WEB CRIPPLING OF HOT-ROLLED BEAMS
AT STIFFENED-SEAT CONNECTIONS

by

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(ABSTRACT)

The current end web crippling design equations are based on a collapse mechanism developed for interior web crippling of short-span plate girders. Yield-line theory and the mechanism solution were used to derive an expression for the web crippling capacity of a plate girder subjected to an interior patch load. The general form of this equation was then calibrated using available data to obtain an equation for end web crippling as well.

This study examines the origin of the equation and identifies inconsistencies which result from using the same assumptions for both types of web crippling. A new collapse mechanism, based on modified assumptions, is used to derive a new equation that applies specifically to end web crippling of hot-rolled sections. In addition, three full-scale tests were conducted on steel-concrete composite beams to verify the accuracy of the proposed equation. Finally, the current web crippling equations and the proposed equation are compared for all available end web crippling data, and conclusions and recommendations are made concerning the validity of the proposed end web crippling equation.
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1.1 Introduction

When steel beams or girders frame into the web of a column, a seat connection, Figure 1.1, is often preferred over a double-angle connection. During erection at a double-angle connection, the beam must remain suspended while the angles are bolted or welded to the web of the column. The seat connection is safer and more efficient since most of the connection can be fabricated in the shop and the beam can rest on the seat while the connection is completed.

Seated connections introduce failure modes that are significantly different than those in other beam-to-column connections. In webs with small depth-to-thickness ratios, the stresses at the toe of the fillet can cause in-plane yielding of the web material. This web yielding can cause the connection to fail immediately or it can lead to web crippling. In webs with larger depth-to-thickness ratios, the web will cripple out-of-plane before it can yield in-plane (Elgaaly, 1983).

Web yielding, erroneously called web crippling, was considered as a potential mode of failure prior to the 1986 American Institute of Steel Construction (AISC) Specification (AISC, 1986) using equations 1.10-8 and 1.10-9 found in the 1978 AISC Specification (AISC, 1978). Equation 1.10-8 applied to interior loads, and was written:
Figure 1.1 Seat connection in the web of a column.
\[
\frac{R}{t_w(N+2k)} \leq 0.75F_y.
\] (1.1)

In this equation, \(t_w\) and \(k\) represent the section properties of the beam for the thickness of the web and the fillet distance, respectively. \(N\) is the bearing length of the seat. \(R\) represents the maximum reaction at the seat end, and \(F_y\) is the yield stress of the steel in the web of the section. Equation 1.10-9 took a similar form, but governed end reactions, such as those studied here:

\[
\frac{R}{t_w(N+k)} \leq 0.75F_y.
\] (1.2)

In addition to these two equations, there was also a check for stability of plate girder webs. Equation 1.10-10 limited the stress at the compression flange of a plate girder restrained against rotation to:

\[
\left[ 5.5 + \frac{4}{(a/h)^2} \right] \frac{10,000}{(h/t_w)^2}, \text{Ksi},
\] (1.3)

where \(a\) represents the panel width and \(h\) represents the height of the section. When the flange was not restrained against rotation, equation 1.10-11 limited this stress to:

\[
\left[ 2 + \frac{4}{(a/h)^2} \right] \frac{10,000}{(h/t_w)^2}, \text{Ksi}.
\] (1.4)

These two equations were based on research by Basler (1961), but they only applied to webs of plate girders, not to the webs of rolled sections.

A new set of equations for web crippling first appeared in the 1986 AISC LRFD Specification (AISC, 1986). The equations previously used for web crippling were retained under a new mode of failure called web yielding. This is a more accurate description for these equations, since they limit the maximum stress in the web relative to
the yield stress of the web. As defined in the 1986 AISC LRFD Specification (AISC, 1986), equation K1-2 limits the nominal interior patch load, \( R_n \), to:

\[
R_n = (5k + N)F_{yw}t_w.
\]  

Equation K1-3 limits the nominal end reaction to:

\[
R_n = (2.5k + N)F_{yw}t_w.
\]  

The strength reduction factor for web yielding is unity.

The new web crippling equations were first developed using yield-line theory by Roberts and Rockey (1979), and limit the applied interior load or end reaction based on the stability of the web rather than the stress being accepted. Equation K1-4 of the 1986 AISC LRFD Specification (AISC, 1986), which applies to interior patch loads, is written:

\[
R_n = 135t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{F_{yw} \frac{t_f}{t_w}},
\]  

where \( d \) represents the depth of the section, and \( t_f \) represents the thickness of the flange.

Equation K1-5 governs end reactions:

\[
R_n = 68t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{F_{yw} \frac{t_f}{t_w}}.
\]

The strength reduction factor for the web crippling limit state is equal to 0.75. While these new equations are more accurately derived to predict web crippling, it can easily be shown that much larger seat connections are required under the new Specifications than were previously required.

The 1989 AISC ASD Specification (AISC, 1989) continued to include both the web yielding and web crippling failure modes as separate design criteria just as the 1986 AISC LRFD Specification (AISC, 1986) had done earlier. The web yielding equations, K1-2 and K1-3 take a different form, but retain the basic concept of limiting the stress at
the toe of the fillet to a percentage of the yield stress of the web. Equations K1-2 and K1-3 are given in the 1989 AISC ASD Specification (AISC, 1989) as follows:

\[
\frac{R}{t_w (N + 5k)} \leq 0.66 F_y, \tag{1.9}
\]

and,

\[
\frac{R}{t_w (N + 2\frac{1}{2}k)} \leq 0.66 F_y. \tag{1.10}
\]

The two web crippling equations take the same form as the web crippling equations above, equation K1-4 is:

\[
R = 67.5 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_y}{t_w}}, \tag{1.11}
\]

and equation K1-5 is:

\[
R = 34 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_y}{t_w}}. \tag{1.12}
\]

Finally, a new, bilinear set of equations for end web crippling based on research conducted by Elgaaly (1991) has been proposed for the 1993 AISC LRFD Specification (AISC, 1993). Equation K1-5a is identical to equation (1.8), above, and applies to connections in which the bearing length, N, is less than or equal to 0.2d. The new equation, K1-5b, is written:

\[
R_n = 68 t_w^2 \left[ 1 + \left( 4 \frac{N}{d} - 0.2 \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_{yw}}{t_w}} t_f. \tag{1.13}
\]

This equation applies to connections in which the bearing length, N, is greater than 0.2d.

Unless stated otherwise, equations K1-4 and K1-5 will refer to those found in the 1986 AISC LRFD Specification (AISC, 1986).
1.2 Background

Since web yielding and/or web crippling can occur either under the point of application of a patch load or over the support, both situations have been examined over the years. Since the moment strength for these specimens must be rather large, short-span plate girders, such as that shown in Figure 1.2, are frequently used for these tests. The bearing stiffeners are placed such that the web does not cripple over the supports before crippling at the midspan.

![Figure 1.2 Typical test for web crippling at the point of application of the load.](image)

Far fewer studies such as the one shown in Figure 1.3, designed to cripple the web over the support, have been conducted. Just as the stiffeners in Figure 1.2 must be placed over the supports, the stiffeners for these tests must be placed at the point of application of the load. Some of these tests have been conducted on rolled sections of at least moderately long spans, but because the research in this area has been limited recently to verification, the design equations for end web crippling are based solely on the mechanism for the midspan tests.
The first experimental studies on web crippling were conducted in the early- to mid-1930's. Ketchum and Draffin (1932) were the first to consider this problem when they tested 66 rolled sections for failure by web crippling over the supports. They concluded that the failures were caused by buckling of the web over the support, and that the maximum allowable stress in the web, \( f \), was most closely approximated by the Carnegie formula:

\[
f = \frac{R}{t_w(N + d/4)}.
\]  

(1.14)

As part of their investigation of web buckling, Lyse and Godfrey (1935) tested six rolled beams specifically for web crippling. Three of the tests were designed to fail over the support, while the other three were designed to fail at the point of application of the load. In all the tests, the webs continued to take load beyond the yield point and eventually failed by web crippling. The authors suggested a design formula for checking the stress, \( f \), at the toe of the fillet:

\[
f = \frac{P}{t_w(N + 2k)},
\]  

(1.15)

where \( P \) is the load at the point of interest.

Most of the studies conducted since the above two have been of midspan web crippling of welded plate girders; however, there are two papers that require specific
mention. Roberts and Rockey (1979) summarized the results of 89 such tests conducted by various other authors and analyzed the design formulas offered to predict the collapse loads due to web crippling under the point of application of the load. Most of these tests were of short-span plate girders, although some were of plate girders as long as approximately 32 ft. Of the most significance, however, was the authors' introduction of their own web crippling formula using yield-line theory, which became the basis for the web crippling equations, K1-4 and K1-5, discussed earlier.

Roberts (1981) then conducted his own series of 26 tests to verify the mechanism solution. He reported very good correlation between the experimental web crippling loads and those predicted by his solution. Finally, he offered a mechanism solution for web yielding and a reduction factor for coexistent bending stresses. The web yielding solution will be discussed later, but the coexistent bending stresses will not be covered.

Roeder and Dailey (1989) conducted a series of seat connection tests on six different rolled sections with three tests being conducted on each section. The beams spanned approximately 8 ft. and were loaded using a single point load placed near the quarter-point closest to the seat. Roeder and Dailey used a bearing plate to distribute the point load and a web stiffener to prevent web crippling beneath the load. In order, each beam was tested using a top angle (as required by AISC for design), an angle in the optional side web location, and with a pair of angles that prevented lateral movement at the top flange but accepted no load. In the first two tests in each series, the web was loaded to "failure" and then unloaded. With the double-angle detail in place, the web was again allowed to fail by web crippling.

In the first two series, the beams yielded under the point load before web crippling could occur. The unstiffened seat used in the final series failed long before web crippling could occur. Based on the three series of tests which failed by web crippling, Roeder and
Dailey concluded that web crippling is a failure mode for some sections even though they suggested that the AISC web crippling equation, K1-5, which is based loosely on the collapse mechanism by Roberts (1981), is too conservative. They also noted that the top or side angle contributed significantly to the strength of the web and emphasized the importance of using one of these details for design.

Elgaaly and Salkar (1990) investigated both midspan and support web crippling using short-span plate girders. Thirteen of the tests were for web crippling over the supports. Based on their results, Elgaaly and Salkar also concluded that the AISC web crippling equations, K1-4 and K1-5, are too conservative, so they offered modifications. They suggested that the equations be multiplied by $\sqrt{t_r / t_w}$ and that the entire equation be modified by a linear function of N/d. Elgaaly (1991) summarized the results of these thirteen tests and included the results of fourteen additional end web crippling tests. Finally, Elgaaly (1991a) made further modifications to the web crippling equation similar to those changes offered by Elgaaly and Salkar (1990).

1.3 Scope of Research

This study was initiated to begin an investigation into the AISC web crippling equations, particularly the one that applies to end reactions, because of recently raised questions (Roeder and Dailey, 1989; Elgaaly and Salkar, 1990; Elgaaly, 1991a). First, the basis for the existing web crippling equations (Roberts, 1981) was analyzed. If discrepancies were to be identified, a strong argument could be made that the equation for end web crippling needs to be changed. In the experimental portion of this project, three tests of stiffened-seat connections were conducted to failure by web crippling. By testing more realistic spans and an actual stiffened-seat connection detail without the top or side angle, it was hoped that more accurate conclusions could be reached regarding web
crippling independent of the contributions of the angle details. Finally, the results of tests conducted by other authors were compared to the predictions of the existing 1986 AISC LRFD design equations (AISC, 1986) for web yielding (K1-3) and web crippling (K1-5), the proposed 1993 AISC LRFD (AISC, 1993) end web crippling equations (K1-5a and K1-5b), and a proposed web crippling equation.
CHAPTER II

PREVIOUS THEORETICAL WORK AND REDERIVATION

2.1 Yield-Line Theory

Yield-line theory has been a limit-state design method for reinforced concrete slabs since its introduction by Ingerslev in the early 1920's and its further development by Johansen in the 1960's (Ahart, 1986). While it is still used primarily in slab design, yield-line theory is applicable to any material that can be modeled as perfectly elastic-plastic. This method for calculating the capacities is simple and straight-forward with emphasis on locating the regions of high stress (Barker, 1991).

Simply stated, yield-line theory assumes that the material being stressed will continue to accept load until it reaches its flexural capacity, when the material yields, forming a plastic hinge, or yield line. The loads redistribute, creating more yield lines, until a yield-line pattern capable of forming a collapse mechanism develops. By properly predicting the yield-line pattern, the sum of the flexural capacities along each of the yield lines can be used to determine the collapse load. However, because yield-line theory is an upper-bound solution method, the choice of the yield-line pattern is crucial. The collapse load calculated from an assumed yield-line pattern will either be correct, if the proper pattern is chosen, or it will be unconservative. There are two basic methods for calculating the collapse load, the equilibrium method and the kinematic method. The equilibrium method is not used in this application; therefore, it will not be discussed here.
The mechanism or kinematic method will be used later in this chapter. In the kinematic method, a yield-line pattern is selected and a virtual displacement, \( \delta \), is introduced into the system. The external work is equal to the work done by the loads moving through the virtual displacement. The internal work is equal to the work done by the slabs along the yield lines as they rotate through an angle, which can be determined by continuity. The principle of virtual work states that the total external work in the system must equal the total internal work in the system, so they may be set equal to each other. As the only unknown in the system, the collapse load can be calculated directly (MacGregor, 1988).

2.2 Web Yielding

As discussed in the Introduction, the equations for web yielding, K1-2 and K1-3, were taken directly from the main web crippling equations, 1.10-8 and 1.10-9, of the 1978 Specification (AISC, 1978). These equations limit the in-plane stress at the toe of the fillet to a value below the yield stress of the web material by assuming that the load is uniformly distributed at the toe of the fillet over a length equal to the bearing length plus a distance related to \( k \), the fillet distance. This added distance is found by projecting an imaginary boundary at a 2½:1 slope from the edge of the bearing up through the fillet to the bottom of the web. As shown in Figure 2.1, the length over which the load is distributed for an end reaction is equal to \((N + 2½k)\). For an interior load, this length is equal to \((N + 5k)\), since there is a stress boundary on either side of the bearing.

Roberts (1981) recognized the effect that an increasingly thick web would have on his web crippling formula. As the web thickness increases, the web's flexural capacity increases by a factor of about 1.5. Taken to the extreme, the web will yield in-plane, he reasoned, before crippling; so he briefly mentioned a mechanism solution for web
yielding using yield-line theory. From the yield-line pattern in Figure 2.2 and the principles discussed in the previous section, the web yielding collapse load can be expressed as:

\[ P_n = 4\sqrt{M_t F_{yw} t_w} + F_{yw} t_w N, \]  

(2.1)

where \( M_t \) represents the yield moment in the flange. However, because this formula is somewhat cumbersome, and because the existing equations have been in place so long, it is unlikely that this formula will gain widespread acceptance.
2.3 Web Crippling at an Interior Patch Load

The derivation of Roberts' (1981) web crippling formula is given below. The model used is a plate girder resisting an interior patch load, which is the case defined under equation K1-4. Following the derivation is the calibration work done to arrive at the equation's final form.

2.3.1 Derivation of Equation K1-4

The assumed collapse mechanism is shown in Figure 2.3. "Dimensions $\alpha$ and $\beta$ define the position of the assumed yield lines in the web and plastic hinges in the flange, and $\theta$ defines the deformation of the web just before collapse (Roberts, 1981)." Under the virtual displacement, $\delta$, at the load point, the rotation of the flanges is $\delta / \beta$ and the rotations of the web are $\delta / (2\alpha \cos \theta)$ and $\delta / (\alpha \cos \theta)$ at the extreme and central yield lines, respectively.

![Diagram of web crippling collapse mechanism at an interior patch load](image)

Figure 2.3 Roberts' (1981) web crippling collapse mechanism at an interior patch load.

