Prebuckling, Buckling, and Postbuckling Response of Segmented Circular Composite Cylinders

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(ABSTRACT)

Discussed is a numerical and experimental characterization of the response of small-scale fiber-reinforced composite cylinders constructed to represent a fuselage design whereby the crown and keel consist of one laminate stacking sequence and the two sides consist of another laminate stacking sequence. This construction is referred to as a segmented cylinder. The response to uniform axial endshortening is discussed. Numerical solutions for the nonlinear prebuckling, buckling, and postbuckling responses are compared to experimental results. Focus is directed at the investigation of two specific cylinder configurations, referred to as axially-stiff and circumferentially-stiff cylinders. The eight-layer stacking sequence for the crown and keel segments is $[\pm45/0]_8$ for the axially-stiff cylinder and $[\pm45/90]_8$ for the circumferentially-stiff cylinder. The eight-layer side laminates are $[\pm45]_{2S}$ in both cylinders. Small-scale cylinders, each having a nominal radius of 5 in., were fabricated on a mandrel by splicing adjacent segments together to form 0.5 in. 12-layer overlaps. Surveys of the inner and outer surfaces of the specimens were taken to characterize overall geometric imperfections and any thickness variation in addition to the overlaps. Finite-element models of both cylinder configurations, including the overlap regions, are developed using the STAGS finite-element code. Perfectly circular cylinder models are considered, as are models which include the geometric imperfection. Clamped boundary conditions are applied which account for the potted ends of the test specimens. A uniform endshortening is applied at one end of the finite-element model, while the other end remains fixed. The endshortening is considered the loading parameter, though the associated axial load is often used to describe the state of loading. Prebuckling predictions show that the segmented cylin-
der response is characterized by the existence of circumferential displacement, and an axial boundary layer accompanied by circumferential gradients in radial displacement. Experimental measurements, taken with strain gages and displacement transducers, confirm these numerical findings. As the endshortening and the associated load, approach the critical, or buckling, values, the response of the cylinders is characterized by wrinkling in the axial direction. In the axially-stiff cylinder, the crown and keel segments wrinkle, while in the circumferentially-stiff cylinder the side segments wrinkle. Experimental images taken from Moire interferometry show this response in the circumferentially-stiff cylinder. For the axially-stiff cylinder the wrinkling is not so evident. Four methods are used to predict the buckling values of endshortening and load for both cylinders, and the four values are in good agreement. The experimentally-measured buckling conditions, however, show that the models overpredict buckling values. For the axially-stiff cylinder, the difference could be due to the fact material failure not included in the model plays a role in the cylinder response. For the circumferentially-stiff cylinder, the difference is definitely due to material failure characteristics not included in the model. The predicted postbuckling response of the segmented cylinders is shown to be dominated by the existence of inward dimples in some or all of the segments. For the axially-stiff cylinder, the postbuckled shape is predicted to consist of a single inward dimple in the crown region and one in the keel region. The predicted postbuckled shape of the circumferentially-stiff cylinder is dominated by two circumferential rings of six inward dimples having the cylinder midlength as a line of anti-symmetry. For the axially-stiff cylinder, the dimpled crown and keel configuration is observed in the experiment but at a load 12% below predicted values. Material failure is present in the postbuckled configuration and it is felt that it occurred just prior to or just after the buckling event. For the circumferentially-stiff cylinder material failure in the linear prebuckling range of response triggered buckling that resembled circumferential rings of dimples, but at a load 31% below predictions. It is felt that the lack of any fibers in the axial direction in any portion of the cylinder was responsible for the behavior of the circumferentially-stiff cylinder. Finally, it is shown that the effect of including the measured imperfections in the model has little observable effect on the circumferentially-stiff cylinder. For the axially-stiff cylinder the inclusion of the imperfections is found to effect the transition from buckling to postbuckling, but ultimately has little effect on postbuckling deformations. The study concludes with recommendations for future activities on this topic.
Dedication

To my mother and father, Audrey R. and Cleveland M. Riddick,
this volume is respectfully dedicated.
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Chapter 1

Introduction

1.1 Background

Composite fuselage structures may well be made in such a way that the laminate stacking sequence varies with circumferential location around the fuselage. The stacking sequence at the top and bottom of the fuselage may be axially stiff to resist axial tensile and compressive loads due to fuselage bending, while the sides may be designed to resist shear loading. This variation in stacking sequence could well be continuous, with the stacking sequence at the top and bottom making a smooth transition to another stacking sequence at the sides. As an alternative, there could be discrete changes in the laminate stacking sequence at specific locations around the fuselage circumference. This would allow the fuselage to be manufactured in circumferential sections, a manufacturing convenience, but would result in a stepwise change in laminate properties at circumferential locations where the different laminates join. Figure 1.1 illustrates a fuselage cross section, idealized as a circular cylinder, with the crown, side, and keel constructed of different laminates. The crown and keel segments subtend the angles $2\theta_c$ and $2\theta_k$, respectively. In reality, there may even be a stiff beam forming the backbone of the keel, and there may be stiffeners at other circumferential locations. For the present work, stiffeners will not be considered. As a result of the discrete change in stiffness, rather unusual response might be expected. In particular, the response of the cylinder to even simple axisymmetric loadings such as axial endshortening and internal pressure could result in variations in the axial, circumferential, and radial displacements with respect to circumferential location. These displacement gradients in the circumferential direction could, in turn, lead to unfavorable stress fields where the segments join. Additionally, the variation in stiffness around the circumference is certain to influence buckling and postbuckling behavior. The research described herein is aimed at understanding these issues. Numerical and experimental results are presented for two specific cylinder configurations.
The behavior of any sized segmented composite cylinder subjected to a variety of loads can be studied numerically. However, for conducting experiments, small-scale cylinders are better for initial studies. The fabrication of such cylinders for the purpose of conducting experiments holds many challenges and has a direct effect on the development of numerical models used to study and understand cylinder behavior. In addition, the method by which loads are applied to the cylinder has an impact on the numerical model. Therefore, this study will focus on a single simple loading of segmented cylinders, namely, axial endshortening. Also, though ultimate interest could well focus on the construction of fig. 1.1, the study here will focus on a simpler situation, namely one where the crown and keel laminates are identical and subtend the same opening angle, $2\theta^*$, i.e., $\theta_k = \theta_c = \theta^*$. The joints will be considered as splices between the laminates of two adjacent segments and will consist of overlapping ends of plies. Each overlap is assumed to span the circumferential angle $\phi$, as shown in fig. 1.2. It is important to realize that manufacturing a small-scale circular segmented cylinder on a mandrel has an influence on the details of the overlap region. Specifically, though the overlap has a greater number of plies than the adjoining segments, and is therefore thicker, the inner radii, $R_i$, of the four segments and the four overlap regions are the same, as indicated in fig. 1.2. This occurs because the cylinder is fabricated from the outer surface of the mandrel outward, independent of the number or plies. As a result, there is an eccentricity, or offset, between the midsurface of the thicker laminate, which constitutes the overlap, and the midsurfaces of the laminates making up the adjacent segments. The eccentricity of the overlap region may have consequences for the overall response. Furthermore, for small-scale cylinders there are issues of how to cut the prepreg and how to deal with the fiber orientations in the layers of the laminates making up the segments.

1.2 Road Map to Remaining Chapters

The next chapter addresses the issue of fabricating the two small-scale cylinders used in this study. As the manner by which axial endshortening is achieved is an important consideration and has a direct influence on modeling philosophy, some of the details of the testing are considered. It is felt that these issues should be examined early, because then the features of the modeling, to be discussed later, will have a basis. It should be noted that while the study is directed at possible aircraft applications, the results may be relevant to other cylindrical structures such as
reusable launch vehicles. Also discussed in Chapter 2 are the geometric imperfections measured for each cylinder.

Chapter 3 discusses the methodology used to analyze the two specific segmented cylinder configurations. Elements of the specimen geometry and properties are highlighted. Details of the finite-element analysis, which is based on the STAGS finite-element code, are presented. Also, the effect of the local eccentricity caused by the overlaps is illustrated.

Chapter 4 discusses the numerical predictions of the response of the two cylinders to axial endshortening. The responses in the linear prebuckling range of endshortening, the geometrically nonlinear range, buckling, and the postbuckling range are discussed by way of spectrum plots of the important response variables superimposed on the deformed finite-element mesh. Predictions are made assuming the cylinders are free of geometric and materials imperfections.

In Chapter 5 the numerical predictions are presented for the responses of the two cylinders to axial endshortening with measured geometric imperfections included. The imperfections are the ones discussed in Chapter 2. The effect of including the actual measured shape on responses in the geometrically linear prebuckling range, the geometrically nonlinear prebuckling range, buckling, and the postbuckling range are discussed. Comparisons with the response of the perfect cylinder discussed in Chapter 4 are made.

Chapter 6 discusses the experimental setup that was used to test the two cylinder specimens fabricated for this study. Details of the instrumentation, data acquisition, and testing apparatus are presented. Also, the test procedure is outlined.

Chapter 7 presents measured and observed experimental responses of the two cylinders. The results from the experiments are compared with the earlier numerical predictions from Chapters 4 and 5. Experimental measurements of deflections and strains are discussed and compared in detail. Also, Moire fringe patterns recorded during the experiment are discussed, as are several photographs of the cylinders after loading them.
Finally, Chapter 8 summarizes the study, presents conclusions, and makes recommendations for future work.
$H = \text{wall thickness}$

Fig. 1.1 - Segmented cylinder construction

Fig. 1.2 - Joint region
Chapter 2

Cylinders Studied

2.1 Details of Cylinder Fabrication

Two cylindrical specimens 10-in. in diameter and 17-in. long were fabricated by hand using 12.0-in. wide unidirectional Hercules AS4/3502 graphite-epoxy prepreg tape. An aluminum mandrel was used to form the cylinders. These cylinders had eight-layer segments and were configured as follows:

The side laminates were the same for each of these cylinders, but the crown and keel laminates were different for each cylinder. The \([\pm 45/0_2]_s\) construction for the crown and keel laminates resulted in a cylinder that was stiffer axially, overall, than the cylinder with the \([\pm 45/90_2]_s\) crown and keel construction. The cylinder descriptions reflect these overall stiffness differences.

To illustrate how these small-scale cylinders were constructed, consider the axially-stiff configuration depicted in fig. 2.1. In this figure the details of the layup in the overlap region are illustrated. Note that the layers are identified by a number and a letter, layer 1 being at the inner radius and layer 8 at the outer radius, with ‘c’ denoting a crown layer and ‘s’ denoting a side layer.

Table 2.1: Details of segmented cylinder configurations

<table>
<thead>
<tr>
<th>cylinder description</th>
<th>crown and keel laminates</th>
<th>side laminates</th>
<th>designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>axially stiff</td>
<td>([\pm 45/0_2]_s)</td>
<td>([\pm 45]_2s)</td>
<td>JCRAXIAL-CYL1</td>
</tr>
<tr>
<td>circumferentially</td>
<td>([\pm 45/90_2]_s)</td>
<td>([\pm 45]_2s)</td>
<td>JCRHOOP-CYL1</td>
</tr>
</tbody>
</table>

The side laminates were the same for each of these cylinders, but the crown and keel laminates were different for each cylinder. The \([\pm 45/0_2]_s\) construction for the crown and keel laminates resulted in a cylinder that was stiffer axially, overall, than the cylinder with the \([\pm 45/90_2]_s\) crown and keel construction. The cylinder descriptions reflect these overall stiffness differences.
There are four overlaps, but the ensuing discussion will be written in the context of one of the overlaps in the crown region, knowing that a similar situation holds for the keel region and the other overlaps. In fact, from this point forward, the discussions will focus only on the crown region and the adjacent segments, the implications being that a similar discussion holds for the keel region. The overlaps were designed to be 0.5-in. in arc length. For a 10-in. diameter cylinder, this translated into $\phi$ of fig. 1.2 being $5.73^\circ$. In the overlap region the crown $[\pm45/0]_s$ laminate and the side $[\pm45]_{2s}$ laminate were spliced together by overlapping the four central $0^\circ$ layers in the crown laminate with the central $\pm45^\circ$ layers in the side laminate, and employing symmetry about the overlap midwall. This created a 12-ply symmetric laminate, namely $[\pm45/0/+45/0/-45]_s$. As stated earlier, during wrapping of the layers onto the mandrel the four extra layers were pushed outward by the mandrel, thereby creating a local eccentricity in the overlap region.

To construct a particular layer, a section of prepreg tape was cut into suitably sized pieces before starting the layup procedure by determining the desired pattern for that layer. The desired pattern was determined by taking into account the 12-in. width of the prepreg tape, the fiber angle of a particular layer, and the fact that as each layer was added, the circumferential distance each layer needed to cover was increased slightly. The two innermost layers were a continuous $+45^\circ$ layer followed by a continuous $-45^\circ$ layer. The two outermost layers were a continuous $-45^\circ$ layer followed by a continuous $+45^\circ$ layer. Figure 2.2(a) shows how the pieces for the innermost and outermost $+45^\circ$ layers, layers 1c/1s and 8c/8s, were cut from the prepreg tape. Figure 2.2(b) shows how these pieces were oriented for lay-up on the mandrel. Note that the numbers in triangles and their orientation on each piece of prepreg are important. When all pieces of prepreg were fit together to form a layer, the numbers in the triangles were all oriented the same. Small triangular pieces had to be trimmed from the ends of particular pieces.

Figures 2.3(a) and (b) illustrate the same details for the innermost and outermost continuous $-45^\circ$ layers, layers 2c/2s and 7c/7s. The edges of the $\pm45^\circ$ layer pieces were butted together, thereby creating seams. These seams were staggered in successive layers so as to minimize the effect of any local stiffness discontinuities that may have occurred. Figures 2.4(a) and (b) illus-
trate the cut-out pattern and layup orientation for the discontinuous central layers which consisted of a +45° layer in the side segment and a 0° layer in the crown segment, layers 3c/3s and 6c/6s. Figure 2.5(a) and (b) illustrate the cut-out pattern and lay-up orientation for the other discontinuous central layers, layers 4c/4s and 5c/5s, which each consisted of a -45° layer in the side segments and a 0° layer in the crown and keel segments. These central layers were stacked according to the overlap detail shown in fig. 2.1. Needless to say, cutting and positioning was easier for the 0° layers than for the ±45° layers.

Before any prepreg was wrapped on the aluminum mandrel, the mandrel was prepared by spraying it with a coat of adhesive. Then a layer of nonporous teflon was wrapped around the mandrel to protect the aluminum surface. Two coats of Frekote™ were applied to the nonporous teflon to ensure that the finished piece could be removed from the mandrel after the cure cycle was completed. The layup procedure thus began and proceeded as follows (refer to fig. 2.1):

**Layer 1** - The continuous +45° ply, layer 1c/1s, was wrapped on the mandrel.

**Layer 2** - The continuous -45° ply, layer 2c/2s, was wrapped over the +45° layer.

**Layer 3** - (a) The 0° piece, layer 3c, was laid on the crown segment, overlapping 0.25-in. into the side segment.
(b) The +45° piece, layer 3s, was laid on the side segment, overlapping 0.25-in into the crown segment, covering the edge of the 0° piece, layer 3c.

Note: The 0.25-in. overlap in these adjacent sections constituted the 0.5-in. overlap region.

**Layer 4** - (a) The 0° piece, layer 4c, was laid on the crown segment, overlapping 0.25-in. into the side segment.
(b) The -45° piece, layer 4s, was laid on the side segments, overlapping 0.25-in. into the crown section, covering the edge of the 0° piece, layer 4c.
Note: At this point the reference surface of fig. 2.1 has been reached. Symmetry of the laminate construction about the reference surface was followed.

**Layer 5** - (a) The -45° piece, layer 5s, was laid on the side segment, overlapping 0.25-in. into the crown segment.

(b) The 0° piece, layer 5c, was laid on the crown segment, overlapping 0.25-in. into the side segment, covering the edge of the -45° piece, layer 5s.

**Layer 6** - (a) The +45° piece, layer 6s, was laid on the side segment, overlapping 0.25-in. into the crown segment, covering the edges of the 0° piece, layer 5c, in the crown segment.

(b) The 0° piece, layer 6c, was laid on the crown segment, overlapping 0.25-in. into the side segment, covering the edge of the +45° piece, layer 6s.

**Layer 7** - The continuous -45° ply, layer 7c/7s, was wrapped over the previous layer.

**Layer 8** - The continuous +45° ply, layer 8c/8s, was wrapped over the previous layer.

After all layers of prepreg were in place, the cylinders were wrapped with layers of porous teflon and bleeder cloth and sealed in an airtight bag. Vacuum ports were attached through the bagging to be used in the curing process. Vacuum hoses were connected to the ports and the cylinders were cured in an autoclave at the manufacturer’s recommended temperature and pressure cycles, while drawing a vacuum. After the cured cylinders were allowed to cool, they were carefully removed from the mandrel by sliding them off one end.

### 2.2 Nondestructive Evaluation

Both cylinders were visually inspected after the fabrication process was completed. The four 1/2-in. overlap sections were clearly visible in the outer surfaces. They appeared to be of uniform width along their length, and no irregularities were observable. The bagging process was successful in minimizing surface wrinkles. The outer surfaces appeared smooth and seemed to indicate good consolidation throughout. Near the end of both cylinders there were deep indentations left by the vacuum port used in the autoclave procedure.
The cylinders were subjected to ultrasonic inspection to evaluate the quality of the specimen. An automated C-scan procedure, shown schematically in Fig. 2.6, was used to check for the presence of inclusions, voids, delaminations, and other signs of poor consolidation. The cylinder to be inspected was placed on a turntable immersed in a water tank. The C-scan technique used employed a pulse-echo method. The scans were conducted such that the inner and outer probes indexed axially after each 360° revolution of the cylinder. Due to the length of the cylinders, the scans were taken in two sections in the axial direction. Overall, the C-scan results indicated satisfactory consolidation in both cylinders. Of particular interest were the overlap regions. It could be seen clearly that for both cylinders the consolidation around these regions was commensurate with that of the overall cylinders.

2.3 Cylinder Machining

The fabrication process produced cylinders with a 5.0-in. nominal radius, each approximately 17.0-in. in length. It was necessary to cut off the imperfections caused by the vacuum ports near the end of each cylinder. The machining was done at ADVEX Corp. in Hampton, VA. The process of machining the cylinders was broken into three steps:

1. The cylinders were cut to a length which corresponded approximately to the test section length plus 1.0-in. on each end.

2. These 1-in. lengths were potted into an epoxy and steel end fitting which prevented brooming of the cylinder ends when subjected to axial compression.

3. The potted ends were machined flat and parallel so the cylinder had a 14-in. overall length.

Details of the three steps are as follows: The approximately 17.0-in. long cylinders were cut to a length of 14.0-in. This corresponded to a 12.0-in. test section with 1.0-in. potted ends. The cutting removed the aforementioned imperfections caused by the vacuum ports. In order to cut the cylinders to length, each was slid back onto the mandrel and secured, with the end portion that was to be cut off extending past the end of the mandrel. The overhang was then cut off, the cylin-
der reversed on the mandrel, and this procedure repeated. The freshly cut ends were sanded to yield a smooth end surface. The cylinders were then potted with HYSOL TE-5647, an aluminum-filled epoxy which used HD-011 hardener. As shown in fig. 2.7, the potting extended inward from each end of the cylinder and was approximately 0.5-in. thick on both the inside and outside of the cylinder wall, forming roughly a 1-in. by 1-in. cross section. The potting in liquid form was actually poured between steel bands, which formed inner and outer rings to back up the epoxy, and was allowed to harden. The potting material extended axially beyond the banded region and, after hardening, was machined so the potting surfaces were flat and parallel to each other. The potting and steel band arrangement resulted in a test length of approximately 12 in. The potting led to end conditions that were approximated as being clamped.

### 2.4 Measurements of the Initial Geometry

In order to characterize the as-fabricated geometry of each specimen, measurements of the geometry were made after the cylinders were potted. Surveys of the inner and outer shell surfaces were taken to characterize any overall geometric imperfections (e.g., out-of-roundness) and any thickness or other local variations. These geometry surveys were composed of measurements taken every 1/8 in. along the axis of the cylinder and every 1° around the circumference. The measurements were made using a Series 300 Brown and Sharpe Validator coordinate measuring machine with a PH-2A contact probe. The assembly produced measurements with an accuracy of 0.006 in. with a repeatability within 0.0003 in. over a 48-in. by 72-in. by 36-in. volume. A schematic of the measuring set-up used is shown in fig. 2.8. Measurements were taken after the specimens were potted. Therefore, surveys were only taken for the unpotted portions, or so-called test sections, which were 12 in. in length. The measurements resulted in inner and outer surface surveys consisting of nearly 70,000 data points for each cylinder. The results of the surface measurements are shown as spectrum plots in figs. 2.9 and 2.10. In these figures the average radius of each surface survey is subtracted from the measured radial position of each point. The average radius was computed from the measured data.
The results for the axially-stiff cylinder in fig. 2.9 show that the initial geometry of the cylinder was dominated by a long wavelength variation in the circumferential direction and practically no variation in the axial direction. In the outer surface variation shown in fig. 2.9(a), the variation due to the overlaps can be seen at the 30°, 150°, 210°, and 330° locations. However this variation is very slight when viewed in the context of the overall variation, which can be viewed as an out-of-roundness. Comparing the outer surface to the inner surface survey shown in fig. 2.9(b) shows the slight variation introduced by the outwardly biased overlap segments did little to influence the overall measured shape. The results for the circumferentially-stiff cylinder shown in fig. 2.10 illustrate that the initial geometry was dominated by a wavelength variation in the circumferential direction that is shorter than for the axially-stiff case. Figure 2.10, also shows that along the axial coordinate, a long wavelength variation was the dominant characteristic of the measured shape. In fig. 2.10(a), which shows the geometry of the outer surface, the overlaps can be seen at 30°, 150°, 210°, and 330°. There were also two axial strips at around 100° and 280° which are visible in the scan of the outer surface shown in fig. 2.10(a). The inner surface survey in fig. 2.10(b) shows the shorter wavelength variation in the circumferential direction, particularly in the vicinity of the x = 12-in. location.

These characteristics of the cylinder geometry could have an influence on the response of the cylinders. However, it is important to understand the response of the cylinders under ideal conditions. That is the subject of the following two chapters.

<table>
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<th>cylinder description</th>
<th>Average Radius, $R_{ave}$ (in.)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>inner surface</td>
</tr>
<tr>
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</tr>
<tr>
<td>circumferentially stiff</td>
<td>4.94</td>
</tr>
</tbody>
</table>
Cylinders Studied

Fig. 2.1 - Cylinder JCAXIAL-CYL1 with overlap detail

Fig. 2.2 - Cutout pattern and layup for continuous +45° layers (layers 1c/1s and 8c/8s)
Fig. 2.3 - Cutout pattern and layup for continuous -45° layers (layer 2c/2s and 7c/7s)

Fig. 2.4 - Cutout pattern and layup for central layers (layers 3c/3s & 6c/6s)

Fig. 2.5 - Cutout pattern and layup for central layers (layers 4c/4s & 5c/5s)
Fig. 2.6 - Schematic of C-scan set-up

Note: Top and bottom surfaces machined flat and parallel.

Fig. 2.7 - Details of potting of cylinder ends
Fig. 2.8 - Schematic of geometry measurement set-up

Fig. 2.9 - Initial geometry measurements for axially-stiff cylinder

(a) Survey of the outer surface

(b) Survey of the inner surface

Fig. 2.9 - Initial geometry measurements for axially-stiff cylinder
Fig. 2.10 - Initial geometry measurements for circumferentially-stiff cylinder
Chapter 3

Analysis Method

3.1 Problem Formulation

The finite-element code STAGS is a special-purpose code designed to study the response of shell structures, especially the geometrically nonlinear response. The code has both static and dynamic capabilities, and has been documented extensively elsewhere [1]. Both capabilities will be used here to study the response of segmented cylinders to axial endshortening. Figure 3.1 shows the geometry and nomenclature of the clamped segmented cylinder of radius \( R \) and length \( L \), where \( R = 5 \) in. and \( L = 14 \) in., as mentioned previously. The axial coordinate is \( x \) and the circumferential arc-length coordinate is \( s \), where \( s^* \), which is equal to the product \( R\theta^* \), denotes the arc length to the center of the joint, or overlap. Here the angle \( \theta^* \) is taken to be 30°. The cylinder is modeled using the STAGS 410 quadrilateral shell element, a popular element in the STAGS library. The element is shown in fig. 3.2. Midplane element displacements are denoted as \( u \), \( v \), and \( w \) in the \( x \)-, \( s \)-, and \( z \)-directions, respectively. The integration points (Gauss points) are denoted by the \( X \)'s. The rotations \( ru \) and \( rv \) represent the rotation of the undeformed \( z \)-axis about the \( x \)- and \( s \)-axes, respectively. The quantity \( rw \) represents the so-called drilling degree-of-freedom and measures rotation about the \( z \)-axis in the \( x \)-\( s \) plane. Figure 3.3 shows the finite element mesh used throughout the study. The overlap regions are evident. Specifically, the cylinder is modeled with 57 elements in the axial direction. The crown and keel each have 12 circumferential elements, the sides 30, and each of the four overlaps 5, for a total of 104 circumferential elements. Other mesh densities were investigated, but the one described here gave results consistent with models utilizing more elements, and execution times were reasonable. Clamped boundary conditions are applied such that \( v \), \( w \), and \( rv \) are zero at both ends of the cylinder; \( u=0 \) at the \( x=L \) end cylinder; and \( u \) is prescribed as \( \Delta \) at the \( x=0 \) end of the cylinder to produce a specific value of endshortening. Only a uniform axial endshortening is considered, i.e., \( \Delta \) is independent of arc-length coordi-
In order to capture the effects of potting the ends of the cylinders, as illustrated in fig. 2.7, in the finite-element model $w$ is set to zero for all the nodes within potted region of the cylinder. The potted region is assumed to extend inward axially 1.0 in. from each end. The cylinder wall configurations considered, namely, JCRAXIAL-CYL1 and JCRHOOP-CYL1, were listed in Table 2.1.

