [K^h + K^e] q = Q - [F] u \]  

(5.30a)

where

\[ M^h = \int_0^L \left[ \begin{array}{c} b_4^h \phi_i^T - (b_4^h + b_{14}^h) \phi_i \phi_i^T \end{array} \right] dz \]  

(5.30b)
Based on the preceding analysis, numerical simulations were carried out to examine the effects of location and size of piezoactuator patch on the control efficiency. The structure to be controlled according to the LQR scheme is subjected to a blast loading, Eq. (5.7) characterized by \( P_m = 500 \text{lb}, \quad d|t_p = 80 \) and \( t_p = 0.03 \text{ sec} \). In Fig. 5.30, the variation of the first mode of the damping factor versus location of piezoactuator, expressed in terms of the parameter \( d \) (pointing the left end of PZT patch from the clamped end) is displayed. From the figure, one can conclude that when the actuator is located closer to the clamped end of the beam, \( \zeta_1 \) increases, whereas \( \zeta_2 \) decreases as the actuator is located towards the free end of beam.

In the preceding numerical examples, no limitations upon the control input voltage \( V(t) \) has been imposed. However, in realistic problems the depoling voltage of 250 volts in current actuator configuration should be considered as a limitation of piezoactuators. In Fig. 5.33, solid lines correspond to the limited voltage, \( V_{\text{max}} = \pm 250 \text{ volts} \), while dotted lines correspond to those in which no limitation was considered. From the figure, in spite of the existence of the limitation on \( V(t) \), except for several fluctuations of the deflection at the beam tip when subjected to a sonic boom, vibration attenuation appears rather satisfactory.

The sensor output voltage is also determined as a function of the sensor position along the beam span when subjected to a triangular pressure pulse. From Fig. 5.34, the sensor voltage is greater in the case of the sensor located near the clamped end, as compared to the case of the sensor located near the free end. This trend can be explained by remarking that the bending moment is larger at the beam root, and as a result, also the strains are larger at that section. In other words, the sensor voltage is greatest for the sensor located at that section where the bending moment is largest. A similar result has been obtained also in Ref. 71.

Furthermore, as far as power consumption is concerned, locations of piezoactuator have a large influence on the power requirements during the suppression of the beam vibration. It is found that as the piezoactuator is located towards the root of the beam (i.e. when control effectiveness increases), the power consumption of the piezoactuator increases as well. Fig. 5.35 displays the time history of the power for two cases of the patch location. The maximum power corresponding to the piezoactuator location...
\( z'' = (0.4 - 0.5)L \) is approximately 33\% of the peak value corresponding to the location \( z'' = (0.1 - 0.2)L \).

Figure 5.36 displays the variation of the control force required during the suppression of the beam subjected to the blast load \( (t_p = 0.03 \text{sec}) \). It is obvious that as the intensity of blast load is decreases, the control force decreases as well. It is also observed that the maximum magnitude of the control force was \( 88.8 [\text{lb}] \) for the simulated loading conditions and beam configurations \( (E/G = 0, AR = 6) \).

Finally, in order to assess the effect played by the inclusion of the time-dependent external excitation in the optimal performance index, several test cases are considered. In Fig. 5.37 the response of a clamped thin-walled beam to a blast pulse is displayed. Whereas the control appears to be extremely efficient in damping out vibrations, the difference in the response when the transient load is included or discarded in the optimal performance index is negligible in the free motion regime (i.e. beyond the instant when the pressure pulse has already swept the structure). However, in the forced motion regime, the predictions based on the discard of the pressure pulse in the optimal performance index tend to overestimate the response quantities obtained via their inclusion, by about 10\%. One can conclude, consequently, that for a reliable design and control of high performance structures, the time-dependent pulses have always to be included in the performance index. Similar conclusions can also be reached from Figs. 5.38 and 5.39 where the cases of a rectangular and sinusoidal pressure pulses have been considered.