The external and internal work are:

$$W_e = P_n \delta,$$

$$W_i = 4 \left[ M_r \left( \frac{\delta}{\beta} \right) \right] + 2 \left[ 4 \left( m_w \beta \left( \frac{\delta}{2\alpha \cos \theta} \right) \right) \right] + 4 \left[ m_w N \left( \frac{\delta}{2\alpha \cos \theta} \right) \right]$$

(2.2)
\[-4\left(\frac{m_w \eta}{2\alpha \cos \theta}\right)\delta\left(\frac{2\delta}{\alpha \cos \theta}\right), \quad (2.3)\]

\[
\frac{4M_f \delta}{\beta} + \frac{4m_w \beta \delta}{\alpha \cos \theta} + \frac{2m_w N \delta}{\alpha \cos \theta} - \frac{2m_w \eta \delta}{\alpha \cos \theta}.
\quad (2.4)
\]

where \( \alpha, \beta, \delta, \) and \( \theta \) are defined in Figure 2.3, \( \eta \) is the length of the web that has yielded below the load, and \( m_w \) represents the yield moment in the web per unit length. In the expression for the internal work, the first two terms represent the work done by the flange at the hinges and by the web along the diagonal yield lines, respectively. The third term represents the work done by the web directly beneath the load. The final term accounts for the portion of the web beneath the load that has already yielded and can no longer resist the load. Equating the external and internal work expressions and solving for \( P_n \) gives:

\[
P_n = \frac{4M_f}{\beta} + \frac{4m_w \beta}{\alpha \cos \theta} + \frac{2m_w N}{\alpha \cos \theta} - \frac{2m_w \eta}{\alpha \cos \theta}.
\quad (2.5)
\]

In order to obtain the true collapse load, the least upper-bound must be found. The collapse load, \( P_n \), may be minimized by setting its derivative with respect to \( \beta \) equal to zero.

\[
\frac{dP_n}{d\beta} = -\frac{4M_f}{\beta^2} + \frac{4m_w}{\alpha \cos \theta} = 0.
\quad (2.6)
\]

Solving for \( \beta \) gives:

\[
\beta^2 = \frac{M_f \alpha \cos \theta}{m_w}, \quad (2.7)
\]

\[
\beta = \sqrt{\frac{M_f \alpha \cos \theta}{m_w}}. \quad (2.8)
\]
Assuming that the moment in the flange varies linearly from the extreme plastic hinge to the inner plastic hinge, and based on the derivation of the stiffness matrix coefficients, the deflection of the flange, $\Delta_f$, can be shown to be:

$$\Delta_f = \frac{M_f \beta^2}{6EI_f}, \quad (2.9)$$

where $E$ is Young's modulus, and $I_f$ is the moment of inertia of the flange alone. From simple geometry, the deflection of the web, $\Delta_w$, is:

$$\Delta_w = 2\alpha(1 - \sin \theta). \quad (2.10)$$

To preserve compatibility, these must be equal.

$$\frac{M_f \beta^2}{6EI_f} = 2\alpha(1 - \sin \theta). \quad (2.11)$$

$$\frac{M_f^2 \alpha \cos \theta}{6EI_f m_w} = 2\alpha(1 - \sin \theta). \quad (2.12)$$

Rearranging, the equation can take the form:

$$\frac{\cos \theta}{1 - \sin \theta} = \frac{12EI_f m_w}{M_f} = \frac{4EF_{yw} t_w^2}{F_{yr} b_r t_r} = H. \quad (2.13)$$

Using the trigonometric identity and the fact that $H >> 1$, $\cos \theta$ can be reduced to the form:

$$\cos \theta = \frac{2H}{1 + H^2} \approx \frac{2}{H}, \quad (2.14)$$

$$\cos \theta = \frac{2}{H} = \frac{2F_{yr}^2 b_r t_r}{4EF_{yw} t_w^2} = \frac{F_{yr}^2 b_r t_r}{2EF_{yw} t_w^2}. \quad (2.15)$$

The $\eta$-term can be eliminated from equation (2.5) because experimental data from tests of interior patch-loaded plate girders show that no significant in-plane web yielding exists below the load. Substituting equations (2.8) and (2.15) into equation (2.5) gives:
\[
P_n = 2\sqrt{2}t_w^2 \sqrt{\frac{E F_y t_f}{\alpha F_y}} \left[ 1 + \frac{E F_y t_w}{8\alpha F_y b_f^2} \left( \frac{t_w}{t_f} \right)^{15} \right] N.
\] (2.16)

Several assumptions must now be made in order to further reduce equation (2.16). First, the yield stresses in the web and the flange can be assumed to be equal. If they are not, the collapse load will decrease with an increase in the yield stress of the flange without a corresponding increase in the web yield stress. Second, Roberts (1981) found in his experimental tests that \( \alpha \), the vertical separation of the yield lines beneath the patch load, could be accurately defined as:

\[
\alpha = 25t_w.
\] (2.17)

These assumptions simplify equation (2.16) to:

\[
P_n = 0.56t_w^2 \sqrt{E F_y} \left( \frac{t_f}{t_w} \right) \left[ 1 + \lambda N \left( \frac{t_w}{t_f} \right)^{15} \right],
\] (2.18)

where:

\[
\lambda = \frac{E t_w}{8\alpha F_y b_f^2}.
\] (2.19)

Finally, since this formula approximates an upper-bound solution, Roberts (1981) reduced the coefficient to one-half, and simplified \( \lambda \) to:

\[
\lambda = \frac{3}{d}.
\] (2.20)

This is a conservative assumption consistent with the available data. Equation (2.18) now becomes:

\[
P_n = 0.5t_w^2 \sqrt{E F_y} \left( \frac{t_f}{t_w} \right) \left[ 1 + \frac{3N}{d} \left( \frac{t_w}{t_f} \right)^{15} \right].
\] (2.21)
Two special limitations of this derivation are important to note as regards this analysis. Roberts and Rockey (1979), with no specific explanation, restricted the bearing length, \( N \), to less than or equal to \( 2\beta \). Roberts (1981) also limited \( N \) to a maximum of 0.2d. The reason for the latter restriction is apparent, and it may be the underlying explanation for the former one as well. If the bearing length is too long, or perhaps too short with respect to the other dimensions, the portion of the flange between the inner plastic hinges will not remain flat. This would alter the assumed mechanism, perhaps creating an unanticipated yield-line pattern.

### 2.3.2 Calibration for Equation K1-4

From the basic derivation above, the results of 89 tests of interior loads were used to calibrate the formula to the web crippling equation found in the AISC Specifications. The coefficient preceding the 1986 AISC LRFD equation was found by assuming a reliability index of 2.5 and a strength reduction factor of 0.75. By including the \( \sqrt{E} \) term in the coefficient and removing the reduction factor, the coefficient 135 was obtained (Elgaaly, 1991a; Galambos, undated). This leaves the 1986 AISC LRFD interior web crippling equation, K1-4 (AISC, 1986), as:

\[
R_n = 135t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_r} \right)^{15} \right] \sqrt{\frac{F_{yw}}{t_w}} \frac{t_r}{t_w}. \tag{1.7}
\]

The 1989 AISC ASD equation was derived directly from the LRFD equation. By assuming an average load factor of 1.5 (LL = 3DL) and using the reduction factor of 0.75, the 1986 AISC LRFD coefficient was multiplied by one half (0.75/1.5 = 0.5) to obtain the 1989 AISC ASD coefficient of 67.5 (Elgaaly, 1991). The 1989 AISC ASD interior web crippling equation, K1-4, then, is written:
\[ R = 67.5 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_{yw}}{t_w}}. \] (1.11)

### 2.4 Web Crippling at an End Reaction

The development of the 1986 AISC LRFD web crippling equation for an end reaction, K1-5 (AISC, 1986), is given below, from the origin to the final calibration. Following this brief discussion, the flaws with this process are examined and a proposed web crippling equation is derived using new initial assumptions.

#### 2.4.1 Equation K1-5

There is no specific derivation for end web crippling in the literature, but equation K1-5 (AISC, 1986) was obtained indirectly through Roberts' (1981) derivation for interior web crippling. Twenty-six of the end web crippling tests from Ketchum and Draffin (1932) and Lyse and Godfrey's (1935) three end web crippling tests were reportedly used for calibration (Elgaaly, 1991a; Galambos, undated), but the coefficients are basically just half those for the interior equations. The 1986 AISC LRFD and 1989 AISC ASD end web crippling equations are:

\[ R_n = 68 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_{yw}}{t_w}}, \] (1.8)

and,

\[ R = 34 t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_{yw}}{t_w}}, \] (1.12)

respectively.
2.4.2 Rederivation for End Web Crippling

The main problem with dividing equation K1-4 by two to obtain equation K1-5 is apparent from Figure 2.4. When the halving of equation K1-4 is traced back to the original collapse mechanism of the yield-line theory, it can easily be seen that the first two terms of the internal work equation (2.4) are correctly halved, since they represent the work done along the diagonal yield lines and the hinges in the flanges. However, the work done beneath the patch load, as represented by the third and fourth terms of the internal work equation, should not be reduced because the full bearing width is able to resist the load.

![Diagram](image)

Figure 2.4 Effect of halving interior web crippling equation.

A secondary problem is that Roberts' (1981) derivation is modeled using a plate girder rather than a rolled section. This creates a minor discrepancy when considering web crippling for rolled sections, particularly concerning the load path through the fillet region. The web-flange interface is generally much stronger in a rolled section than in a
plate girder because of the fillet. This factor should somehow be taken into account in the design equation.

Finally, it must be recognized that most of the data available to calibrate equation K1-5 is incomplete, or the tests were conducted without proper care being given to the test configuration. Also, the majority of the tests, which were conducted by Ketchum and Draffin (1932), were of light I-beams, not the standard rolled sections commonly used for construction today. Such incomplete and perhaps misleading data should not be used as the sole basis for a design equation.

The collapse mechanism for end web crippling of rolled sections is given in Figure 2.5. This solution also uses yield-line theory and the principle of virtual work, and is based on the derivation by Roberts (1981) given previously, so it will not be included in its entirety here. For the complete rederivation see Appendix A.

![Yield Lines](image)

Figure 2.5 Collapse mechanism for end web crippling.

The expressions for external and internal work are:

\[ W_e = R_n \delta, \]  

(2.22)
\[ W_i = 2 \left[ M_f \left( \frac{\delta}{\beta} \right) \right] + 4 \left[ (m_w \beta) \left( \frac{\delta}{2\alpha \cos \theta} \right) \right] + 2 \left[ (m_w c) \left( \frac{\delta}{2\alpha \cos \theta} \right) \right] + 2 \left[ (m_w \eta) \left( \frac{\delta}{2\alpha \cos \theta} \right) \right], \]

\[ (2.23) \]

\[ = \frac{2M_f \delta}{\beta} + \frac{2m_w \beta \delta}{\alpha \cos \theta} + \frac{2m_w c \delta}{\alpha \cos \theta} - \frac{2m_w \eta \delta}{\alpha \cos \theta}, \]

\[ (2.24) \]

where,

\[ c = N + k. \]

\[ (2.25) \]

This relationship is similar to the one present in the web yielding equations, and represents the same type of stress boundary. Setting the work equations equal and solving for the collapse load gives:

\[ R_n = \frac{2M_f}{\beta} + \frac{2m_w \beta}{\alpha \cos \theta} + \frac{2m_w c}{\alpha \cos \theta} - \frac{2m_w \eta}{\alpha \cos \theta}. \]

\[ (2.26) \]

Reducing this formula to its simplest, unfactored form leaves:

\[ R_n = 24t_w^2 \left[ \sqrt{F_y \frac{t_f}{t_w} + 48 \left( \frac{c}{b_f} \right) \left( \frac{t_w}{t_f} \right)} \right]. \]

\[ (2.27) \]

In order to evaluate the validity of the proposed end web crippling equation for hot-rolled sections, three full-scale tests were performed. The test configurations are described in Chapter III, Test Details.
CHAPTER III

TEST DETAILS

Three full-scale tests of end web crippling were conducted in the Structures and Materials Research Laboratory at Virginia Polytechnic Institute and State University. The details concerning the design, set-up, and loading procedure for these tests, designated SC-i, are given below. SC indicates that a seated connection was used, and i represents the test number, 1 through 3. SC-2 and SC-3 were tests conducted on the opposite ends of the same beam. For a complete list of all test details, see Appendix B.

3.1 Configuration Selection

The choice of the test configuration can be important to the quality and applicability of the results obtained. The three major parameters for these tests were the beam cross-section, the span, and the initial load points. In each case, every effort was made to simulate typical design situations in the selection of each parameter.

3.1.1 Beam Cross-Section Selection

This experimental program was initiated over a concern for the web crippling strength of beams at seated connections in steel-frame buildings. Therefore, beams of 14 and 16 in. nominal depth were used. These are depths most often used as floor beams, and are also in the range used in other test series (Roeder and Dailey, 1989; Elgaaly, 1991).
The actual cross-section was selected based on its vulnerability to web crippling. This vulnerability has been shown to be closely related to its depth-to-thickness ratio (Elgaaly, 1983). Since the actual depth changes very little within a given series (i.e., W16's), the web thickness was the dominant parameter used in selecting the cross-section. Web crippling is much more likely to occur in sections with thinner webs than in sections with thicker webs; it will also occur at much smaller loads. Therefore, the lightest cross-sections in each series were selected. SC-1 used a W16 × 26 section; SC-2 and SC-3 used a W14 × 22 section.

Also of importance to the selection of the beam cross-section is the nominal yield strength. For each test a nominal yield strength of 50 Ksi was specified, since this grade of steel is becoming more popular in commercial construction. The actual yield strengths for the webs in the vertical direction for the three tests were in the range of 59 to 61 Ksi.

3.1.2 Span Selection

As with the cross-section of the beam, the span chosen was based loosely on the typical span used in construction for the particular cross-section. In general the span of the beam in feet is approximately equal to two times the depth of the section in inches, or L/24. This criterion was met for SC-1, which spanned 34 ft. from centerline-to-centerline of the supports. However, the moment capacity under the point loads controlled for both SC-2 and SC-3. SC-2 spanned 19 ft., and SC-3, which could not use the end portion of the span tested in SC-2, measured only 16 ft. between support centerlines.
3.1.3 Initial Load Point Selection

Each test used just two point loads to facilitate application and control of the load. These points were initially located near the quarter-points and were symmetric with respect to the midspan. The selection process for these initial load points was based on the end rotation under the two point loads.

The generic expression for the end rotation due to a uniform load was set equal to the end rotation due to two symmetric load points at an unknown distance from each end of the beam. By equating the total load on each system, the distance from the centerline of the support to the load point was found that gave the same end rotation for each system under the same total load. The expression for this distance is:

\[
x = \frac{L(3 - \sqrt{3})}{6} \approx 0.211L.
\]

where \(x\) is the distance from the centerline of the support to the nearest point load, and \(L\) is the span length.

The initial load points for SC-1 were located 7 ft., 2 in. from the centerline of each support. For SC-2, the distance, \(x\), was equal to 4 ft., 0 in. This method was not used for SC-3, however. In this test, the initial load points were located 3 ft., 0 in. from the support centerlines.

3.2 Test Set-Up

Each of the three tests used a concrete-steel composite beam, with similar materials and construction procedures. All three slabs were constructed using 81 in. wide, cold-formed steel deck; 2½ in. tall, 3/4 in. diameter shear studs; and 3500 psi nominal strength concrete with wire mesh for temperature and shrinkage control. The concrete slab began approximately 10 in. from the end of the steel beam to avoid
interference with the flanges of the steel column to which the connection was attached. Specially-shaped, 5 in. tall, cold-formed steel pour-stop was used around the perimeter of the slab as form-work and remained in place throughout testing.

The test end of the beam rested on a stiffened-seat connection attached to the web of a 9 ft. tall W14×145 column of grade A36 steel. The seat was intentionally over-designed to insure that web crippling controlled the failure of the connection. Figure 3.1 shows a schematic of the seat that was used for all three tests. The two bolt holes aligned with holes in the bottom flanges of the beams. Two 3/4 in. bolts were used in SC-1 to stabilize the beam during testing, a detail used in the field as protection during erection. Two, ½ in. bolts with washers were used in SC-2 and SC-3 due to edge distance limitations in the narrower flange of the W14×22 section. The locations of the holes provided a 3½ in. bearing length and ½ in. clear distance between the end of the beam and the web of the column.

![Diagram](image)

Figure 3.1 Seat detail used in all three full-scale tests.
Several important details were required to insure the safety and stability at each end of the test beam, particularly at the column and the connection. First, a 3/4 in. steel plate was welded to each end of the column. The bottom plate served as a base plate, and the top plate had a 3½ in., square pattern of 1-1/16 in. bolt holes. This detail enabled the column to connect to an existing steel load frame, which was bolted to the laboratory reaction floor. A hot-rolled angle, connected to the top of this load frame and to the bottom of an adjacent one, prevented the entire frame-column unit from rotating toward the beam. Similarly, wood blocking, tightly wedged between the back of the column and a hot-rolled channel section bolted to the back of the load frame, prevented the column from rotating out from under the test beam. A lateral brace mechanism was bolted to each side of the load frame and attached to an anchor bolt drilled into the concrete slab. This mechanism allows vertical movement, but prevents all movement in the direction perpendicular to the span. It, therefore, prevented instability similar to the commonly used top or side stabilizing angle, but accepted no load, allowing a true measurement of the web crippling load capacity of the beam web. Only one lateral brace mechanism was used in SC-1; two were used in SC-2 since experience from SC-1 suggested it might be required. However, space limitations prevented the use of more than one lateral brace mechanism in SC-3. Figure 3.2 shows the configuration of the test end of SC-1 at its initial load location.

The opposite end of the beam rested on two, 1 in. plates and a 4 in. roller assembly. This was all supported by an existing steel stand, which consisted of a deep beam with a central web stiffener spanning, perpendicular to the test span, between two stub columns bolted into the reaction floor. Two winch systems, one attached to each end of the stand and to the top flange of the test beam, prevented substantial rotation during installation and testing. And finally, since the entire test was symmetric about the
midspan of the test beam, the same load had to be resisted at each end. Therefore, the web of the test beam had to be prevented from crippling at the opposite end before failure at the desired end. This was accomplished using hot-rolled angles placed between the top and bottom fillets and welded to the web at the centerline of the roller.

The two point loads were applied using hydraulic rams attached to load frames, and located at the load points. The load frames were similar to the one used to hold the column in place, and permitted adjustment sufficient to apply a point load at almost any location along the span. The loads were applied through a 1 in. steel plate and a 1 in. neoprene pad centered on the load point and the web of the test beam. Figure 3.3 shows an end view of SC-1, which includes the load apparatus and the support at the opposite end.

3.3 Instrumentation

There were six parameters measured throughout the testing process on each beam: applied load, midspan deflection, support settlement at the opposite end, test-end rotation,
vertical strain along each side of the web at the centerline of the seat, and longitudinal strain at or near the beam midspan. The instruments measuring each of these parameters were connected to, and the data was recorded with, a PC-based data acquisition system.

A 500 Kips capacity load cell at each load point measured the applied loads. The load cells were located between the hydraulic ram and the load frame and were secured in position between two, 1 in. steel plates. The load being applied to the web at the seated connection was determined using statics for a simple beam.

Five displacement transducers were used to measure the midspan deflection, support settlement, and test-end rotation. One of these measured the midspan deflection. Two more measured the support settlement. A metal bar was attached across the support beam so that the ends extended equally over the sides of the support beam. One
transducer connected to each end of this bar, and the support settlement was assumed to be the average of these two measured deflections. The final two displacement transducers measured the test-end rotation. Metal bars were attached across the top and bottom flanges so that the ends extended beyond the flanges of the column. The transducers were attached horizontally to an independent column and connected to one of the bars. They measured the horizontal movement of the beam at the flanges, and, by dividing the difference in their deflections by their separation, the end rotation was calculated using small angle theory.