The material properties for a layer of graphite-epoxy are assumed as follows:

$$
\begin{align*}
E_1 &= 18.85 \text{ Msi} \\
E_2 &= 1.407 \text{ Msi} \\
G_{12} &= 0.725 \text{ Msi} \\
\nu_{12} &= 0.300 \\
layer \text{ thickness} &= 0.00550 \text{ in.}
\end{align*}
$$

(3.1)

In both cases, the wall thickness $H$ of the crown, side, and keel segments are equivalent to eight layers of graphite-epoxy, namely, 0.044 in. The overlap region formed by splicing the two adjacent segments has 12 layers, resulting in a thickness of 0.066 in. Figure 3.4 shows details of the overlap for the cylinder designated JCRAXIAL-CYL1. The eccentricity as depicted in fig. 3.4 is two layer thicknesses, or 0.011 in. This eccentricity is the radial distance between the reference surface of the overlap and the reference surface of the crown, keel, and side segments.

### 3.2 Effect of Local Eccentricity (Overlap)

Because the overlap regions are 50% thicker than the crown, keel, or side segments, they act as light stiffeners, and eccentric ones at that. Eccentric stiffeners can produce localized effects that should be understood. Before proceeding, it is important to understand exactly how the overlap is modeled, and to understand the influence of the overlap on the response of the cylinder. Figure 3.5(a) shows schematically how two eight-layer laminates, specifically the crown and side laminates, and the 12-layer overlap laminate would be arranged if there was no eccentricity in the overlap construction. Within the finite-element model the midplane reference surfaces of the laminates would coincide and the material properties would suddenly change from those of the eight-layer crown to those of the 12-layer overlap and then again to the eight-layer side. This model is referred to as a locally concentric model. Figure 3.6(a) shows how the overlap is actually modeled. Specifically, the midplanes of the two eight-layer laminates are not aligned with the centerplane of the 12-layer laminate, as shown in fig. 3.4. The material properties change discontinuously, and there is the issue of eccentricity caused by the lack of alignment of the mid-
plane reference surfaces. This model is referred to as the locally eccentric model. The influence of these two overlap modeling schemes is considered in a simple problem. Two cylinders, one with each overlap model, are subjected to a circumferentially-uniform axial endshortening $\Delta$. Otherwise, the ends of the cylinders are free to deform, both radially and circumferentially. This condition could be referred to as greased ends [2]. Both cylinders have the axially-stiff construction and are analyzed using a STAGS model. An overall axial compressive strain of 1000$\mu\varepsilon$ is applied. This strain level is considered small enough that a linear analysis is sufficient for predicting the response. For both models, the radial displacements, $w$, are shown in figs. 3.5(b) and 3.6(b), and the inplane shear force resultants, $N_{xy}$, are shown in fig. 3.5(c) and 3.6(c). The radial displacement for the locally concentric model, fig. 3.5(b), is characterized by an overall pattern much like that of a infinitely-long segmented-stiffness cylinder under uniform endshortening as discussed in [3], namely, the radial displacement varies smoothly with the circumferential coordinate, the crown and keel segments displacing outward more than the sides. For this finite-length cylinder there are some boundary layer effects at the ends, but even these vary smoothly in the circumferential direction and there are no localized effects. From fig. 3.5(c) it is seen that $N_{xy}$ is quite uniform over much of the cylinder, but at the overlaps near the ends of the cylinders, increased values occur. On the other hand, for the locally eccentric model, the radial displacements in fig. 3.6(b) show the effect of the eccentricity. Here, there is evidence of local deformations in the radial displacement close to the ends of the cylinder at the overlaps. This effect can be characterized as a local inward pillowing. When the ends of the cylinder are constrained to have zero radial displacement, as they are when potting is used to secure the ends of the cylinders, this tendency to pillowing inward is resisted and a local bending stress state develops. Also, related to a local stress state, as shown in fig. 3.6(c), the shear force resultant $N_{xy}$ takes on nonzero values at the ends of the cylinder in the overlap region, more so than for the locally concentric overlap. The force resultant $N_{xy}$ represents an inplane effect of the overlap eccentricity, while the inward dimpling represents an out-of-plane effect. Both are involved, even though the loading is what could be considered inplane, because of the local eccentricity. Viewed alternatively, the local geometric eccentricity can also be viewed as the overlap region having a bending-stretching stiffness matrix, while the side and crown laminates do not. The STAGS model for the perfectly circular cylinder uses one reference surface for the entire cylinder, namely a circular cylinder with mean radius $R$,
and the various laminates are defined relative to that reference surface. Thus, in the case of the locally eccentric cylinder, while the midsurface of the eight-layer laminates coincides with the reference surface, the midsurface of the 12-layer overlap laminates does not, and thus the laminates can be considered as unsymmetric, leading to components of the bending-stretching stiffness matrix. This means an inplane loading can lead to both inplane and out-of-plane responses. This is more in evidence at the ends of the cylinder than away from the ends. Away from the ends the natural circular cylinder geometry prevents a nonaxisymmetric response - i.e., even a generally unsymmetrically laminated nonsegmented cylinder would remain circular under an endshortening. In the next chapter, when the response of the potted cylinder to axial endshortening is considered in detail, it will be seen that the eccentricity of the overlaps or, alternatively, the bending-stretching terms in the stiffness matrix, results in identifiable characteristics.

The next chapter presents results from the STAGS finite-element code analyses of the two specific cylinders being discussed.
Fig. 3.1 - Geometry and nomenclature for analysis of segmented cylinder

Fig. 3.2 - STAGS 410 quadrilateral shell element
Fig. 3.3 - Finite-element mesh of segmented cylinder

Fig. 3.4 - Overlap details showing eccentricity
Chapter 3. Analysis Method

Fig. 3.5 - Segmented cylinder with locally concentric overlap

(a) Geometry of locally concentric overlap

(b) Radial displacement

(c) Shear force resultant

Fig. 3.6 - Segmented cylinder with locally eccentric overlap

(a) Geometry of locally eccentric overlap

(b) Radial displacement

(c) Shear force resultant
Chapter 4

Predicted Response of Segmented Composite Cylinders

4.1 Introduction

The results from finite-element analyses of segmented axially-stiff and circumferentially-stiff cylinders will be presented in this chapter. The analysis will be discussed in terms of considering the endshortening $\Delta$ the loading parameter. Figures 4.1(a) and 4.1(b) show the normalized load vs. endshortening response for the axially- and circumferentially-stiff cylinders, respectively. The load is the overall axial load $P$ associated with a given level of endshortening. The normalizing factors are the values of endshortening and the associated axial load at buckling. These figures can be considered as a guide to the discussion that follows. The endshortening $\Delta$ is applied starting from a state of no deformation ($\Delta = 0$) and is increased monotonically to reach buckling. The buckling point is labeled in each portion of fig. 4.1. The region of the response between $\Delta = 0$ and the buckling point is referred to as the prebuckling response. The prebuckling response initiates from $\Delta = 0$ as a linear relationship between load and endshortening. This linear relationship occurs in the region labeled ‘linear prebuckling’ on both plots. As $\Delta$ increases monotonically toward buckling, the load vs. endshortening relationship takes on a nonlinear character. Thus the region just before buckling in both plots is labeled ‘nonlinear prebuckling’. As the value of $\Delta$ is increased further, the cylinder buckles. Further increases result in postbuckling response. Accordingly, the analyses will be presented and discussed in three parts:

I. prebuckling response
   A. geometrically linear range
   B. geometrically nonlinear range
II. buckling response
III. postbuckling response
The overlap will be idealized as the locally eccentric configuration shown in fig. 3.6(a). It should be noted that the drop in load from the buckling point to the postbuckling path is a dynamic event and will be modeled as such. As points of reference for the discussion, and as will be discussed later, the buckling values of endshortening and the associated axial load are as follows: for the axially-stiff case, $\Delta_{cr} = 0.0351$ and $P_{cr} = 19,810$ lbs.; for the circumferentially-stiff case, $\Delta_{cr} = 0.1087$ and $P_{cr} = 32,000$ lbs.

4.2 Prebuckling Response

4.2.1 Geometrically Linear Range

In this section the results for axially-stiff and circumferentially-stiff cylinders subjected to an overall axial compressive strain of $1000 \mu \varepsilon (\Delta/L = 0.014/14)$ will be discussed. Figures 4.2, 4.3, and 4.4 show the predicted displacement responses for both cylinders. The displacements have been normalized by the laminate thickness $H$, which corresponds to the 8-layer thickness of the crown/keel and side laminates. The displacement responses are shown as spectrum plots mapped onto the deformed finite-element meshes. The normalized axial displacements plots shown in figs. 4.2(a) and 4.2(b) reflect an overall linear distribution of $u$ with respect to the axial coordinate $x$, and very little variation with respect to the circumferential coordinate $s$. Recall that with the analysis here, the end at $x=L$ is restrained from moving axially and the end at $x=0$ moves $\Delta$, which is a positive number. As seen in figs. 4.2(a) and (b), the condition of $\Delta$ being independent of circumferential location forces the axial displacement along the entire length to be quite independent of circumferential location, despite the variation in stiffness with circumferential position. Also, the axial displacement response for both cylinders is much the same, indicating that the axial displacement response is entirely a kinematic issue, with no dependence on material properties.

The normalized circumferential displacement spectrum plots are shown in figs. 4.3(a) and 4.3(b). As is well known, a uniform cylinder constructed of a symmetric balanced laminate and subjected to axial endshortening has no circumferential displacements. As can be seen in figs. 4.3(a) and (b), this is not the case for segmented construction, as there are circumferential displacements for both cylinders. The greatest circumferential displacement occurs at the
cylinder midlength \((x = L/2)\) at the overlap location \((\theta = \pm 30^\circ)\). The circumferential displacement goes to zero at the cylinder boundaries due to the clamped end conditions there. The spectrum plot in fig. 4.3(a) for the axially-stiff cylinder shows that circumferential displacement at the \(\theta = +30^\circ\) overlap has a negative value and at the \(\theta = -30^\circ\) overlap location has a positive value. This means that both of those overlaps move toward the crown as the cylinder is compressed axially. The plot of normalized circumferential displacement vs. circumferential location at midlength shown in fig. 4.5(a) shows this characteristic and also shows that there are large circumferential gradients in the circumferential displacement near the overlaps. Also, there are sign reversals of the circumferential displacement over relatively small spans of the circumference at the side locations \((\theta = 90^\circ \text{ and } 270^\circ)\). Due to symmetry, the maximum and minimum displacements and the gradients are also present in the keel and the associated overlaps. The information gained from figs. 4.3(a) and 4.5(a) indicate that the axially-stiff crown and keel sections are not expanding circumferentially as much as the sides due to the relative values of Poisson’s ratios for those segments. More will be said of this shortly. The circumferentially-stiff cylinder displays the same circumferential deformation pattern as the axially stiff cylinder, as evidenced by the spectrum and line plots shown in figs. 4.3(b) and 4.5(b). However, the maximum value of the circumferential deformation for the circumferentially-stiff cylinder is slightly greater than in the axially-stiff cylinder.

Studies conducted by Riddick and Hyer [3] indicate that the presence of circumferential displacements in segmented circular cylinders is due to a mismatch in Poisson’s ratio between the crown, keel, and side segments. For an infinitely long cylinder, the circumferential displacement is directly proportional to the mismatch of Poisson’s ratios. For the axially-stiff cylinder the ratio of Poisson’s ratio of the crown segment to that of the side segments is 0.885. For the circumferentially-stiff cylinder the ratio is 0.264, indicating a greater mismatch. Accordingly, the circumferentially-stiff cylinder should have considerably greater circumferential displacements than the axially-stiff cylinder. However, as also shown in ref. 3, restricting the \(v\)-displacement to be zero at the ends, due to clamping, greatly alters the circumferential displacements of a finite-length cylinder relative to an infinite-length cylinder.
Therefore, there is not as much difference in the circumferential displacements between these two finite-length segmented cylinders as there would be for infinite-length segmented cylinders.

Figures 4.4(a) and (b) show the normalized radial displacement spectrum plots for both cylinders. In general, the radial displacement response of a circular cylinder with clamped end conditions subjected to axisymmetric loadings is characterized by a bending boundary layer in the axial direction at the ends of the cylinder, and spatially uniform radial displacements near midspan, the so-called membrane region. For the segmented construction, this description fits the radial displacement response, except it is not uniform with circumferential position. The plot of normalized radial displacement vs. circumferential location at the midlength in fig. 4.6(a) shows this nonuniformity of radial displacement with respect to circumferential location in the axially-stiff cylinder. The \( w \)-displacement at midlength is greatest in the crown (\( \theta = 0^\circ \)) and decreases with increasing \( \theta \). For \( \theta \) just beyond the overlap position the radial displacement has decreased by about 50\%. As \( \theta \) increases further, the displacement increases again to a local maximum at the side location (\( \theta = 90^\circ \)) that is close in value to the displacement in the crown. This pattern reverses as the keel (\( \theta = 180^\circ \)) is approached. The radial displacement spectrum plot for the circumferentially-stiff cylinder exhibits different behavior. Here the maximum radial displacement occurs at the side and the radial displacement at the crown is about 40\% less. Figure 4.6(b) illustrates the radial displacement at midlength for the circumferentially-stiff cylinder and shows this 40\% difference. In both cylinders, the overlaps cause an inward pillowing effect in the radial displacement. Interestingly, however, fig. 4.6 shows that the minimum radial displacement with respect to circumferential location is not centered at the overlap, i.e., \( \theta = \pm 30^\circ \).

In composite cylinders, the length of the bending boundary layer is dependent on the layup \([4,5]\). Figure 4.7(a) shows plots of normalized radial displacement vs. normalized axial location at different circumferential locations for the axially stiff cylinder. The normalized radial displacement vs. normalized axial location responses shown are for \( \theta = 0^\circ \), the axial centerline of the crown segment, and \( \theta = 90^\circ \), the axial centerline of a side segment. The layup dependence of the boundary layer length can be observed here by noting the difference between the crown \([\pm 45/0_2]_s\)
and the side \([\pm 45]_2\) laminates. Figure 4.7(b) shows the same normalized radial displacement vs. circumferential location response for the other cylinder. A notable difference in boundary layer length can also be observed between the crown \([\pm 45/90]_2\) and the side \([\pm 45]_2\) laminates. Figures 4.7(a) and (b) emphasize a characteristic seen in figs. 4.6(a) and (b), namely the difference in the midlength maximum radial displacements in the crown and sides. With the axially-stiff cylinder the midlength displacement at the crown and side are about equal. For the circumferentially-stiff case the 40\% difference is evident, the side displacing more radially. Figure 4.7(b) also shows that in the crown region for the circumferentially-stiff case there is about a factor of two greater displacement at midlength \((x/L = 0.5)\) compared to the displacement at the end of the boundary layer at the locations \(x/L = 0.15\) and 0.85. In the side region, the displacement at midlength is close to that at the end of the boundary layer. This is also the case for the side and crown locations of the axially-stiff cylinder, fig. 4.7(a).

The application of \(\Delta\) that is independent of circumferential location introduces an overall state of compression in the cylinder. This applied displacement, combined with the segmented construction, produces an interesting variation of stress resultants, i.e., force and moment resultants, around the cylinder. These are depicted in figs. 4.8-4.15. The axial force resultant \(N_x\) for the axially-stiff cylinder subjected to an axial compressive strain of 1000\(\mu\varepsilon\) is shown in fig. 4.8(a) in the form of a spectrum plot. In this figure the force resultant has been normalized by the average axial force resultant for the entire cylinder, so the quantity shown is given by

\[
\frac{N_x}{\frac{P}{2\pi R}}
\]

where \(P\) is the compressive axial load due to \(\Delta\). Using the absolute value in the denominator preserves the signs in the spectrum plots. For the axially-stiff case, \(P = 8,280\) lb in compression, while for the circumferentially-stiff case, \(P = 4,390\) lb in compression. Because for the axially-stiff case the crown and keel segments are stiffer axially than the side segments, the value of \(N_x\) is significantly greater in magnitude in the crown than in the side segments. The overlap regions are even stiffer, resulting in an even larger compressive value of \(N_x\). In particular, the ratio of \(N_x\) in
the crown to $N_x$ in the sides is about 4:1. Tables 4.1 and 4.2 provide a comprehensive summary of the laminate stiffnesses ($A$s, $B$s, and $D$s), laminate engineering properties, the thickness $H$, the arc length $S$, and the load proportioning for the various segments of both cylinders, including the overlap segments. Basically, away from the ends, it can be assumed that the circumferential force resultant $N_y$ is negligible and, as a result, the axial force resultant $N_x$ is given approximately by

$$N_x = \frac{\bar{A}_{11} \Delta}{L} \quad (4.2)$$

where $\bar{A}_{11}$ is the reduced axial stiffness of a particular segment given by

$$\bar{A}_{11} = A_{11} - \frac{A_{11}^2}{A_{22}} \quad (4.3)$$

This quantity is also reported in Table 4.2. With a compressive $\Delta/L = 1000 \mu \varepsilon$, the values of $N_x$ away from the ends can be predicted quite easily. Multiplying the value of $N_x$ in a particular segment by the arc length of the segment gives the axial load supported by that segment, $P_{seg}$. Adding the loads in all the segments gives the total load, from which the proportion of the total load each segment supports, $\% P_{seg}$, can be computed. These calculations are also given in Table 4.2 and it is seen that the proportion of total axial load taken by the overlap regions in the axially-stiff cylinders is about 14%. It is important to note in Table 4.1 that two values are given for the $B$s and $D$s for the overlap segments. The values in parentheses are the values computed using the geometric midsurface of the 12-layer overlap as the reference surface. Since the laminates in the overlap segments are symmetric, the $B$s are zero. The values not in parentheses are the values computed using the midsurface of the 8-layer crown, keel, and side laminates as the reference surface. This surface was used in the finite-element analyses. Since this represents an eccentricity for the 12-layer laminates, the values of the $B$ matrix and the $D$ matrix are altered. The values of the $B$s in the overlap region are the values of the corresponding $A$s multiplied by level of the eccentricity, namely two layer thicknesses, or $2h$. Unlike a general unsymmetric laminates, the $B$s are not independent stiffnesses in the present situation.
Figure 4.8(b) shows the spectrum of normalized $N_x$ in the circumferentially-stiff cylinder. Here also the stiffer crown and keel segments carry more $N_x$ than the less stiff side segments, but the contrast is not as great as it is for the axially stiff crown, as the ratio of axial stiffness in the crown to the axial stiffness in the sides is closer to 1:1 than the roughly 4:1 of the axially-stiff case. Again, the $N_x$ in the overlaps is largest in magnitude. Calculations for $\%P_{seg}$ given in Table 4.2 show that the proportion of axial load taken by the overlap regions in the circumferentially-stiff cylinder is about 12%.

Figures 4.9(a) and (b) show the character of the circumferential force resultant $N_y$, normalized as in eq. 4.1, for the axially-stiff and circumferentially-stiff cylinders, respectively. For both cases, $N_y$ is negligible away from the potted boundaries (being basically zero) and is highest in magnitude within the potted boundaries. In the boundary layers just outside the potted boundaries, the magnitude of $N_y$ is about one-quarter of what it is within the potting, and considerably less than the value of the axial force resultant there. The values of $N_y$ in the boundary regions are negative and there is virtually no dependence of $N_y$ on the circumferential location except at the very end of the overlap within the potting.

As noted before, the bending boundary layer length due to clamped end conditions is layup dependent. The segmented construction requires the coincidence of characteristically different boundary layers at the overlap, such as the boundary layers at $\theta=0^\circ$ and $\theta=90^\circ$ for the axially-stiff cylinder shown in fig. 4.7(a). In order to maintain the integrity of the cylinder at the overlaps, shear stresses are required, both inplane and through the thickness. The shear force resultant $N_{xy}$ is a measure of the inplane shear stress. The distributions of $N_{xy}$, normalized as in eq. 4.1, for the axially-stiff and circumferentially-stiff cylinders are shown in figs. 4.10(a) and (b), respectively. Note the fact that at the overlaps, within and near the potted ends, there are high values of the inplane shear force resultants, while they are practically zero everywhere else. The high levels of the shear force resultant, though much lower in magnitude than the axial resultant, could lead to material failure as the value of $\Delta$ increases into the buckling and postbuckling range.
Figures 4.11(a) and (b) show the characteristics of the axial bending moment resultant \( M_x \) for the axially-stiff and circumferentially-stiff cylinders, respectively. In the figures the moment resultants have been normalized to yield

\[
\frac{M_x}{\frac{P}{2\pi R}} \frac{H}{2} \quad (4.4)
\]

Note that throughout most of each cylinder the moment resultants are zero, but they have a large value in the overlap region. In particular, the value is largest just outside the potting in the boundary layer region of the overlap. Away from the overlaps there is a slight boundary layer effect in both cylinders, where the value of \( M_x \) varies when moving in the axial direction beginning just outside the potting, and ending, effectively, at the end of the bending boundary layer region. The value of \( M_x \) is large in the overlap region because of the eccentricity of these regions. In the boundary there are additional bending effects due to the clamping. Viewed differently, there is a value of \( M_x \) in the overlap regions because of the existence of the bending-stretching coupling term \( B_{11} \).

The circumferential stress resultant \( M_y \) is virtually zero throughout the cylinder in both cylinders, as shown in figs. 4.12(a) and (b), where the values have been normalized as in eq. 4.4. In both cases, however, \( M_y \) is non-zero in the boundary layer and takes on even larger values in the overlap regions of the boundary layer. The values of \( M_y \) in the overlap boundary layers are the same order of magnitude as the values of \( M_x \) there.

The twisting moment resultant \( M_{xy} \) is virtually zero throughout much of the cylinder, as shown in figs. 4.13(a) and (b), where the normalization of eq. 4.4 has again been used. A boundary layer effect gives rise to a band of non-zero values of \( M_{xy} \), particularly in the overlap region. However, even where the values of \( M_{xy} \) are large, the values of this moment resultant are smaller than the axial and circumferential moment resultants.
Finally, the out-of-plane shear force resultants $Q_x$ and $Q_y$ are shown in figs. 4.14 and 4.15. The values have been normalized as in eq. 4.1 so they can be directly compared in magnitude to the inplane force resultants of figs. 4.8-4.10. For a clamped non-segmented circular cylinder, $Q_x$ would reflect the force required to enforce the $w=0$ conditions at the ends of the cylinder. Elsewhere $Q_x$ would be zero and $Q_y$ would be very small, if not exactly zero. In the case of the segmented cylinder, $Q_x$ serves the same role and would be proportional to the axial bending stiffness of the particular segment, and thus changes magnitude from segment to segment. On the otherhand, since the boundary layers in the various segments have the tendency to have different characteristic lengths, the existence of $Q_y$, like $N_{xy}$, serves to keep the segments and overlaps together. Therefore, the peak values of $Q_y$ are at the overlaps in the boundary layer, where there is the mismatch in the boundary layers. The $Q_y$ values are generally zero elsewhere. Note that the sign of $Q_y$ changes from one side of the overlap to the other. Note also that the values of $Q_x$ and $Q_y$ are much smaller than force resultants $N_x$ and $N_y$, and somewhat comparable to $N_{xy}$, but there are no fibers to resist the out-of-plane shear force resultants, as is the case for the inplane resultants. Failure due to excessive values of $Q_x$ and $Q_y$ is a possibility, even though they are small in magnitude compared to the other force resultants.
Table 4.1: Laminate stiffnesses of the segmented cylinder configurations

<table>
<thead>
<tr>
<th>Laminate Stiffnesses</th>
<th>axially stiff [±45/0₂]ₙ</th>
<th>overlap (axial/side) [±45/0/45/0/-45]ₙ</th>
<th>side [±45]₂s</th>
<th>overlap (circ./side) [±45/90/45/90/-45]ₙ</th>
<th>circ. stiff [±45/90₂]ₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>inplane (lb/in)</strong></td>
<td></td>
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<tr>
<td>A₁₁</td>
<td>550,000</td>
<td>683,000</td>
<td>26,600</td>
<td>29,700</td>
<td>164,000</td>
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<tr>
<td>A₁₂</td>
<td>110,200</td>
<td>211,000</td>
<td>20,200</td>
<td>211,000</td>
<td>110,200</td>
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<tr>
<td>A₂₂</td>
<td>164,000</td>
<td>297,000</td>
<td>266,000</td>
<td>683,000</td>
<td>550,000</td>
</tr>
<tr>
<td>A₁₆</td>
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<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
</tr>
<tr>
<td>A₆₆</td>
<td>123,400</td>
<td>23,100</td>
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<td>23,100</td>
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<td><strong>coupling (lb)</strong></td>
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<td>B₁₁</td>
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<td>(0) 7,510</td>
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<td>(0) 3,260</td>
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</tr>
<tr>
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</tr>
<tr>
<td>B₂₂</td>
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<td>(0) 3,260</td>
<td>0</td>
<td>(0) 7,510</td>
<td>0</td>
</tr>
<tr>
<td>B₁₆</td>
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<td>0</td>
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<td>0</td>
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<td>(0) 2,540</td>
<td>0</td>
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<tr>
<td><strong>bending (lb-in)</strong></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>D₁₁</td>
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<td>(208) 290</td>
<td>42.8</td>
<td>(122.1) 158.0</td>
<td>38.7</td>
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<tr>
<td>D₁₂</td>
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<td>(89.6) 115.1</td>
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<td>(89.6) 115.1</td>
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<td>D₂₂</td>
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<td>42.8</td>
<td>(208) 290</td>
<td>54.3</td>
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<td>D₁₆</td>
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<td>11.69</td>
<td>(23.4) 23.4</td>
<td>8.77</td>
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<td>11.69</td>
<td>(23.4) 23.4</td>
<td>8.77</td>
</tr>
<tr>
<td>D₆₆</td>
<td>31.0</td>
<td>(96.8) 125.0</td>
<td>34.7</td>
<td>(96.8) 125.0</td>
<td>31.0</td>
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Table 4.2: Laminate properties of the segmented cylinder configurations

<table>
<thead>
<tr>
<th>Laminate Properties</th>
<th>axially stiff $[\pm 45/0_2]_s$</th>
<th>overlap (axial/side) $[\pm 45/0/45/0/-45]_s$</th>
<th>side $[\pm 45]_2s$</th>
<th>overlap (circ./side) $[\pm 45/90/45/90/-45]_s$</th>
<th>circ. stiff $[\pm 45/90_2]_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{A}_{II}$ (lb/in)</td>
<td>476,000</td>
<td>532,000</td>
<td>112,200</td>
<td>236,000</td>
<td>141,900</td>
</tr>
<tr>
<td>$E_x$ (Msi)</td>
<td>10.82</td>
<td>8.07</td>
<td>2.55</td>
<td>3.51</td>
<td>3.22</td>
</tr>
<tr>
<td>$E_y$ (Msi)</td>
<td>3.22</td>
<td>3.51</td>
<td>2.55</td>
<td>8.07</td>
<td>10.82</td>
</tr>
<tr>
<td>$v_{xy}$</td>
<td>0.672</td>
<td>0.711</td>
<td>0.760</td>
<td>0.310</td>
<td>0.200</td>
</tr>
<tr>
<td>$G_{xy}$ (Msi)</td>
<td>2.81</td>
<td>3.45</td>
<td>4.89</td>
<td>3.45</td>
<td>2.81</td>
</tr>
<tr>
<td>$H$ (in)</td>
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<td>0.0660</td>
<td>0.0440</td>
<td>0.0660</td>
<td>0.0440</td>
</tr>
<tr>
<td>$S$ (in)</td>
<td>4.74</td>
<td>0.500</td>
<td>9.97</td>
<td>0.500</td>
<td>4.74</td>
</tr>
<tr>
<td>$P_{seg}$ (lb)</td>
<td>2,250</td>
<td>266</td>
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<td>118</td>
<td>672</td>
</tr>
<tr>
<td>$%P_{seg}$</td>
<td>28.8</td>
<td>3.40</td>
<td>14.3/27.6*</td>
<td>2.91</td>
<td>16.6</td>
</tr>
</tbody>
</table>

* The value of the first number is the $\%P_{seg}$ supported by the laminate in the side segment in the axially-stiff cylinder. The value of the second number is the $\%P_{seg}$ supported by the laminate in the side segment in the circumferentially-stiff cylinder.
Fig. 4.1 - Load vs. endshortening relations

(a) axially-stiff cylinder

(b) circumferentially-stiff cylinder
Fig. 4.2 - Axial displacement contours, linear range
(a) axially stiff (b) circumferentially stiff

Fig. 4.3 - Circumferential displacement contours, linear range
(a) axially stiff (b) circumferentially stiff

Fig. 4.4 - Radial displacement contours, linear range
(a) axially stiff (b) circumferentially stiff
Fig. 4.5 - Circumferential displacement vs. circumferential location at midlength, linear range

(a) axially stiff
(b) circumferentially stiff

Fig. 4.6 - Radial displacements vs. circumferential location at midlength, linear range

(a) axially stiff
(b) circumferentially stiff

Fig. 4.7 - Radial displacement vs. axial location, linear range

(a) axially stiff
(b) circumferentially stiff
Fig. 4.8 - Axial force resultant $N_x$, linear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.8 - Axial force resultant $N_x$, linear range
Fig. 4.9 - Circumferential force resultant $N_y$, linear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.9 - Circumferential force resultant $N_y$, linear range
Fig. 4.10 - Shear force resultant $N_{xy}$, linear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.10 - Shear force resultant $N_{xy}$, linear range
Fig. 4.11- Axial bending moment resultant $M_x$, linear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.11- Axial bending moment resultant $M_x$, linear range
Fig. 4.12 - Circumferential bending moment resultant $M_y$, linear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.12 - Circumferential bending moment resultant $M_y$, linear range
Fig. 4.13- Twist moment resultant $M_{xy}$ linear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.13- Twist moment resultant $M_{xy}$ linear range
Fig. 4.14 - Transverse shear resultants for axially-stiff cylinder, linear range

(a) axial shear component, $Q_x$

(b) circumferential shear component, $Q_y$

Fig. 4.14 - Transverse shear resultant for axially-stiff cylinder, linear range
(a) axial shear component, $Q_x$

(b) circumferential shear component, $Q_y$

Fig. 4.15 - Transverse shear resultants for circumferentially-stiff cylinder, linear range
4.2.2 Geometrically Nonlinear Range

As $\Delta$ increases toward the critical, or buckling value, the load vs. endshortening relations, fig. 4.1, become slightly nonlinear. Specifically, just before buckling, the load vs. endshortening relations soften somewhat. This indicates that geometric nonlinearities are important. In that context, then, in this section the results for axially-stiff and circumferentially-stiff cylinders subjected to an overall axial compressive strain of 2390$\mu$ε and 7730$\mu$ε, respectively, will be discussed. These values of axial strain are within 5% of the strain at buckling, and therefore cylinder response would be considered to be in the geometrically nonlinear range. The total axial loads for these two strain levels are 19,320 lbs. and 31,900 lbs., respectively. Figures 4.16-4.18 show the predicted displacement responses for both cylinders. The displacements have again been normalized by the laminate thickness $H$. Compared to the geometrically linear case of figs. 4.2-4.4 discussed in the last section, there is wrinkling of certain segments of the cylinder in the axial direction. This is most evident in fig. 4.18, which shows the radial displacement response.

As with the response in the geometrically linear range discussed in the last section, the normalized axial displacements plots shown in figs. 4.16(a) and (b) reflect an overall linear distribution of $u$ with respect to axial coordinate $x$, and very little variation with respect to the circumferential coordinate $s$. Recall again that with the analysis here, the end at $x=L$ is restrained from moving axially and the end at $x=0$ moves $\Delta$. The condition of $\Delta$ being independent of circumferential location provides an overwhelming effect, forcing the axial displacement to be independent of circumferential position, despite the variation in stiffness with circumferential position in a particular cylinder and the overall differences in axial stiffness between the two cylinders.

The normalized circumferential displacement spectrum plots are shown in figs. 4.17(a) and (b). Like the response in the geometrically linear range, the greatest circumferential displacement occurs for both cylinders at cylinder midlength ($x = L/2$) and at the overlap location ($\theta = \pm30^\circ$). The spectrum plot in fig. 4.17(a) for the axially-stiff cylinder shows that circumferential displacement at the $\theta = +30^\circ$ overlap has a negative value and at the $\theta = -30^\circ$ overlap location has a positive value. The plot of normalized circumferential displacement vs. circumferential location at the midlength for the axially-stiff cylinder, shown in fig. 4.19(a), is similar to the
response in the linear range, fig. 4.5(a), except for magnitude, and depicts high circumferential gradients in the circumferential displacement near the overlaps and sign reversals over relatively small spans of the circumference at the side locations. The geometrically nonlinear response for the circumferentially-stiff cylinder in figs. 4.17(b) and 4.19(b) displays the same general circumferential displacement pattern as in the geometrically linear range, and it is similar to the axially-stiff case. At the sides, as seen by comparing figs. 4.5(b) and 4.19(b), there are some differences, however.

Figures 4.18(a) and (b) show the normalized radial displacement spectrum plots for both cylinders. Compared to the response in the geometrically linear range, figs. 4.4(a) and (b), there is a significant difference. For the axially-stiff cylinder, fig. 4.18(a), there is wrinkling of the crown, while the sides are much less wrinkled. For the circumferentially-stiff cylinder, fig. 4.18(b), there is wrinkling of the sides, while the crown is much less deformed. Why certain segments wrinkle and others do not is a function of the portion of the axial load being reacted by each segment and the bending stiffness of the segment. For the axially-stiff cylinder, at least in the geometrically linear range, the crown and keel each react about 30% of the total axial load (see Table 4.2), while the sides each react about 14%. For the circumferentially-stiff case, the sides again react the majority of the total axial load. Specifically, the sides each react about 28% of the total axial load and the crown and keel each react about 17%. This means that as the endshortening increases and the cylinder nears buckling, the segments reacting a greater proportion of the load tend to buckle first. This, of course, assumes the bending stiffnesses of the segments are such that the radial deformations can develop into the short wavelength wrinkle pattern.

As a result of the wrinkling of certain segments and not others, there is an even greater variation of the radial displacements with respect to the circumferential coordinates than in the geometrically linear range of response. This wrinkling of both cylinders and the difference of radial displacement with circumferential position for the axially-stiff cylinder can be readily observed by examining the normalized radial displacement vs. the normalized axial location at different circumferential locations shown in fig. 4.20(a). The normalized radial displacement vs. normalized axial location responses shown are for $\theta=0^\circ$, the axial centerline of the crown.
segment, and $\theta=90^\circ$, the axial centerline of a side segment. The extent of wrinkling can be observed by noting the differences in the responses of the crown and the side laminates. Figure 4.20(b) shows the same normalized radial displacement vs. normalized axial location response for the circumferentially-stiff cylinder. Compared to figs. 4.7(a) and (b), which show the same normalized radial displacement vs. normalized axial location relations for the linear range of response, an alternative interpretation of the wrinkling is that the boundary layer in the crown of the axially-stiff cylinder engulfs one-half of the length of the cylinder. A similar interpretation can be made for the sides of the circumferentially-stiff cylinder. Regardless of the interpretation, fig. 4.20(a) shows that for the axially-stiff cylinder the average radial displacement of the wrinkled crown has about the same magnitude as the side, while for the circumferentially-stiff case, the crown has much smaller overall radial displacements. This is basically the same situation as for the linear range of response.

The plot of normalized radial displacement vs. circumferential location at the midlength in fig. 4.21(a) shows the nonuniformity of radial displacement, $w$, with respect to $\theta$ in the axially-stiff cylinder. Although the greatest values of radial displacement are in the crown segment, the $w$-displacement at midlength is actually greatest at the side. This is not evident in fig. 4.21(a). Because of the symmetry of the longitudinal waves with respect to cylinder midspan exhibited in the radial displacement (see fig. 4.20(a)), the trace of $w$-displacement in the crown at the midlength begins in a low-point, or trough. After the overlap position, as $\theta$ increases, the radial displacement decreases by about 50% relative to the value in the crown. As $\theta$ increases further, the displacement increases again to a local maximum at the mid-arc of the side segment that is somewhat greater in value than the displacement in the crown ($\theta = 90^\circ$). This pattern reverses as the keel is approached ($\theta=180^\circ$). For the circumferentially-stiff cylinder, the maximum radial displacement occurs at the side and the radial displacement of the crown is about 50% less. The high circumferential gradients due to these various effects may lead to high circumferential strains and bending stresses.

The axial force resultant $N_x$ for the axially-stiff cylinder is shown in fig. 4.22(a) in the form of a spectrum plot. In this figure the stress resultant is normalized as in eq. 4.1. Compared to
the response in the geometrically linear range, figs. 4.8(a) and (b), there is not much overall difference. The wrinkling of the crown, however, causes $N_x$ to vary, particularly in the crown with $x$, more for the nonlinear range than for the linear range. The ratio of $N_x$ in the crown to $N_x$ in the side is about 4:1 for both the linear and nonlinear ranges. For the circumferentially-stiff case, fig. 4.22(b), compared to the linear range, fig. 4.8(b), $N_x$ varies somewhat in the side due to the wrinkling there. The ratio of $N_x$ in the crown to that in the side is about 1:1, as it was for the linear range. In both cylinders, the value of $N_x$ in the overlap is high and about the same as for the linear range.

Figures 4.23(a) and (b) show the character of the circumferential stress resultant $N_y$ for the nonlinear response, normalized as in eq. 4.1, for the axially-stiff and circumferentially-stiff cylinders, respectively. In the unwrinkled segments, $N_y$ is negligible away from the potted boundaries and is highest in magnitude within the potted boundaries. In the segments where wrinkling is apparent, the variation in $N_y$ is notable. Along the longitudinal waves, $N_y$ varies from positive values on the crests of the waves to negative values in the troughs. The average value, however, is close to zero. The largest positive and negative values $N_y$ in the wrinkled segments occur near the ends of the cylinder, in the boundary layer. Note that the locally high negative values of $N_y$, combined with the overall compressive state introduced by increasing $\Delta$, creates the potential for biaxial compression in the cylinder wall that could give rise to stability problems.

As noted before, bending boundary layer length due to clamped end conditions is layup dependent. The distributions of $N_{xy}$, normalized as in eq. 4.1, for the axially-stiff and circumferentially-stiff cylinders are shown in figs. 4.24(a) and (b), respectively. Note the fact that at the overlaps, within and near the potted ends, there are high values of the inplane shear force resultants, while they are practically zero everywhere else. Even the deformations of the wrinkled segments do very little to generate an inplane shear force resultant. Note that the high shear force resultants near the boundaries are larger than their counterpart for the linear range of response shown in figs. 4.10(a) and (b).
Figures 4.25(a) and (b) show the characteristics of the axial bending moment resultant, $M_x$, for the axially-stiff and circumferentially-stiff cylinders, respectively. In the figures the moment resultants have been normalized as in eq. 4.4. As with the linear range of response, for both cylinders these moment resultants are dominated by their values in the overlap regions. In the unwrinkled segments there is a slight boundary layer effect in both cylinders, where the values of $M_x$ range from negative to positive moving in the axial direction, beginning at the end of the potting and ending effectively at the end of the bending boundary layer region. The wrinkled segments show a variation of the $M_x$ value from slightly positive at the crests to slightly negative at the troughs along the axial direction.

The circumferential bending moment resultant, $M_y$, in both cylinders is shown in figs. 4.26(a) and (b), where the values have been normalized as in eq. 4.4. In both cases, $M_y$ in the unwrinkled segments is virtually zero throughout the segments. Non-zero values in the boundary layer take on even larger values in the overlap regions of the boundary layer. The wrinkled segments show crest-to-trough variation from slightly positive to slightly negative. The same characterization can be applied to the inplane moment resultant $M_{xy}$, as shown in figs. 4.27(a) and (b). A boundary layer effect gives rise to a band of non-zero values of $M_{xy}$. It is interesting to note that there are highly localized values of $M_{xy}$ in the overlap and away from the boundary in the circumferentially-stiff case.

Finally, the out-of-plane shear force resultants $Q_x$ and $Q_y$ are shown in figs. 4.28 and 4.29. The values have been normalized by the quantity in eq. 4.1 so they can be directly compared in magnitude directly to the inplane force resultants of figs. 4.22-4.24. Like for the linear range of response, figs. 4.14 and 4.15, figs. 4.28 and 4.29 show that the values of $Q_x$ and $Q_y$ are much smaller than shear force resultants $N_x$ and $N_y$. For $Q_y$ for the circumferentially-stiff case, there is a local value in the overlap that is higher than the linear case. As there are no fibers to resist are the out-of-plane shear stress resultants, as is the case for the inplane resultants, failure due to excessive values of $Q_x$ and $Q_y$ is a possibility.
Fig. 4.16 - Axial displacement contours, nonlinear range

Fig. 4.17 - Circumferential displacement contours, nonlinear range

Fig. 4.18 - Radial displacement contours, nonlinear range
Fig. 4.19- Circumferential displacement vs. circumferential location at midlength, nonlinear range

(a) axially stiff
(b) circumferentially stiff

Fig. 4.20 - Radial displacement vs. axial location, nonlinear range

(a) axially stiff
(b) circumferentially stiff

Fig. 4.21 - Radial displacement vs. circumferential location at the midlength, nonlinear range

(a) axially stiff
(b) circumferentially stiff
Fig. 4.22 - Axial force resultant \( N_x \), nonlinear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.22 - Axial force resultant \( N_x \), nonlinear range
Fig. 4.23 - Circumferential force resultant $N_y$, nonlinear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.23 - Circumferential force resultant $N_y$, nonlinear range
Fig. 4.24 - Shear force resultant $N_{xy}$, nonlinear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.24 - Shear force resultant $N_{xy}$, nonlinear range
Fig. 4.25 - Axial bending moment resultant $M_x$, nonlinear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.25 - Axial bending moment resultant $M_x$, nonlinear range
Fig. 4.26 - Circumferential bending moment resultant $M_y$, nonlinear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.26 - Circumferential bending moment resultant $M_y$, nonlinear range
Fig. 4.27 - Twist moment resultant $M_{xy}$, nonlinear range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.27 - Twist moment resultant $M_{xy}$, nonlinear range
Fig. 4.28- Transverse shear resultants for axially-stiff cylinder, nonlinear range

(a) axial shear component, $Q_x$

(b) circumferential shear component, $Q_y$

Fig. 4.28- Transverse shear resultants for axially-stiff cylinder, nonlinear range
Fig. 4.29 - Transverse shear resultants for circumferentially-stiff cylinder, nonlinear range
4.3 Buckling Response

The past sections have shown that as the endshortening increases, the deformation pattern of the segmented stiffness cylinders evolves. To review: at low values of endshortening, the cylinders have a short boundary layer region, where the displacements vary rapidly with axial coordinate, and a longer central region where the response does not vary as rapidly with the axial coordinate. At higher levels of endshortening, the character of the longer central region changes to one where there are a number of wrinkles in the axial direction in some segments. Which segments are wrinkled depends on whether the axially-stiff or circumferentially-stiff cylinder is being considered. In either case, as $\Delta$ is increased, a critical value is reached at which the structure becomes unstable and can not support the increased compressive load level (see fig 4.1). The load corresponding to this value of $\Delta$ is referred to as the buckling load, as denoted in fig. 4.30. Buckling of a structure, or alternatively, the loss of stability of the structure, is defined as a condition of equilibrium whereby the addition, or introduction, of small deformations to an existing state of deformation of a structure does not result in any change in the second variation of the total potential energy of the structure. These slightly altered states of deformation are also in equilibrium. Stated differently, for the buckling condition, a structure can have multiple states of deformation, each one being in equilibrium, and the states do not differ significantly from each other and all result in the structure having the same total potential energy. An important question associated with buckling is the state of deformation to which the small deformations are added. Strictly speaking, for the problem at hand, the small deformations should be added to the state determined by the geometrically nonlinear analysis, which was discussed in the last section. Formally, this would be referred to as a buckling analysis relative to a geometrically nonlinear equilibrium state. Often, however, buckling conditions are computed relative to simpler states of deformation. For cylinders this has historically been the case because of the complexity of geometrically nonlinear analyses. However, the existence of advanced finite-element analysis codes has somewhat relieved this constraint. Despite this, many researchers still find it convenient to compute results, conduct parametric studies, and make comparisons of buckling response using a prebuckling state that can be computed by less advanced analyses. For a cylinder, one of these simpler prebuckling states is a membrane state, whereby the bending effects in the boundary layer are ignored and the only stress resultants assume to exist are the inplane ones. For a more complicated analysis, the response in the geometrically linear range, as discussed in section 4.2.1, is assumed to be the pre-
buckling state. The STAGS finite-element code is capable of this latter buckling analysis, referred to as a linear buckling analysis, and it is capable of a geometrically nonlinear analysis. Furthermore, STAGS has the capability to assess the stability of the nonlinear equilibrium state by computing the eigenvalues of the tangent stiffness matrix associated with that state of equilibrium. If one eigenvalue is negative, the structure is considered unstable. The key to using this feature in STAGS of computing the eigenvalues of the tangent stiffness matrix is to find the lowest level of endshortening, or lowest level of load, that leads to a zero eigenvalue. This condition represents the transition from stability to instability. Often considerable iteration is required to find the condition leading to an eigenvalue being exactly zero. If found, the condition is the best estimate of stability. Here an effort was made to find this condition. As stated in the introduction to this chapter, the buckling values of endshortening for the axially-stiff and circumferentially-stiff cylinders are taken to be 0.0351 and 0.1087 in., respectively. The associated loads are 19,810 and 32,000 lbs. These values are summarized in Table 4.3 under the entry ‘Nonlinear’ and are the values as determined by the zero eigenvalue condition. In Table 4.3 the value of axial strain at buckling is also included. As a comparison, a linear buckling analysis indicates that for the lowest mode, the buckling displacement for the axially-stiff and circumferentially-stiff cylinders are 0.0333 and 0.1009 in., respectively. The buckling loads are 19,700 and 31,700 lb. These results are summarized in Table 4.3. The load and endshortening predictions using the linear analysis are very close to those associated with the zero eigenvalues, demonstrating that the use of a simpler prebuckling state can give accurate results for this case. Figures 4.31(a) and 4.31(b) show the buckling displacements for the lowest mode from the linear buckling analysis for both the axially-stiff and circumferentially-stiff cylinders, respectively. Upon observation, it can be noted that in both cases the buckling displacements reflect the character of radial displacement patterns of the nonlinear prebuckling responses shown in fig. 4.18. For the axially-stiff cylinder the buckling displacements show longitudinal waviness in the crown and keel segments and relatively undeformed side segments. For the circumferentially-stiff cylinder the buckling displacements show longitudinal waviness in the side segments and relatively undeformed crown and keel segments.

As alternative predictions, consider the buckling conditions as predicted by using a geometrically nonlinear prebuckling analysis at 85% of the endshortening value given by the zero eigenvalue condition. Figure 4.32 shows the buckled shapes for these computations for both the
axially-stiff and circumferentially-stiff cylinders and Table 4.3 summarizes the endshortening and load levels predicted. It is seen that the buckling displacements are very similar to those predicted by the linear buckling analysis, which makes them similar to the displacements shown in fig. 4.18 for the geometrically nonlinear prebuckling analysis. The values of endshortening and load are also close to the zero eigenvalue condition. As a matter of interest, the results of similar calculations obtained using a prebuckling state at 95% of the zero eigenvalue conditions are shown in Table 4.3. Overall, for each cylinder the four estimates of the buckling endshortening values and loads are very close.

<table>
<thead>
<tr>
<th>Cylinder description</th>
<th>Analysis type</th>
<th>$\Delta_{cr}$ (in)</th>
<th>$P_{cr}$ (lb)</th>
<th>$\langle \varepsilon_x^o \rangle_{cr}$ (µε)</th>
</tr>
</thead>
<tbody>
<tr>
<td>axially stiff</td>
<td>Nonlinear</td>
<td>0.0351</td>
<td>19,810</td>
<td>2,510</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>0.0333</td>
<td>19,700</td>
<td>2,380</td>
</tr>
<tr>
<td></td>
<td>Nonlinear EVP 85%</td>
<td>0.0400</td>
<td>19,820</td>
<td>2,850</td>
</tr>
<tr>
<td></td>
<td>Nonlinear EVP 95%</td>
<td>0.0346</td>
<td>19,960</td>
<td>2,470</td>
</tr>
<tr>
<td>circumferentially stiff</td>
<td>Nonlinear</td>
<td>0.1087</td>
<td>32,000</td>
<td>7,760</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>0.1009</td>
<td>31,700</td>
<td>7,210</td>
</tr>
<tr>
<td></td>
<td>Nonlinear EVP 85%</td>
<td>0.1083</td>
<td>32,700</td>
<td>7,740</td>
</tr>
<tr>
<td></td>
<td>Nonlinear EVP 95%</td>
<td>0.1107</td>
<td>33,000</td>
<td>7,910</td>
</tr>
</tbody>
</table>
(a) axially-stiff cylinder

(b) circumferentially-stiff cylinder

Fig. 4.30 - Load vs. endshortening relations
Fig. 4.31 - Linear buckling solutions (mode 1)
Fig. 4.32 - Nonlinear buckling analysis initiated at 85% predicted buckling load
4.4 Postbuckling Response

When the endshortening is increased beyond the buckling value, the cylinder deformations depart significantly from those observed up to the point of buckling, and the axial load supported suddenly decreases. The cylinder deformations can consist of large localized inward and outward radial displacements, not necessarily the periodic wrinkling observed at lower endshortening levels. The transition from the state just prior to buckling, through buckling, and to this state of large localized deformations occurs suddenly, often leading to audible emissions during experiments. As most of the large localized deformations are inward, this transition state is sometimes referred to as snap through and it is often referred to in the literature as the collapse state. It is a dynamic event and must be treated as such.

Figure 4.33 shows the load vs. endshortening relationships resulting from a nonlinear finite-element analysis of both the axially-stiff and circumferentially-stiff cylinders. This figure is essentially a repeat of fig. 4.1, which was shown earlier for the purpose of an introduction. Consider by way of example the results for the axially-stiff cylinder shown fig. 4.33(a), and note that the buckling condition is labeled A. The buckling condition is one condition in the transition from stable prebuckling equilibrium to stable postbuckling equilibrium, point B. With increased endshortening beyond point B to point C, the load level increases again. The fact that a circular cylinder is capable of sustaining loading after collapse is attributed to postbuckling stiffness formed by the interaction of the membrane stiffness and the bending stiffness.

The endshortening analyses performed to this point in the discussion to obtain the prebuckling results shown in the previous sections has been a static geometrically nonlinear finite-element analysis. Continuing with a static analysis to pass from the nonlinear prebuckling condition through the buckling condition and into the postbuckling regime represents a great challenge numerically. To avoid these difficulties, and to more closely represent what happens physically, for the present study, the transient dynamic approach was taken. It was used for both the axially-stiff and circumferentially-stiff cylinder finite-element models to pass from the point A to point B. Below is an outline of the steps taken, assuming that the configuration of the cylinder is unstable at point A, which is arrived at by a nonlinear static analysis.
Step 1: Begin the dynamic transient analysis by incorporating damping into the finite element model.

The goal of the transient analysis is to find the dynamic response of the cylinder using a nonlinear finite-element analysis to solve the equations of motion given by

\[
[M] \ddot{u}(t) + [C] \dot{u}(t) + \{f(\lambda, u(t))\} = 0
\]  

(4.5)

where \([M]\) and \([C]\) denote the mass and damping matrices, respectively, and \(u(t)\) is the matrix of time-dependent nodal displacements. A dot above a variable denotes a derivative with respect to time. The term \(f(\lambda, u(t))\) represents a set of nonlinear functions of the nodal degrees-of-freedom and the load parameter \(\lambda\) which describe the structural stiffness and the external loads. By definition, the tangent stiffness matrix, \([K]\), is given by

\[
[K] = \frac{\partial f}{\partial u}
\]  

(4.6)

Note, the equations of motion govern the dynamic behavior of the structure, but if the load is applied slowly, as in the case of a static analysis, the velocity and acceleration terms can be neglected. To represent damping in the dynamic equations, STAGS employs Rayleigh’s proportional damping of the form

\[
[C] = \alpha [M] + \beta [K]
\]  

(4.7)

where \(\alpha\) and \(\beta\) are mass and stiffness damping factors, respectively. Assuming, the dynamic response to be dominated by a particular frequency of vibration \(\nu\), \(\alpha\) and \(\beta\) can be calculated by the following relations:
where $\zeta$ is the non-dimensional viscous damping factor and $\nu$ is the first fundamental frequency. The value for $\nu$ is obtained from a linear vibration analysis at zero load performed using STAGS. For the present study, Table 4.4 shows the values of $\nu$, $\alpha$, and $\beta$ for both axially-stiff and circumferentially-stiff cylinders.