Strain gauges placed vertically at the centerline of the seat connection, along each face of the web measured the vertical strains in the web. The strain gauges assigned to channels 11 through 17 measured the strain on one face of the web. The gauge assigned to channel 14 was placed at the mid-depth of the section. The remaining gauges were evenly spaced above and below it; those in channels 11 through 13 above, and those in channels 15 through 17 below the mid-depth. In SC-1, the gauges were spaced at 2 in., center-to-center. Since SC-2 and SC-3 used shallower beams, the gauges had to be spaced at just 1-3/4 in. The strain gauges on the other face were aligned the same way, and were assigned to channels 21 through 27 with channel 24 assigned to the gauge at mid-depth. Figure 3.4 shows the locations of the strain gauges on one face of the test end for SC-1.

The longitudinal strains at or near the midspan were measured using an additional eight strain gauges, but they were only used as a guide to detect yielding in the beam during excessive loading. These strain gauges were placed at the midspan for SC-1 and SC-2, but since the end 3 ft. was eliminated from SC-3, the gauges were 1½ ft. from the beam's midspan for the final test. This was relatively unimportant, however, since yielding of the beam was also monitored with a load vs. midspan deflection graph.
3.4 Test Procedure

All channels were initially set equal to zero and recorded. Following the placing of the concrete slab, all of the measurements were again recorded, allowing the stress in the web and the deflection of the beam caused by the weight of the concrete to be included in the behavior of the system. The initial zero reading and the effect of the concrete on the beam was not available for test SC-3 since the slab was in place for test SC-2 before test SC-3 was instrumented.

Following the 28-day curing period for the concrete slab, the system was again prepared for testing. For each test, four basic load cycles were conducted, the target load for each cycle being based on the predicted web crippling capacity from the 1986 AISC LRFD Specification (AISC, 1986). The first cycle was to apply a pre-load of 10% of the predicted web crippling capacity of the beam before the load was released. The purpose of the pre-load was to verify the predicted stiffness of the system and to insure that the testing apparatus was functioning properly. The next two cycles were conducted to 50%
and 75% of the predicted web crippling capacity before the load was once again removed. The final cycle continued until the beam failed by web crippling at the seat end.

The load on each hydraulic ram was initially increased at an increment of 1 Kip. Whenever the target load for a given cycle was reached, the load was removed at approximately 5 Kip increments to zero ram load. Then, for the next load cycle, the load was reapplied at 2 Kips to the maximum recorded load from the previous cycle, and then the load increment was returned to 1 Kip. This procedure allowed sufficient time to detect and correct any problems that might arise during the loading process. Each of the three full-scale tests was conducted using the same test procedure. The results of these tests are given in Chapter IV, Test Results, Observations, and Discussion.
CHAPTER IV

TEST RESULTS, OBSERVATIONS, AND DISCUSSION

4.1 Test Results

The locations of the load points for the test configurations described in the previous chapter are given in Table 4.1. Also listed are the maximum end reactions that the web resisted at each load location. For each test, the end reaction given at the final load location is the failure load based on the weight of the concrete slab and the last observed set of applied loads. Included in this section is a discussion of the stain behavior of the webs under each load location for each of the three full-scale tests. For a review of the strain gauge positions, see the Section 3.2, Test Set-Up. For a review of the plots obtained from these three tests, see Appendix C.

4.1.1 Results for Test SC-1

At load location #1, the vertical strains measured at the seat displayed a definite pattern. The strains measured by the top two gauges on one face (channels 11 and 12) experienced increasingly greater tensile strains with increased load from the beginning of the load application to the largest end reaction at this load location, which was approximately 40 Kips (Figures C.11 and C.16). The strain gauge at channel 13 initially recorded increasing tensile strains, but this trend reversed at about 20 Kips and the strains began to approach compression (Figure C.21). The remaining strain gauges on that face
Table 4.1 Position and maximum end reaction for each load location.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Load Location</th>
<th>Distance between Load Point and Support Centerline</th>
<th>Maximum End Reaction (Kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-1</td>
<td>#1</td>
<td>7 ft., 2 in.</td>
<td>39.9</td>
</tr>
<tr>
<td></td>
<td>#2</td>
<td>5 ft., 2 in.</td>
<td>55.5</td>
</tr>
<tr>
<td></td>
<td>#3</td>
<td>4 ft., 2 in.</td>
<td>68.0</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>3 ft., 8 in.</td>
<td>68</td>
</tr>
<tr>
<td>SC-2</td>
<td>#1</td>
<td>4 ft., 0 in.</td>
<td>44.0</td>
</tr>
<tr>
<td></td>
<td>#2</td>
<td>3 ft., 0 in.</td>
<td>53</td>
</tr>
<tr>
<td>SC-3</td>
<td>#1</td>
<td>3 ft., 0 in.</td>
<td>48</td>
</tr>
</tbody>
</table>

all recorded compressive strains, which increased linearly with the load and increased from the mid-depth to the bottom flange.

The strain gauges on the other face recorded a much different behavior. Those for channels 21 through 26 recorded compressive strains of the same relative magnitude throughout the loading cycle. The gauge in channel 27, nearest the bottom flange, measured compressive strains 50 to 100% greater than those recorded by the six other gauges on that face (Figure C.41). When considered together, the strain patterns on both faces of the web indicate a possible reverse curvature in the web.

As shown by the load vs. midspan deflection graph of Figure 4.1, the beam began to yield before the predicted and actual web crippling collapse loads were reached. In this case, the load was gradually released in approximately 5 Kip increments, and the load points were moved toward the supports. This increased the loading capacity by decreasing the midspan moment under a given load. The load was reapplied at 2 Kip increments to the highest load previously recorded. The increment was then returned to 1 Kip. Similar load vs. midspan deflection plots were observed later in test SC-1 (Figures C.2 and C.3) and once again in test SC-2 (Figure C.48). This procedure was repeated each time that the web began to yield before web crippling could occur.
At load location #2, the strains measured on one face exhibited behavior similar to that from load location #1. The strains on the other face, however, displayed a much more linear behavior. The compressive strains at a given load level were least nearest the top flange and increased somewhat linearly through the top six strain gauges. The strains measured at channel 27 were still much greater, usually about twice the strain measured by the gauge immediately above it (Figure C.42). The overall result of this change was a dramatic reduction in the S-curve effect from the previous load location.

Also worth noting in this loading cycle is that a total end reaction of approximately 56 Kips was resisted by the web. The strains measured at the bottom gauge position on each face approached or exceeded the yield strain of the steel (Figure C.42). As a result, there were substantial permanent sets in the strain readings at these two positions after the ram loads were released to move the load points. On one face, this set was almost 1000 με; in the north face, it was just over 600 με. This permanent set
affected all future strain readings at these positions and made it impossible to assign accurate stress values.

At load location #3, additional changes in the strain behaviors were observed. The strains on one face began to behave as they had before, but at higher loads, all of the strains were in the compression zone. They also displayed linearity in the top six locations with the bottom-most gauge recording a much higher strain, well beyond yielding at over 12,000 με (Figure C.43). Above 55 Kips on the other face, the strains measured by the top five gauges began to reverse into tension while the bottom two gauges continued to record increasingly higher compressive strains with each load increment. The gauge nearest the bottom flange ceased recording after it reached a compressive strain of almost 16,000 με (Figure C.43). The strains no longer exhibited a reverse curvature in the web, but instead suggested a curvature in the direction in which the web would eventually buckle.

The web did not resist any more load at load location #4 than it had at location #3. In fact, the web finally crippled at an end reaction of about 68 Kips during the final load cycle (Figure C.4), which was almost the exact value that it had already resisted. As expected, then, the behavior of the web along each face was virtually identical at the final two load locations.

The failure in this test was extremely quick and dramatic. While the final load increment was being applied, the web of the beam crippled immediately. A small lateral displacement at the top flange caused the concrete slab to hit the column of the load frame, which was separated from the slab by about an inch. This very loud noise masked any sound made by the web itself, but it was quite obvious that the web had crippled. The largest end reaction that the web resisted just before collapse was approximately 68 Kips.
4.1.2 Results for Test SC-2

The strains along both faces of the web in SC-2 behaved much differently than those in SC-1. They also behaved much differently than one another. On one face, the strain gauge in the position nearest the bottom flange recorded linearly until the strain reached yielding under the first load location (Figure C.72). From that point, the strain fluctuated inconsistently for the remainder of the test, regardless of the load being resisted by the web (Figure C.74). The remaining strain gauges on that face exhibited a linear behavior up to an end reaction of approximately 32 Kips under load location #1. After this point, the strains in the middle of the web, at channels 12 through 15 (Figures C.57, C.60, C.63, and C.66), began to increase beyond the strain reading in channel 16 (Figure C.69), but without approaching the yield strain and without initiating a permanent set in the strain once the load was removed. This pattern continued under load location #2. Finally, in the last reading before failure, the strains were largest in the three middle positions, at channels 13 through 15 (Figures C.61, C.64, and C.67). The strain readings at channels 12 (Figure C.58) and 16 (Figure C.70) were about equal, and the reading at channel 11 (Figure C.55) was the least. The gauge at channel 17 had become unreliable much earlier after it recorded strains well beyond yielding.

Unfortunately, the strain gauges at channels 23 and 27 on the other face of the web were not functioning properly. This did not allow for a complete analysis of the strain behavior along this face; however, several brief observations were possible. First, the strains in the top half of the web never increased much in compression, and by the time the end reaction had reached 34 Kips, the strains above mid-depth had all become tensile or were approaching the tension zone (Figures C.54, C.57, and C.63). The gauge in channel 25 behaved linearly in the compression zone throughout load location #1 and up to 46 Kips under load location #2 (Figures C.66 and C.67). Beyond 46 Kips, however,
the strain readings reversed into tension and continued to rise linearly through the tension region until failure. Finally, the strain gauge at channel 26 behaved very similarly to the strain gauge at channel 16, directly opposite it on the other face of the web. It increased linearly in compression and was still in compression, but still below yielding, at failure (Figures C.69 and C.70).

Unlike the failure of the previous test, which occurred rather violently, immediately upon applying additional load, the failure in SC-2 was much less dramatic. There was no immediate indication of failure after applying the final load increment. Instead, the web seemed to have resisted the additional end reaction. However, after approximately 10 seconds, the web slowly began to cripple and made a very soft creaking noise. The highest observed applied load indicated that the web was resisting approximately 53 Kips just prior to failure.

4.1.3 Results for Test SC-3

The strain behavior in SC-3 was also different, and much simpler, than in either of the previous tests. All of the strain measurements behaved linearly, for the most part, until they exceeded the yield strain or until failure. The strain gauges nearest the bottom flange recorded linear compressive strains until the steel began to yield in that region at approximately 34 Kips (Figure C.93). In the top half of the web, one face of the web was in compression, while the other face was in tension (Figures C.81, C.83, and C.85). A similar pattern was observed between mid-depth and the bottom-most gauge position, except that the compression and tension sides were reversed (Figures C.87, C.89, and C.91). This suggests that a severe reverse curvature was present in the web before failure, which was confirmed by observation in the final stages of loading. Despite the reverse curve prior to failure, however, the crippling of the web was still in single curvature.
The failure of SC-3 was even slower and quieter than that of SC-2. Following the addition of the final load increment, the web again appeared to have successfully resisted the end reaction for several seconds. After this brief pause, however, the yield lines began to enlarge and the web crippled. The crippling was very slow and seemed quite smooth, and there was very little sound as the web buckled. The highest observed load indicated that the web resisted approximately 48 Kips prior to failure.

4.2 Post-Test Observations

Table 4.2 lists the predicted and measured values for several parameters, shown in Figure 4.2, relevant to this study. These include the measured locations of all of the yield lines and the ultimate collapse load. The predicted values are from equations found in the derivation in Appendix A and use the measured section and material properties found in Appendix B.

The general appearance of each yield-line pattern is very similar to the model used to predict the ultimate collapse load of the web. A set of three parallel yield lines project horizontally from the end of the beam at the seat. Once beyond the length of the seat, they begin to merge to a point near the bottom flange at some distance from the end of the seat. In the triangular region formed by these yield lines, the bottom flange of the beam is also visibly deformed. The vertical separations of the yield lines directly above the seat were measured along the end of the beam. The separation of the bottom two yield lines, $\alpha_1$, was between 1/4 and 1/3 of the overall beam depth. The top yield line extended horizontally along the bottom of the fillet at the top flange. This suggests that an expression can be given for the separation of the top two yield lines, $\alpha_2$, in terms of the separation of the bottom two yield lines:
Table 4.2 Predicted and measured values from the three full-scale tests.

<table>
<thead>
<tr>
<th></th>
<th>SC-1</th>
<th>SC-2</th>
<th>SC-3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>predicted</td>
<td>measured</td>
<td>predicted</td>
</tr>
<tr>
<td>$\alpha_1$ in.</td>
<td>6.5</td>
<td>5.0</td>
<td>5.7</td>
</tr>
<tr>
<td>$\alpha_2$ in.</td>
<td>6.5</td>
<td>7.0</td>
<td>5.7</td>
</tr>
<tr>
<td>$c$ in.</td>
<td>4.6</td>
<td>3.5</td>
<td>4.4</td>
</tr>
<tr>
<td>$\beta$ in.</td>
<td>1.2</td>
<td>31.0</td>
<td>1.2</td>
</tr>
<tr>
<td>$R_s$ Kips</td>
<td>64.9</td>
<td>68</td>
<td>49.5</td>
</tr>
</tbody>
</table>

Figure 4.2 Measured dimensions for the three full-scale tests.

\[
\alpha_2 = T - \alpha_1, \quad (4.1)
\]

where $T$ is the distance between the fillets of the beam section.

The distance, $c$, was not as easily measured. Along the bottom of the bottom flange of all three tests, the flange bent exactly 3½ in. from the edge of the beam, the exact length of the seat. The distance along the top edge of the fillet is more critical though, since it indicates the horizontal length of the yield line in the web. However, an accurate measurement of this distance is not possible due to the nature of the crippling and the locations of the yield lines. Therefore, the length of the seat, 3½ in., has been reported as the $c$-distance.
The distance, $\beta$, is also difficult to measure accurately. The distortion of the bottom flange makes a true horizontal measurement difficult. Also, the triangular region formed by the yield lines in SC-1 was very long, and the curvatures faded out gradually, making the measurement even more difficult. These minor points were taken into account in order to get a rough estimate of the length, $\beta$, for SC-1. In SC-2 and SC-3, however, the triangular yield-line patterns were more well defined and the $\beta$-lengths were much easier to measure.

The estimates for the measured web crippling collapse loads have already been discussed at length. The weight of the concrete slab was added to the highest observed ram loads to give an estimate of the largest end reaction resisted by the web for each test. In all cases, of course, this estimate is only a few hundred pounds larger than the highest recorded end reaction, since the load increments near failure were so small. Table 4.3 shows the comparison between the experimental failure loads for the three full-scale tests and the predicted values from the 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b), and the proposed web crippling equation.

4.3 Discussion

First, it is noted that the webs behaved almost exactly as expected. The correlation between the predicted and the actual values for the yield-line patterns and collapse loads were quite close, despite some of the variations pointed out in the last section. To further illustrate this point, Figures 4.3 and 4.4 show each face of the crippled web of SC-1 just after failure, and Figure 4.5 shows the end views of tests SC-1 and SC-3 after failure.
Table 4.3 Summary of the three full-scale tests.
(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K1-3 (Kips)</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>SC-1</td>
<td>W16x26</td>
<td>68</td>
<td>97.00</td>
</tr>
<tr>
<td>SC-2</td>
<td>W14x22</td>
<td>53</td>
<td>78.97</td>
</tr>
<tr>
<td>SC-3</td>
<td>W14x22</td>
<td>48</td>
<td>78.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Average</strong></td>
<td></td>
</tr>
</tbody>
</table>

(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Controlling Equation</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>SC-1</td>
<td>W16x26</td>
<td>K1-5b</td>
<td>59.66</td>
</tr>
<tr>
<td>SC-2</td>
<td>W14x22</td>
<td>K1-5b</td>
<td>48.81</td>
</tr>
<tr>
<td>SC-3</td>
<td>W14x22</td>
<td>K1-5b</td>
<td>48.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Average</strong></td>
<td>1.07</td>
</tr>
</tbody>
</table>
Figure 4.3 One face of the web of SC-1 just after failure.

Figure 4.4 Opposite face of the web of SC-1 just after failure.
Figure 4.5 End views of the crippled webs of tests SC-1 and SC-3.
The variation in the vertical separations of the yield lines over the seat actually is not surprising. Roberts (1981) first introduced the estimate of $25t_w$ as the vertical separation of the yield lines, $\alpha$, based on tests he conducted on midspan tests of plate girders. He also recommended that a maximum value of $1/6$ of the depth of the section be imposed on this parameter, but the webs of plate girders are much more slender. As a result, the web crippling failure will be much more localized near the flange in a plate girder. In rolled sections such as those tested for this study, the quantity $25t_w$ is very close to half of the depth of the beam. Therefore, the bottom two yield lines, in the case of end web crippling, should be separated by a predictable distance, and the separation of the top two yield lines will take up the remainder of the available T-distance.

Now, assume that the parameters $\alpha_1$ and $\alpha_2$ are once again set equal to each other. A good assumption for the average separation, $\bar{\alpha}$, would seem to be:

$$\bar{\alpha} = \frac{T}{2}.$$  \hspace{1cm} (4.2)

However, since the original assumption of $25t_w$ is approximately equal to $T/2$ for the sections tested in this study, and since the final equation is much simpler if $\alpha$ is a function of the web thickness; Roberts' (1981) assumption for the separation of the horizontal yield lines will still be used.