### Table 4.4: Damping parameters for transient analysis of segmented stiffness cylinders

<table>
<thead>
<tr>
<th>cylinder description</th>
<th>$\nu$ (sec$^{-1}$)</th>
<th>$\alpha$ (sec$^{-1}$)</th>
<th>$\beta \times 10^{-5}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>axially stiff</td>
<td>898</td>
<td>735</td>
<td>3.06</td>
</tr>
<tr>
<td>circumferentially stiff</td>
<td>870</td>
<td>694</td>
<td>3.24</td>
</tr>
</tbody>
</table>

Step 2: Apply a “small” inward pressure load over a specific “short” time interval. That is, apply a small inward pressure pulse while holding the endshortening fixed.

The fixed endshortening condition simulates what would happen in a displacement-controlled experiment. In order to initiate the dynamic response, a slight perturbation to the cylinder must be applied when the cylinder is in a statically unstable equilibrium state at or near point A. The perturbation is taken to be a live pressure load, applied inwardly. In effect, the transient analysis allows the load vs. endshortening solution to move from an unstable equilibrium condition near point A to the postbuckling equilibrium path at point B. The magnitude of the live pressure load

$$\alpha = 2\pi \nu \zeta \quad \text{(4.8)}$$

$$\beta = \frac{\zeta}{2\pi \nu} \quad \text{(4.9)}$$
and the relatively small time interval over which it is applied have to be chosen carefully. The load has to be just large enough to initiate the dynamic response in the cylinder, but also small enough to ensure that the result of the transient analysis is the equilibrium configuration having the lowest potential energy. For the axially-stiff cylinder, a 1-psi load was applied, in ramp-up/ramp-down fashion, over a $1.00 \times 10^{-2}$ second time interval. For the circumferentially-stiff cylinder, a 2-psi load was applied in the same fashion over a $2.00 \times 10^{-2}$ second time interval. Other pressure levels and time intervals were examined and gave similar results. Ideally, no pressure pulse would be needed, as simply initiating the dynamic analysis would allow a structure to move away from an unstable configuration. However, this may take a number of integration steps. To accelerate the move to a stable equilibrium state, the pressure pulse was used.

**Step 3**: Begin the transient dynamic solution using a numerical time integrator.

The pressure pulse acts as the excitation needed to initiate the dynamic analysis starting at point A. Time equal to zero for the dynamic analysis begins at the start of the pressure pulse. The equations of motion are solved using the numerical time integrator in STAGS. Considering the load vs. endshortening relations shown in fig. 4.33, the line between A and B represents the transient portion of the analysis.

**Step 4**: Integrate forward in time while monitoring the kinetic energy until it dissipates to an insignificant level compared to the maximum level. Terminate the transient analysis.

After the pressure pulse initiates the transient response, the kinetic energy of the underdamped system initially peaks and then dissipates with each time step in the analysis. Over time, the cylinder appears to be converging to a specific configuration and the kinetic energy becomes smaller and smaller. The specific configuration is considered the stable postbuckling equilibrium configuration.
Step 5: Use the load relaxation technique to make the transition from the dynamic analysis to the static analysis.

With the load relaxation technique, the inertia and damping terms in the governing equations are multiplied by a scalar factor. The influences of the inertia and damping terms are slowly decreased (relaxed) by solving the equilibrium equations a number of times, gradually letting the scalar factor approach zero, thereby eventually phasing out the dynamic terms in the equilibrium equations.

Step 6: Restart the static analysis.

The static analysis of the cylinder is re-initiated by using the results from the load relaxation calculations to essentially begin a new static nonlinear finite-element analysis. The endshortening is slowly increased and the path from B to C is found.

In the paragraphs to follow, the postbuckling response of both cylinders is discussed. The response corresponds to a point between points B and C in fig. 4.33. Specifically, for the axially-stiff cylinder, the postbuckling axial compressive strain is 3,010 µε and the postbuckling axial compressive load is 17,360 lbs. For the circumferentially-stiff cylinder, the postbuckling axial compressive strain is 11,300 µε and the postbuckling axial compressive load is 16,370 lbs.

It should be noted in fig. 4.33 that the vertical line going from point A to point B extend below point B. This reflects the fact that during the transient analysis for a fixed level of endshortening, the load level oscillates around stable point B as the solution converges with time to point B. The load goes lower than point B, then higher, then lower, etc., in a damped oscillating fashion.

Figures 4.34 - 4.36 show the predicted postbuckling displacement responses for both cylinders. The displacements have been normalized, in accordance with pervious sections, by laminate thickness \( H \). The spectrum plots for the radial displacement contours, fig. 4.36, show clearly that in both cases the displacement responses are dominated by diamond-shaped inward...
dimples. The overall dimpling varies between the two cylinder constructions. In the axially-stiff case, the displacement pattern is dominated by a single diamond-shaped inward dimple in the crown and keel segments that are bounded by the overlap segments. The circumferentially-stiff case is dominated by two rows of inward dimples, also diamond-shaped, which ring the cylinder’s middle, forming an anti-symmetric pattern. The two circumferential rings of dimples run along lines at \( x = 3L/8 \) and \( x = 5L/8 \). These circumferential rings of dimples consist of six inward dimples each. The ring at \( x = 3L/8 \) is made up of dimples which are contained in the main segments of the cylinder: two in each side segment and one each in the crown and keel segments, for a total of six dimples. The ring of dimples at \( x = 5L/8 \) displays an alternate symmetry: one dimple in each of the side segments and one dimple each centered on each overlap, again for a total of six dimples. These two rings of dimples have as a line of anti-symmetry the circumference at \( x = L/2 \).

Figure 4.34 shows the axial displacements. In both cases there are relatively small gradients in the axial or circumferential directions near the ends of the cylinder. In the axially-stiff cylinder, fig. 4.34(a), away from the ends, the crown and keel segments show gradients of the axial displacement in both directions. The sides of the axially-stiff configuration show a gradient in the axial displacement in the axial direction and barely any in the circumferential direction, reminiscent of the axial displacement pattern shown in 4.16(a) for the response in the geometrically nonlinear range. The axial displacement pattern for the circumferentially-stiff cylinder shown in fig. 4.34(b) reflects gradients in the axial and circumferential directions concentrated around the middle of the cylinder. The circumferential displacements shown in fig. 4.35 for both cylinders are effectively zero everywhere except around the dimples. There the normalized values indicate circumferential displacement values on the order of one laminate thickness. In the axially-stiff cylinder shown in fig. 4.35(a), the circumferential displacement pattern reflects that the dimple is causing the crown and keel segments to stretch circumferentially away from the trough of the dimple toward the ridges in the outward normal displacement in the overlap at \( x = L/2 \). The circumferential displacement pattern of fig. 4.35(b) indicates the same stretching away from the troughs of the dimples toward the ridges for the circumferentially-stiff cylinder. For the radial displacement configuration of the circumferentially-stiff cylinder shown in fig. 4.36(b), these ridges are seen as zones of positive
radial displacement and the troughs of the dimples are negative radial displacement. Note that for the circumferentially-stiff cylinder the normalized value of the radial displacement in the inward dimple is over 9 times the laminate thickness. For the axially-stiff cylinder, shown in fig. 4.36(a), the inward dimples in the crown and keel segments are on the order of 7 times the laminate thickness. The formation of ridges in the overlap due to circumferential displacement contributes to the large outward radial displacement present at \( x = L/2 \) along the overlaps.

Line plots in figs. 4.37-4.39 provide a closer look at some of the details of the dimpled cylinder configurations that result from snap through, and also further characterize the postbuckling response. Figure 4.37 shows line plots of normalized circumferential displacement as a function of circumferential location for both cylinders. In the case of the axially-stiff cylinder, this line is plotted around the middle of the cylinder, i.e., at \( x = L/2 \). Noting from the spectrum plot shown in fig. 4.35(a), the circumferential displacement in the axially-stiff cylinder is practically zero everywhere except near the edges of the dimples. The line plot of fig. 4.37(a) shows the circumferential displacement to reach maxima in the crown and keel segments between the overlaps. In the case of the circumferentially-stiff cylinder, the line plot for normalized circumferential displacement as a function of circumferential location is plotted around the circumference at about \( x = 3L/8 \). This is done in order to capture the maxima in the circumferential displacements. Noting the spectrum plot of fig. 4.35(b), the circumferential displacement in the circumferentially-stiff cylinder around the circumference at \( x = L/2 \) is practically zero. The line plot in fig. 4.37(b) at \( x = 3L/8 \) shows the circumferential displacement going from maxima to minima over relatively short circumferential distances. This would indicate high gradients in the circumferential direction, a cause for concern.

Figure 4.38 shows normalized radial displacement as a function of normalized axial location. For each cylinder the character along two lines is plotted: one along the center of the crown segment and another along the center of the side segment. These two lines capture the character of the radial response due to the symmetry or anti-symmetry present in the cylinder configurations. Figure 4.38(a) shows the radial displacement in the side segment of the axially-stiff cylinder to have nearly the same character as the nonlinear prebuckling response shown in fig. 4.20(a), a response characterized by a slight axial boundary layer and very little gradient in the axial direc-
tion in the membrane region. The line plot along the center of the crown segment shows no significant boundary layer effect and an overwhelming inward dimpling in the center of the crown. For the circumferentially-stiff cylinder, fig. 4.38(b) shows the anti-symmetry of the radial displacement response in the crown and side segments. The most significant effect in the result shown in fig. 4.38(b) is that, unlike the prebuckled response in the geometrically nonlinear range for the circumferentially-stiff cylinder, the postbuckled cylinder shows significant radial displacement in the crown, keel, and sides, rather than just in the sides.

Figure 4.39 shows line plots of normalized radial displacement as a function of circumferential location in both cylinders. In fig. 4.39(a) the character of the normalized radial displacement along $x = L/2$ is plotted for the axially-stiff cylinder. Note that the large inward dimples in the crown and keel segments are over 7 times the laminate thickness. In fig. 4.39(b) the normalized radial displacement along the line at $x = 3L/8$ is plotted. Like the spectrum plot of fig. 4.36(b), this line plot shows drastic swings between maxima and minima over relatively short circumferential distances. Note again that the inward dimples are over 9 times the laminate thickness. Here, the large value of the inward radial displacement accompanied by the steep gradients in circumferential direction are also a cause for concern in the context of failure.

The displacement results discussed so far indicate that the postbuckling response in both cylinders is dominated by diamond-shaped inward dimples whose radial displacement is quite significant in value. The stress resultants for the segmented cylinders are affected greatly by this dimpling. The snap through that occurs in postbuckling, allowing the cylinders to deform into these dimpled configurations, causes a redistribution of the loading in the cylinder wall. This redistribution is required in order to satisfy the stability and equilibrium considerations which govern the behavior of the cylinder. A discussion of the overall stress state of the cylinder in postbuckling essentially is a discussion of the effect of the dimpling present due to snap through on the redistribution of load in the cylinder.

The axial force resultant $N_x$ for the axially-stiff cylinder in postbuckling is shown in fig. 4.40(a). The spectrum plot shows the maximum compressive value of $N_x$ value occurring at the corners of the dimples at $x = L/2$ just inside the overlaps. In an overall sense, the contouring of
fig. 4.40(a) shows that the overlaps support more of the axial compressive load than the other segments in the cylinder. In fact, \(N_x\) is small along the centerline of the crown and keel segments. Comparing fig. 4.40(a) and the geometrically nonlinear prebuckling axial force resultant spectrum plots in fig. 4.22(a), it is seen there is a significant redistribution of \(N_x\) and a change in the range of values. The range of normalized \(N_x\) in the geometrically nonlinear prebuckling range shown in fig. 4.22(a) is compressive 0.551 to 2.39. In postbuckling, the range is 0.315 to 5.07. The load bearing capacity of the crown and keel segments has been diminished due the dimpling. The axial force resultant spectrum plot for the circumferentially-stiff cylinder is shown in fig. 4.40(b). A path of maximum compressive value of \(N_x\) runs along the overlap from \(x = 0\). At around \(x = 3L/8\) the path splits and follows the ridges formed between adjacent inward dimples. This path of maximum compressive \(N_x\) becomes more diffuse as it reaches \(x = L\). It follows the ridges in the overlaps between dimples at \(x = 3L/8\), passes \(x = L/2\) on the ridges, and diminishes in value around \(x = 5L/8\) to proceed to \(x = L\). The axial force resultant also has a high compressive value in the troughs of dimples that are centered on overlaps at \(x = 5L/8\). Even though the cylinder is under the influence of endshortening, positive values of normalized \(N_x\) exist in some of the ridges between dimples, particularly in the ridges on the overlaps. The load redistribution in the circumferentially-stiff cylinder can be put into context by considering fig. 4.22(b). The nonlinear prebuckling response is somewhat segmented and certainly more uniform, reflecting the geometry and stiffness distribution of the cylinder. In postbuckling the response is quite nonuniform, with highly localized compressive values.

Spectrum plots for normalized circumferential force resultant \(N_y\) for both the axially- and circumferentially-stiff cylinders are shown in fig. 4.41. Recall from discussion of the circumferential displacement shown in fig. 4.35 that there is a circumferential stretching effect in the troughs of the dimples. In the axially-stiff cylinder, shown in fig. 4.41(a), there are regions of high compressive \(N_y\) in the corners (at the ends) of the dimple, just inside of the overlap. There is also appreciable tensile \(N_y\) around the perimeter of the dimple. The existence of high compressive \(N_y\), in combination with compressive \(N_x\), gives rise to concerns about biaxial compression occurring in the corners (at the ends) of the dimple, near the overlaps. To discuss load redistribution, consider fig. 4.23(a), which shows the nonlinear prebuckling spectrum plot of \(N_y\) for the axially-stiff
cylinder. The nonlinear prebuckling distribution of $N_y$ depends upon the wrinkling present in the crown and keel segments and has a maximum compressive value of 0.64 that occurs near the ends of the cylinder. In postbuckling, the maximum compressive value occurs at $x = L/2$ with a value around 1.42, a value more than double the nonlinear prebuckling value. Considering the normalized circumferential force resultant for the circumferentially-stiff cylinder in fig. 4.41(b), it is seen that there are rings of high compressive values of $N_y$ near $x = 3L/8$ and $5L/8$. At $x = L/2$ there is a ring of high tensile $N_y$, indicating a stretch of the circumference along the line at $x = L/2$. The maximum compressive $N_y$ is at the corners of the small diamond-shaped dimples which ring the circumference. Interestingly, the pattern of $N_y$ shows no dependence on the segmented construction. The similarity of the pattern of normalized $N_y$ in the multiple dimples of the circumferentially-stiff cylinder to that of the single dimple of the axially-stiff case seems to indicate $N_y$ is more dependent on circumferential displacement than the segmented construction. The redistribution in the pattern of $N_y$ for the circumferentially-stiff cylinder can be observed by comparing the pattern for the postbuckling condition of fig. 4.41(b) with the nonlinear prebuckling pattern shown in fig. 4.23(b). In prebuckling, the distribution of $N_y$ is dependent on the wrinkled pattern in the side segments, reaching maximum compressive values in the potted region near the joint of the overlap and side segments. The range of the nonlinear prebuckling normalized circumferential force resultant is from -1.35 to 0.45, while the postbuckling range is from -2.79 to 2.30.

The normalized shear force resultant $N_{xy}$ for the both the axially- and circumferentially-stiff cylinders is shown in fig. 4.42. For the axially-stiff case shown in fig. 4.42(a), the shear force resultant is practically zero everywhere except at the point where the corners of the inward dimples in the crown and keel segments meet the overlaps near $x = L/2$. The shear force resultant reaches maximum and minimum values on either side of the $x = L/2$ location on the overlap. Overall, the normalized shear stress resultant is smaller than the maximum values for $N_x$ and $N_y$. However, due to the load redistribution after snap through, in postbuckling the ratio of the maximum shear force resultant to maximum normalized compressive axial force resultant is around 1:5, while the same ratio in nonlinear prebuckling is around 1:14. For the circumferentially-stiff cylinder shown in fig. 4.42(b), the normalized shear force resultant is practically zero everywhere except along the edges of inward dimples and on the ridges between dimples. As in the axially-
stiff cylinder, the ratio of the maximum normalized shear stress resultant to maximum normalized compressive axial stress decreases significantly in postbuckling to a value of about 1:3 compared to a value of about 1:10 for nonlinear prebuckling.

It is important to keep in mind that because of equilibrium requirements, gradients of $N_x$ in the axial direction require there be gradients of $N_{xy}$ in the circumferential direction. Therefore, though $N_{xy}$ may be zero in some region of the cylinder, say, away from the dimples and ridges, as $N_x$ changes with axial position, $N_{xy}$ cannot not remain zero and, in fact, will change the most rapidly with circumferential position when $N_x$ changes the most rapidly with axial position. Likewise, a gradient of $N_y$ in the circumferential direction requires there be a gradient of $N_{xy}$ in the axial direction. Thus the spectrum plots for $N_{xy}$ are coupled with the spectrum plots for $N_x$ and $N_y$, particularly the change in the spectrum plots for $N_x$ and $N_y$ with spatial location.

Figures 4.43-4.45 show spectrum plots of the moment resultants $M_x$, $M_y$, and $M_{xy}$ for the postbuckled axially- and circumferentially-stiff cylinder configurations. In general, moment resultants correspond to bending in the structure, i.e., changes in midsurface curvature. In prebuckling, figs. 4.25-4.27, the bending corresponds mostly to wrinkling in the crown, side, or keel segments. In postbuckling, the dimpled configurations of fig. 4.36 result in a spatial redistribution and an increase in magnitude of the moment resultants relative to the nonlinear prebuckling state. These trends increase the tendency of the structure to fail. For the axially-stiff cylinder, a spectrum plot of the normalized axial bending moment resultant $M_x$ is shown in fig. 4.43(a). The maximum positive value of $M_x$ occurs in the corners of the diamond-shaped inward dimples in the crown and keel segments, just inside the overlaps. The sign of $M_x$ changes just outside the dimple, and where the corner of the dimple meets the overlap the moment attains its maximum positive value. Clearly there are high bending gradients in the axial direction as a result of the dimple. A large negative value of the moment resultant occurs near the end of the cylinder in the overlaps at the end of the potting. The maximum negative value of $M_x$ in nonlinear prebuckling, shown in fig. 4.25(a), is 1.96. The maximum negative value in postbuckling is 2.92. The bending pattern of the
postbuckled cylinder reflects more bending activity and higher magnitudes. It should be noted that the value of $M_x$ in the entire overlap is high, as it was in prebuckling.

For the circumferentially-stiff cylinder, the spectrum plot of normalized $M_x$ is shown in fig. 4.43(b). Maximum negative values occur in the troughs of the overlaps centered on the stiffer overlap segments. Large positive values occur on the ridges between the dimples. For the circumferentially-stiff cylinder the maximum negative value for normalized $M_x$ in postbuckling is about 7. The maximum positive value for normalized $M_x$ in the nonlinear prebuckling spectrum plot shown in fig. 4.25(b) is 2.09. Note that the magnitude of $M_x$ in the overlap region away from the dimple does not standout, as it does for the nonlinear prebuckling condition, fig. 4.25(b).

Spectrum plots of normalized circumferential bending moment resultant $M_y$ for both the axially-stiff and circumferentially-stiff cylinders are shown in fig. 4.44. In the axially-stiff case shown in fig. 4.44(a), maximum negative values of $M_y$ occur in the troughs of the inward dimples in the crown and keel regions. Maximum positive values of $M_y$ occur in the overlaps at $x = L/2$, just outside the dimples, where the corners of the diamond-shaped dimples meet the overlaps. This close proximity of circumferential bending moment resultants of opposite sign suggest the presence of high stress gradients.

The spectrum plot for normalized $M_y$ for the circumferentially-stiff cylinder is shown in fig. 4.44(b). In general, there are high negative values of $M_y$ in the troughs of the dimples. The maximum negative value of $M_y$ occurs in the troughs of the dimples that fall on the overlaps. Maximum positive values of $M_y$ occur on the ridges between the dimples. Compared to the nonlinear prebuckling case, fig. 4.26, the extreme values of $M_y$ for both the axially-stiff and circumferentially-stiff cylinders are considerable higher, particularly for the circumferentially-stiff case. Also, the pattern of normalized $M_y$ in both cylinders, along with other elements of the response, indicate that the postbuckling response of the circumferentially-stiff cylinder is not driven by the segmented construction of the cylinder.
The spectrum plots for the normalized twist moment resultant $M_{xy}$ for both the axially-stiff and circumferentially-stiff cylinder are shown in fig. 4.45. In the axially-stiff cylinder, shown in fig. 4.45(a), it can be seen that the nonzero values in $M_{xy}$ are isolated to be within the dimples, with maximum positive and negative values occurring at the corners of the dimple, just inside the overlaps at $x = L/2$. The range of normalized $M_{xy}$ in the postbuckling response is from -0.791 to 1.35. In the prebuckling response for the same cylinder, shown in fig. 4.27(a), the range of normalized $M_{xy}$ is from -0.27 to 0.29. For the circumferentially-stiff cylinder, shown in fig. 4.45(b), the maximum positive and negative values of normalized $M_{xy}$ occur in the ridges, both on the overlap and away from the overlaps. For the circumferentially-stiff cylinder, the range of normalized $M_{xy}$ in the postbuckling response is from -4.51 to 3.09. For the nonlinear prebuckling response, shown in fig. 4.27(b), the range is from -0.297 to 0.357. The postbuckling response induces high values in normalized $M_{xy}$ compared to the nonlinear prebuckled state. This increased twisting moment could increase the tendency toward structural failure due to shear. It must be kept in mind that equilibrium conditions also require that when the value of $M_x$ or $M_y$ change with spatial location, the value of $M_{xy}$ must also change. Therefore, as it was for $N_x$, $N_y$, and $N_{xy}$, for a given cylinder, the spectrum plots for $M_x$, $M_y$, and $M_{xy}$ are related.

Plots of the normalized transverse shear resultants $Q_x$ and $Q_y$ are shown for the axially-stiff cylinder in fig. 4.46. The normalized axial transverse shear component $Q_x$ is shown in fig. 4.46(a). The maximum positive and negative values of $Q_x$ in postbuckling are nearly three times the values in nonlinear prebuckling, shown in fig. 4.28(a). Note extreme values occur within the dimples and at the points in the overlaps where there is a change in curvature due to the edges of the dimples meeting the overlaps. Moving in the axial direction through the dimple, the sign of $Q_x$ changes. The normalized values of $Q_y$ for the axially-stiff cylinder are nearly two times the nonlinear prebuckling values. The normalized $Q_y$ maximum positive and negative values occur at the points where the corners of the diamond-shaped inward dimples meet the overlaps at $x = L/2$. Moving in the circumferential direction through the dimple, the sign of $Q_y$ changes. These plots of the normalized shear components show very dramatically that the areas of concern for structural failure in postbuckling for the axially-stiff cylinder could be at the overlaps at $x = L/2$. 

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Figure 4.47 shows the spectrum plots for normalized transverse shear resultant $Q_x$ and $Q_y$ for the circumferentially-stiff cylinder. The values in the range of normalized $Q_x$ in the postbuckling response are nearly four times larger than the values in the nonlinear prebuckling response. The pattern of $Q_x$ seems to show some sign changes and a degree of anti-symmetry when moving through the dimples in the axial direction. However as these patterns of anti-symmetry intersect the overlaps, they are broken up and lead to maximum values of $Q_x$ in the overlaps. The normalized values of $Q_y$ for the circumferentially-stiff cylinder in postbuckling, fig. 4.47(b), have a larger range than the range in the nonlinear prebuckling response for the same cylinder. High values occur near the ends of the dimples at the overlaps. The plots for the normalized transverse shear resultants for the circumferentially-stiff cylinder, along with other evidence, seem to indicate that the failure of this cylinder will initiate at the overlaps along the line at either $x = 3L/8$ or $x = 5L/8$.

4.5 Summary

A discussion of the predicted response of segmented-stiffness axially-stiff and circumferentially-stiff cylinders subjected to uniform axial endshortening has been presented. In the linear prebuckling range the response of both cylinders is characterized by the existence of circumferential displacement and circumferential gradients in the radial displacement. It has been shown that the greatest outward radial displacement in the axially-stiff cylinder is in the crown segment. The greatest outward radial displacement for the circumferential cylinder is in the side segments. The distribution of the normalized axial force resultant also shows directly the effect of the segmented-stiffness construction. In the axially-stiff cylinder, the value of $N_x$ is higher in the crown and keel segments than in the side segments. In the circumferentially-stiff cylinder, the value of $N_x$ in the side segments is higher than in the crown and keel segments. For both cases, the value of $N_x$ in the overlap segments is highest overall. Because of eccentricity, the value of $M_x$ is not zero in the overlaps away from the ends. The linear prebuckling response is also characterized by an axial boundary layer effect in the radial displacement response. Joining different layups side-by-side to form the cylinders causes a mismatch in the radial responses near the ends of segmented-stiffness cylinders. Shear forces are created in the cylinder to overcome this mismatch. The nor-
malized components of $N_{xy}$, $M_{xy}$, and $Q_y$ provide evidence that there are shear effects in and around the overlap region near the ends to overcome the mismatch due to the boundary layer effect in the radial displacement.

For the nonlinear prebuckling range the results for the axially-stiff and circumferentially-stiff cylinders were presented for overall axial strain values close to predicted buckling values. For both cylinders, the response in the nonlinear prebuckling range is characterized by waviness in the axial direction. For the axially-stiff cylinder, the waviness occurs in the crown and keel segments. For the circumferentially-stiff cylinder, the waviness occurs in the side segments. The effect of the longitudinal waviness causes the load distribution established in the linear prebuckling range to change somewhat. The normalized axial force resultant maintains its segmented distribution, with the overlap segments having the highest overall values of $N_x$ for both cylinders. The normalized components of $N_y$, $M_x$, and $Q_x$ show the effect of the longitudinal waviness on the load distribution.

Four methods were used to compute the value of endshortening at buckling. There is good agreement among all four methods for both cylinders. For a slight increase in endshortening beyond the buckling value, cylinder behavior changes dramatically. As discussed, the behavior of the cylinder for this condition is dynamic and was treated as such. The dynamic event was a transition between two statically stable conditions, namely prebuckling and postbuckling.

The response of both cylinders in the postbuckling range is characterized by patterns of inward dimples. In the axially-stiff cylinder, a single inward dimple centered in the crown and keel segments dominates the displacement response. For the circumferentially-stiff cylinder, the displacement response is dominated by two circumferential rings of six inward dimples. Maximum predicted values of inward radial displacements in the postbuckling range are on the order of 7 to 10 laminate thicknesses. The values of all shear resultants are high at the edges of the dimples or on the ridges between the dimples. All indications are that if loaded into the postbuckling range, the cylinders are very likely to fail at the edges of a dimple or at the ridges between dimples. The former situation would most likely be the case for the axially-stiff cylinder, while the
latter situation would be the case for the circumferentially-stiff cylinder.

While the results presented in this chapter are interesting and important, the geometry of the cylinder has been idealized as being perfectly round. In practice, geometric imperfections are always present in cylindrical elements. Here, the different coefficients of thermal expansion of the segments result in distortions during cool-down from the cure temperature. The overlaps are not perfectly constructed, nor are plys perfectly butted together. Likewise, one or more of the layers could be misaligned by a degree or two relative to the intended orientation. These influences could have an impact on the results just presented. As such, the next chapter addresses the role of the measured imperfections on the response of the cylinder.
Figure 4.33 - Load vs. endshortening relations

(a) axially stiff

(b) circumferentially stiff

Figure 4.33 - Load vs. endshortening relations
Fig. 4.34 - Axial displacement contours, postbuckling range

(a) axially stiff
(b) circumferentially stiff

Fig. 4.35 - Circumferential displacement contours, postbuckling range

(a) axially stiff
(b) circumferentially stiff

Fig. 4.36 - Radial displacement contours, postbuckling range

(a) axially stiff
(b) circumferentially stiff
Fig. 4.37 - Circumferential displacements vs. circumferential location near midlength, postbuckling range

Fig. 4.38 - Radial displacements vs. axial location, postbuckling range

Fig. 4.39 - Radial displacements vs. circumferential location near midlength, postbuckling range
Fig. 4.40 - Axial force resultant $N_x$, postbuckling range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.40 - Axial force resultant $N_x$, postbuckling range
Fig. 4.41 - Circumferential force resultant $N_y$, postbuckling range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.41 - Circumferential force resultant $N_y$, postbuckling range
Fig. 4.42 - Shear force resultant \( N_{xy} \), postbuckling range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.42 - Shear force resultant \( N_{xy} \), postbuckling range
Fig. 4.43 - Axial bending moment resultant $M_x$, postbuckling range

(a) axially stiff

(b) circumferentially stiff
Fig. 4.44 - Circumferential bending moment resultant $M_y$, postbuckling range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.44 - Circumferential bending moment resultant $M_y$, postbuckling range
Fig. 4.45 - Twist moment resultant $M_{xy}$, postbuckling range

(a) axially stiff

(b) circumferentially stiff

Fig. 4.45 - Twist moment resultant $M_{xy}$, postbuckling range
Fig. 4.46 - Transverse shear resultants for axially-stiff cylinder, postbuckling range

(a) axial shear component, $Q_x$

(b) circumferential shear component, $Q_y$

Fig. 4.46 - Transverse shear resultants for axially-stiff cylinder, postbuckling range
Fig. 4.47 - Transverse shear resultants for circumferentially-stiff cylinder, postbuckling range

(a) axial shear component, $Q_x$

(b) circumferential shear component, $Q_y$

Fig. 4.47 - Transverse shear resultants for circumferentially-stiff cylinder, postbuckling range
Chapter 5

Influence of Geometric Imperfections on the Predicted Response

5.1 Introduction

As discussed in Chapter 2, the initial geometries of both the axially-stiff and circumferentially-stiff cylinder specimens were surveyed and recorded. Recall, this was accomplished by measuring the radial location of the inner and outer surfaces as a function of axial and circumferential position. The inner and outer geometry surveys of both cylinders are shown in figs. 2.9 and 2.10. Table 2.2 records the average cylinder radii that constitute the actual shapes of both cylinders. In order to model the response of the actual specimens, the geometries of the inner surfaces shown in figs. 2.9(b) and 2.10(b) were introduced into the finite-element models. The geometry surveys were taken at 97 axial locations at 1/8-in. intervals and at 360 circumferential locations at one-degree intervals, for a total of 33,480 data points in each inner and outer surface survey. Recall that the finite-element model in fig. 3.3 depicts the cylinder midsurface as a mesh containing 57 nodal locations axially and 94 nodal location circumferentially, for a total of 5,358 nodes. The circumferential nodes are distributed as follows: 12 nodes along the arc-length of the crown and keel segments, 30 nodes along the arc-length of the side segments, and 5 nodes along the arc length of each of the overlap segments. An interpolation scheme was devised using Mathematica™ to map the measured geometry of the inner surface from the 33,480 data points onto the 5,358 nodal locations of the finite-element mesh. Only the inner surface geometry was used because the inner survey adequately represented the long wavelength imperfection present in both cylinders. At a given axial and circumferential location, the difference between the inner and outer radial measurements represented the thickness of the cylinder at that location. The wall thickness measurements could have been used to develop a finite-element model that would have
also represented thickness variations as a function of axial and circumferential coordinates. This would have, in turn, required variations in material properties that presumably would have occurred due to the volume fraction variations which led to the thickness variations. Variations in material properties could have been computed by a simple micromechanics scheme. However, the thickness variations were dominated, as would be expected, by the overlaps regions. As the overlap regions were already being modeled, only the long wavelength deviations from a perfectly round cylinder were considered, and the inner surface surveys captured these deviations. The new unloaded shape resulting from applying the interpolation scheme was used as the basis for a non-linear quasi-static analysis.

Figure 5.1 shows the load vs. endshortening relations for both the axially-stiff, fig. 5.1(a), and circumferentially-stiff, fig. 5.1(b), cylinders. Note in both plots the dotted line represents the load vs. endshortening relation for the perfect cylinder discussed in Sections 4.2 - 4.4. The solid line in both figs. 5.1(a) and (b) represents the load vs. endshortening relation for the cylinder which includes the measured geometry of the inner survey, namely, the imperfect cylinder. Recall from Section 4.1, endshortening was applied to the perfect cylinder by the increasing the loading parameter $\Delta$ from a state of no deformation, for which $\Delta = 0$, through linear and nonlinear prebuckling ranges up to buckling and beyond into postbuckling. The drop in load between buckling and the postbuckling response was modeled as a dynamic event. The imperfect cylinders were loaded and studied numerically in the same fashion. Namely, the response in the linear prebuckling range, the nonlinear prebuckling range, and the postbuckling range of endshortening was studied. The values of endshortening and the associated loads and overall strain levels used to represent these three ranges of loading, and the counterpart values for the perfect cylinder case, are given in Table 5.1. In Table 5.1, the numbers in parenthesis under the entries for the imperfect cylinder are the ratios of the value of that entry for the imperfect cylinder relative to the value of that entry for the perfect cylinder. The fact that all ratios are close to unity indicates comparisons between the perfect and imperfect cylinders are being made for similar conditions. The following sections present a comparison of the cylinder response of the perfect and imperfect cylinders in the following order:
I. prebuckling response
   A. linear prebuckling range
   B. nonlinear prebuckling range
II. buckling response
III. postbuckling response

As in the previous chapter, the results are presented primarily as contours of normalized radial displacements mapped onto deformed finite-element meshes. As points of reference, and as will be discussed shortly, the value of $P_{cr}$ for the imperfect axially-stiff cylinder is 18,580 lb, while that for the circumferentially-stiff cylinder is 31,700 lb. The values of $\Delta_{cr}$ are 0.0326 and 0.1073 in., respectively. The corresponding values of overall strain are 2330$\mu$ε and 7660$\mu$ε, respectively.

5.2 Prebuckling Response

5.2.1 Geometrically Linear Range

The plots in fig. 5.1 indicate that the load vs. endshortening relation for both the axially-stiff and circumferentially-stiff cylinder specimens begins with a linear character. For both cylinders the linear imperfect and perfect cylinder responses are indistinguishable. As a means of comparison, and as was done previously, the radial displacements resulting from an overall compressive axial endshortening of 1000$\mu$ε will be considered for both cylinders to study the linear prebuckling response. The contours of radial displacement for the perfect axially-stiff and circumferentially-stiff cylinders at this strain level are shown and discussed in Section 4.2.1. The contours of radial displacement for the perfect and imperfect axially-stiff cylinders at an overall axial compressive strain level of 1000$\mu$ε are shown in fig. 5.2. The perfect cylinder in figure 5.2(a) is distinguished by a symmetric radial displacement response. By that it is meant that the crown and keel have nearly identical deformation response, and the sides have nearly identical deformation response. Figure 5.2(b) shows a lack of symmetry in the radial displacement response in comparison to the perfect cylinder response shown in fig 5.2(a). Table 5.1 shows that the imperfect axially-stiff cylinder has a slightly lower value of $P$ for the same overall axial compressive strain level of 1000$\mu$ε. Table 5.2 shows the effect of the including the actual measured geometry in the model on the percentage of load each segment of the cylinder supports, namely,
the laminate property $\%P_{seg}$ described in section 4.2.1. The lack symmetry of the imperfect axially-stiff case is clear. The values of $\%P_{seg}$ in the crown and keel are nearly the same; they have a range of values in the overlaps; and in the sides they vary by almost 5%.

Radial displacement contours for the perfect and imperfect circumferentially-stiff cylinders are shown in fig. 5.3. Upon close inspection it can be seen that while the cylinders have the same over axial compressive strain applied, and while they both exhibit the previously defined symmetric radial displacement response to the applied endshortening, there is a slight effect of the including the actual measured geometry in the model. There is a slight difference in the appearance of the boundary layers of the imperfect cylinder compared to those of the perfect cylinder. Table 5.1 shows that for the imperfect case, the axial load $P$ is slightly lower. Table 5.2 shows the effect of the actual measured geometry on $\%P_{seg}$ for the circumferentially-stiff case. Here there is less of an overall effect compared to the axially-stiff case. However, there is some lack of symmetry reflected.

### 5.2.2 Geometrically Nonlinear Range

Again considering fig. 5.1, for the axially-stiff and circumferentially-stiff cylinders, beyond the linear prebuckling range, $\Delta$ is increased and the cylinder is loaded into the nonlinear prebuckling range. For both cylinders there is a just a very slight difference between load vs. endshortening relations for the perfect and imperfect cylinders in the nonlinear range. For the axially-stiff response shown in fig. 5.1(a), the slopes of the perfect and imperfect cases begin to differ slightly as $\Delta$ increases into the nonlinear prebuckling range. As $\Delta$ approaches its critical, or buckling, value the character of the load vs. endshortening relation begins to soften somewhat, just as in the perfect cylinder case. In the circumferentially-stiff case, fig. 5.1(b) shows the imperfect case differs just slightly over a considerable portion of the nonlinear prebuckling range.

Radial displacement contours for the perfect and imperfect axially-stiff cylinders in the nonlinear prebuckling range are shown in fig. 5.4. In the case of the perfect cylinder, the axial endshortening value is within 5% of the predicted critical endshortening for the perfect axially-stiff cylinder. For the imperfect cylinder, the endshortening value is within 5% of the predicted
critical endshortening for the imperfect cylinder. Focusing on the entries for the nonlinear prebuckling of the axially-stiff cylinder in Table 5.1, though the endshortening values for both the perfect and imperfect cylinders are within 5% of their respective critical values, the ratio of the imperfect to perfect endshortening values is 0.906, while ratios of the strains and loads are 0.907 and 0.906, respectively. These numbers indicate that the inclusion of the actual measured geometry acts to increase the degree of nonlinearity in the response. The nonlinear prebuckling response of the perfect cylinder shown in fig. 5.4(a) is discussed in detail in Section 4.2.2. It is characterized by wrinkling in the crown and keel segments and, more importantly in the context of the present discussion, its symmetry. The nonlinear prebuckling response of the imperfect cylinder shown in fig. 5.4(b) is characterized by noticeable wrinkling in the keel segment (visible in the interior of the cylinder) but not so much in the crown, and a more pronounced bending boundary layer in the axial direction in the crown segment. The imperfect cylinder response indicates a bias in the radial displacement response, and thereby all other displacement responses, bought about by the inclusion of the actual measured geometry in the model. An overall lack of symmetry characterizes the nonlinear prebuckling response of the imperfect axially-stiff cylinder. The range of normalized radial displacement values for both the perfect and imperfect cylinders is between 0.025 and 0.386.

Focusing on the entries in Table 5.1 for nonlinear prebuckling of the circumferentially-stiff cylinder, the ratio of the endshortening value for the imperfect cylinder versus the perfect cylinder is 0.975. The ratios of strains and loads are 0.975 and 0.980, respectively. These ratios indicate that in the case of the circumferentially-stiff cylinder, there is very little effect on the nonlinear prebuckling response resulting from including the actual measured geometry. The radial displacement contours for both the perfect and imperfect circumferentially-stiff cylinders are shown in fig. 5.5. The perfect cylinder response, also discussed in detail in Section 4.2.2, is characterized by wrinkling in the side segments and overall symmetry. The imperfect cylinder response shown in fig. 5.5(b) is characterized as well by wrinkling in the side segments, but not as much for the perfect case. In addition, the response for the imperfect case displays a lack of symmetry evidenced most clearly by the inward local maximum in radial displacement in the side segment adjacent to the overlap at 210° visible in the interior of the cylinder. According to the symmetry observed in the case of the perfect cylinder, a matching inward local maximum should
be visible in the side segment along the overlap at $30^\circ$. There is no such matching feature. The range in normalized radial displacement values for both the perfect and imperfect cases is 0.073 to 1.122.

### 5.3 Buckling Response

Focusing again on fig. 5.1, in the load vs. endshortening relationships for both the axially-stiff and circumferentially-stiff cases, $\Delta$ increases from a state of no deformation through prebuckling, linear and nonlinear, until it reaches a critical or buckling point. These buckling points are labeled for the imperfect cylinder for both the axially-stiff and circumferentially-stiff cases. The load vs. endshortening relation for the axially-stiff cylinder shown in fig. 5.1(a) shows what appears to be two buckling points. In terms of the nonlinear quasi-static analysis, it is most accurate to define the first point of loss of stability as the buckling point. All the response after this point is correctly referred to as postbuckling response.

Table 5.3 shows a comparison of the buckling results of the perfect and imperfect cylinders for both the axially-stiff and circumferentially-stiff cylinder configurations. The numbers in parentheses represent the ratios of critical values for strain, load, and endshortening for the imperfect cylinders relative to those same critical variables for the perfect cylinder. For the linear buckling prediction, Table 5.3 shows that the critical value for the axially-stiff imperfect cylinder is 6\% less than the perfect cylinder. For circumferentially-stiff cylinder, the linear prediction for the imperfect cylinder is less than 1\% greater than for the perfect cylinder. Therefore, in the case of the axially-stiff cylinder, inclusion of the measured geometry has some effect on the linear buckling prediction, while in the circumferentially-stiff case there is little to no effect.

The buckling shape for the lowest mode predicted by the linear analysis for both the imperfect axially-stiff and circumferentially-stiff cylinders are shown in fig. 5.6. The buckling shape for the imperfect axially-stiff cylinder, shown in fig. 5.6(a), is characterized by wrinkling in the keel segment but not in the crown. Recall from Section 4.3, the linear buckling mode shape predicted for the perfect axially-stiff cylinder shown in fig. 4.31 is characterized by wrinkling in both the crown and keel segments. The linear buckling shape for the imperfect circumferentially-
stiff cylinder is shown in fig. 5.6(b). The predicted shape for this case is characterized by wrinkling in the side segments, with the side corresponding to $\theta = 270^\circ$ having a more pronounced wrinkling pattern. Again, comparing this result to the predicted linear buckling shape for the perfect circumferentially-stiff cylinder shown in fig. 4.31 indicates that the bias in the wrinkling is bought about by including the actual measured geometry in the model.

Note, in fig. 5.1(a), the load vs. endshortening relation shows that the critical endshortening value for the imperfect axially-stiff cylinder occurs at a normalized value of just over 0.9. This reflects the numbers in Table 5.3. It should be noted that while there is a non-negligible effect on predicted buckling values in the imperfect circumferentially-stiff cylinder, the effect in the imperfect axially-stiff cylinder is greater.

5.4 Postbuckling Response

Referring again to fig. 5.1, beyond $\Delta_{cr}$, the response of both perfect cylinders enters the postbuckling range. The transition from buckling to postbuckling is modeled dynamically by the transient analysis procedure described in Section 4.4. It was established in Section 4.4 that the response of segmented perfect cylinders is generally characterized by dimpling at $\Delta_{cr}$. The axial load drops suddenly from $P_{cr}$ to a load satisfying the conditions for a secondary state of structural equilibrium. After reaching a point of secondary equilibrium, the value of $\Delta$ increases along a path which reflects lower inplane stiffness compared to the stiffness along the primary equilibrium path. This is due to the fact that in the dimpled condition a significant amount of the inplane load is reacted by inplane bending stiffness, namely, $D_{11}$ and $D_{12}$. This general character of the postbuckling response of the perfect segmented cylinders established in Section 4.4 is also reflected in both load vs. endshortening relations shown in fig. 5.1 for the imperfect cylinders. In the response of the imperfect axially-stiff cylinder shown in fig. 5.1(a), the load at point $A'$ reflects a 9.6% drop in load. As $\Delta$ is increased beyond the value of $\Delta$ at point $A'$, the load increases along the path $A'B$ to a second point of loss of stability at point $B$. At point $B$, the transient analysis procedure from Section 4.4 was again implemented. The result is the path from point $B$ to point $C$ reflecting another sudden drop in load. The drop from point $B$ to point $C$
reflects a 11.07% drop in load. The total load drop from the original buckling point, or initial point of loss of stability, to point $C$ is 15.94%. In a local sense, point $C$ can be considered a tertiary point of structural equilibrium. In an overall sense, the postbuckling response between the buckling point and point $C$ can be considered intermediate stages of buckling, and all response beyond point $C$ can be considered the true postbuckling range. Recall, in the perfect axially-stiff cylinder, the drop in load from buckling to postbuckling is 19.87%. As fig. 5.1(a) shows, the load-endshortening relation of the perfect cylinder somewhat bounds the load and endshortening responses along path $ABC$ in the imperfect axially-stiff case. The point $A'$ represents an interesting transitional position in the response of the imperfect axially-stiff cylinder. The shape at point $A'$, shown in fig. 5.7, is characterized by an inward dimple in the keel and relatively flat crown and side segments. As $\Delta$ increases along path $A'B$, the inward dimple in the keel grows deeper, wrinkles develop in the axial direction in the crown, and the sides remain relatively undeformed. Finally at point $C$, dimpling occurs in the crown to match the inward dimple in the keel. The load vs. endshortening relation for the imperfect axially-stiff cylinder in fig 5.1(a) shows that as $\Delta$ is increased beyond the value at point $C$, a postbuckling path develops along a slope similar to the perfect cylinder, but with lower load values than the perfect cylinder response. The postbuckled shapes for both perfect and imperfect axially-stiff cylinders at a point beyond point $C$ are shown in fig. 5.8. The postbuckling strain, load, and endshortening values for the shapes shown in fig. 5.8(b) are the entries for the imperfect axially-stiff cylinder in Table 5.1. Comparing the postbuckled shapes of the perfect cylinder and the imperfect cylinder shown in fig. 5.8, it can be seen that the shapes are the same, except for the fact that the radial displacement is slightly greater in the case of the imperfect cylinder.

In the case of the circumferentially-stiff cylinder, the load vs. endshortening relation shown in fig. 5.1(b) shows that the inclusion of the actual measured geometry seems to have slightly decreased the load bearing capacity in the postbuckling range. However, the slopes of the imperfect and perfect postbuckling paths are virtually identical. The postbuckled shape for the imperfect circumferentially-stiff cylinder is shown in comparison to the postbuckled perfect circumferentially-stiff cylinder in fig. 5.9. The shapes and the range of radial displacements appear to be identical. The ratios in Table 5.1 indicate that the load in the postbuckling range for the circumferentially stiff cylinder is effected a little more than the strains and endshortening values.
However, overall, including the actual measured geometry in the circumferentially-stiff cylinder model has little effect on the postbuckling response.

5.5 Summary

Overall, it can be seen that the influence of the geometric imperfection on the circumferentially-stiff cylinder is minimal. The imperfection in the axially-stiff case changes the manner in which the cylinder dimpled, the dimple in the keel occurring before the crown. In the far postbuckling response, this segmented dimpling had little influence.

The next chapter begins a discussion of the experimental aspects of this study. While some aspects were discussed earlier in connection with the actual segmented construction and the potting, the next chapter discusses the experimental setup to compress the cylinders axially, and describes the instrumentation and experimental procedure.
Table 5.1: Values of endshortening and load used to study segmented cylinders, measured geometry included

<table>
<thead>
<tr>
<th>Cylinder description</th>
<th>Load Range</th>
<th>Cylinder Geometry</th>
<th>$\Delta$ (in)</th>
<th>$P$ (lb)</th>
<th>$\bar{\varepsilon}_x^c$ (µε comp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>axially stiff</td>
<td>Linear prebuckling</td>
<td>Perfect</td>
<td>0.014</td>
<td>8,280</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.014 (1.000)</td>
<td>8,200</td>
<td>1,000 (0.990)</td>
</tr>
<tr>
<td></td>
<td>Nonlinear prebuckling</td>
<td>Perfect</td>
<td>0.0335</td>
<td>19,320</td>
<td>2,390</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.0304 (0.906)</td>
<td>17,520</td>
<td>2,170 (0.906)</td>
</tr>
<tr>
<td></td>
<td>Postbuckling</td>
<td>Perfect</td>
<td>0.0421</td>
<td>17,360</td>
<td>3,010</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.0422 (1.003)</td>
<td>17,000</td>
<td>3,020 (1.003)</td>
</tr>
<tr>
<td>circumferentially stiff</td>
<td>Linear prebuckling</td>
<td>Perfect</td>
<td>0.014</td>
<td>4,390</td>
<td>1,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.014 (1.000)</td>
<td>4,370</td>
<td>1,000 (0.994)</td>
</tr>
<tr>
<td></td>
<td>Nonlinear prebuckling</td>
<td>Perfect</td>
<td>0.1082</td>
<td>31,900</td>
<td>7,730</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.1054 (0.975)</td>
<td>31,300</td>
<td>7,530 (0.975)</td>
</tr>
<tr>
<td></td>
<td>Postbuckling</td>
<td>Perfect</td>
<td>0.1582</td>
<td>16,370</td>
<td>11,300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.1597 (1.009)</td>
<td>16,540</td>
<td>11,410 (1.009)</td>
</tr>
<tr>
<td>Cylinder description</td>
<td>Segment description</td>
<td>Circum location</td>
<td>%P_{seg}</td>
<td>%P_{seg}</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>----------------------</td>
<td>----------------</td>
<td>----------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Perfect</td>
<td>Imperfect</td>
<td></td>
</tr>
<tr>
<td><strong>axially stiff</strong></td>
<td>overlap</td>
<td>-30°</td>
<td>3.36</td>
<td>2.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>crown</td>
<td>0°</td>
<td>29.9</td>
<td>28.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>overlap</td>
<td>+30°</td>
<td>3.36</td>
<td>2.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>side</td>
<td>+90°</td>
<td>14.4</td>
<td>15.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>overlap</td>
<td>+150°</td>
<td>3.36</td>
<td>2.90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>keel</td>
<td>+180°</td>
<td>29.9</td>
<td>28.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>overlap</td>
<td>+210°</td>
<td>3.36</td>
<td>2.90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>side</td>
<td>+270°</td>
<td>14.4</td>
<td>16.06</td>
<td></td>
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<tr>
<td><strong>circumferentially stiff</strong></td>
<td>overlap</td>
<td>-30°</td>
<td>2.87</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>crown</td>
<td>0°</td>
<td>16.6</td>
<td>15.81</td>
<td></td>
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<tr>
<td></td>
<td>overlap</td>
<td>+30°</td>
<td>2.87</td>
<td>2.26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>side</td>
<td>+90°</td>
<td>27.7</td>
<td>29.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>overlap</td>
<td>+150°</td>
<td>2.87</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>keel</td>
<td>+180°</td>
<td>16.6</td>
<td>15.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td>overlap</td>
<td>+210°</td>
<td>2.87</td>
<td>2.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>side</td>
<td>+270°</td>
<td>27.7</td>
<td>29.7</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.3: Buckling conditions for segmented cylinders, measured geometry included

<table>
<thead>
<tr>
<th>Cylinder description</th>
<th>Analysis type</th>
<th>Cylinder Geometry</th>
<th>$\Delta_{cr}$ (in)</th>
<th>$P_{cr}$ (lb)</th>
<th>$\left(\varepsilon_x\right)^*_{cr}$ (με comp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nonlinear</td>
<td>Perfect</td>
<td>0.0351</td>
<td>19,810</td>
<td>2,510</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.0326 (0.930)</td>
<td>18,580 (0.938)</td>
<td>2,330 (0.930)</td>
</tr>
<tr>
<td>axially stiff</td>
<td>Linear</td>
<td>Perfect</td>
<td>0.0333</td>
<td>19,700</td>
<td>2,380</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.0314 (0.944)</td>
<td>18,530 (0.941)</td>
<td>2,240 (0.944)</td>
</tr>
<tr>
<td></td>
<td>Nonlinear</td>
<td>Perfect</td>
<td>0.1087</td>
<td>32,000</td>
<td>7,760</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.1072 (0.987)</td>
<td>31,700 (0.989)</td>
<td>7,660 (0.987)</td>
</tr>
<tr>
<td>circumferentially stiff</td>
<td>Linear</td>
<td>Perfect</td>
<td>0.1009</td>
<td>31,700</td>
<td>7,210</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imperfect</td>
<td>0.1018 (1.009)</td>
<td>31,900 (1.007)</td>
<td>7,270 (1.009)</td>
</tr>
</tbody>
</table>
Fig. 5.1 - Load vs. endshortening relations, perfect and imperfect cylinders

(a) axially-stiff cylinder

(b) circumferentially-stiff cylinder

Fig. 5.1- Load vs. endshortening relations, perfect and imperfect cylinders
Fig. 5.2 - Radial displacement contours, linear range, perfect and imperfect axially-stiff cylinders
(a) perfect cylinder

(b) imperfect cylinder

Fig. 5.3 - Radial displacement contours, linear range, perfect and imperfect circumferentially-stiff cylinders
Fig. 5.4 - Radial displacement contours, nonlinear range, perfect and imperfect axially-stiff cylinders
Fig. 5.5 - Radial displacement contours, nonlinear range, perfect and imperfect circumferentially-stiff cylinders
Fig. 5.6 - Linear buckling solutions (mode 1), imperfect cylinders
Fig. 5.7 - Radial displacement contours, postbuckling range, imperfect axially-stiff cylinder at $A'$
Chapter 5. Influence of Geometric Imperfections on the Predicted Response

Fig. 5.8 - Radial displacement contours, postbuckling range, perfect and imperfect axially-stiff cylinders

(a) perfect cylinder

(b) imperfect cylinder
Fig. 5.9 - Radial displacement contours, postbuckling range, perfect and imperfect circumferentially-stiff cylinders
Chapter 6

Experimental Set-up

To further study the response of small-scale segmented circular composite cylinders to axial endshortening, and to provide a comparison with the numerical predictions, the axially-stiff and circumferentially-stiff cylinder specimens described in Chapter 2 were tested. The following sections detail the experimental apparatus, the data acquisition and instrumentation, and the experimental procedure.