There were also discrepancies in the horizontal lengths of the yield lines from the predicted values. For instance, a clear measurement of the horizontal length of the yield lines over the seat could not be made. The bottom flange did bend at the edge of the seat, but the more critical measurement is still the length of the yield lines, not the location of the bend in the bottom flange. For this reason, the original assumption that the yield lines extend a distance, $k$, from the end of the seat will also be used for now.
Much more of a problem is the discrepancy between the predicted and experimental values of $\beta$. The actual length of the triangular region measured on the beam section itself is very similar to the expectation based on the model; however, the predicted value for this length is just over 1 in. The reason for this very small prediction is not clear, but nevertheless, the predicted values for the web crippling collapse load were very close to those obtained in the experiment. In fact, the predicted values were even a little on the conservative side.

Another note that should be discussed concerns the angle, $\theta$, created by the web just prior to collapse. The values obtained from the prediction equation suggest that the web made an angle of approximately $88.5^\circ$ with the horizontal, or just $1.5^\circ$ with the vertical, for all three tests. This illustrates that despite the strength of the steel in the web, very little deformation is permitted in the web before failure will occur.

It should also be pointed out that in all three of the tests, the strains measured at the bottom gauge position on both faces of the web yielded long before web crippling. In the tests conducted on plate girders by Roberts (1981), he reported that no yielding occurred along the flange, so he was able to ignore the effect of the $\eta$-term in the work equation, (2.4). Since significant yielding occurred in these tests long before web crippling, the $\eta$-term should somehow be considered in the formulation of the design equation. More investigation should be done on this point, but for now the $\eta$-term will be ignored in the design equation until a more definitive answer is available on how significant of an effect the yielding along the fillet has on the collapse load.

Finally, since SC-2 and SC-3 were tests of opposite ends of the same beam, it was expected that the failure loads would be about the same. However, SC-3 failed approximately 5 Kips earlier than SC-2. Several factors may have caused this to occur,
such as initial deformations in the web; but the difference between these failure loads is not so great as to call the tests into question.
CHAPTER V

EVALUATION OF THE PROPOSED WEB Crippling EQUATION

To further evaluate the proposed web crippling equation given in Chapter II and derived in Appendix A, its predictions must be compared, along with the predictions from the 1986 AISC LRFD design equations (AISC, 1986) for web yielding (K1-3) and web crippling (K1-5) and the predictions from the proposed 1993 AISC LRFD (AISC, 1993) web crippling equations (K1-5a and K1-5b), to the actual failure loads from data obtained in other research. The four sets of data available are from Ketchum and Draffin (1932), Lyse and Godfrey (1935), Roeder and Dailey (1989), and a collection of tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a). Not all of the required data is available for each test, but whenever possible, the actual measured values are used in the evaluations. In cases where the section properties are not provided, the available nominal section properties are used. In cases where the yield stress of the steel is not provided, an estimate is made based on other available data or the grade of steel specified. Each set of data is examined separately below, and several general observations are made at the conclusion of the chapter. For a more complete list of the reported and nominal values from each of the test programs, see Appendix D.

5.1 Ketchum and Draffin, 1932

Ketchum and Draffin (1932) conducted 66 tests on Jones and Laughlin junior beams to study the effect of web crippling on light I-beam sections. Eight of these tests,
however, were tested with an overhang beyond the bearing block and will not be considered. Twenty-six of the remaining 58 tests were loaded by a single point load distributed through a loading block at the midspan of the beam. These tests consisted of two distinct series, series F and part of series K. Series F examined 6 in., 10 in., and 12 in. deep beams whose total lengths were 12 in., 20 in., and 24 in., respectively. Therefore, the shear spans for the series F tests were between half of the depth and the full depth of the beam. The series K tests in this loading configuration (K33-K48, excluding those with overhang) used 10 in. deep beams with shear spans equal to 10 in. A general test set-up representing all of the midspan-loaded tests is illustrated in Figure 5.1.

The remaining 32 tests (K1-K32) were loaded by two point loads placed symmetrically about the midspan. These load points were located such that the shear span was equal to half of the depth of the beam. The test set-up for the two-point-loaded tests is illustrated in Figure 5.2.

Tables 5.1, 5.2, and 5.3 show the comparisons between the experimental failure loads for the three sets of data and the predicted values from the 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b), and the proposed web crippling equation. The predicted values are based on the measured thickness of the web at mid-depth, the measured bearing length, and an average yield stress reported elsewhere in their report, entitled "Strength of Light I-Beams." This average yield stress was selected because it represents a sampling from the type of beams provided by the manufacturer at the time of the tests. The nominal dimensions, as reported by AISC (1953), were used for the remaining variables needed in the various equations. These values are included in Appendix D.
Figure 5.1 Set-up for midspan-loaded series F and K tests by Ketchum and Drafín (1932).

Figure 5.2 Set-up for two-point-loaded series K tests by Ketchum and Drafín (1932).
Table 5.1 Summary of series F tests by Ketchum and Draffin (1932).

(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

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<th>Experimental Failure Load Predicted Failure Load</th>
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Average 1.69 1.79 1.16
Standard Deviation 0.31 0.24 0.22
(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

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<th>Experimental Failure Load Predicted Failure Load</th>
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Table 5.2 Summary of midspan-loaded series K tests by Ketchum and Draffin (1932).
(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

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|                | Average                        | 1.71                    | 1.86       | 1.13             |
| Standard Deviation | 0.56                           | 0.29                    | 0.40       |
(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

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Table 5.3 Summary of two-point-loaded series K tests by Ketchum and Draffin (1932).

(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

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Average: 1.47, 1.70, 1.02
Standard Deviation: 0.33, 0.32, 0.30
(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

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<td>K1-5b</td>
<td>15.19</td>
<td>24.91</td>
</tr>
<tr>
<td>K9 Jr. 10</td>
<td>20.31</td>
<td>K1-5a</td>
<td>14.65</td>
<td>18.65</td>
</tr>
<tr>
<td>K10 Jr. 10</td>
<td>17.96</td>
<td>K1-5a</td>
<td>14.48</td>
<td>18.34</td>
</tr>
<tr>
<td>K11 Jr. 10</td>
<td>27.85</td>
<td>K1-5a</td>
<td>16.88</td>
<td>26.17</td>
</tr>
<tr>
<td>K12 Jr. 10</td>
<td>23.98</td>
<td>K1-5a</td>
<td>16.88</td>
<td>26.17</td>
</tr>
<tr>
<td>K13 Jr. 10</td>
<td>45.13</td>
<td>K1-5b</td>
<td>21.65</td>
<td>38.52</td>
</tr>
<tr>
<td>K14 Jr. 10</td>
<td>41.80</td>
<td>K1-5b</td>
<td>24.09</td>
<td>44.44</td>
</tr>
<tr>
<td>K15 Jr. 10</td>
<td>33.22</td>
<td>K1-5b</td>
<td>25.51</td>
<td>47.72</td>
</tr>
<tr>
<td>K16 Jr. 10</td>
<td>35.48</td>
<td>K1-5b</td>
<td>25.84</td>
<td>48.77</td>
</tr>
<tr>
<td>K17 Jr. 10</td>
<td>58.11</td>
<td>K1-5b</td>
<td>31.44</td>
<td>62.32</td>
</tr>
<tr>
<td>K18 Jr. 10</td>
<td>56.75</td>
<td>K1-5b</td>
<td>31.44</td>
<td>62.32</td>
</tr>
<tr>
<td>K19 Jr. 10</td>
<td>43.05</td>
<td>K1-5b</td>
<td>30.02</td>
<td>59.01</td>
</tr>
<tr>
<td>K20 Jr. 10</td>
<td>49.90</td>
<td>K1-5b</td>
<td>30.96</td>
<td>61.20</td>
</tr>
<tr>
<td>K21 Jr. 12</td>
<td>24.91</td>
<td>K1-5a</td>
<td>16.15</td>
<td>19.61</td>
</tr>
<tr>
<td>K22 Jr. 12</td>
<td>24.33</td>
<td>K1-5a</td>
<td>16.15</td>
<td>19.61</td>
</tr>
<tr>
<td>K23 Jr. 12</td>
<td>29.51</td>
<td>K1-5a</td>
<td>19.45</td>
<td>30.68</td>
</tr>
<tr>
<td>K24 Jr. 12</td>
<td>25.72</td>
<td>K1-5a</td>
<td>19.00</td>
<td>29.70</td>
</tr>
<tr>
<td>K25 Jr. 12</td>
<td>54.54</td>
<td>K1-5b</td>
<td>27.49</td>
<td>51.75</td>
</tr>
<tr>
<td>K26 Jr. 12</td>
<td>57.01</td>
<td>K1-5b</td>
<td>33.18</td>
<td>66.38</td>
</tr>
<tr>
<td>K27 Jr. 12</td>
<td>46.16</td>
<td>K1-5b</td>
<td>28.48</td>
<td>54.85</td>
</tr>
<tr>
<td>K28 Jr. 12</td>
<td>69.80</td>
<td>K1-5b</td>
<td>38.55</td>
<td>80.21</td>
</tr>
<tr>
<td>K29 Jr. 12</td>
<td>69.78</td>
<td>K1-5b</td>
<td>42.75</td>
<td>91.14</td>
</tr>
<tr>
<td>K30 Jr. 12</td>
<td>46.68</td>
<td>K1-5b</td>
<td>35.22</td>
<td>72.49</td>
</tr>
<tr>
<td>K31 Jr. 12</td>
<td>56.38</td>
<td>K1-5b</td>
<td>43.33</td>
<td>92.58</td>
</tr>
</tbody>
</table>

Average: 1.61
Standard Deviation: 0.28
Since the tests conducted by Ketchum and Druffin (1932) represent a large percentage of the end web crippling test data available, much can be learned about the proposed equation from a careful examination of these data. From the series F tests, which represent the widest range of tests parameters, it is noted that the proposed equation predicts the actual failure load much more closely than any of the AISC design equations. The average prediction for the series F tests is 16% conservative, and the proposed equation predicts an unconservative failure load for only two of the tests in this series, F21 and F22. These two unconservative results for the proposed equation can not be adequately explained other than that many of the actual parameters are not used in the prediction equation; however, it may be significant that they were identical test configurations.

The midspan-loaded series K tests indicate a trend relating to the bearing-length-to-depth ratio, as shown by the graph of Figure 5.3. In this series of tests, the beam section and the shear span were held constant while the bearing length was varied from one-tenth to one-half of the depth of the beam. This trend suggests that the failure loads are not accurately predicted as this ratio becomes large. This is an observation made by Roberts (1981) when he derived the original web crippling equation. He pointed out that, as the bearing length becomes too large with respect to the depth of the section, the failure mode would be altered, and any prediction equation based on the assumption that the beam remains flat at the loading point would be unconservative. He recommended that a value of 0.2d be used as the upper-bound for the bearing length when using the design equation.

Finally, it must be noted that the proposed equation predicts primarily unconservative failure loads for the two-point-loaded tests, particularly as the bearing-length-to-depth ratio increases. This can be explained by the test configuration for these
tests, in which the shear span was held constant at half of the depth of the section. Not only is this a loading condition that very rarely, if ever occurs, but by applying all of the load at such a short distance from the bearing block, additional direct compression stresses are introduced which may cause the section to fail in a mode other than web crippling, e.g. web buckling.

5.2 Lyse and Godfrey, 1935

In conjunction with a paper that they published on web buckling, Lyse and Godfrey (1935) tested six beams for web crippling, three of them for web crippling at the support. Not much is known about the configurations of these three tests except for the bearing lengths used and that the ends were prevented from twisting through the use of steel bars welded to the top flange of the beams. The bars were inserted into two vertical
slots in a steel plate welded to the bottom flange of the beam. The top flange was thus able to move vertically without permitting lateral movement which would cause failure due to lateral instabilities in the top flange. The authors also noted that the results for test number CT-3 were suspect because the bearing plate used was too thin. They reasoned that this created a stress concentration above the spherical bearing block at the support that caused a premature failure based on the full bearing length.

Table 5.4 shows the comparison between the experimental failure loads and the predicted failure loads for the 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b), and the proposed web crippling equation. The predicted failure loads are based on the measured bearing length, web thickness, and steel yield stress, and the nominal dimensions as reported by AISC (1953), except for the flange thickness. An average flange thickness was used since the section tested had tapered flanges, similar to the current AISC S-shapes.

Since very little is known about these tests, little can be concluded from the data. The predicted failure loads for the 1986 AISC LRFD web crippling equation, the proposed 1993 AISC LRFD web crippling equation, and the proposed web crippling equation are all quite accurate, but the predictions for CT-1 and CT-5 by the proposed equation are more consistent than either of the AISC web crippling design equations. The lower ratios for CT-3 tend to verify the authors' note that a thin bearing plate may have caused a premature failure in the web.

5.3 Roeder and Dailey, 1989

Roeder and Dailey (1989) tested various beam sizes for end web crippling at seated connections using a test configuration similar to the one shown in Figure 5.4. The
Table 5.4 Summary of tests by Lyse and Godfey (1935).

(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K1-3 (Kips)</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>CT-1 B22x8½</td>
<td>101.0</td>
<td>126.15</td>
<td>103.85</td>
</tr>
<tr>
<td>CT-3 B22x8½</td>
<td>115.6</td>
<td>168.43</td>
<td>119.22</td>
</tr>
<tr>
<td>CT-5 B22x8½</td>
<td>104.8</td>
<td>117.88</td>
<td>98.81</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Controlling Equation</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>CT-1 B22x8½</td>
<td>101.0</td>
<td>K1-5a</td>
<td>103.85</td>
</tr>
<tr>
<td>CT-3 B22x8½</td>
<td>115.6</td>
<td>K1-5b</td>
<td>121.85</td>
</tr>
<tr>
<td>CT-5 B22x8½</td>
<td>104.8</td>
<td>K1-5a</td>
<td>98.81</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
beams tested were light W8's, W10's, W12's, and W14's. All were loaded at the quarter-point of an 8 ft. span, and each beam was also tested to "failure" using three different angle details. First, the beam was tested with a stabilizing angle welded to the top flange. Next, the top angle was removed and the beam was tested with a stabilizing angle welded into the side of the web. Finally, the beam was tested with a double-angle detail, which allowed vertical movement and accepted no load, but prevented lateral instability of the web.

![Diagram of beam testing setup](image)

Figure 5.4 Set-up for tests by Roeder and Dailey (1989).

The three lightest sections tested, W8 \(\times 10\), W12 \(\times 14\), and W12 \(\times 22\), all yielded in flexure under the load point before web crippling could occur. The next three sections tested, W12 \(\times 26\), W14 \(\times 22\), and W14 \(\times 26\), all failed by web crippling at the seat under each of the three angle details, except for the W12 \(\times 26\) with the top angle detail, which failed by local flange buckling. The final four tests, which were tested with unstiffened angle seats rather than the stiffened seats used for all of the previous tests, would not accept additional load at the support once the seat angle began to rotate under the reaction.
Table 5.5 shows a comparison of the experimental failure loads and the predicted failure loads for the 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b), and the proposed web crippling equation for the three sections that failed by web crippling using the double-angle detail. The results for the tests which used the double-angle detail are the only ones considered because the top and side angle details cannot be compared fairly to the rest of the available data. The angles clearly accept some of the reaction, and, therefore, the load that can be resisted by these tests will be disproportionate to the other tests. No information was provided on the measured section dimensions or yield stress of the steel except that grade 50 steel was specified for all of the tests. The predicted failure loads, therefore, are based on the nominal section dimensions and a yield stress of 50 Ksi.

No useful conclusions can be drawn from these tests for several reasons. First, the sections were reportedly tested to failure by web crippling twice before being tested with the double-angle detail. Permanent damage to the web could easily have been caused by either of the first two tests which would have lowered the web crippling capacity for the third test in the sequence. Decreases of 16%, 5%, and 23% in the failure load from the test with the angle welded to the web to the test with the double-angle detail occurred in the three tests. Certainly, some of this decrease was due to the fact that the double-angle detail could not carry any load, but some of it may also have been due to permanent damage to the web. Also, since no measured section or material properties were reported, the predictions do not reflect the actual beams tested. Finally, as observed in the three full-scale tests conducted for this study, the damage caused by end web crippling for hot-rolled beams occurs along the full depth of the section. The angles used to prevent lateral movement in the web may have adversely affected the web crippling capacity by altering
Table 5.5 Summary of tests with double angle detail by Roeder and Dailey (1989).

(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K1-3 (Kips)</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>1 W12x26</td>
<td>28.25</td>
<td>65.41</td>
<td>45.92</td>
</tr>
<tr>
<td>2 W14x22</td>
<td>46.45</td>
<td>65.41</td>
<td>44.04</td>
</tr>
<tr>
<td>3 W14x26</td>
<td>36.52</td>
<td>74.51</td>
<td>54.46</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Controlling Equation</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>1 W12x26</td>
<td>28.25</td>
<td>K1-5b</td>
<td>47.25</td>
</tr>
<tr>
<td>2 W14x22</td>
<td>46.45</td>
<td>K1-5b</td>
<td>45.00</td>
</tr>
<tr>
<td>3 W14x26</td>
<td>36.52</td>
<td>K1-5b</td>
<td>55.44</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the yield-line pattern in the web. Any potential effect this had would have been relatively minor; but if the web was not allowed to develop the proper yield-line pattern, then the proposed equation can not be adequately evaluated.