6.1 Set-up and Instrumentation

A photograph of the set-up to compress the cylinders axially is shown in fig. 6.1. Points of interest in the photograph are labeled by number and are described in the captions below the photograph. The top and bottom loading platens are labeled numbers 1 and 2, respectively. The specimen, labeled number 3, was mounted between the loading platens to be tested in axial compression. One of the overlaps is clearly visible, aligned with the vertical axis of the portion of the specimen shown. Referring to fig.1.1, this is the overlap at 210°. The endshortening was introduced and controlled by bringing the bottom platen up toward the top platen using a hydraulic system below the floor. The top loading platen had four adjustable bolts, one at each corner. One of these bolts is labeled number 4 in fig. 6.1. A schematic of the test set-up is depicted in fig. 6.2, showing the potted cylinder specimen mounted between the loading platens and indicating the loading direction. Here, the loading platen adjustment bolts in the top loading platen are shown. Also shown in the schematic is the loading platen pivot which, along with the platen adjustment bolts, allowed for the horizontal alignment of the top platen with the specimen. Surface strains were measured with pairs of back-to-back strain gages distributed over the inner and outer surfaces of the specimen. Wire connectors running from each strain gage carried the signal to the
data acquisition system. In the schematic of fig. 6.2 it can be seen that the bottom of the specimen did not contact the bottom loading platen. Instead, it rested on top of a 1-in. thick, flat steel ring, which had a 12-in. outer diameter and an 8-in. inner diameter. The features labeled as interior surface strain gage connection routers in fig. 6.2 were wide grooves machined into the bottom surface of the flat steel ring to allow wire connectors from the strain gages on the inner surface of the cylinder to pass to the outside of the cylinder specimen and connect to the data acquisition system. In fig. 6.1 these connection routers are labeled number 6. The strain gage pattern used for both specimens is shown in figs. 6.3 and 6.4. In these figures the cylinders are unrolled and viewed as if flat. Note the orientation of the \( x \)- and \( \theta \)-directions. Figure 6.4 shows the numbering scheme used to identify the strain gages. The legend at the lower left of each figure identifies the three different style gages used to measure strain responses. The point labeled “(0/0)” at the left center of fig. 6.3 is at the center midlength axial location \( x = L/2 \) of the crown segment, and corresponds to the point on the line at 0\(^\circ\) (360\(^\circ\)) where the back-to-back 0-90 two-arm rosette numbered 39, 40, 41, and 42 in fig. 6.4 are located. Noting fig. 6.4, all gages on the outside surface of the specimens are identified by an odd number. The strain gage types and locations were chosen based on the numerical predictions in Chapters 4 and 5. All strain gages were from the Vishay Measurements Group [6]. Single axis back-to-back axial strain gages mounted at the 0\(^\circ\) (360\(^\circ\)), 90\(^\circ\), 180\(^\circ\), and 270\(^\circ\) locations in figs. 6.3 and 6.4 were CEA-06-187UW-350 gages. These gages were located near the potting at either end of the test section; the \( x = 1 \)-in. and \( x = 13 \)-in. locations. They were intended to measure cylinder wall midsurface membrane and bending strains in the axial direction within the boundary layer region of the radial displacement pattern. The boundary layer effect in the radial displacement was present in both cylinders in the linear and nonlinear prebuckling response shown in figs. 4.4 and 4.18. All back-to-back 0-90 two-arm rosette gages mounted around the circumference at \( x = L/2 \) were CEA-06-125UT-250 gages. These gages were mounted at the midlength axial locations of the inner and outer surfaces of the crown, keel, and side segments for the purpose of measuring axial and circumferential midsurface membrane and bending strains, particularly in postbuckling, as shown in figs. 4.36(a) and (b). The axial gages at cylinder midlength and in the boundary layers were also used to assess the uniformity of loading around the circumference of the cylinder, and from one end to the other.
Back-to-back three-arm rosette gages were used near the overlap at 30° to capture the shear effect predicted by the analysis and shown in figs. 3.6, 4.10, 4.24, and 4.42. Because of size requirements, two types of back-to-back three-arm rosette gages were used. The gages labeled 1, 2, 3, 4, 5, and 6 in fig. 6.4 are back-to-back three-arm rosette CAE-06-062UR-350 gages. This type of three-arm rosette was chosen because it is small enough to hopefully measure shear within the boundary layer region. The gages labeled 7, 8, 9, 10, 11, and 12 were back-to-back three-arm rosette CAE-06-125-UR-350 gages. This nominal-sized gage was chosen to measure shear just outside the boundary layer.

Axial, circumferential, and radial displacements taken with respect to the specimen coordinate axis were measured using direct current differential transformers (DCDTs). The DCDTs, each measuring a different displacement, were located by number 5 in fig. 6.1. Axial motion of the bottom load platen was measured by DCDTs positioned at three corners of the top load platen, as shown schematically in fig. 6.5. The three axial displacement measurements could be decomposed into an axial translation and two rotations, thus fully characterizing the motion of the loading platen during the test. The axial load was measured using a 120-kip capacity load frame. From axial load and DCDT measurements, a load-endshortening relation could be established.

Before discussing the DCDTs measuring the circumferential and radial displacements, it is important to understand some details of the shadow Moire set-up. Shadow Moire interferometry was used for both cylinders to qualitatively observe deformation patterns during the tests. Figure 6.5 shows the segmented construction of the cylinder and the location of the Moire interferometry measurements of the cylinder. To enhance the shadows produced by the Moire grating, the portion of each cylinder observed by the Moire interferometry set-up was painted white. The wide hatch marks in the arc from 0° to 30° represent the portion of the crown segment that was painted white. The thinner hatch marks in the arc from 30° to 90° represent the portion of the side segment that was painted white. Figure 6.6 shows the grating for shadow Moire interferometry, along with the specimen, mounted in the load frame. Behind the grating was nearly a quarter of the cylinder encompassing half the crown segment, the overlap at 30°, and half the side segment, as described above.
The placement of DCDTs measuring circumferential and radial displacement was carefully coordinated with the Moire interferometry set-up in order to fit all the measuring devices into the available space. Circumferential displacements were measured at three locations, as shown schematically in fig. 6.7. The circumferential measurements were made by bonding wooden tabs to the cylinder on the overlap segments at the midlength axial locations at the circumferential locations shown in fig. 6.7. A DCDT was the positioned at a right angle to the tab in order to measure the motion of the tab, thereby measuring the circumferential motion of the specimen’s surface at the point where the tab was bonded. The overlap locations were chosen because the circumferential displacements in the prebuckling range were predicted to be largest there (See figs. 4.3 and 4.17). Note that due to the Moire interferometry location shown in figs. 6.5 and 6.6, a circumferential DCDT measurement could not be made at the overlap at 30°.

Radial displacements were measured using DCDTs at four midlength axial locations around the circumference of the cylinder. The radial DCDT set-up is shown schematically in fig. 6.8. The radial displacement measurements are made at 0°, 90°, 180°, and 270°. These locations were chosen for the radial DCDTs because the predicted postbuckling response shows distinct features in the radial displacement patterns of both cylinders at these points (See fig. 4.36). The arc subtended by the Moire interference grating shown in fig. 6.6 had to actually be fixed just short of 90° to allow radial DCDTs to be placed at the 0° and 90° locations. The horizontally-oriented DCDTs visible on both sides of the grating in the photograph in fig. 6.6 are positioned at 0° (background right) and 90° (foreground left).

All electrical signals from the strain gages, DCDTs, and load output were recorded using a digital data acquisition system. Each test was recorded using audio-visual equipment. Still photos were taken before, during, and after testing.

6.2 Procedure

Before each test, all instrumentation was attached to the data acquisition system and synchronized to establish a condition of zero response. Also, a load balancing procedure was used to ensure that load would be applied to the cylinder as uniformly as possible. This was accomplished
by adjusting the horizontal alignment of the top loading platen. In order to align the top loading platen properly, a specimen was mounted between the loading platens, as shown in fig. 6.1, and loaded axially to approximately 4 kips while monitoring the axially-aligned back-to-back strain gages located around the circumference at the midlength axial location of the cylinder specimen. The strain gage pairs monitored are numbered in fig. 6.4 as 17 and 18 at 270°, 25 and 26 at 180°, 33 and 34 at 90°, and 41 and 42 at 360°(0°). The alignment of the top loading platen was adjusted with the bolts at the four corners of the platen shown in figs. 6.1 and 6.2. The alignment was adjusted by small increments until strains measured at the four circumferential locations reflected a uniform load application. Once the load balancing procedure was completed, the cylinder was unloaded and the instrumentation was synchronized a final time to re-establish the zero response condition. Just before the cylinder was loaded, Moire interferometry grating and lights were set-up, along with a video camera, a still photography camera, and a microphone. Loading of the cylinder then began. The load rate started at a relatively high value and decreased to a lower value as the axial load approached the predicted buckling value. Data collected included the following: (1) load rate introduction, (2) axial load, (3) displacement readings from axially-, circumferentially-, and radially-oriented DCDTs, (4) strain gage measurements, (5) Moire interferometry video, (6) audio track recording during testing, and (7) still photography, before, during, and after testing.

6.3 Apparatus

The tests were conducted in the Mechanics and Durability Branch at NASA-Langley Research Center in Hampton, VA. The specimens were loaded axially using a Baldwin-Tate Emery load frame. The load introduction was controlled by a SATEK model MK3 controller. The strain gages were installed by the Modern Machine and Tool Company of Newport News, VA. All data was sampled at one-second time intervals and recorded on a data acquisition system called MODCOMP[7].
1. top loading platen
2. bottom loading platen
3. specimen
4. loading platen adjustment bolts
5. DCDTs
6. inner surface strain gage connection routers

Fig. 6.1 - Typical test set-up
Fig. 6.2 - Schematic of the test set-up
Jaret C. Riddick

Chapter 6. Experimental Set-up

1. back-to-back axial strain gage
2. back-to-back 0-90 two arm rosette
3. back-to-back three arm rosette

Notes:
L ~ 12.0 in.
Point (0/0) is labeled on each cylinder

Fig. 6.3 - Strain gage pattern detailing gage location and orientation
Top view of specimen mounted in load frame

Fig. 6.5 - Axial DCDT placement and Moire interferometry location

Fig. 6.6 - Shadow Moire interferometry grating with specimen mounted in load frame
Fig. 6.7 - Circumferential DCDT placement

Fig. 6.8 - Radial DCDT placement
Chapter 7

Experimental Results and Comparison with Numerical Predictions

This chapter presents the results from the DCDT and strain gage measurements and compares them with the numerical predictions of Chapters 4 and 5. Additionally, to provide an overall view of cylinder response, selected photographs of the Moire pattern during testing and the cylinder after testing are included.

7.1 Axially-Stiff Cylinder

7.1.1 Overall response

During the testing of the axially-stiff cylinder, as the endshortening of the cylinder was increased, there was no noticeable response until the load was greater than 16,000 lb. With no endshortening - i.e., no load applied - the Moire fringe pattern showed what could be referred to as an initial pattern due to lack of a perfect geometric match between the curved Moire grid and the cylinder, as shown in fig. 7.1. As the endshortening increased, this fringe pattern began to change only slightly. At 16,000 lb, fig. 7.2, there was no significant change in the fringe pattern. As photographs were being taken only periodically, by the time the next photograph was taken, fig. 7.3, the first drop in load had occurred and the load was 14,000 lb. The drop in load was felt to be due to dimpling in the crown and keel segments. Since both the crown and keel were not observed with the Moire fringe set-up, it was not possible to tell what the sequence of events was and whether one dimple occurred before the other, or whether dimpling occurred simultaneously in the crown and keel. As the endshortening increased, the load level went up slightly and the fringe pattern of fig. 7.3 deepened to that shown in fig. 7.4. This fringe pattern and the load level were stable with increased endshortening, so the test was stopped. The deformations of the crown region of the cylinder with the load held at a level of 11,800 lb and the Moire grid removed are
shown in fig. 7.5. (Note the quarter of the cylinder in the crown and side segments painted white to enhance the Moire pattern, as indicated in fig. 6.5) The dimple was outlined with black and white markers for later identification when the cylinder was unloaded. The marker lines are visible in fig. 7.5. Some material failure was visible at the left and right edges of the dimple. Figure 7.6 shows the deformations outlined in the keel region of the cylinder. There was material failure at the edges of the dimple, particularly where the edges of the dimple intersected with the overlap. This is visible in fig. 7.6.

In general, the axially-stiff cylinder behaved as predicted. Dimpling occurred in the crown and keel, at the midlength location, as predicted in fig. 4.36(a). The sides remained unperturbed. As material failure was not part of the predictions, correlation between the observed failure and predictions was not possible. Although dimpling did occur, it occurred at a lower load than predicted. It is not clear whether this was due to some material failure preceding dimpling, and thereby softening the cylinder, or if dimpling was a strictly elastic phenomenon and it simply occurred at a lower load than predicted, and then material failure occurred. If the latter situation was the case, then it should be noted that the failure locations coincided with predicted locations of maximum postbuckling force and moment resultants in figs. 4.40 - 4.47. Specifically, maximum levels were predicted to occur on or near the ridges of the dimples. Dimpling will be discussed more in the ensuing paragraphs.

### 7.1.2 Details of response

Axial DCDTs positioned in the manner shown in fig. 7.7(a) were used to make experimental measurements of the endshortening of the axially-stiff segmented cylinder. Figure 7.7(b) shows the experimental measurements from the axial DCDTs in comparison with the predicted load vs. endshortening relation for the imperfect axially-stiff cylinder from the STAGS finite-element analysis of Chapter 5. Since the STAGS analysis is ideal, all locations around the circumference at the ends of the cylinder have the same axial displacement. Therefore, there is only one line representing the STAGS analysis. As can be seen, there is reasonable correlation between the STAGS prediction and the displacement measured by the three DCDTs. As was seen in Chapter 5, the load vs. endshortening relation for the imperfect cylinder shows that the finite-element analysis predicts loading to a critical point, at which the keel dimples, triggering a drop in load. As the
endshortening increases beyond the critical value, the load increases to a second critical point where the crown dimples to match the keel, triggering a second load drop. With increased endshortening beyond the second load drop, load increases along a path having a slightly decreased slope compared to the primary load path. The DCDT measurements show the cylinder was loaded to a critical point, after which the load dropped. Note that in the experimental data, the load dropped along a diagonal rather than vertical line. The maximum load reached was 16,570 lb. The load then dropped to a value of about 14,000 lb. After the load dropped, with increased endshortening the load increased along a path with a slope less than that of the primary path. Beyond a certain value of endshortening, there was a series of small load drops and increases as the endshortening continued. Finally, the test was stopped when it appeared the cylinder was going to crush due to material failure. The lack of perfect displacement control with the loading fixture was somewhat responsible for the load not dropping along a vertical line. Material failure could also have contributed to this behavior. Those issues aside, the experimentally measured buckling load was 16,570 lb, whereas the predicted load for the first dimpling was 18,580 lbs. This represents an 12% overprediction of buckling capacity. To be noted in fig. 7.7(b) is the fact that all three DCDT traces do not lie in a single line, indicating that there could have been some circumferential nonuniformity of endshortening due to the lower or upper load platen not remaining parallel to its initial position. Also, there is an indication of what could have been slack in the loading system. The displacement traces do not emanate from the origin in a straight line, rather the traces are curved as they leave the origin. Sometimes in the reporting of experimental data, the data are adjusted by eliminating the initial loading portion and shifting the data so the extrapolation back to zero load passes through the origin. This was not done here.

The predicted response of the segmented cylinder is characterized, in part, by the existence of circumferential displacement. The STAGS analyses of Chapters 4 and 5 have predicted the circumferential displacement to be greatest at the cylinder midlength point of the overlap segments. Negative circumferential displacement was predicted at the overlaps at $\theta = 30^\circ$ and $210^\circ$; positive circumferential displacement was predicted at $\theta = 150^\circ$ and $330^\circ$. In the experiments, the circumferential displacement was measured at the center midlength points of the overlaps at $150^\circ$, $210^\circ$, and $330^\circ$ by DCDTs positioned in the manner shown in fig. 7.8(a). Figure 7.8(b) shows the
circumferential displacement measured in the experiment, along with the circumferential response in the imperfect cylinder predicted by the finite-element analysis as a function of the load for a maximum load level below buckling. As fig. 7.8(b) shows, the experimental measurements confirmed the existence of circumferential displacements. Further, the character of the experimental measurements agreed to a significant degree with the predictions of the finite-element analysis, namely, the measured circumferential displacement values at 210° were negative and the circumferential displacement values measured at 150° and 330° were positive. While the magnitudes of the measured displacements at 210° and 330° agreed with the predictions, the value measured at 150° was greater than predicted.

Experimental radial displacement measurements were taken by DCDTs positioned at cylinder midlength in the manner shown in fig. 7.9(a). Figure 7.9(b) shows the radial displacements measured at the midlength points of the centers of the crown, keel, and side segments, along with predictions from the STAGS analysis. The predictions of Chapters 4 and 5 showed that the radial displacement response of the segmented axially-stiff cylinder was characterized, in part, by circumferential gradients in the linear prebuckling range where the values of the outward displacement were highest in the crown and keel segments. In the postbuckling range, the predicted response was characterized by single inward dimples in the crown and keel segments accompanied by undeformed side segments. Overall, this figure shows the existence of inward dimples in the crown and keel, and none in the sides. A close examination of fig. 7.9(b) in the predicted linear prebuckling load range around 8,000 lb shows that the outward radial displacements in the crown and keel are slightly higher than those in the sides, indicating a circumferential gradient in radial displacement. The data in this figure thus confirm the prediction of the STAGS analysis. As indicated in fig. 4.36, albeit at a higher load level, the STAGS predictions for displacements at 0° and 180° at midlength were in excess of 7 laminate thicknesses. Figure 7.9(b) shows inward displacements on the order of 10 laminate thicknesses occurred in the experiments. Again, the load level for which dimpling occurred was within 12% of the predicted level.

An important aspect of the experiments should be mentioned at this point. The data acquisition system recording the voltage from the DCDTs, strain gages, and the load cell was sampling
once per second. From the STAGS transient analysis it was predicted that the dimpling event in the keel of the imperfect axially-stiff cylinder took but 0.05 sec. (refer to fig. 5.7). The time between dimpling in the keel and dimpling in the crown would depend on the rate of endshortening. In the STAGS analysis an increase in endshortening was quasi-static with no inertia effects, so time was not a real parameter after the transient analysis for the first dimple was terminated. In the experiments, however, if one dimple occurred before the other, since the load frame was not perfectly rigid, nor was it enforcing pure displacement control, it is very likely that the time between the two dimples, if they were not simultaneous, was less than the one second sampling interval. The perturbation of the cylinder and load frame system initiated by the occurrence of the first dimple would more than likely have triggered the second dimple, even though in the STAGS analysis the two dimples were separate events. A careful look at the radial displacement data in fig. 7.9(b) just prior to the load drop shows that the DCDT in the crown (0°) began its excursion to a large inward displacement slightly before the DCDT in the keel did. The STAGS analysis predicted that, due to the imperfection in the cylinder, the keel would dimple first, just the opposite. That is troubling, but then as the lower platen of the load frame was moving upward compressing the cylinder, it or the upper platen, or both, were most likely not remaining parallel with their initial positions. As mentioned earlier, fig. 7.7(b) shows this could have happen. If the either platen did not remain parallel with its initial position, then the crown could have been forced to dimple before the keel simply due to platen motion. All these statements could be interpreted as detracting statements regarding the experiment and correlation between predictions and experiments. However, overall, the axially-stiff cylinder behaved much as predicted. Taken together, the experimental measurements taken by the DCDTs shown in figs. 7.7(b), 7.8(b), and 7.9(b) show that the axial endshortening caused the cylinder to be loaded compressively to a critical point. As the endshortening was increased from a state of no deformation, circumferential displacements occurred. Before the endshortening reached its critical value, the radial displacements in the crown and keel segments were on the order of the measured circumferential displacements. Suddenly, at the critical point of load-endshortening, dimples developed in the crown and keel, and the load dropped. As the endshortening was increased, the radial displacement measurements dominated the displacement responses measured by the DCDTs. Circumferential displacements were evident, and the crown and keel dimpled, while the sides remained relatively undeformed. The strain data to be discussed next reinforce these conclusions.
The strains at midlength, both as measured and as predicted by the STAGS analysis, are shown in the next few figures. Two types of strains are shown. One is the average, at a given circumferential position, of the strains at the inner and outer surfaces of the cylinder (back-to-back strains). This strain is interpreted to be the membrane strain at the geometric midsurface of the cylinder wall and will be referred to simply as “strain”, or “shear strain”, as the case may be. The second strain is the difference in the strains at the inner and outer surfaces. This strain is a measure of curvature of the geometric midsurface multiplied by the cylinder wall thickness, a dimensionless curvature. This strain will be referred to as “bending strain”, or “twisting strain”, as the case may be. Specifically, fig. 7.10 shows the axial strains at the midlength axial location at 0° and 180° (crown and keel). The small diagram to the right in the figure identifies the 0, M, L and 0°, 90°, 180°, and 270° locations where the strains were measured in these experiments (see fig. 6.4). Figure 7.11 shows these same axial strains at 90° and 270° (sides). The axial strain in all segments was increasingly negative with increasing endshortening up to the critical load value. Until dimpling or material failure, the axial strains at all circumferential locations should have been very similar. Away from the ends of the cylinder, these strains should basically have been a measure of the compressive value of $\Delta L/L$. A comparison with fig. 7.7 confirms they were. Note here that the predicted axial strain values in both figs. 7.10 and 7.11 coincide up to about 14,000 lb. The measured prebuckling axial strain values in the crown and keel, fig. 7.10, showed a spread from the initial loading that increased up to buckling. The measured prebuckling axial strain values in the sides, fig. 7.11, appeared to coincide up to about 6,000 lb, at which point they began to show a slight spread up to buckling. The spread in measured prebuckling axial strain indicates that the axial stress around the circumference of the cylinder was less symmetric than predicted. The lack of symmetry in the prebuckling axial stress was important because the more highly-loaded segment would likely buckle first. As the load reached the critical value, the axial strain in the crown (EX0M) showed a softening relationship with load; a sign of the ensuing dimpling. The sudden dimpling in the crown and keel rendered these segments incapable of supporting their share of the axial compressive load, causing a load redistribution in which the side segments and overlaps reacted much of the compressive load. This load redistribution caused the measured axial strain in the crown and keel to go to zero, while the values of axial strain measured in the sides became increasingly negative. After dimpling, as the endshortening was increased, based on
experimental data presented in fig. 7.10, the axial strain in the center of the crown and keel remained at relatively small positive values. To be noted in fig. 7.10, it appears that the midlength axial strain measured in the keel at 180° was larger in magnitude than the strain in the crown at 0°. Similarly, the midlength axial strain in the side at 270° was slightly larger in magnitude than the strain at the side at 90°. These slight differences in the strains from circumferential location to circumferential location reinforce the notion that in the experiment there was somewhat of a circumferential nonuniformity of loading. Also, as with the displacement traces of fig. 7.7(b), the lack of some of the strain responses having a perfectly linear relation near zero endshortening - i.e., some of the experimental traces are curved when they leave the origin - again indicates there was some degree of slack in the loading system that could have slightly skewed the loading.

The circumferential strains at the 0° and 180° locations (crown and keel) are shown in fig. 7.12, while these same strains for the 90° and 270° locations (sides) are shown in fig. 7.13. Both the strains as predicted by the STAGS analysis and those measured in the experiments are shown. Due to the Poisson effect, at low load levels the circumferential strains were expected to be positive in response to the axial endshortening, which caused negative axial strains. The circumferential strains reflected the material properties of the various laminates, and the strains in the crown and keel did not have to be the same as those in the sides. As the endshortening was increased from zero, the circumferential strain was indeed positive in all segments and increased linearly with load level. As the endshortening approached the critical value, the measured circumferential strain vs. load relationships in the crown and keel segments, ES0M and ES180M, respectively, began to reverse sign. The load vs. strain relationships in the side segments, ES90M and ES270M, continued to increase linearly as the load approached the critical value. At the critical point, the load drop due to dimpling of the crown and keel segments caused the measured circumferential strains ES0M and ES180M to suddenly become negative in value. At the same time, the values of ES90M and ES270M increased as the load dropped. The load redistribution that resulted from the dimpling of the crown and keel segments caused the circumferential strains in the sides to become suddenly more tensile as the axial strains suddenly became more compressive. A close examination of the strains from the experiments in fig. 7.12 shows the strain in the crown exhibited a more gradual change to compressive values (the line more gradually curves toward zero after reaching
its maximum tensile value) than the strain in the keel. The strain in the keel suddenly jumped to a large negative value from almost its maximum positive value, whereas the strain in the crown did not suddenly jump until the strain was very close to zero. This characteristic is felt to again be an indication that the crown began to dimple before the keel did, and once the crown dimpled, it triggered dimpling in the keel. The straight line segments in these two strains traces as they become negative is due to the one second sampling rate. The strains changed so fast in the experiment that to connect two adjacent strain values, measured one second apart, results in the straight lines connecting these two adjacent values.

The axial bending strains in the crown and keel at the midlength axial location are shown in fig. 7.14, while those in the sides are shown in fig. 7.15. These figures also support the notion that the crown and keel dimpled, while the sides did not. Also, there is further evidence that the crown began to dimple before the keel. To explain: Based on the predictions of the STAGS analysis, the back-to-back gages at midlength were placed so as to be at the bottom of the dimple. If due to imperfections or other anomalies with the testing they were not exactly at the bottom of the dimples, then they would be close to the bottom. When dimpling occurred, these gages should have registered a large change in curvature, going essentially from the state of no curvature to a large negative value, negative based on the coordinate system being used and the definition of curvature in the STAGS analysis. This is essentially what happened with the gages in the crown and keel. No significant curvature was registered until the load reached just over 16,000 lb. Then the curvature suddenly became negative. At the same time, looking at fig. 7.15, it can be seen that there was little change in the curvature of the sides throughout the loading range. (Note the difference in scales of the horizontal axes in figs. 7.14 and 7.15.) From a close examination of the measured bending strains at 0° and 180° in fig. 7.14, it is seen that again the curvature in the crown began to gradually change to a negative value before the curvature in the keel jumped to a negative during the time interval of one second. The circumferential bending strains of figs. 7.16 and 7.17 also showed this character. Dimpling is a two-dimensional phenomenon, and the circumferential gages near the center of the dimple measured the rapid change to a negative curvature in the circumferential direction when the dimpling event began.

The axial strains at the two boundary layer locations, denoted as 0 and L, at circumferen-
ential locations 0° and 180°, and 90° and 270°, are shown in figs. 7.18 and 7.19, respectively. These strains should have been similar to the axial strains at cylinder midlength, except for the fact that at the boundaries the cylinder were restrained from expanding in the radial direction and the circumferential displacement was restrained to be zero, so the circumferential strain was zero there. At cylinder midlength, the cylinder was free to deform, so as was seen in figs. 7.12 and 7.13, the circumferential strains were non-zero there. For a given level of axial load, the restraint on the circumferential strain at the boundary resulted in a lower level of axial strain there than at the midlength location. To explain: At the potting, the \( w = 0 \) condition implied that \( N_y \) would be not equal to zero there. Away from the potting, \( w \) was unrestricted. Therefore, the value of \( N_y \) tended toward zero. The constitutive relations for stress resultants \( N_x \) and \( N_y \) are given by

\[
N_x = A_{11} \varepsilon_x^o + A_{12} \varepsilon_s^o \quad (7.1)
\]

\[
N_y = A_{12} \varepsilon_x^o + A_{22} \varepsilon_s^o \quad (7.2)
\]

Considering the conditions on radial displacement, along with the above given expressions for the stress resultants, it can be shown that away from the potting

\[
\varepsilon_x^o = \frac{N_x A_{22}}{A_{11} A_{22} - A_{12}^2} \quad (7.3)
\]

Near the \( w = 0 \) condition, i.e., at the potting,

\[
\varepsilon_x^o = \frac{N_x A_{22} - N_y A_{12}}{A_{11} A_{22} - A_{12}^2} \quad (7.4)
\]

As a result, the absolute value of axial strain would be smallest where the condition \( w = 0 \) exists. A quick glance at fig. 7.11 and 7.19 clearly shows this was the case for the side segments, and a
close comparison of figs. 7.10 and 7.18 show this was true for the crown and keel segments as well. As with the axial strains at midlength, the compressive strains in the boundary layer of the crown and keel increased in magnitude and then, due to dimpling, rapidly decreased in magnitude and even changed sign with increased endshortening. The strains in the side segments increased in magnitude, reflecting the increased load within the side segments due to dimpling in the crown and keel.

Figures 7.20 and 7.21 show the axial bending strains that were measured in the boundary layers. In the boundary layers the radial displacement changed rapidly with axial position, and thus the axial curvatures were large in magnitude and even changed sign. Referring to fig. 6.4, the measurements shown in figs. 7.20 and 7.21 were made with strain gages 0.187 in. long. The intent of these gages was to monitor circumferential uniformity of the loading, and compare the loading situation at one end of the cylinder with the other. As can be seen from figs. 7.20 and 7.21, the strains at the various circumferential locations and at both ends of the cylinder displayed uniformity to the degree indicated by data in the previous figures. To accurately measure bending strains in the boundary layer with gages this long was questionable. However, the bending strains as measured by these gages are shown in figs. 7.20 and 7.21 and are compared with the STAGS predictions. The comparison is not good. However, it should be recalled that strain gages register an average of the strain along their length. Though there could be high strain magnitudes, the strains could change sign over a relatively short distance in the direction of the gage, and the average would register as zero, or certainly less than the maximum magnitude. That appears to have been the case here. To remedy this situation, the STAGS results could have been averaged and compared with the measured results. Since this study was not focusing on boundary layer strains, this approach was not pursued. If boundary layer strains were of more interest, much smaller gages or an optical technique would have been an option for measuring the strains.

In the segmented construction, laminates having characteristically different responses were joined at the overlap segments to form the cylinder. For this reason, the response at the overlaps was of particular interest. Recall, the three-arm rosettes were mounted next to the overlap because that region was predicted to have high levels of shear strain in the prebuckling range of loading. This can be observed in fig. 4.24. The increased level of shear strain was felt to be due to
mismatches in laminate material properties in that region. In order to measure the strain response at the overlap, three-arm rosette gages were placed in the crown segment along the overlap at 30°. Both rosette gages were bonded next to the overlap, with one rosette bonded against the potting at the end at \( x = L \) of the test section, and the second rosette 1 in. away from the potting at the end at \( x = L \). The strain gages making up the rosettes were smaller than the gages used elsewhere on the cylinder with the hopes of being able to measure strains as near the overlaps and potted end as possible, and to measure the strains in that region with as little averaging as possible. The strains from the rosette nearest the potting (at \( x = L \)) were given the suffix label B1, while the strains from the rosette 1 in. away from the potting (\( x = L - 1 \) in.) were given the suffix label B2. Figure 7.22, in the inset to the right of the figure, illustrates this notation, and shows the axial strain measured at the two locations. The axial strain was measured at the potting by back-to-back gages denoted as 5 and 6 in fig. 6.4. The back-to-back gage pair which measured the axial strain 1 in. into the test section is denoted as 11 and 12 in the same figure. Figure 7.22 shows the axial strain measurements from the rosette gages at the potting (EX30B1) and 1 in. away from the potting (EX30B2), along with predictions from the STAGS analysis. The plots show that due to the potting restraint the axial strain closest to the potting, EX30B1, was smaller in magnitude than the strain measured 1 in. away, EX30B2. The comparison between the STAGS prediction and the experiment is good.

Now considering the circumferential strains measured by the three-arm rosettes, fig. 7.23 shows the circumferential strain at the potting, ES30B1, and 1 in. away, ES30B2, along with the STAGS predictions. In addition to the \( w = 0 \) condition that existed at the potting, a \( v = 0 \) condition was also enforced at the potting. Due to the kinematic relationship that governs circumferential strain in circular cylinders, when \( w = 0 \) and \( v = 0 \), there is no mechanism to generate circumferential strain. Figure 7.23 shows that at the potting, the prebuckling circumferential strains measured in the experiment and predicted by the STAGS analysis confirm that there was no significant magnitude to the circumferential stain component. One inch away, where the circumferential and radial displacements were unrestricted, fig. 7.23 shows that a circumferential strain existed. Comparisons between predictions and experiments were good. The shear strain component shown in fig. 7.24 for both the experiment and the STAGS prediction is the strain component commonly denoted \( \gamma_{xs} \). Of note in fig. 7.24 is the fact that before dimpling, the character of the experimental measurements and the STAGS predictions showed reasonable agreement, both in the sign of the
shear strain, and the magnitude.

The bending strain measurements from the rosette gages reflected response in the boundary layer near the potting. For the axial bending strain shown in fig. 7.25, the experimental value at the potting is denoted by KX30B1, and 1 in. away as KX30B2. These results show that the curvature changed from a negative value at KX30B1 to a positive value at KX30B2. This change in curvature characterized the boundary layer response. While the signs of the axial bending strains agreed with the STAGS predictions, the magnitudes were not in exact agreement. This may have been due to the fact that the finite size of the strain gages measured an average curvature for a very localized phenomenon.

The measured circumferential bending strains shown in fig. 7.26 did not show particularly good agreement with predictions. Again, the finite size of the gage could have been an issue, as could have the magnitude of the strains being measured. Finally, the measured twisting strains in fig. 7.27, showed good agreement with the STAGS predictions. The sign difference at B1 and B2 was captured by the predictions.

In summary, it can be said that, overall, the axially-stiff cylinder behaved as predicted. Circumferential displacements were recorded, as predicted, and the crown and keel dimpled after a critical value of endshortening was reached. The magnitudes of some of the predicted responses were not as predicted, but reasons have been given for this. Attention now turns to the results from the circumferentially-stiff cylinder.

7.2 Circumferentially-Stiff cylinder

7.2.1 Overall response

In terms of overall response during testing, the circumferentially-stiff cylinder behaved differently than the axially-stiff cylinder, and differently than the predictions regarding the dimpling behavior. These differences can be attributed to material behavior that was not modeled. The initial (no-load) Moire fringe pattern is shown in fig. 7.28. As the applied load approached 18,000 lb there was a noticeable change in the fringe pattern. This change indicated recordable deforma-
tions were occurring in the side of the cylinder between the 60° and 90° locations. Symmetry considerations indicate deformations were most likely also occurring in the other side segment, but these were not being monitored with Moire grids. These side segment deformations are shown in fig. 7.29 and are visible in the left half of the Moire pattern as a rippling of the fringes. At a load level of 19,800 lb, illustrated in fig. 7.30, these deformations had increased, and at a load level of 21,000 lb, as shown in fig. 7.31, they had increased even more. At a load of just greater than 21,000 lb, the cylinder failed dramatically. Inspection of the cylinder indicated that material in the crown failed, near or at the overlap at the 60° location, about one-quarter of the way from the lower end of the cylinder. Material failure was also evident in the overlap in that region. Figure 7.32 shows the deformation pattern after the load had dropped to 2,400 lb. Material failures occurred at other locations, namely in the keel and side near the 210° overlap, and in the side near the 270° location. Both of these failures occurred away from the boundaries between the bottom loading platen and the cylinder midlength location. A suspected scenario is as follows: With increased endshortening from the no-load condition, but below the buckling load, the sides began to experience increased radial deformations. As a result, some of the axial load was transferred from the sides to the crown and keel, and the overlaps. As shown in fig. 4.22(b), the axial stress resultants in the overlaps were higher than elsewhere. Though the overlaps were thicker, for the circumferentially-stiff cylinder there were no 0° fibers in the overlaps, or elsewhere for that matter. If failure had occurred in the overlap, then the crown, keel, and sides would have had to react more load and perhaps would have buckled at a lower overall load level than predicted. With buckling occurring, increasing the load would have led to more material failure, particularly in and around the buckle pattern. The region of material failure near the 30° overlap is seen in fig. 7.33. The white paint sprayed on the cylinder to enhance the Moire pattern was chipped and peeled away in the region of failure. The largest region of chipping and peeling was right on the overlap, which is faintly visible in the photograph. The regions outlined with black marker were the regions showing large deformations under load, and the plus and minus indicate the regions which deformed outward and inward, respectively. The black marker was applied by maintaining the load and removing the Moire grid to have access to the deformed region. As the Moire pattern was sensitive to the slope of the deformed surface, comparison of the fringe pattern in fig. 7.32 with the outlined regions in fig. 7.33 indicate the increased fringe density occurred above the
minus sign and below the plus sign at the 30° location. Note there was some material failure where this high fringe density occurred. Figure 7.34 shows outlines, with white marker, of the regions in the other side segment and in the keel that exhibited large deformations under load. Plus and minus signs are visible, as are regions of material failure.

It is interesting to note that although material failure occurred and the load drop took place at a much lower load than predicted in Chapters 4 and 5, the deformation characteristics outlined in figs. 7.33 and 7.34 do have features in common with the earlier predictions. Specifically figs. 4.36(b) and 5.9(a) and (b) show anti-symmetric rings of dimples at locations 3L/8 and 5L/8 from one end of the cylinder, with the midlength circumference a line of anti-symmetry. The deformation patterns shown in figs. 7.33 and 7.34 suggest an anti-symmetric of ring dimples and ridges. The line of anti-symmetry, visible where the white paint is chipped and peeled away in fig. 7.33, is a circumference located between the cylinder midlength and the bottom of the cylinder.

In summary, axial compression of the circumferentially-stiff cylinder resulted in material failure, accompanied by buckling deformations, at a load just over 21,000 lb. The load at which the material failure and buckling deformations occurred was considerably below the predicted buckling load of 31,700 lb. Unlike the axially-stiff cylinder specimen, the circumferentially-stiff specimen showed none of the postbuckling behavior of the prediction. In fact, the specimen response was characterized more by material failure than by buckling response.

7.2.2 Details of response

The motion of the bottom load platen was measured by three DCDTs positioned according to fig. 7.35(a). The measured and predicted endshortening vs. load relations for the circumferentially-stiff cylinder are shown in fig. 7.35(b). The load drop from the point of material failure and buckling is not included. The load at which material failure and subsequent buckling occurred experimentally was only 69% of the load predicted by the STAGS analysis. The measured buckling axial displacement was 25% lower than predicted. The figure also shows softening of the experimentally-measured relations as the maximum load was reached. In fact, the slopes of the experimentally-measured endshortening vs. load relations were never really constant. Rather, the
relations showed a continual softening characteristic of laminates dominated by ±45 deg. layers. This was a material characteristic not modeled. Interestingly, the measurements of the three DCDTs coincided, indicating that compared to the axially-stiff cylinder, the platens seemed to have moved parallel to their original positions.

The positions of the DCDTs measuring circumferential displacement are shown in fig. 7.36(a). Figure 7.36(b) shows the circumferential displacement measurements made. The measurement of the circumferential displacements of the overlaps was not particularly successful. The DCDTs measuring the circumferential displacement at 210° (DCDT71) and 330° (DCDT72) malfunctioned, or perhaps the tabs bonded to the cylinder for the purpose of measuring the circumferential displacements disbonded. From the data, it appeared all three DCDTs measured a negative circumferential displacement, while the predictions indicated either a positive or negative circumferential displacement, depending on the particular overlap. For loads greater than about 14,000 lb, only one DCDT was functioning. Though the tabs were bonded to the cylinder on the overlaps somewhat away from the region of visible material failure, there could have been sites of less serious failure near where the tabs were bonded. These failure could have caused the tabs to disbond.

Radial displacements were measured at the midlength locations of 0°, 90°, 180°, and 270° by DCDTs positioned according to fig. 7.37(a). As seen in fig. 7.37(b), though the signs were predicted correctly, the radial displacements measured at the crown, side, and keel locations were less than the predictions. Around the predicted linear prebuckling load level of 4,400 lb, displacements at the crown (DCDT1) and keel (DCDT3) are considerably less than those measured at the sides (DCDT2 and DCDT4), confirming the existence of circumferential gradients in radial displacement. The trend toward a sign reversal after 16,000 lb, of course, was not predicted and is a result of the material failure and buckling at a lower than predicted load level. The peak outward radial displacement measured just before the trend toward a sign reversal was about 35% of the cylinder wall thickness and occurred in the sides. After the sign reversal, the peak inward radial displacement was about 10% of the wall thickness. The DCDTs were not at the buckle locations, so these displacements do not represent those in the regions outlined in figs. 7.33 and 7.34. Compared to the large radial displacements of the axially-stiff cylinder in the dimples, the measured
radial displacements of the circumferentially-stiff cylinder were quite small.

The axial strains at the cylinder midlength location in the crown and keel are shown in fig. 7.38. Until the cylinder began to soften after a load of about 15-16,000 lb due to unmodeled material behavior, the correlation between predictions and measurements was good. The closeness of the traces representing the measurements for the crown and keel indicated that the platens remained fairly parallel with their original orientations, as indicated by the fact the two traces are almost coincident. Due to imperfections, the two traces would not necessarily ever be coincident. This is shown by the STAGS predictions, which show some difference in the two axial strains even for ideal platen movement. The axial strains at cylinder midlength in the two side segments are shown in fig. 7.39. It is interesting to note that the strains in both sides began to decrease just prior to the sudden load drop. So, considering figs. 7.38 and 7.39, the strain increased in the crown and keel, in a softening fashion just prior to the load drop, while the strain began to decrease in the sides. This indicates the sides began to unload, due to buckling or material failure or both, and the crown and keel load increased, an indication of load transfer. Here, the spread in the measured values of axial strain indicating lack of symmetry of the axial stress distribution in the linear prebuckling load range was considerably less noticeable than in the axially-stiff cylinder. Around 4,400 lb in the sides, fig. 7.39, the axial strain measurements were quite consistent. However, in the crown and keel, fig. 7.38, there was some spread, indicating less than symmetric axial load distribution. In light of the axial DCDT measurements which indicate that compared to the axially-stiff cylinder, the platens seemed to have moved parallel to their original positions, the apparent lack of symmetry in axial load distribution would likely be attributed to local material failures in the crown and keel or the imperfect cylinder geometry.

The strains at cylinder midlength in the circumferential direction, which were induced by Poisson effects, are illustrated in fig. 7.40 for the crown and keel and in fig. 7.41 for the sides. Since the Poisson’s ratios were not the same for the crown, keel, and side laminates, even though the axial strains were similar in magnitude in all segments, the circumferential strains did not have to be the same, and they were not. As expected, compressive axial strains yielded tensile circumferential strains. The measured values in the sides were greater, reflecting higher Poisson’s ratio in the sides. The STAGS predictions and the experimental measurements correlated quite well.
The axial bending strains in the crown and keel, both as measured and as predicted, are shown in fig. 7.42. Recall, the bending strains are defined to be the difference in strains between the outer and inner back-to-back gages. Since no buckling or material failure occurred exactly where these gages were located, as was the case for the axially-stiff cylinder, the bending strains in the crown and keel were quite small. However, since there were deformations occurring in the crown and keel due to material softening, material failure, and buckling due to load transfer, the measured bending strains were considerably larger than those predicted. The signs of the predicted and measured strains, however, were in agreement. Attesting to the smallness of bending effects in the crown and keel, it can be seen that the measured axial bending strains were about a factor of 10 less than the measured axial strains in fig. 7.38.

Until softening began at about 18,000 lb, the measured axial bending strains in the sides, fig. 7.43, were similar in magnitude but opposite in sign relative to the predicted values. These strains were greater than a factor of 10 less than the midsurface circumferential strains in fig. 7.39.

The circumferential bending strains at cylinder midlength are shown in figs. 7.44 and 7.45, for the crown and keel, and sides, respectively. Until softening began, the magnitudes of the measured and predicted strains were similar. It is not clear why the signs differed for all four measured values relative to the predicted values. These circumferential bending strains were small compared to the circumferential strains, figs. 7.40 and 7.41. The measured and predicted strains were less than 100\(\mu\varepsilon\) for the crown and keel in fig. 7.44, and the predicted strains were less than 100\(\mu\varepsilon\) and the measured were less than 250\(\mu\varepsilon\) for the sides in fig. 7.45.

The fact that most of the measured bending strains had signs opposite the predictions can possibly be attributed to strain gage location effects. As mentioned, bending effects were quite small at the strain gage sites and could have easily changed sign within a short distance in the axial direction, even toward the midlength of the cylinder and particularly in the presence of imperfections.

The axial strain in the two boundary layers in the crown and keel segments are shown in
fig. 7.46. These strains for the side segments are shown in fig. 7.47. The correlation between the experiment and the predictions was good, as was the case for all the axial strain measurements.

The measured and predicted axial bending strains in the boundary layers are illustrated in figs. 7.48 and 7.49. The predicted bending effects were significantly greater than the measured values. As with the axially-stiff cylinder, the bending strains changed sign rapidly with axial position away from the end of the cylinder. The strain gages measured the average values of the strains along their lengths and this is reflected in the comparison between measurements and predictions. Even the basic character of the measured bending strains in the sides was quite different than that of the predicted strains, though their magnitudes were similar before the experiment was terminated. Initially, the measured strains were considerably less than the predicted strains. The predicted strains were almost linear with increased load, but the measured strains were quite non-linear. This implies there was material softening or failure in the side boundary layers. Failure, however, was not observed in the boundary layer regions.

The axial strains in the two rosettes are shown in fig. 7.50. Correlation between predictions and measurements for the axial strains was good. The measured circumferential strains are shown in fig. 7.51 and they correlated well with predictions. To be noted is the large increase in circumferential strain in the B2 rosette relative to the B1 rosette. The restraint at the boundary was responsible for this difference. The effect of the boundary restraint decreased rapidly away from the boundary. This was seen to be the case for the axially-stiff cylinder.

The axial and circumferential bending strains as measured and predicted for the rosettes at locations B1 and B2 are shown in figs. 7.52 and 7.53. Correlation with the STAGS predictions was poor. Reasons for the poor correlation have been given before for bending strains in the boundary layers. The $45^\circ$ gages failed to register any strains. For this reason the shear and twist strains are not shown at locations B1 and B2 for the circumferentially-stiff cylinder.

Overall, the correlation between predictions and measurements for the circumferentially-stiff cylinder was poor. The correlation was poor because the cylinder exhibited material failure and the predictions did not include this effect. However, correlation was also poor in that in some
cases the signs of various measured responses were not the same as predicted, even for low load levels, a situation where material behavior was not a factor.

7.3 Material Failure

Since material failure was an important component of the response of the axially-stiff cylinder and dominated the response of the circumferentially-stiff cylinder, comments regarding failure are warranted. First, it should be noted that failure did not appear to occur at the ends of the cylinders. Failure could have occurred near the ends, and often does in many cylinders, because of the increased value of the force and moment results in the boundary layer adjacent to the potting. The (a) portion of figs. 4.22 - 4.29 showed that in the nonlinear prebuckling range, elevated values of the resultants were present in or near the overlap in the area of the potting for the axially-stiff cylinder. These elevated values did not seem to be a major problem because the cylinder did not appear to fail in those locations and moved to the postbuckled configuration predicted in figs. 4.36 and 5.8. However, buckling did occur at a lower than predicted load level. An examination of (a) portion of figs. 4.22 - 4.29 shows that for the axially-stiff cylinder, over the length of the overlap and over the length of the crown, the force and moment resultants increased and decreased in value in accordance with the wrinkling present. It is entirely possible that a slight material flaw, not visible to the eye nor detected as being significant by the C-scan, combined with the increased level of stress associated with the wrinkling-induced force and moment resultants, caused a localized material failure in the nonlinear prebuckling range. That material failure softened the cylinder and lowered its resistance to buckling, resulting in a lower-than-predicted buckling load.