5.4 Elgaaly and Salkar, 1990; Elgaaly, 1991; and Elgaaly, 1991(a)

The final group of data is a collection gathered from several articles and reports written about a test program on web crippling at the University of Maine (Elgaaly and Salkar, 1990; Elgaaly, 1991; Elgaaly, 1991a). Among the data reported were those from two sets of tests conducted to examine end web crippling. Each of these tests was conducted on light rolled sections ranging from $W12 \times 14$ to $W21 \times 50$ sections. All of the tests were loaded by a single point load located at the midspan of the beam, and the top flanges of the beams were left unsupported against lateral movement. The first series of 13 tests was conducted on short beams with length-to-depth ratios ranging from 1.58 to 1.77. Therefore, the shear spans were all less than $3/4$ of the depth of the beam for these 13 tests. The second series of tests was conducted on $W12 \times 16$ sections, exclusively. The length-to-depth ratio was increased to 3.00 and held constant through most of the tests, but the bearing-length-to-depth ratio was increased gradually from 0.20 to 0.60.

Tables 5.6 and 5.7 show the comparisons of the experimental and predicted failure loads for the current 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b), and the proposed web crippling equation for the two sets of tests, respectively. The predicted failure loads for the first set of tests are all based on the measured section and material properties; however, these values were not reported for the second set of tests (Elgaaly, 1991), so the nominal section properties and the mill certificate for the steel yield stress were used for the predicted failure loads for this set of tests.
Table 5.6 Summary of first set of tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a).

(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K1-3</td>
<td>K1-5</td>
</tr>
<tr>
<td>1 W12x14</td>
<td>28.25</td>
<td>43.78</td>
<td>31.96</td>
</tr>
<tr>
<td>2 W12x14</td>
<td>46.45</td>
<td>68.34</td>
<td>43.21</td>
</tr>
<tr>
<td>3 W12x14</td>
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<td>95.81</td>
<td>53.64</td>
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<tr>
<td>4 W14x22</td>
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<td>63.82</td>
<td>45.37</td>
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<td>5 W14x22</td>
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<td>88.56</td>
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<td>6 W14x22</td>
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<td>133.13</td>
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<tr>
<td>7 W16x31</td>
<td>67.50</td>
<td>90.89</td>
<td>58.77</td>
</tr>
<tr>
<td>8 W16x31</td>
<td>64.75</td>
<td>158.58</td>
<td>70.16</td>
</tr>
<tr>
<td>9 W16x31</td>
<td>127.00</td>
<td>190.64</td>
<td>85.12</td>
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<tr>
<td>10 W18x35</td>
<td>73.25</td>
<td>117.09</td>
<td>75.30</td>
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<tr>
<td>11 W18x35</td>
<td>99.00</td>
<td>175.14</td>
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<tr>
<td>12 W21x50</td>
<td>127.50</td>
<td>169.79</td>
<td>114.25</td>
</tr>
<tr>
<td>13 W21x50</td>
<td>180.00</td>
<td>240.70</td>
<td>129.55</td>
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<td></td>
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</tr>
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<td>0.66</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.09</td>
<td>0.18</td>
</tr>
</tbody>
</table>
(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load</th>
<th>Predicted Failure Load</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Controlling Equation</td>
<td>K1-5 (Kips)</td>
<td>Proposed (Kips)</td>
</tr>
<tr>
<td>1 W12x14</td>
<td>28.25</td>
<td>K1-5a</td>
<td>31.96</td>
<td>40.35</td>
</tr>
<tr>
<td>2 W12x14</td>
<td>46.45</td>
<td>K1-5b</td>
<td>46.98</td>
<td>67.40</td>
</tr>
<tr>
<td>3 W12x14</td>
<td>64.75</td>
<td>K1-5b</td>
<td>60.99</td>
<td>88.92</td>
</tr>
<tr>
<td>4 W14x22</td>
<td>46.00</td>
<td>K1-5a</td>
<td>45.37</td>
<td>44.82</td>
</tr>
<tr>
<td>5 W14x22</td>
<td>58.50</td>
<td>K1-5b</td>
<td>55.20</td>
<td>63.45</td>
</tr>
<tr>
<td>6 W14x22</td>
<td>95.50</td>
<td>K1-5b</td>
<td>79.90</td>
<td>99.66</td>
</tr>
<tr>
<td>7 W16x31</td>
<td>67.50</td>
<td>K1-5a</td>
<td>58.77</td>
<td>53.28</td>
</tr>
<tr>
<td>8 W16x31</td>
<td>64.75</td>
<td>K1-5b</td>
<td>74.34</td>
<td>73.90</td>
</tr>
<tr>
<td>9 W16x31</td>
<td>127.00</td>
<td>K1-5b</td>
<td>93.85</td>
<td>109.95</td>
</tr>
<tr>
<td>10 W18x35</td>
<td>73.25</td>
<td>K1-5a</td>
<td>75.30</td>
<td>72.51</td>
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<td>11 W18x35</td>
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<td>K1-5b</td>
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<td>127.50</td>
<td>K1-5a</td>
<td>114.25</td>
<td>116.52</td>
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<tr>
<td>13 W21x50</td>
<td>180.00</td>
<td>K1-5b</td>
<td>138.08</td>
<td>165.32</td>
</tr>
</tbody>
</table>

|               | Average                          | 1.07                     | 0.95         |
|               | Standard Deviation               | 0.15                     | 0.18         |
Table 5.7 Summary of the second set of tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a).
(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load</th>
<th>Predicted Failure Load</th>
</tr>
</thead>
<tbody>
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<td>K1-3 (Kips)</td>
<td>K1-5 (Kips)</td>
<td>Proposed (Kips)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>K1-3</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>K1-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Proposed</td>
</tr>
<tr>
<td>14 W12x16</td>
<td>36.52</td>
<td>40.61</td>
<td>34.52</td>
<td>44.90</td>
</tr>
<tr>
<td>15 W12x16</td>
<td>35.02</td>
<td>40.61</td>
<td>34.52</td>
<td>44.90</td>
</tr>
<tr>
<td>16 W12x16</td>
<td>43.52</td>
<td>46.31</td>
<td>37.21</td>
<td>51.85</td>
</tr>
<tr>
<td>17 W12x16</td>
<td>46.10</td>
<td>52.01</td>
<td>39.90</td>
<td>58.81</td>
</tr>
<tr>
<td>18 W12x16</td>
<td>44.98</td>
<td>52.01</td>
<td>39.90</td>
<td>58.81</td>
</tr>
<tr>
<td>19 W12x16</td>
<td>52.30</td>
<td>57.70</td>
<td>42.60</td>
<td>65.76</td>
</tr>
<tr>
<td>20 W12x16</td>
<td>50.60</td>
<td>63.40</td>
<td>45.29</td>
<td>72.72</td>
</tr>
<tr>
<td>21 W12x16</td>
<td>55.05</td>
<td>63.40</td>
<td>45.29</td>
<td>72.72</td>
</tr>
<tr>
<td>22 W12x16</td>
<td>52.40</td>
<td>69.10</td>
<td>47.99</td>
<td>79.67</td>
</tr>
<tr>
<td>23 W12x16</td>
<td>48.16</td>
<td>74.80</td>
<td>50.68</td>
<td>86.63</td>
</tr>
<tr>
<td>24 W12x16</td>
<td>53.05</td>
<td>74.80</td>
<td>50.68</td>
<td>86.63</td>
</tr>
<tr>
<td>25 W12x16</td>
<td>49.90</td>
<td>80.49</td>
<td>53.37</td>
<td>93.58</td>
</tr>
<tr>
<td>26 W12x16</td>
<td>56.70</td>
<td>86.19</td>
<td>56.07</td>
<td>100.54</td>
</tr>
<tr>
<td>27 W12x16</td>
<td>57.60</td>
<td>86.19</td>
<td>56.07</td>
<td>100.54</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Controlling Equation</td>
<td>K\textsubscript{1-5}</td>
</tr>
<tr>
<td>14 W12x16</td>
<td>36.52</td>
<td>K1-5a</td>
<td>34.52</td>
</tr>
<tr>
<td>15 W12x16</td>
<td>35.02</td>
<td>K1-5a</td>
<td>34.52</td>
</tr>
<tr>
<td>16 W12x16</td>
<td>43.52</td>
<td>K1-5b</td>
<td>38.11</td>
</tr>
<tr>
<td>17 W12x16</td>
<td>46.10</td>
<td>K1-5b</td>
<td>41.70</td>
</tr>
<tr>
<td>18 W12x16</td>
<td>44.98</td>
<td>K1-5b</td>
<td>41.70</td>
</tr>
<tr>
<td>19 W12x16</td>
<td>52.30</td>
<td>K1-5b</td>
<td>45.29</td>
</tr>
<tr>
<td>20 W12x16</td>
<td>50.60</td>
<td>K1-5b</td>
<td>48.88</td>
</tr>
<tr>
<td>21 W12x16</td>
<td>55.05</td>
<td>K1-5b</td>
<td>48.88</td>
</tr>
<tr>
<td>22 W12x16</td>
<td>52.40</td>
<td>K1-5b</td>
<td>52.48</td>
</tr>
<tr>
<td>23 W12x16</td>
<td>48.16</td>
<td>K1-5b</td>
<td>56.07</td>
</tr>
<tr>
<td>24 W12x16</td>
<td>53.05</td>
<td>K1-5b</td>
<td>56.07</td>
</tr>
<tr>
<td>25 W12x16</td>
<td>49.90</td>
<td>K1-5b</td>
<td>59.66</td>
</tr>
<tr>
<td>26 W12x16</td>
<td>56.70</td>
<td>K1-5b</td>
<td>63.25</td>
</tr>
<tr>
<td>27 W12x16</td>
<td>57.60</td>
<td>K1-5b</td>
<td>63.25</td>
</tr>
</tbody>
</table>

Average 1.01 0.69

Standard Deviation 0.11 0.11
While the proposed equation predicted the failure loads for the first set of tests rather well on the whole, quite a few of the predictions were unconservative, particularly for the three W12 × 14 sections. Once again, the question of direct compression failures, mentioned in connection with the Ketchum and Draffin (1932) tests earlier, could be a potential cause for the unconservative predictions, since the length-to-depth ratios were all rather small.

In the second set of tests, the 1986 AISC LRFD and the proposed 1993 AISC LRFD web crippling equations (K1-5) appear to predict the failure loads very well, while the proposed equation predicts unconservative failure loads; but a further examination of the data reveals that this is not the case. First, the actual section and material properties were not provided. Since the measured values are invariably larger than the nominal values for these tests (compare the data for the first set of data), the predicted failure loads would all be somewhat greater. This would likely result in even more unconservative predictions by both web crippling equations. However, the author pointed out that "Due to the higher L/d ratio of these specimens and since the compression flange was not laterally braced, lateral-torsional buckling was observed, particularly in the longer specimens... (Elgaaly, 1991)." Apparently, in the shorter spans, such as in the first set of tests, the top flange did not experience excessive lateral instability, but as the length-to-depth ratio became larger, the instability manifested itself in a lateral-torsional buckling before web crippling could occur. Because the beams in the second series of tests were not allowed to fail by web crippling, the results of these 14 tests can not be accurately compared to the other tests reported.
5.5 Results of Tests SC-4 and SC-5

Two additional tests, designated SC-4 and SC-5, were conducted as part of this project in an attempt to prove that the web crippling capacity of a beam is reduced if the top flange is not braced in the lateral direction. Because this failure mode was most evident in the tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a) on W12 × 16 sections whose top flanges were not laterally braced, the two specimens used for SC-4 and SC-5 were taken from the same W12 × 16 beam. The top flange was laterally braced using a lateral brace mechanism similar to the ones discussed in Chapter III, Test Details. If the proposed web crippling equation accurately predicts the failure load for these two tests, the premature failures in the tests by Elgaaly (1991) could reasonably be attributed to lateral instabilities in the top flange of the beam.

The specimens were loaded to failure using a single point load, distributed through a 1 in. thick bearing plate. The failure mode was web crippling at the stiffened-seat connection. Web stiffeners were placed under the load point and above the support at the opposite end of the beam to prevent web crippling in these two locations. Because it did not appear that framing the connection into the web of a W14 × 145 column had had any significant effect on the three full-scale tests conducted earlier, the connection for these two tests was framed into the flange of that same W14 × 145 column. This allowed easier access to the connection and permitted the use of potentiometers to measure the horizontal deformation of the web of the beam during testing. Table 5.8 lists the spans and the locations of the load points for these two tests. The measured section and material properties for these tests can be found in Appendix B.

Table 5.9 shows the comparison of the experimental and predicted failure loads for the current 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b),
Table 5.8 Configurations for tests SC-4 and SC-5.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Distance between Support Centerlines</th>
<th>Distance from Centerline of Seat to Load Poin!</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-4</td>
<td>5 ft., 6 in.</td>
<td>1 ft., 3½ in.</td>
</tr>
<tr>
<td>SC-5</td>
<td>4 ft., 6 in.</td>
<td>1 ft. 10 in.</td>
</tr>
</tbody>
</table>

and the proposed web crippling equation for these two tests. It is clear from this comparison that by laterally bracing the top flange of the beam and using the measured section and material properties, the proposed web crippling equation, as well as the current 1986 AISC LRFD and the proposed 1993 AISC LRFD web crippling equations (K1-5), accurately predicts the failure load for the W12 × 16 sections.

As mentioned earlier, the deformation of the web was also measured with three potentiometers for these two tests. They were placed at the mid-depth of the section and at the two quarter-points of the depth. Channel 91 recorded the deformation at the top quarter-point, channel 92 recorded the mid-depth deformation, and channel 93 recorded the bottom quarter-point deformation. Figure 5.5 shows the deformations recorded by these potentiometers for test SC-5. It appears that despite the lateral brace mechanism, there was still some slight movement at the top flange as the load was applied in the beginning of the test; but once the reaction reached about 12 Kips, there was very little movement before the web crippling failure above 33 Kips. According to the model, the web is able to rotate a few degrees about its yield lines before failure occurs, so some lateral movement is possible, and that may be what is seen in this plot. This lateral movement may be revealed in the final two data points before failure in the potentiometer at channel 93, which happened to be located very near the central yield line of the web crippling yield-line pattern.
Table 5.9 Summary of tests SC-4 and SC-5.

(a) Comparison between the 1986 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K1-3 (Kips)</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>SC-4 W12x16</td>
<td>33.88</td>
<td>50.04</td>
<td>33.86</td>
</tr>
<tr>
<td>SC-5 W12x16</td>
<td>33.43</td>
<td>50.04</td>
<td>33.86</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.67</td>
<td>0.99</td>
</tr>
</tbody>
</table>

(b) Comparison between the proposed 1993 AISC LRFD equations and the proposed web crippling equation.

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Experimental Failure Load (Kips)</th>
<th>Predicted Failure Loads</th>
<th>Experimental Failure Load Predicted Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Controlling Equation</td>
<td>K1-5 (Kips)</td>
</tr>
<tr>
<td>SC-4 W12x16</td>
<td>34</td>
<td>K1-5b</td>
<td>33.98</td>
</tr>
<tr>
<td>SC-5 W12x16</td>
<td>33</td>
<td>K1-5b</td>
<td>33.98</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>0.99</td>
</tr>
</tbody>
</table>
5.6 Observations

The evaluation of this additional data has revealed several points regarding the proposed web crippling equation. First, and perhaps most important, this equation accurately predicts the web crippling failure loads for a large majority of the data available. Several exceptions have been revealed, but in most cases the test configurations or the lack of needed data prevented a fair comparison. These inconsistencies in the testing and reporting for the tests discussed are listed in Tables 5.10 and 5.11. Aside from the tests with incomplete data, the two major categories of tests in which the proposed equation predicted unconservative failure loads are: tests with small shear spans and tests conducted without laterally bracing the top flange of the beam. In the first case, it is suggested that a premature failure occurs by another failure mode, such as web buckling, when direct compression stresses are present at the connection. In the latter case, tests
SC-4 and SC-5 have shown that by laterally bracing the top flange near the connection, the web crippling failure load can indeed be reached by the web.
<table>
<thead>
<tr>
<th>Test Series</th>
<th>Test Designation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ketchum &amp; Draffin (1932)</td>
<td>Series F</td>
<td>Only measured section property provided is web thickness. Web yield stress used was an average of those provided for other beams in the report by Ketchum and Draffin (1932). Top flange was not laterally braced.</td>
</tr>
<tr>
<td></td>
<td>K33-K48</td>
<td>Only measured section property provided is web thickness. Web yield stress used was an average of those provided for other beams in the report by Ketchum and Draffin (1932). Top flange was not laterally braced. K37-K40, K43, and K44 are excluded due to overhang beyond the end of the beam. Bearing length was increased to as much as 0.5d in K45 and K46.</td>
</tr>
<tr>
<td></td>
<td>K1-K32</td>
<td>Only measured section property provided is web thickness. Web yield stress used was an average of those provided for other beams in the report by Ketchum and Draffin (1932). Top flange was not laterally braced. Shear span was decreased to 0.5d.</td>
</tr>
<tr>
<td></td>
<td>CT-1, -3, -5</td>
<td>Only measured section property provided is web thickness. Sections tested had tapered flanges. The bearing plate in CT-3 was too thin, which caused a stress concentration over the support.</td>
</tr>
</tbody>
</table>
Table 5.11 Comments on the data obtained from recent research.