For the circumferentially-stiff cylinder, failure in the prebuckling range definitely did occur. Evidence suggested that the failure occurred part way between the end of the cylinder and the midlength location. The load level was low enough that the cylinder could have been in the linear prebuckling range. The (b) portion of figs. 4.8 - 4.15 showed that for the circumferentially-stiff cylinder in the linear prebuckling range, elevated values of the resultants were present in the boundary layer regions all around the circumference. Some stress resultants were particularly high within or near the overlaps. The elevated values of the resultants at these locations apparently were not a problem. There did not appear to be a bending- or compression-related failure at
or near an overlap in either boundary layer region. There did not appear to be a shear-related failure in those regions either. A close look at figs. 4.8(b) and 4.11(b) reveals that partway between the ends and the midlength, elevated values of force resultants $N_x$ and $M_x$ occur. There are not specific locations of concentrations, like there is with the wrinkling associated with the nonlinear pre-buckling state of the (b) portion of figs. 4.22 - 4.29, but the values are elevated in the linear range. Also, in Chapter 5, though the effects were not large, it was seen that the geometric imperfection caused localized variations in the displacement response not present in the perfect cylinder. The geometric imperfections would also influence the local nature of the force and moment resultants, particularly peak values. Since, as mentioned before, there were no fibers in the axial direction for the circumferentially-stiff cylinder, these increased levels of the resultants, either the global levels as seen in figs. 4.8(b) and 4.11(b), or local levels due to geometric imperfections, could have combined, perhaps with a material flaw, to cause material to fail. These hypotheses can only be substantiated by a more thorough analysis of cylinder response, one that includes failure theories and allows for damage to initiate and progress as the load level increases.
Fig. 7.1 - Initial Moire fringe pattern, axially-stiff cylinder
Fig. 7.2 - Moire fringe pattern, axially-stiff cylinder at 16,000 lb
Fig. 7.3 - Moire fringe pattern, axially-stiff cylinder at 14,000 lb (after load drop)
Fig. 7.4 - Moire fringe pattern, axially-stiff cylinder in postbuckling
Fig. 7.5 - Deformation pattern, axially-stiff cylinder with load held at 11,800 lb
Fig. 7.6 - Keel dimpling, axially-stiff cylinder
Fig. 7.7 - Experimental measurement of axial displacement, axially-stiff cylinder
Fig. 7.8 - Experimental measurement of circumferential displacement, axially-stiff cylinder
Fig. 7.9 - Experimental measurement of radial displacement, axially-stiff cylinder

(a) Radial DCDT placement and identification

(b) Radial displacement vs. load
Fig. 7.10 - Crown and keel midlength axial strain vs. load, axially-stiff cylinder

Fig. 7.11 - Side midlength axial strain vs. load, axially-stiff cylinder
Fig. 7.12 - Crown and keel midlength circumferential strain vs. load, axially-stiff cylinder

Fig. 7.13 - Side midlength circumferential strain vs. load, axially-stiff cylinder
Fig. 7.14 - Crown and keel midlength axial bending strain vs. load, axially-stiff cylinder

Fig. 7.15 - Side midlength axial bending strain vs. load, axially-stiff cylinder
Fig. 7.16 - Crown and keel midlength circumferential bending strain vs. load, axially-stiff cylinder

Fig. 7.17 - Side midlength circumferential bending strain vs. load, axially-stiff cylinder
Fig. 7.18 - Crown and keel boundary layer axial strain vs. load, axially-stiff cylinder

Fig. 7.19 - Side boundary layer axial strain vs. load, axially-stiff cylinder
Fig. 7.20 - Crown and keel boundary layer axial bending strain vs. load, axially-stiff cylinder

Fig. 7.21 - Side boundary layer axial bending strain vs. load, axially-stiff cylinder
Fig. 7.22 - Axial strain vs. load, axially-stiff cylinder at 30° overlap

Fig. 7.23 - Circumferential strain vs. load, axially-stiff cylinder at 30° overlap

Fig. 7.24 - Shear strain vs. load, axially-stiff cylinder at 30° overlap
Fig. 7.25 - Axial bending strain vs. load, axially-stiff cylinder at 30° overlap

Fig. 7.26 - Circumferential bending strain vs. load, axially-stiff cylinder at 30° overlap

Fig. 7.27 - Twisting strain vs. load, axially-stiff cylinder at 30° overlap
Fig. 7.28 - Initial Moire fringe pattern, circumferentially-stiff cylinder
Fig. 7.29 - Moire fringe pattern, circumferentially-stiff cylinder at 18000 lb
Fig. 7.30 - Moire fringe pattern, circumferentially-stiff cylinder at 19,800 lb
Fig. 7.31 - Moire fringe pattern, circumferentially-stiff cylinder at 21,000 lb
Fig. 7.32 - Moire fringe pattern, circumferentially-stiff cylinder at 2,400 lb (after load drop)
Fig 7.33 - Material failure near overlap at 30°, circumferentially-stiff cylinder
Fig. 7.34 - Deformation pattern, circumferentially-stiff cylinder near overlap at 210°
Fig. 7.35 - Experimental measurement of axial displacement, circumferentially-stiff cylinder

(a) Axial DCDT placement and identification

(b) Load vs. axial displacement
Fig. 7.36 - Experimental measurement of circumferential displacement, circumferentially-stiff cylinder
Fig. 7.37 - Experimental measurement of radial displacement, circumferentially-stiff cylinder

(a) Radial DCDT placement and identification

(b) Load vs. radial displacement
Fig. 7.38 - Crown and keel midlength axial strain vs. load, circumferentially-stiff cylinder

Fig. 7.39 - Side midlength axial strain vs. load, circumferentially-stiff cylinder
Fig. 7.40 - Crown and keel midlength circumferential strain vs. load, circumferentially-stiff cylinder

Fig. 7.41 - Side midlength circumferential strain vs. load, circumferentially-stiff cylinder
Fig. 7.42 - Crown and keel midlength axial bending strain vs. load, circumferentially-stiff cylinder

Fig. 7.43 - Side midlength axial bending strain vs. load, circumferentially-stiff cylinder
Fig. 7.44 - Crown and keel midlength circumferential bending strain vs. load, circumferentially-stiff cylinder

Fig. 7.45 - Side midlength circumferential bending strain vs. load, circumferentially-stiff cylinder
Fig. 7.46- Crown and keel boundary layer axial strain vs. load, circumferentially-stiff cylinder

Fig. 7.47 - Side boundary layer axial strain vs. load for circumferentially-stiff cylinder
Fig. 7.48 - Crown and keel boundary layer axial bending strain vs. load, circumferentially-stiff cylinder

Fig. 7.49 - Side boundary layer axial strain vs. load, circumferentially-stiff cylinder
Fig. 7.50 - Axial strain vs. load, circumferentially-stiff cylinder at 30° overlap

Fig. 7.51 - Circumferential strain vs. load, circumferentially-stiff cylinder at 30° overlap
Fig. 7.52 - Axial bending strain vs. load, circumferentially-stiff cylinder at 30\(^\circ\) overlap

Fig. 7.53 - Circumferential bending strain vs. load, circumferentially-stiff cylinder at 30\(^\circ\) overlap
Chapter 8

Summary, Conclusions, and Recommendations for Future Work

8.1 Introduction

This chapter concludes a numerical and experimental investigation of the response of segmented circular composite cylinders subjected to axial endshortening. As was stated in the introduction in Chapter 1, composite fuselage structures may well be built in such a way that laminate stacking sequence varies with circumferential location. As, a manufacturing convenience, tailored sections of the fuselage could be built separately and joined to form a circular structure. Because of the stepwise changes in laminate properties, responses to even simple axisymmetric loadings were expected to result in unusual responses. Specifically, axial endshortening was expected to result in variations in axial, circumferential, and radial displacements with respect to circumferential, as well as axial, location. In addition, the variation in stiffness around the circumference was certain to influence the buckling and postbuckling behavior. In the literature, stiffened fuselage structures have been treated extensively. For this study, stiffeners were not considered, and the characterization of the resulting numerical predictions and experimental results constitute a unique contribution to the literature in the area of stability of circular shells. A summary of the study and its conclusions as well as recommendations for future work are presented below.

8.2 Summary

Two segmented cylindrical specimens formed of four segments 10 in. in diameter and 17 in. long were fabricated by hand using 12.0 in. wide unidirectional Hercules AS4/3502 graphite epoxy prepreg tape. The cylinders were layed up by hand on an aluminum mandrel. The study focused on two cylinder configurations, referred to as axially-stiff and circumferentially-stiff, where the crown and keel laminates were identical and subtend the same opening angle of 60°.
The four eight-layer segments were configured as follows: For the axially stiff cylinder, the lamination sequence for the crown and keel was \([\pm 45/0_2]_S\) and that for the sides was \([\pm 45]_{2S}\), where 0° is the axial direction. For the circumferentially-stiff cylinder, the lamination sequence for the crown and keel was \([\pm 45/90_2]_S\) and the side laminates were the same as for the axially-stiff cylinder. The stiffness of the crown and keel segments reflected the nomenclature used to identify the two cylinders. The cylinders were formed by splicing adjacent segments to form four overlaps which each covered 0.5 in. arclengths. The overlap segments were constructed to be symmetric 12-layer laminates. In constructing the cylinders, the extra layers in the overlaps were pushed outward by the mandrel, thereby creating a local eccentricity in the overlap region. For the axially-stiff case, the overlap lamination sequence was \([\pm 45/0/+45/0/-45]_S\), while for the circumferentially-stiff case, the lamination sequence was \([\pm 45/90/+45/90/-45]_S\). The radial offset of each overlap was two layers.

To construct a particular layer, a section of prepreg tape was cut into suitably sized pieces before starting the layup procedure by determining the desired pattern for that layer. Before any prepreg was wrapped on the mandrel, the mandrel was prepared by spraying it with adhesive. A layer of nonporous teflon was wrapped around the mandrel to protect the aluminum surface. Frekote™ was applied to the nonporous teflon to ensure that the finished cylinder could be removed from the mandrel after the cure cycle was completed. After all layers of prepreg were in place, the cylinders were wrapped with layers of porous teflon and bleeder cloth and sealed in an airtight bag. The cylinders were cured in an autoclave at the manufacturer’s recommended temperature and pressure cycles, while drawing a vacuum. After curing, the cylinders were allowed to cool to room temperature and the carefully removed from the mandrel by sliding them off one end.

Both cylinders were visually inspected after the fabrication process was completed. The four 0.5-in. overlaps were clearly visible on the outer surfaces. The outer surfaces appeared smooth and seemed to indicate good consolidation throughout. The cylinders were also subjected to ultrasonic inspection to evaluate the quality of the specimen. An automated C-scan procedure was used to check for the presence of inclusions, voids, delaminations, and other signs of poor
consolidation in both cylinders. Overall, the C-scan results indicated satisfactory consolidation in both cylinders. Of particular interest in both cylinders were the overlaps, which showed consolidation commensurate with that of the overall cylinders.

The fabrication process produced cylinders with a 5.0-in nominal radius, each approximately 17.0 in. in length. The cylinders were cut to a length which corresponded approximately to the 12-in. test section length plus 1.0 in. on each end. These 1.0-in. lengths were potted into an epoxy and steel end fitting to prevent brooming of the cylinder ends when subjected to axial compression. The potted ends were machined flat and parallel so the cylinder had a 14.0-in. length.

In order to characterize the as-fabricated geometry of each specimen, measurements of the geometry were made after the cylinders were potted. Therefore, surveys were only taken between the potted portions, or so-called test sections, which were 12.0 in. in length. Surveys of the inner and outer shell surfaces were taken to characterize any overall geometric imperfection (e.g., out-of-roundness) and any thickness or other local variations. These geometry surveys were composed of measurements taken every 0.125 in. along the axis of the cylinder and every 1° around the circumference. The initial geometry of the axially-stiff cylinder was dominated by a long wavelength variation in the circumferential direction and practically no variation in the axial direction. The measured geometry of the circumferentially-stiff cylinder was dominated by a wavelength variation in the circumferential direction that was shorter than for the axially-stiff case, particularly at the $x = 12.0$-in. location.

Both the static and dynamic capabilities of the STAGS finite-element code were used to study the response of the segmented cylinders to axial endshortening. The two segmented cylinder configurations were modeled as clamped cylinders using the STAGS 410 quadrilateral element. The cylinders were modeled with 57 elements in the axial direction. The crown and keel each had 12 circumferential elements, the sides 30, and each of the four overlaps 5, for a total of 104 circumferential elements. Clamped boundary conditions were applied such that $v$, $w$, and $rv$ were zero at both ends of the cylinder; $u = 0$ at the $x = L$ end of the cylinder; and $u$ is prescribed as $\Delta$ at the $x = 0$ end of the cylinder to produce a specific endshortening. The potted region, assumed to extend inward axially 1.0 in. from each end, was also taken into account in the model. In both
the axially-stiff and circumferentially-stiff cylinder configurations, the wall thickness of the crown, side, and keel segments was equivalent to eight layers of 0.044 in. The overlaps had 12 layers, resulting in a thickness of 0.066 in. The eccentricity developed due to forming the cylinder on a mandrel was the radial distance between the reference surface of the overlap and adjacent segments.

Because the overlaps were 50% thicker than the crown, side, and keel segments, they acted as mild stiffeners, and eccentric ones at that. The effect of these eccentric overlap segments was investigated in order to confirm the need to include this feature in the finite-element model. It was found that including the local eccentricity in the finite-element model caused inward pillowing in the radial displacement, and gave rise to the development of shear stress resultant \( N_{xy} \). It was concluded that these effects were due to the fact that the misalignment of the reference surfaces between the overlaps and adjacent segments led to nonzero members in the bending-stretching matrix \( B_{ij} \).

The predicted response of the two segmented cylinder configurations was first presented for an idealized round geometry. The analysis was discussed in terms of considering the endshortening \( \Delta \) the loading parameter. To obtain the predicted response, the endshortening \( \Delta \) was applied starting from a state of no deformation (\( \Delta = 0 \)) and increased monotonically to reach buckling. The response from \( \Delta = 0 \) to buckling was referred to as the prebuckling response. This prebuckling initiated from \( \Delta = 0 \) as a linear relationship between load and endshortening. This linear relationship was referred to as the linear prebuckling range. As \( \Delta \) was monotonically increased toward buckling, the load vs. endshortening relationship took on a nonlinear character. Thus just before buckling the load vs. endshortening relationship was said to be in the nonlinear prebuckling range. As the value of \( \Delta \) was increased further, the cylinder buckled. Increasing \( \Delta \) beyond the buckling resulted in postbuckling response. Increasing \( \Delta \) beyond the buckling value changed cylinder behavior dramatically. The transition from prebuckling to postbuckling, a dynamic event, was modeled using transient analysis. Briefly, the steps in the transient analysis were as follows:

1. The dynamic transient analysis was initiated by incorporating damping into the finite-element model.
2. A “small” inward pressure load was applied over a specific “short” time interval.
3. The transient dynamic solution was begun using a numerical time integrator.
4. Time was moved forward by integrating while monitoring the kinetic energy until it dissipated to an insignificant level compared to the initial level. The transient analysis was then terminated.
5. A load relaxation technique was used to make the transition from the transient analysis back to the quasi-static analysis.
6. The quasi-static analysis was restarted.

Because geometric imperfections are always present in cylindrical elements, the effect of imperfections on the response of these segmented configurations was investigated. The actual measured geometries of the inner surfaces of the cylinders were introduced into the finite-element models. An interpolation scheme was devised using Mathematica™ to map the actual measured geometry onto the finite-element mesh. The new unloaded shape resulting from applying the interpolation scheme, considered an imperfect cylinder, was used as the basis for a nonlinear quasi-static analysis.

In order to further study the response of small-scale segmented composite cylinders subjected to axial endshortening, the two specimens were tested. The load introduction was controlled by a SATEK model MK3 controller. The specimens were loaded axially using a Baldwin-Tate Emery 120-kip capacity load frame. Axial, circumferential, and radial displacements taken with respect to the specimen coordinate axis were measured using DCDTs. Axial motion of the bottom load platen was measured by DCDTs positioned at three corners of the top load platen, in order to establish a load-endshortening relation. Circumferential displacements were measured by positioning DCDTs at right angles to wooden tabs bonded to the cylinder on the overlap segment at the axial midlength location. These DCDTs measured the motion of the tabs, and thereby the displacement of the specimen at the axial midlength where it was predicted to be largest. Radial displacement was measured using DCDTs at the centers of the crown, side, and keel segments in an effort to capture distinct features predicted numerically. Surface strains were measured with pairs of back-to-back strain gages distributed over the inner and outer surfaces of the specimens. The strain gages were mounted on the circumference at the axial midlength of the test section of each cylinder and on the circumferences at the boundaries of the test section at $x = 1$ in. and $x = 13$
in. These gages measured the surface strain response during the test at predicted areas of interest. Their placement was also crucial to the load balancing procedure used to ensure that load was applied to the cylinder as uniformly as possible. Two three arm rosette gages were positioned in the crown segment along side the overlap at $30^\circ$ the $x = 13$-in and $x = 12$-in. locations. Three-arm rosettes were chosen to measure the shear strain predicted numerically. Shadow Moire interferometry was used on the portion of each cylinder between $0^\circ$ and $90^\circ$ for both cylinder specimens to qualitatively observe deformation patterns during the tests. Selected photographs of the Moire fringe patterns were presented. All electrical signals from strain gages, DCDTs, and load output were recorded using a digital data acquisition system called MODCOMP. All data was sampled at a rate of one cycle per second. Test results were presented as load vs. displacement relations, load vs. strain relations, photographs of Moire interferometry fringe patterns, and photographs of the specimens, which failed during testing.

8.3 Conclusions

Following is a discussion of conclusions regarding the numerical and experimental investigation of the response of segmented circular composite cylinders subjected to uniform axial end-shortening.

8.3.1 Linear Prebuckling Range

The numerically-predicted response of both segmented cylinder configurations studied was characterized, in part, by the existence of circumferential displacement. The finite-element analyses of Chapter 4 predicted the circumferential displacement to be largest at the cylinder midlength point of the overlap segments. Negative circumferential displacement was predicted at the overlaps at $\theta = 30^\circ$ and $210^\circ$; positive at $\theta = 150^\circ$ and $330^\circ$. In both cases, the experimental measurements confirmed the existence of circumferential displacement. Further, in the case of the axially-stiff cylinder, the character of the experimental measurements agreed to a significant degree with the numerical predictions. (Note here that the Moire interferometry set-up made it impossible to measure the circumferential displacements at $30^\circ$.) The magnitudes of the experimentally measured displacements for the axially-stiff cylinder at $210^\circ$ and $330^\circ$ agreed with the
predictions, but the values measured at 150° were greater than predicted. In the case of the circumferentially stiff cylinder, the measurement of the circumferential displacement was not particularly successful. The DCDTs measuring circumferential displacement in the circumferentially-stiff cylinder at 210° and 330° malfunctioned. Prior to malfunctioning and contrary to the predicted response, the data indicated that all three DCDTs measured a negative circumferential displacement. It is believed that localized material failures may have caused the tabs to disbond, rendering the DCDTs of little value.

The numerical responses of both segmented cylinder configurations studied were also characterized by circumferential gradients and an axial boundary layer in the radial displacement. For the perfect axially-stiff cylinder case, the analysis predicted the greatest outward radial displacement to be in the crown segment, and the identical keel segment. In the imperfect axially-stiff model, the symmetry of the predicted response was affected, but the circumferential gradients in the radial displacement response persisted and the boundary layer effect remained pronounced. Experimental measurements for the axially-stiff cylinder specimen showed slightly greater outward radial displacement in the crown and keel than the side segments at a load level of about 8000 lb, thus confirming the existence of a circumferential gradient in radial displacement in the linear prebuckling range. For the perfect circumferentially-stiff cylinder configuration, larger outward radial displacements were predicted in the side segments than the crown and keel. The inclusion of the actual measured geometry in the imperfect circumferentially-stiff model had no significant effect on the radial displacement response. In the predicted linear prebuckling range of loading, the experimentally-measured radial displacement in the sides were higher than in the those measured in the crown and keel. This confirmed the predicted existence of circumferential gradients in radial displacement. A boundary layer in the radial displacement response was also predicted as a consequence of the clamped boundary conditions. The decrease in the absolute value axial strain measurements near the boundaries of the test section in both cylinder specimens crown, side, and keel segments confirmed the boundary layer effect. Furthermore, the rosettes yielded measurements which reflected a change in curvature which characterized the boundary layer.
In the linear prebuckling range, a segmented distribution of $N_x$ corresponding to laminate properties of each cylinder was predicted. The implications of these predicted load distributions were that ultimately the more highly-loaded segments would buckle first. Conclusions concerning buckling will be addressed later. However, comments concerning aspects of the measured axial strain distribution are worth noting here. While it was not possible to measure the $N_x$ distribution experimentally, axial strain measurements provided indications that the predicted $N_x$ distribution was confirmed experimentally. In the perfect axially-stiff cylinder, the predicted values of $N_x$ were higher in the identical crown and keel segments than in the sides. Including the actual measured geometry affected the symmetry of the $N_x$ distribution. The measured axial strain in the axially-stiff cylinder indicated similar strains in all segments, as expected, but reflected a lack of symmetry indicating as predicted a lack of symmetry in the distribution of $N_x$. However, the predicted lack of symmetry showed a higher compressive strain in the crown, and the measured response showed a higher compressive strain in the keel. This was likely due to the load platens not remaining parallel to their initial positions during testing. For the perfect circumferentially-stiff cylinder, the $N_x$ in the side segments was predicted to be higher than in the crown and keel, and including the actual measured geometry had no significant effect. The experimental measurements of axial strain in the circumferentially-stiff cylinder reflected a segmented distribution of $N_x$ in the linear prebuckling range that correlated with the predictions.

Predicted values of normalized $N_{xy}$, $M_{xy}$, and $Q_y$ showed there were shear effects in and around the overlap region near the potted ends of both the axially-stiff and circumferentially-stiff cylinders. The shear components were generated to overcome the mismatch due to the boundary layer effect in the radial displacement. Rosette gages in the crown segment, along side the overlap at $30^\circ$ at the $x = 13$-in. and $x = 12$-in. locations, measured shear strain in the linear prebuckling range. As was seen, the correlations between measurements and predictions were good.

**8.3.2 Nonlinear Prebuckling Range**

For both the axially-stiff and circumferentially-stiff cylinders, the nonlinear prebuckling response was characterized by wrinkling, or waviness, in the axial direction. For the perfect axi-
ally-stiff cylinder, wrinkling occurred in the crown and keel segments identically. For the imperfect axially-stiff cylinder, the radial displacement response was characterized by noticeable wrinkling in the keel segment but not so much in the crown, with a more pronounced boundary layer effect in the crown. For the perfect circumferentially-stiff cylinder, the wrinkling occurred in the side segments. There was little effect in the response of the imperfect circumferentially-stiff cylinder resulting from including the actual measured geometry. The Moire interferometry for the circumferentially-stiff cylinder showed a pattern between 60° and 90° that could be attributed to wrinkling developing in the axial direction.

For both cylinders, the load distribution predicted for the linear prebuckling response changed somewhat due to the effect of the wrinkling. However, the normalized value of $N_x$ maintained its segmented distribution, with the highest overall values in the overlaps.

### 8.3.3 Buckling Response

For the case of the perfect cylinder, four methods were used to compute the value of end-shortening at buckling and the associated buckling load. Overall, for both the axially-stiff and circumferentially-stiff cylinders the four estimates obtained showed good agreement. For the zero-eigenvalue condition, the critical load and endshortening predicted for the imperfect axially-stiff cylinder were 7% lower than for the perfect axially-stiff cylinder predictions. The difference between the predicted buckling load and the measured buckling load for the axially-stiff cylinder represented a 12% overprediction in buckling capacity. Here, also the critical endshortening is overpredicted by 15%. Lack of perfect displacement control with the loading fixture and localized material failure not modeled were believed to have contributed to these overpredictions. For the circumferentially-stiff cylinder, the predicted critical values of load and endshortening were 1% lower than the perfect circumferentially-stiff cylinder. The experimentally measured buckling load indicated a 69% overprediction of the buckling capacity. The critical endshortening was overpredicted by 25%. The circumferentially-stiff specimen suffered material failures just prior to buckling. In fact, the experimental response is characterized more by material failure than buckling. Since the material failure was not modeled, it is not possible to accurately predict the
buckling values in the circumferentially-stiff cylinder. It was suggested, however, that modeling to account for material failure would be an approach to begin to understand the problem.

8.3.4 Postbuckling Response

The numerical prediction for the postbuckling response was characterized by a dimpling at $\Delta_{cr}$. The axial load dropped suddenly from $P_{cr}$ to a load which satisfied the conditions for a secondary state of structural equilibrium. After having reached a point of secondary equilibrium, the value of $\Delta$ was predicted to increase along a path which reflected lower inplane stiffness compared to the stiffness along the primary equilibrium path. The response of the perfect axially-stiff cylinder, predicted a 19.87% load drop to a secondary equilibrium configuration, a configuration which was dominated by a single inward dimple in the crown and keel segments. The imperfection in the axially-stiff case changed the manner in which the cylinder dimpled. The keel was predicted to dimple first following a 9.7% drop in load, then the crown following loading along a secondary load path and a subsequent 11.07% drop in load. The total load drop predicted was 15.94%. The experimental postbuckling response showed a 15.6% load drop, along a diagonal, rather than vertical, line thus confirming the predicted load drop in postbuckling. In the experiments, if one dimple occurred before the other, it most likely happened in less than the one second time interval. However, there was evidence that just before buckling the crown began its inward displacement before the keel. The variation in cylinder stiffness that would have led to keel buckling first in the model assumed a uniform endshortening independent of circumferential coordinate. However, it was possible that the bottom and top load platen did not remain parallel to their original position, and this led to the crown buckling before the keel. For the perfect circumferentially-stiff cylinder, a 34% load drop was predicted. The postbuckled shape was dominated by a two rings of 6 inward dimples which have as a line of anti-symmetry the circumference at the cylinder midlength. The effect of the imperfection on the circumferentially-stiff cylinder was minimal. In the experiment, the material failure in the circumferentially-stiff cylinder prevented any postbuckling response. However, two rings of inward dimples were observed to characterize the response at failure.
8.4 Recommendations for Future Work

The goal of the present study has been to investigate numerically and experimentally the response of segmented composite cylinders subjected to uniform axial endshortening. Following is a discussion of relevant topics for future work based on the findings of this study.

At a given axial and circumferential location, the difference between the inner and outer radial measurements represent the thickness of the cylinder at that location. The wall thickness measurements could be used to develop a finite-element model that would also represent thickness variations as a function of axial and circumferential coordinates. This would, in turn, require variations in material properties that presumably would occur due to the volume fraction variations which led to the thickness variations. Variations in material properties could be computed by a simple micromechanics scheme.

The comparison of the experimental and numerical results brought to the forefront the issue of material failure. For this reason it is necessary to incorporate failure analysis into the modeling of the segmented cylinder to achieve better predictions of the response, particularly of the buckling, or critical, values in the presence of localized material failure. A finite-element model that takes into account material failure, perhaps even progressive failure, would be useful.

Other combinations of laminates should be explored as well, particularly those having 0° layers in all segments. With the goal of designing fuselage structures which take advantage of the tailorability of composites, a parametric study could perhaps lead to an optimal set of laminate combinations to be used to develop a more extensive experimental program for the purpose of developing more robust modeling techniques.

Finally, there are areas outside the scope of this study having to do with the response of segmented cylinders which should be addressed. The response of segmented cylinders to internal pressure loading, another simple axisymmetric load, should be investigated in order to characterize the failure mechanisms activated by the pressure loading. Ultimately, combined loadings of axial endshortening and internal pressure would be of interest to designers. The response of segmented cylinders to bending should be investigated. This is a non-axisymmetric loading problem.
In fact there are several non-axisymmetric loadings which could be investigated, such as the response of segmented cylinder to any nonuniform axial endshortening, including the slight non-axisymmetric loading such as was encountered during testing of the axially-stiff cylinder. It would also be useful to investigate the free thermal expansion of the segmented cylinder. Finally, a consistent set of semi-closed form solutions for the buckling, vibration, failure, and perhaps aeroelastic responses of segmented cylinders would be of use.
References


6. Vishay Measurements Group, Inc., P. O. Box 27777, Raleigh, NC 27611-77777

7. MODCOMP, Inc., 1650 West McNabb Road, Fort Lauderdale, FL 33309-1088
Bibliography


Vita

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