<table>
<thead>
<tr>
<th>Test Series</th>
<th>Test Designation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roeder &amp; Dailey (1989)</td>
<td>1-3</td>
<td>No measured section or material properties were provided.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beams were tested to &quot;failure&quot; three separate times.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Double-angle detail may have prevented the proper yield-line pattern from developing.</td>
</tr>
<tr>
<td>Elgaaly &amp; Salkar (1990), Elgaaly (1991), and Elgaaly (1991a)</td>
<td>1-13</td>
<td>Top flange was not laterally braced.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beams were tested with small shear spans.</td>
</tr>
<tr>
<td></td>
<td>14-27</td>
<td>Top flange was not laterally braced.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Used nominal section properties and mill certificate for the yield stress.</td>
</tr>
</tbody>
</table>
CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1 Summary

This study describes the results of research conducted at Virginia Polytechnic Institute and State University and sponsored by the American Institute of Steel Construction. The objective of this project was to investigate the current 1986 AISC LRFD end web crippling equation (K1-5) as it applies to hot-rolled beams. The first step in this evaluation was to study the equation's origin, which was a derivation first presented by Roberts and Rockey (1979). Using the principle of yield-line theory and the mechanism solution, they derived an equation to predict the web crippling capacity of plate girders for interior patch loads based on an assumed collapse mechanism. By altering the initial assumptions to more accurately reflect end web crippling of hot-rolled beams, it was possible to derive a new expression for this failure mode:

\[ R_n = 24 t_w^2 \left[ \frac{t_f}{t_w} \sqrt{\frac{F_y}{t_w}} + 48 \left( \frac{e}{b_f} \right) \left( \frac{t_w}{t_f} \right) \right]. \]  

(2.27)

The complete derivation is presented in Appendix A.

In conjunction with this investigation, three full-scale tests were conducted. These tests were performed on the webs of hot-rolled beams that were incorporated into steel-concrete composite slabs. The beam end being tested framed into a stiffened-seat connection located in the web of a W14 \times 145 column, and the system was loaded by two
point loads placed symmetrically about the midspan. The results of these three tests and the comparisons with the current 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations, the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b), and the proposed end web crippling equation are given in Chapter IV, Test Results, Observations and Discussion. The 1986 AISC LRFD web yielding (K1-3) and web crippling (K1-5) equations are given as:

\[ R_n = (2.5k + N)F_{yw}tw, \]  \hspace{1cm} (1.6)

and,

\[ R_n = 68t_w^2 \left[ 1 + 3 \left( \frac{N}{d} \right) \left( \frac{tw}{tf} \right)^{15} \right] \sqrt{F_{yw} \frac{tf}{tw}}, \]  \hspace{1cm} (1.8)

respectively. The proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b) are a bilinear set of equations. Equation K1-5a applies to bearing lengths less than 0.2d, and has the same form as equation (1.8), above. Equation K1-5b applies to bearing lengths greater than or equal to 0.2d, and is written:

\[ R_n = 68t_w^2 \left[ 1 + \left( \frac{4N}{d} - 0.2 \right) \left( \frac{tw}{tf} \right)^{15} \right] \sqrt{F_{yw} \frac{tf}{tw}}. \]  \hspace{1cm} (1.13)

Finally, the results from four other sets of data obtained from end web crippling tests were used to evaluate the accuracy of the various prediction equations given above. These data were obtained from research conducted by Ketchum and Draffen (1932), Lyse and Godfrey (1935), Roeder and Dailey (1989), and Elgaaly and Salkar (1990), Elgaaly (1991) and Elgaaly (1991a). The results of this evaluation are presented in Chapter V, Evaluation of the Proposed Web Crippling Equation.
6.2 Conclusions

Based on the investigation into the origin of the existing design equation, the results of the three full-scale tests presented, and the evaluation of the historical data, the following conclusions are made. First, there are several significant differences between the model used for the current 1986 AISC LRFD web crippling equation (K1-5) and the actual failure mode. In the model used for the existing equation, half of the horizontal yield lines are eliminated from consideration because the end web crippling equation is simply half of that for interior patch loads. Also not considered in the existing model is the contribution to the web crippling capacity made by the fillet distance in a hot-rolled section. The contributions from these two sources must be considered in order to develop an accurate prediction equation.

Second, many of the results obtained from other research can not be used due to insufficient data; however, based on the three full-scale tests, SC-1 through SC-3, the two additional tests performed on the W12 × 16's, SC-4 and SC-5, and the results from the first series of tests by Elgaaly and Salkar (1990), the proposed end web crippling equation does accurately predict the web crippling capacities, as shown by Figure 6.1. The results from Elgaaly and Salkar (1990) were included with the five tests from this study because all of the required section and material properties were reported by the authors; however, it must be remembered that these additional thirteen tests were conducted over very short shear spans and without laterally bracing the top flange of the beam at the connection. The average for these eighteen tests is 0.97 with a standard deviation of 0.15.

Figures 6.2 and 6.3 show the comparison between the predicted capacities for the proposed end web crippling equation and the proposed 1993 AISC LRFD web crippling equations (K1-5a and K1-5b). It can be seen that below a bearing length of about 2½ in., the web crippling capacity of these sections is reduced compared to the proposed 1993
AISC LRFD design equations, but it must be remembered that the minimum bearing length for a 3/4 in. bolt is 2 in., due to the minimum edge distance requirements, in order to secure the beam to the seat.

Finally, based on many of the tests presented, it is clear that the method of testing a beam for end web crippling has a significant effect on its web crippling capacity. Most of the tests conducted by Ketchum and Draffin (1932) and Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a) were loaded by point loads placed very near the connection. This reduced the shear span and may have induced direct compression stresses, causing failures by modes other than web crippling, such as web buckling. Also of some concern is the method of laterally bracing the top flange of the beam at the connection. In the tests by Roeder and Dailey (1989) the top flange was ultimately braced with two angles which may have interfered with the yield-line patterns in the web, and in the tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a) the top
Figure 6.2 Comparison of the proposed equations for a W14 × 22 beam with $F_y = 50$ Ksi.

Figure 6.3 Comparison of the proposed equations for a W16 × 26 beam with $F_y = 50$ Ksi.
flanges were left unbraced altogether. These factors should be considered when evaluating the results from such tests.

6.3 Recommendations

6.3.1 Design Recommendations

Based on the information presented, the proposed end web crippling equation, equation (2.27), should be adopted for end web crippling. This equation predicts an accurate, conservative collapse load for configurations such as those studied in this project. Using the existing factors of safety for ASD and LRFD, the new equation will allow for a sufficiently conservative solution, yet one that will provide enough capacity that end web crippling will rarely control the design of the beam. Furthermore, as proven by Roeder and Dailey (1989), an additional factor of safety is provided by the standard stabilizing angles that are typically used for design, but whose strengths are not included as part of this, or any other, investigation.

6.3.2 Future Research

While the equation developed provided extremely good results for the tests conducted, additional research is recommended for the following areas:

1. Determine a more accurate prediction equation for $\alpha$, the vertical separation of the horizontal yield lines above the reaction.

2. Investigate the discrepancies between the predicted and the experimental values for $\beta$, the horizontal length of the diagonal yield lines.
3. Investigate Roberts’ (1981) seat length restriction \( (N \leq 0.2d) \). The results of the tests by Ketchum and Draffin (1932) seem to indicate that the yield-line pattern may be altered as the bearing-length-to-depth ratio approaches half of the depth of the web.

4. Investigate the possible contributions to the web crippling capacity by the fillets in a hot-rolled section. When the internal work equation was derived, the moment of inertia of the flange alone was considered. If the fillet area is included, the yield moment in the bottom flange should increase, producing a corresponding increase in the web crippling capacity.

5. Measure strains along the bottom yield line to determine the extent of yielding. If the steel in the web yields along a significant portion of the web near the toe of the fillet, then the design equation may need to be modified to account for this reduction.

6. Measure the out-of-plane deformation of the web during testing in order to plot a deflected shape. This could help to answer questions about the yield-line pattern, such as the separations and lengths of the yield lines. It could also indicate when, during the loading cycle, that the web begins to cripple and how much additional load the web can resist once web crippling has begun. In order to accurately record this deformation, a stable lateral bracing device must be used; otherwise, the deformation of the web will be masked by slight lateral movements of the top flange. A possible solution to this problem may be the lateral bracing method used by Lyse and Godfrey (1935).
REFERENCES

· Ahart, S. G. and Barker, R. M. (1986). Yield-Line Analysis and Experimental Study of Reinforced Concrete Slabs Containing Openings, Virginia Polytechnic Institute and State University, Blacksburg, VA.


· Barker, R. M. (1991). Class Notes: Advanced Reinforced Concrete, Virginia Polytechnic Institute and State University, Blacksburg, VA.


APPENDIX A

REDERIVATION FOR END WEB CRIPPLING EQUATION

The assumed collapse mechanism for web crippling at the support is shown in Figure A.1. The dimensions $\alpha$ and $\beta$ represent the locations of the yield lines in the web of the section in the undeformed state and the horizontal distance between the plastic hinges in the flange, respectively. The angle, $\theta$, represents the rotation of the web at the yield lines just before collapse. Under the virtual displacement, $\delta$, at the support, the rotation of the flange is $\delta/\beta$ and the rotations of the web are $\delta/(2\alpha \cos \theta)$ and $\delta/(\alpha \cos \theta)$ at the extreme and central yield lines, respectively.

![Diagram showing components of the collapse mechanism](image)

Figure A.1 Collapse mechanism for end web crippling.

The external work is:

$$W_e = R_n \delta.$$  \hspace{1cm} (2.22)

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The internal work equation is composed of four terms. The first two represent the work done by the flange at the hinges and by the web along the diagonal yield lines, respectively. The third term represents the work done by the web directly above the reaction. The final term accounts for the portion of the web above the reaction that has already yielded and can no longer resist the load.

\[ W_i = 2 \left[ M_f \left( \frac{\delta}{\beta} \right) \right] + 4 \left[ (m_w \beta \left( \frac{\delta}{2\alpha \cos \theta} \right) \right] + 4 \left[ (m_w c \left( \frac{\delta}{2\alpha \cos \theta} \right) \right] + 4 \left[ (m_w \eta \left( \frac{\delta}{2\alpha \cos \theta} \right) \right], \]  
(2.23)

\[ = \frac{2M_f \delta}{\beta} + \frac{2m_w \beta \delta}{\alpha \cos \theta} + \frac{2m_w c \delta}{\alpha \cos \theta} - \frac{2m_w \eta \delta}{\alpha \cos \theta}. \]  
(2.24)

The dimension, \( c \), represents the length of the horizontal yield lines, and is:

\[ c = N + k. \]  
(2.25)

The length of these yield lines is extended beyond the bearing because the web is stronger at the toe of the fillet. This situation is somewhat analogous to web yielding in which the stress is extended at a 2½:1 slope to the top of the fillet. In this case, however, the slope is projected at 1:1. Equating the external and internal work expressions and solving for \( R_n \) gives:

\[ R_n = \frac{2M_f}{\beta} + \frac{2m_w \beta}{\alpha \cos \theta} + \frac{2m_w c}{\alpha \cos \theta} - \frac{2m_w \eta}{\alpha \cos \theta}. \]  
(2.26)

In order to obtain the true collapse load, the least upper-bound must be found. The collapse load, \( R_n \), may be minimized by setting its derivative with respect to \( \beta \) equal to zero.

\[ \frac{dR_n}{d\beta} = -\frac{4M_f}{\beta^2} + \frac{4m_w}{\alpha \cos \theta} = 0. \]  
(A.6)
Solving for $\beta$ gives:

$$\beta^2 = \frac{M_r \alpha \cos \theta}{m_w},$$

(2.7)

$$\beta = \sqrt{\frac{M_r \alpha \cos \theta}{m_w}}.$$  (2.8)

To obtain the expression for the deflection in the flange the model shown in Figure A.2 is used to represent the conditions in the flange just prior to web crippling collapse. The moment in the flange is assumed to vary linearly from the extreme plastic hinge to the inner plastic hinge, and the end moments are assumed to be the plastic moments of the flanges. Finally, it must be assumed that the flange maintains continuity in slope, and, therefore, is horizontal at the ends, a and b. The deflection in the flange can be derived using the slope-deflection and moment-area methods.

![Figure A.2 Model for the derivation of the deflection of the flange.](image)

Using the positive, counter-clockwise sign convention, the moments can be expressed as:

$$\overline{M}_a = M_a + \hat{M}_a,$$  \hspace{1cm} (A.1)

$$\overline{M}_b = M_b + \hat{M}_b,$$  \hspace{1cm} (A.2)

where $\hat{M}_a$ and $\hat{M}_b$ represent the fixed-end moments caused by member loads, but, since there are none, these are equal to zero. The end moments, then, can be written:

$$\overline{M}_a = M_a = \frac{2EI_f}{\beta} \left(2\theta_a + \theta_b - 3\psi_{ab}\right),$$  \hspace{1cm} (A.3)
\[ \bar{M}_b = M_b = \frac{2EI_f}{\beta} (\theta_a + 2\theta_b - 3\psi_{ab}), \quad \text{(A.4)} \]

where \( \psi_{ab} \) is the rotation of the segment \( ab \), which is:

\[ \psi_{ab} = -\frac{\Delta_f}{\beta}. \quad \text{(A.5)} \]

Since the rotations at the ends are both equal to zero due to continuity, the moments are equal to each other.

\[ \bar{M}_a = \bar{M}_b = M_f = \frac{2EI_f}{\beta} \left[ -3 \left( -\frac{\Delta_f}{\beta} \right) \right] = \frac{6EI_f\Delta_f}{\beta^2}. \quad \text{(A.6)} \]

Solving for the deflection in the flange gives:

\[ \Delta_f = \frac{M_f\beta^2}{6EI_f}. \quad \text{(2.9)} \]

Similarly, the deflection in the web may be derived from simple geometry, as shown in Figure A.3.

![Figure A.3 Model for the derivation of the deflection in the web.](image)

\[ \sin \theta = \frac{\alpha - \Delta_w / 2}{\alpha}, \quad \text{(A.7)} \]
\[
\sin \theta = 1 - \frac{\Delta_w}{2\alpha}.
\]  \hspace{1cm} (A.8)

\[
\frac{\Delta_w}{2\alpha} = 1 - \sin \theta
\]  \hspace{1cm} (A.9)

\[
\Delta_w = 2\alpha (1 - \sin \theta)
\]  \hspace{1cm} (2.10)

To preserve compatibility, these must be equal.

\[
\frac{M_f \beta^2}{6EI_f} = 2\alpha (1 - \sin \theta).
\]  \hspace{1cm} (2.11)

\[
\frac{M_f^2 \alpha \cos \theta}{6EI_f m_w} = 2\alpha (1 - \sin \theta).
\]  \hspace{1cm} (2.12)

Rearranging, the equation takes the form:

\[
\frac{\cos \theta}{1 - \sin \theta} = \frac{12EI_f m_w}{M_f^2} = \frac{12E \left( \frac{1}{12} b_f t_f^3 \right) \left( \frac{1}{4} F_{yw} t_w^2 \right)}{\left( \frac{1}{4} F_{yr} b_f t_f^2 \right)^2} = \frac{4EF_{yw} t_w^2}{F_{yr}^2 b_f t_f}.
\]  \hspace{1cm} (A.10)

Using the trigonometric identity,

\[
\cos \theta = \frac{2H}{1 + H^2},
\]  \hspace{1cm} (A.11)

where,

\[
H = \frac{4EF_{yw} t_w^2}{F_{yr}^2 b_f t_f}.
\]  \hspace{1cm} (A.12)

The value, H, is very large with respect to unity for all reasonable quantities in its definition; therefore,

\[
\cos \theta \approx \frac{2}{H} = \frac{2F_{yr}^2 b_f t_f}{4EF_{yw} t_w^2} = \frac{F_{yr}^2 b_f t_f}{2EF_{yw} t_w^2}.
\]  \hspace{1cm} (A.13)
The \( \eta \)-term is eliminated from equation (A.5) because no substantial experimental data exist to prove that significant in-plane web yielding occurs above the reaction. Substituting equations (2.8) and (A.13) into equation (2.26) gives:

\[
R_n = \frac{t_w^2}{\sqrt{2}} \left[ \frac{E F_{yw} t_f}{\alpha F_{yf}} \left[ 1 + \frac{2 E F_{yw} t_w}{\alpha F_{yf} b_f^2} \left( \frac{t_w}{t_f} \right)^{15} \right] \right].
\]  

(A.14)

Several assumptions must now be made to further reduce equation (A.14). First, if the yield stresses in the web and flange are not assumed equal, the collapse load will decrease with an increase in the yield stress of the flange without a corresponding increase in the web yield stress. To avoid this situation, it is assumed that:

\[
F_y = F_{yw} = F_{yf}.
\]  

(A.15)

Second, Roberts' (1981) assumption for the vertical separation of the yield lines may be used since it appears to be consistent with the tests conducted in this study as well. Based on his experiments he concluded that \( \alpha \), the vertical separation of the yield lines above the reaction, could be accurately defined as:

\[
\alpha = 25 t_w.
\]  

(2.17)

Finally, Young's modulus can be substituted into the equation as 29,000 Ksi, provided that consistent units are used throughout. Including these simplifications and rearranging equation (A.14) into a form similar to the AISC design equations leaves:

\[
R_n = 24t_w^2 \left[ 1 + \frac{48}{\sqrt{F_y}} \left( \frac{c}{b_f} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{F_y}{t_w}} t_f.
\]  

(A.16)

This equation can also be written as:

\[
R_n = 24t_w^2 \left[ \sqrt{F_y} + 48 \left( \frac{c}{b_f} \right) \left( \frac{t_w}{t_f} \right)^{15} \right] \sqrt{\frac{t_f}{t_w}},
\]  

(A.17)
or,

\[ R_n = 24t_w^2 \left[ \sqrt{\frac{F_y}{b_t}} \frac{t_f}{t_w} + 48 \left( \frac{c}{b_f} \right) \left( \frac{t_w}{t_f} \right) \right]. \]  \hspace{1cm} (2.27)

Finally, it should be emphasized that the suggestions Roberts (1981) makes regarding the limitations of the \( N \)- (or in this case, \( c \))- distance may still have some validity and should not be ignored. For completeness, these recommendations are:

1. The bearing length should not be less than twice the \( \beta \)-distance and,

2. The bearing distance should not be greater than 0.2\( d \).

Roberts (1981) maintains that these limitations are necessary to insure a conservative solution to the web crippling collapse load.
APPENDIX B

TEST SYNOPTSES

Following are the measured data for the three full-scale and the two supporting tests conducted under this research project at Virginia Polytechnic Institute and State University. The yield stresses in the steel were obtained from the 0.2% offset method, and the vertical web yield stress was used to calculate the predicted web crippling failure loads.
Test SC-1

Connection Data:

Stiffened seat in the web of a W14×145 column.
Bearing Length \( N \) (in.) = 3.5
Set-Back (from column flange to beam end) \( s \) (in.) = 0.5
Seat Width \( w \) (in.) = 6.0

Beam Data:

Section W16×26; A572 Grade 50
Length (between support centerlines) \( L \) (ft.) = 34.0
Yield Stresses:
Vertical Web \( F_{yw} \) (ksi) = 60.6
Longitudinal Web \( F_{yw} \) (ksi) = 49.6
Longitudinal Top Flange \( F_{yft} \) (ksi) = 59.3
Longitudinal Bottom Flange \( F_{ybt} \) (ksi) = 58.8
Depths:
North Face \( d_N \) (in.) = 15.88
Web \( d \) (in.) ≈ 15.81
South Face \( d_S \) (in.) = 15.69
Flange Width \( b_f \) (in.) = 5.44
Web Thickness \( t_w \) (in.) = 0.260
Flange Thickness \( t_f \) (in.) = 0.336
Fillet Distance (Nominal) \( k \) (in.) = 1-1/16
Moment of Inertia (Nominal) \( I \) (in.\(^4\)) = 301

Slab Data:

Deck Profile 2 in. × 12 in.
Slab Width \( b_s \) (in.) = 80
Slab Thickness \( t_s \) (in.) = 5
Concrete Compressive Strength (test day) \( f'_c \) (ksi) = 5300
Number of Shear Studs \( N_s \) = 30
Diameter of Shear Studs \( d_s \) (in.) = 0.75
Height of Shear Studs \( h_s \) (in.) = 3.5
Effective Moment of Inertia \( I_{eff} \) (in.\(^4\)) = 1070
Tests SC-2 and SC-3

**Connection Data:**

Stiffened seat in the web of a W14×145 column.

Bearing Length N (in.) = 3.5

Set-Back (from column flange to beam end) s (in.) = 0.5

Seat Width w (in.) = 6.0

**Beam Data:**

Section W14×22; A572 Grade 50

Length (between support centerlines):

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-2</td>
<td>L (ft.) = 19.0</td>
</tr>
<tr>
<td>SC-3</td>
<td>L (ft.) = 16.0</td>
</tr>
</tbody>
</table>

Yield Stresses:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Web</td>
<td>F_{yvw} (Ksi) = 60.9</td>
</tr>
<tr>
<td>Longitudinal Web</td>
<td>F_{ylw} (Ksi) = 59.8</td>
</tr>
<tr>
<td>Longitudinal Top Flange</td>
<td>F_{ytf} (Ksi) = 55.6</td>
</tr>
<tr>
<td>Longitudinal Bottom Flange</td>
<td>F_{ybf} (Ksi) = 55.8</td>
</tr>
</tbody>
</table>

Depth d (in.) = 13.75

Flange Widths:

<p>| | |</p>
<table>
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<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Flange</td>
<td>b_{ft} (in.) = 4.75</td>
</tr>
<tr>
<td>Bottom Flange</td>
<td>b_{fb} (in.) = 4.69</td>
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<tr>
<td>Web Thickness</td>
<td>t_w (in.) = 0.228</td>
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<tr>
<td>Flange Thickness</td>
<td>t_f (in.) = 0.338</td>
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<tr>
<td>Fillet Distance (Nominal)</td>
<td>k (in.) = 7/8</td>
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<tr>
<td>Moment of Inertia (Nominal)</td>
<td>I (in.⁴) = 199</td>
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</table>

**Slab Data:**

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<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck Profile</td>
<td>2 in. × 12 in.</td>
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<tr>
<td>Slab Width</td>
<td>b_s (in.) = 80</td>
</tr>
<tr>
<td>Slab Thickness</td>
<td>t_s (in.) = 5</td>
</tr>
<tr>
<td>Concrete Compressive Strength (test day)</td>
<td>f'_c (psi) = 6300</td>
</tr>
<tr>
<td>Number of Shear Studs</td>
<td>N_s = 20</td>
</tr>
<tr>
<td>Diameter of Shear Studs</td>
<td>d_s (in.) = 0.75</td>
</tr>
<tr>
<td>Height of Shear Studs</td>
<td>h_s (in.) = 3.5</td>
</tr>
<tr>
<td>Effective Moment of Inertia</td>
<td>I_{eff} (in.⁴) = 668</td>
</tr>
</tbody>
</table>
Tests SC-4 and SC-5

**Connection Data:**

Stiffened seat in the flange of a W14×145 column.

- Bearing Length: $N$ (in.) = 2.5
- Set-Back (from column flange to beam end): $s$ (in.) = 0.5
- Seat Width: $w$ (in.) = 6.0

**Beam Data:**

<table>
<thead>
<tr>
<th>Section</th>
<th>W12×16; A36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (between support centerlines)</td>
<td></td>
</tr>
<tr>
<td>SC-4</td>
<td>$L$ (ft.) = 5.25</td>
</tr>
<tr>
<td>SC-5</td>
<td>$L$ (ft.) = 4.25</td>
</tr>
<tr>
<td>Yield Stresses:</td>
<td></td>
</tr>
<tr>
<td>Vertical Web</td>
<td>$F_{yvw}$ (Ksi) = 56.9</td>
</tr>
<tr>
<td>Longitudinal Web</td>
<td>$F_{ylw}$ (Ksi) = 57.1</td>
</tr>
<tr>
<td>Longitudinal Top Flange</td>
<td>$F_{ytf}$ (Ksi) = 55.2</td>
</tr>
<tr>
<td>Longitudinal Bottom Flange</td>
<td>$F_{ybf}$ (Ksi) = 54.7</td>
</tr>
<tr>
<td>Depths:</td>
<td></td>
</tr>
<tr>
<td>North Face</td>
<td>$d_N$ (in.) = 11.91</td>
</tr>
<tr>
<td>Web</td>
<td>$d$ (in.) = 11.97</td>
</tr>
<tr>
<td>South Face</td>
<td>$d_S$ (in.) = 12.03</td>
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<tr>
<td>Flange Width</td>
<td>$b_f$ (in.) = 4.00</td>
</tr>
<tr>
<td>Web Thickness</td>
<td>$t_w$ (in.) = 0.201</td>
</tr>
<tr>
<td>Flange Thickness</td>
<td>$t_f$ (in.) = 0.287</td>
</tr>
<tr>
<td>Fillet Distance (Nominal)</td>
<td>$k$ (in.) = 3/4</td>
</tr>
<tr>
<td>Moment of Inertia (Nominal)</td>
<td>$I$ (in.⁴) = 103</td>
</tr>
</tbody>
</table>
APPENDIX C

PLOTS FOR TESTS SC-1, SC-2, AND SC-3

Following are the plots for all of the data obtained from the three full-scale tests. The plots included are those for midspan deflection, seat end rotation, vertical strain at each pair of strain gauge locations, and the strain along each face of the web at various reaction values. The pair of strain gauges plotted together are those that are at the same distance from the centerline of the web but on opposing faces of the web. Where the values for a particular strain gauge are not given, the strain gauge had somehow become defective or it had exceeded its measuring capacity.
Figure C.1 Midspan deflection at load location #1 for test SC-1.

Figure C.2 Midspan deflection at load location #2 for test SC-1.
Figure C.3 Midspan deflection at load location #3 for test SC-1.

Figure C.4 Midspan deflection at load location #4 for test SC-1.
Figure C.5 Midspan deflection for test SC-1.
Figure C.6 Seat end rotation at load location #1 for test SC-1.

Figure C.7 Seat end rotation at load location #2 for test SC-1.
Figure C.8 Seat end rotation at load location #3 for test SC-1.

Figure C.9 Seat end rotation at load location #4 for test SC-1.
Figure C.10 Seat end rotation for test SC-1.
Figure C.11 Vertical strains at channels 11 & 21 at load location #1 for test SC-1.

Figure C.12 Vertical strains at channels 11 & 21 at load location #2 for test SC-1.
Figure C.13 Vertical strains at channels 11 & 21 at load location #3 for test SC-1.

Figure C.14 Vertical strains at channels 11 & 21 at load location #4 for test SC-1.
Figure C.15 Vertical strains at channels 11 & 21 for test SC-1.
Figure C.16 Vertical strains at channels 12 & 22 at load location #1 for test SC-1.

Figure C.17 Vertical strains at channels 12 & 22 at load location #2 for test SC-1.
Figure C.18 Vertical strains at channels 12 & 22 at load location #3 for test SC-1.

Figure C.19 Vertical strains at channels 12 & 22 at load location #4 for test SC-1.
Figure C.20 Vertical strains at channels 12 & 22 for test SC-1.
Figure C.21 Vertical strains at channels 13 & 23 at load location #1 for test SC-1.

Figure C.22 Vertical strains at channels 13 & 23 at load location #2 for test SC-1.
Figure C.23 Vertical strains at channels 13 & 23 at load location #3 for test SC-1.

Figure C.24 Vertical strains at channels 13 & 23 at load location #4 for test SC-1.
Figure C.25 Vertical strains at channels 13 & 23 for test SC-1.
Figure C.26 Vertical strains at channels 14 & 24 at load location #1 for test SC-1.

Figure C.27 Vertical strains at channels 14 & 24 at load location #2 for test SC-1.
Figure C.28 Vertical strains at channels 14 & 24 at load location #3 for test SC-1.

Figure C.29 Vertical strains at channels 14 & 24 at load location #4 for test SC-1.
Figure C.30 Vertical strains at channels 14 & 24 for test SC-1.
Figure C.31 Vertical strains at channels 15 & 25 at load location #1 for test SC-1.

Figure C.32 Vertical strains at channels 15 & 25 at load location #2 for test SC-1.
Figure C.33 Vertical strains at channels 15 & 25 at load location #3 for test SC-1.

Figure C.34 Vertical strains at channels 15 & 25 at load location #4 for test SC-1.
Figure C.35 Vertical strains at channels 15 & 25 for test SC-1.
Figure C.36 Vertical strains at channels 16 & 26 at load location #1 for test SC-1.

Figure C.37 Vertical strains at channels 16 & 26 at load location #2 for test SC-1.
Figure C.38 Vertical strains at channels 16 & 26 at load location #3 for test SC-1.

Figure C.39 Vertical strains at channels 16 & 26 at load location #4 for test SC-1.
Figure C.40 Vertical strains at channels 16 & 26 for test SC-1.
Figure C.41 Vertical strains at channels 17 & 27 at load location #1 for test SC-1.

Figure C.42 Vertical strains at channels 17 & 27 at load location #2 for test SC-1.
Figure C.43 Vertical strains at channels 17 & 27 at load location #3 for test SC-1.

Figure C.44 Vertical strains at channel 17 at load location #4 for test SC-1.
Figure C.45 Vertical strains at channels 17 & 27 for test SC-1.
Figure C.46 Vertical compressive strains along one face of the web for test SC-1.

Figure C.47 Vertical compressive strains along the other face of the web for test SC-1.
Figure C.48 Midspan deflection at load location #1 for test SC-2.

Figure C.49 Midspan deflection at load location #2 for test SC-2.
Figure C.50 Midspan deflection for test SC-2.
Figure C.51 Seat end rotation at load location #1 for test SC-2.

Figure C.52 Seat end rotation at load location #2 for test SC-2.
Figure C.53 Seat end rotation for test SC-2.
Figure C.54 Vertical strains at channels 11 & 21 at load location #1 for test SC-2.

Figure C.55 Vertical strains at channels 11 & 21 at load location #2 for test SC-2.
Figure C.56 Vertical strains at channels 11 & 21 for test SC-2.
Figure C.57 Vertical strains at channels 12 & 22 at load location #1 for test SC-2.

Figure C.58 Vertical strains at channels 12 & 22 at load location #2 for test SC-2.
Figure C.59 Vertical strains at channels 12 & 22 for test SC-2.
Figure C.60 Vertical strains at channel 13 at load location #1 for test SC-2.

Figure C.61 Vertical strains at channel 13 at load location #2 for test SC-2.
Figure C.62 Vertical strains at channel 13 for test SC-2.
Figure C.63 Vertical strains at channels 14 & 24 at load location #1 for test SC-2.

Figure C.64 Vertical strains at channels 14 & 24 at load location #2 for test SC-2.
Figure C.65 Vertical strains at channels 14 & 24 for test SC-2.
Figure C.66 Vertical strains at channels 15 & 25 at load location #1 for test SC-2.

Figure C.67 Vertical strains at channels 15 & 25 at load location #2 for test SC-2.
Figure C.68 Vertical strains at channels 15 & 25 for test SC-2.
Figure C.69 Vertical strains at channels 16 & 26 at load location #1 for test SC-2.

Figure C.70 Vertical strains at channels 16 & 26 at load location #2 for test SC-2.
Figure C.71 Vertical strains at channels 16 & 26 for test SC-2.
Figure C.72 Vertical strains at channel 17 at load location #1 for test SC-2.

Figure C.73 Vertical strains at channel 17 at load location #2 for test SC-2.
Figure C.74 Vertical strains at channel 17 for test SC-2.
Figure C.75 Vertical compressive strains along one face of the web for test SC-2.

Figure C.76 Vertical compressive strains along the other face of the web for test SC-2.
Figure C.77 Midspan deflection at load location #1 for test SC-3.

Figure C.78 Midspan deflection for test SC-32.
Figure C.79 Seat end rotation at load location #1 for test SC-3.

Figure C.80 Seat end rotation for test SC-3.
Figure C.81 Vertical strains at channels 11 & 21 at load location #1 for test SC-3.

Figure C.82 Vertical strains at channels 11 & 21 for test SC-3.
Figure C.83 Vertical strains at channels 12 & 22 at load location #1 for test SC-3.

Figure C.84 Vertical strains at channels 12 & 22 for test SC-3.
Figure C.85 Vertical strains at channels 13 & 23 at load location #1 for test SC-3.

Figure C.86 Vertical strains at channels 13 & 23 for test SC-3.
Figure C.87 Vertical strains at channels 14 & 24 at load location #1 for test SC-3.

Figure C.88 Vertical strains at channels 14 & 24 for test SC-3.
Figure C.89 Vertical strains at channels 15 & 25 at load location #1 for test SC-3.

Figure C.90 Vertical strains at channels 15 & 25 for test SC-3.
Figure C.91 Vertical strains at channels 16 & 26 at load location #1 for test SC-3.

Figure C.92 Vertical strains at channels 16 & 26 for test SC-3.
Figure C.93 Vertical strains at channels 17 & 27 at load location #1 for test SC-3.

Figure C.94 Vertical strains at channels 17 & 27 for test SC-3.
Figure C.95 Vertical compressive strains along one face of the web for test SC-3.

Figure C.96 Vertical compressive strains along the other face of the web for test SC-3.
APPENDIX D

DATA FOR END WEB CRIPLING TESTS BY OTHER AUTHORS

Following is the available data for the end web crippling tests by the various other authors. In addition to the measured data provided, the nominal section properties are also given for each section. Whenever available, the measured section and material properties were used to calculate predicted failure loads. Where these properties were not available, the nominal section properties, or a reasonable approximation of the steel yield stress, were used. Figure D.1 shows the basic cross-section used by Ketchum and Draffin (1932) and Lyse and Godfrey (1935).

Figure D.1 Sample cross-section for beams tested in the 1930's.
Table D.1 Section and material properties for the series F tests by Ketchum and Draffin (1932).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Measured Dimensions</th>
<th>Nominal Section Dimensions</th>
<th>Average $F_y$ (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L (in.)</td>
<td>N (in.)</td>
<td>$t_w$ (in.)</td>
</tr>
<tr>
<td>F3</td>
<td>Jr. 6</td>
<td>10.00</td>
<td>1.92</td>
</tr>
<tr>
<td>F4</td>
<td>Jr. 6</td>
<td>10.00</td>
<td>1.92</td>
</tr>
<tr>
<td>F5</td>
<td>Jr. 6</td>
<td>10.80</td>
<td>1.20</td>
</tr>
<tr>
<td>F6</td>
<td>Jr. 6</td>
<td>10.80</td>
<td>1.20</td>
</tr>
<tr>
<td>F1</td>
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<td>Jr. 10</td>
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<td>1.20</td>
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<tr>
<td>F21</td>
<td>Jr. 12</td>
<td>21.00</td>
<td>2.96</td>
</tr>
<tr>
<td>F22</td>
<td>Jr. 12</td>
<td>21.00</td>
<td>2.96</td>
</tr>
<tr>
<td>F23</td>
<td>Jr. 12</td>
<td>22.10</td>
<td>1.92</td>
</tr>
<tr>
<td>F24</td>
<td>Jr. 12</td>
<td>22.10</td>
<td>1.92</td>
</tr>
<tr>
<td>F25</td>
<td>Jr. 12</td>
<td>22.80</td>
<td>1.20</td>
</tr>
<tr>
<td>F26</td>
<td>Jr. 12</td>
<td>22.80</td>
<td>1.20</td>
</tr>
<tr>
<td>F27</td>
<td>Jr. 12</td>
<td>20.80</td>
<td>1.20</td>
</tr>
<tr>
<td>F28</td>
<td>Jr. 12</td>
<td>18.50</td>
<td>2.96</td>
</tr>
</tbody>
</table>
Table D.2 Section and material properties for the midspan-loaded series K tests by Ketchum and Dräffen (1932).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Measured Dimensions</th>
<th>Nominal Section Dimensions</th>
<th>Average $F_y$ (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L$ (in.)</td>
<td>$N$ (in.)</td>
<td>$t_w$ (in.) middle</td>
</tr>
<tr>
<td>K33</td>
<td>21.00</td>
<td>1.00</td>
<td>0.161 0.172 middle</td>
</tr>
<tr>
<td>K34</td>
<td>21.00</td>
<td>1.00</td>
<td>0.163 0.172 middle</td>
</tr>
<tr>
<td>K35</td>
<td>21.75</td>
<td>1.75</td>
<td>0.159 0.163 middle</td>
</tr>
<tr>
<td>K36</td>
<td>21.75</td>
<td>1.75</td>
<td>0.161 0.164 middle</td>
</tr>
<tr>
<td>K41</td>
<td>23.50</td>
<td>3.50</td>
<td>0.164 0.150 middle</td>
</tr>
<tr>
<td>K42</td>
<td>23.50</td>
<td>3.50</td>
<td>0.164 0.151 middle</td>
</tr>
<tr>
<td>K45</td>
<td>25.00</td>
<td>5.00</td>
<td>0.161 0.153 middle</td>
</tr>
<tr>
<td>K46</td>
<td>25.00</td>
<td>5.00</td>
<td>0.161 0.164 middle</td>
</tr>
</tbody>
</table>
Table D.3 Section and material properties for the two-point-loaded series K tests by Ketchum and Draffin (1932).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Measured Dimensions</th>
<th>Nominal Section Dimensions</th>
<th>Average Fy (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L (in.)</td>
<td>N (in.)</td>
<td>(t_w) (in.)</td>
</tr>
<tr>
<td>K1 Jr. 6</td>
<td>23.50</td>
<td>0.50</td>
<td>0.114</td>
</tr>
<tr>
<td>K2 Jr. 6</td>
<td>23.50</td>
<td>0.50</td>
<td>0.115</td>
</tr>
<tr>
<td>K3 Jr. 6</td>
<td>23.00</td>
<td>1.00</td>
<td>0.113</td>
</tr>
<tr>
<td>K4 Jr. 6</td>
<td>23.00</td>
<td>1.00</td>
<td>0.114</td>
</tr>
<tr>
<td>K5 Jr. 6</td>
<td>22.00</td>
<td>2.00</td>
<td>0.112</td>
</tr>
<tr>
<td>K6 Jr. 6</td>
<td>22.00</td>
<td>2.00</td>
<td>0.116</td>
</tr>
<tr>
<td>K7 Jr. 6</td>
<td>21.00</td>
<td>3.00</td>
<td>0.112</td>
</tr>
<tr>
<td>K8 Jr. 6</td>
<td>21.00</td>
<td>3.00</td>
<td>0.116</td>
</tr>
<tr>
<td>K9 Jr. 10</td>
<td>29.00</td>
<td>1.00</td>
<td>0.160</td>
</tr>
<tr>
<td>K10 Jr. 10</td>
<td>29.00</td>
<td>1.00</td>
<td>0.159</td>
</tr>
<tr>
<td>K11 Jr. 10</td>
<td>28.25</td>
<td>1.75</td>
<td>0.159</td>
</tr>
<tr>
<td>K12 Jr. 10</td>
<td>28.25</td>
<td>1.75</td>
<td>0.159</td>
</tr>
<tr>
<td>K32 Jr. 10</td>
<td>27.00</td>
<td>3.00</td>
<td>0.158</td>
</tr>
<tr>
<td>K13 Jr. 10</td>
<td>26.50</td>
<td>3.50</td>
<td>0.159</td>
</tr>
<tr>
<td>K14 Jr. 10</td>
<td>26.50</td>
<td>3.50</td>
<td>0.163</td>
</tr>
<tr>
<td>K31 Jr. 10</td>
<td>26.00</td>
<td>4.00</td>
<td>0.158</td>
</tr>
<tr>
<td>K15 Jr. 10</td>
<td>25.00</td>
<td>5.00</td>
<td>0.161</td>
</tr>
<tr>
<td>K16 Jr. 10</td>
<td>25.00</td>
<td>5.00</td>
<td>0.161</td>
</tr>
<tr>
<td>K29 Jr. 10</td>
<td>25.00</td>
<td>5.00</td>
<td>0.158</td>
</tr>
<tr>
<td>K30 Jr. 10</td>
<td>25.00</td>
<td>5.00</td>
<td>0.160</td>
</tr>
<tr>
<td>Test Designation</td>
<td>Measured Dimensions</td>
<td>Nominal Section Dimensions</td>
<td>Average $F_y$ (Ksi)</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------</td>
<td>---------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td></td>
<td>$L$ (in.)</td>
<td>$t_w$ (in.)</td>
<td>$m$ (in.)</td>
</tr>
<tr>
<td>K17</td>
<td>33.00</td>
<td>0.171</td>
<td>0.177</td>
</tr>
<tr>
<td>K18</td>
<td>33.00</td>
<td>0.171</td>
<td>0.177</td>
</tr>
<tr>
<td>K19</td>
<td>32.00</td>
<td>0.172</td>
<td>0.177</td>
</tr>
<tr>
<td>K20</td>
<td>32.00</td>
<td>0.170</td>
<td>0.169</td>
</tr>
<tr>
<td>K21</td>
<td>31.00</td>
<td>0.186</td>
<td>0.193</td>
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<tr>
<td>K22</td>
<td>30.00</td>
<td>0.187</td>
<td>0.187</td>
</tr>
<tr>
<td>K23</td>
<td>30.00</td>
<td>0.181</td>
<td>0.181</td>
</tr>
<tr>
<td>K24</td>
<td>29.00</td>
<td>0.175</td>
<td>0.169</td>
</tr>
<tr>
<td>K25</td>
<td>28.00</td>
<td>0.185</td>
<td>0.193</td>
</tr>
<tr>
<td>K26</td>
<td>28.00</td>
<td>0.171</td>
<td>0.169</td>
</tr>
<tr>
<td>K27</td>
<td>28.00</td>
<td>0.186</td>
<td>0.193</td>
</tr>
<tr>
<td>K28</td>
<td>28.00</td>
<td>0.185</td>
<td>0.193</td>
</tr>
</tbody>
</table>
### Table D.5 Section and material properties for the tests by Lyse and Godfrey (1935).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Measured Dimensions</th>
<th>Nominal Section Dimensions</th>
<th>Calc. Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N (in.)</td>
<td>t_w (in.)</td>
<td>F_y (Ksi)</td>
</tr>
<tr>
<td>CT-1 B22 × 8½</td>
<td>3.5</td>
<td>0.230</td>
<td>49.0</td>
</tr>
<tr>
<td>CT-3 B22 × 8½</td>
<td>5.5</td>
<td>0.230</td>
<td>50.0</td>
</tr>
<tr>
<td>CT-5 B22 × 8½</td>
<td>3.5</td>
<td>0.255</td>
<td>44.7</td>
</tr>
</tbody>
</table>
Table D.6 Section and material properties for the tests by Roeder and Dailey (1989).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>N (in.)</th>
<th>$t_w$ (in.)</th>
<th>$t_r$ (in.)</th>
<th>d (in.)</th>
<th>$b_r$ (in.)</th>
<th>k (in.)</th>
<th>$F_y$ (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 W12 x 26</td>
<td>3.5</td>
<td>0.230</td>
<td>0.380</td>
<td>12.22</td>
<td>6.490</td>
<td>7/8</td>
<td>50</td>
</tr>
<tr>
<td>2 W14 x 22</td>
<td>3.5</td>
<td>0.230</td>
<td>0.335</td>
<td>13.74</td>
<td>5.000</td>
<td>7/8</td>
<td>50</td>
</tr>
<tr>
<td>3 W14 x 26</td>
<td>3.5</td>
<td>0.255</td>
<td>0.420</td>
<td>13.91</td>
<td>5.025</td>
<td>15/16</td>
<td>50</td>
</tr>
</tbody>
</table>
Table D.7 Section and material properties for the first set of tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>t_w (in.)</th>
<th>t_r (in.)</th>
<th>d (in.)</th>
<th>b_r (in.)</th>
<th>N (in.)</th>
<th>L (in.)</th>
<th>F_y (Ksi)</th>
<th>k (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 W12 x 14</td>
<td>0.201</td>
<td>0.215</td>
<td>11.88</td>
<td>4.03</td>
<td>2.38</td>
<td>21.0</td>
<td>53.2</td>
<td>11/16</td>
</tr>
<tr>
<td>2 W12 x 14</td>
<td>0.202</td>
<td>0.214</td>
<td>12.00</td>
<td>4.05</td>
<td>4.80</td>
<td>21.0</td>
<td>51.9</td>
<td>11/16</td>
</tr>
<tr>
<td>3 W12 x 14</td>
<td>0.199</td>
<td>0.215</td>
<td>11.94</td>
<td>4.06</td>
<td>7.16</td>
<td>21.0</td>
<td>54.2</td>
<td>11/16</td>
</tr>
<tr>
<td>4 W14 x 22</td>
<td>0.237</td>
<td>0.329</td>
<td>13.81</td>
<td>5.12</td>
<td>2.75</td>
<td>24.0</td>
<td>54.5</td>
<td>7/8</td>
</tr>
<tr>
<td>5 W14 x 22</td>
<td>0.230</td>
<td>0.331</td>
<td>13.82</td>
<td>5.14</td>
<td>5.53</td>
<td>24.0</td>
<td>49.9</td>
<td>7/8</td>
</tr>
<tr>
<td>6 W14 x 22</td>
<td>0.242</td>
<td>0.334</td>
<td>13.82</td>
<td>5.11</td>
<td>8.29</td>
<td>24.0</td>
<td>52.5</td>
<td>7/8</td>
</tr>
<tr>
<td>7 W16 x 31</td>
<td>0.263</td>
<td>0.431</td>
<td>15.94</td>
<td>5.64</td>
<td>3.19</td>
<td>26.9</td>
<td>57.6</td>
<td>1-1/8</td>
</tr>
<tr>
<td>8 W16 x 31</td>
<td>0.248</td>
<td>0.414</td>
<td>15.94</td>
<td>5.49</td>
<td>6.37</td>
<td>26.9</td>
<td>69.6</td>
<td>1-1/8</td>
</tr>
<tr>
<td>9 W16 x 31</td>
<td>0.262</td>
<td>0.430</td>
<td>15.94</td>
<td>5.50</td>
<td>9.56</td>
<td>26.9</td>
<td>58.8</td>
<td>1-1/8</td>
</tr>
<tr>
<td>10 W18 x 35</td>
<td>0.294</td>
<td>0.429</td>
<td>17.75</td>
<td>6.05</td>
<td>3.55</td>
<td>30.0</td>
<td>62.6</td>
<td>1-1/8</td>
</tr>
<tr>
<td>11 W18 x 35</td>
<td>0.294</td>
<td>0.426</td>
<td>17.75</td>
<td>6.08</td>
<td>7.10</td>
<td>30.0</td>
<td>60.1</td>
<td>1-1/8</td>
</tr>
<tr>
<td>12 W21 x 50</td>
<td>0.362</td>
<td>0.525</td>
<td>20.94</td>
<td>6.62</td>
<td>4.19</td>
<td>33.1</td>
<td>62.8</td>
<td>1-5/16</td>
</tr>
<tr>
<td>13 W21 x 50</td>
<td>0.350</td>
<td>0.525</td>
<td>20.94</td>
<td>6.62</td>
<td>8.37</td>
<td>33.1</td>
<td>59.0</td>
<td>1-5/16</td>
</tr>
</tbody>
</table>
Table D.8 Section and material properties for the second set of tests by Elgaaly and Salkar (1990), Elgaaly (1991), and Elgaaly (1991a).

<table>
<thead>
<tr>
<th>Test Designation</th>
<th>Measured Dimensions</th>
<th>Nominal Section Dimensions</th>
<th>( F_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( N ) (in.)</td>
<td>( L ) (in.)</td>
<td>( t_w ) (in.)</td>
</tr>
<tr>
<td>14</td>
<td>W12 \times 16</td>
<td>2.40</td>
<td>36.0</td>
</tr>
<tr>
<td>15</td>
<td>W12 \times 16</td>
<td>2.40</td>
<td>36.0</td>
</tr>
<tr>
<td>16</td>
<td>W12 \times 16</td>
<td>3.00</td>
<td>36.0</td>
</tr>
<tr>
<td>17</td>
<td>W12 \times 16</td>
<td>3.60</td>
<td>36.0</td>
</tr>
<tr>
<td>18</td>
<td>W12 \times 16</td>
<td>3.60</td>
<td>36.0</td>
</tr>
<tr>
<td>19</td>
<td>W12 \times 16</td>
<td>4.20</td>
<td>36.0</td>
</tr>
<tr>
<td>20</td>
<td>W12 \times 16</td>
<td>4.80</td>
<td>36.0</td>
</tr>
<tr>
<td>21</td>
<td>W12 \times 16</td>
<td>4.80</td>
<td>36.0</td>
</tr>
<tr>
<td>22</td>
<td>W12 \times 16</td>
<td>5.40</td>
<td>42.0</td>
</tr>
<tr>
<td>23</td>
<td>W12 \times 16</td>
<td>6.00</td>
<td>48.0</td>
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<tr>
<td>24</td>
<td>W12 \times 16</td>
<td>6.00</td>
<td>42.0</td>
</tr>
<tr>
<td>25</td>
<td>W12 \times 16</td>
<td>6.60</td>
<td>48.0</td>
</tr>
<tr>
<td>26</td>
<td>W12 \times 16</td>
<td>7.20</td>
<td>42.0</td>
</tr>
<tr>
<td>27</td>
<td>W12 \times 16</td>
<td>7.20</td>
<td>42.0</td>
</tr>
</tbody>
</table>

\(^{1}\)"The web material yield stress was obtained from the mill certification as 43.2 Ksi (Elgaaly, 1991)."
APPENDIX E

NOMENCLATURE

E    Young’s modulus (29,000 Ksi)

Fy   Yield stress in the steel (Ksi)

F_{ybf}   Yield stress in the bottom flange (Ksi)

F_{yf}   Yield stress in the flange (Ksi)

F_{ylw}   Longitudinal yield stress in the web (Ksi)

F_{ytf}   Yield stress in the top flange (Ksi)

F_{yvw}   Vertical yield stress in the web (Ksi)

F_{yw}   Yield stress in the web (Ksi)

H    Arbitrary variable assigned for simplicity

I    Nominal moment of inertia of the beam cross-section (in.⁴)

I_{eff}    Effective moment of inertia of the composite slab (in.⁴)

I_{f}    Moment of inertia of the flange (in.⁴)

L    Span (ft.)

M_{a,b}   Moment from the slope-deflection method at ends a and b (in.-Kips)

\bar{M}_{a,b}   End moment at ends a and b (in.-Kips)

\hat{M}_{a,b}   Fixed-end moment at ends a and b (in.-Kips)
$M_f$  Yield moment in the flange (in.-Kips)

$N$  Seat length (in.)

$N_s$  Total number of shear studs

$P$  Load at the flange (Kips)

$P_n$  Nominal web crippling failure load (Kips)

$R$  End reaction (Kips)

$R_n$  Nominal web crippling failure reaction (Kips)

$T$  Clear distance between the toes of the top and bottom fillets (in.)

$W_e$  External work (in.-Kips)

$W_i$  Internal work (in.-Kips)

$a$  Panel width (in.)

$b_F$  Width of the flange (in.)

$b_{fb}$  Width of the bottom flange (in.)

$b_{ft}$  Width of the top flange (in.)

$b_S$  Width of the concrete slab (in.)

$c$  Length of the horizontal yield lines in end web crippling (in.)

$d$  Depth of the section (in.)

$d_{N}$  Depth of the section at the North face (in.)

$d_S$  Depth of the section at the South face (in.)

$d_s$  Diameter of the shear studs (in.)

$f$  Stress at the toe of the fillet (Ksi)

$f'_c$  Compressive stress in concrete (psi)

$h$  Panel height (in.)
\( h_s \) Height of the shear studs (in.)
\( k \) Fillet distance (in.)
\( m_w \) Yield moment in the web per unit length (in.-Kips/in.)
\( s \) Set-back from the web or flange of the column to the end of the beam (in.)
\( t_f \) Thickness of the flange (in.)
\( t_s \) Thickness of the concrete slab (in.)
\( t_w \) Thickness of the web (in.)
\( w \) Seat width (in.)
\( x \) Distance from the support centerline to the load point (ft., in.)
\( \Delta_f \) Deflection of the flange just prior to collapse (in.)
\( \Delta_w \) Deflection of the web just prior to collapse (in.)
\( \alpha \) Vertical separation of the horizontal yield lines in the web of the beam (in.)
\( \bar{\alpha} \) Average vertical separation of the horizontal yield lines in the web (in.)
\( \alpha_1 \) Vertical separation of the bottom two horizontal yield lines in the web (in.)
\( \alpha_2 \) Vertical separation of the top two horizontal yield lines in the web (in.)
\( \beta \) Horizontal length of the diagonal yield lines in the web of the beam (in.)
\( \delta \) Virtual displacement (in.)
\( \eta \) Length of web that can not resist any moment due to yielding of the steel (in.)
\( \lambda \) Ratio assigned by Roberts (1981) as 3/d
\( \theta \) Rotation of the web just prior to collapse (radians)
\( \theta_{a,b} \) End rotation at ends a and b (radians)
\( \psi_{ab} \) Rigid-body rotation of segment \( \overline{ab} \) (radians)
VITA

Cameron L. Bryant was born in Glyndon, Maryland on August 20, 1969. He graduated from Franklin High School in Reisterstown, Maryland in 1987, and received his Bachelor of Science in Civil Engineering from Virginia Polytechnic Institute and State University in May, 1991. He entered the structures graduate program of the Charles E. Via Department of Civil Engineering in the fall of 1991 to pursue a Master of Science in Civil Engineering.

Cameron L. Bryant