PREDICTION OF FLOOR VIBRATION RESPONSE USING THE FINITE ELEMENT METHOD

by

Michael J. Sladki

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APPROVED:

________________________________________
Thomas M. Murray, Chairman

________________________________________
Raymond H. Plaut

________________________________________
Mehdi Setareh

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Michael J. Sladki

Dr. Thomas M. Murray, Chairman
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(ABSTRACT)

Several different aspects of floor vibrations were studied during this research. The focus of the research was on developing a computer modeling technique that will predict the fundamental frequency of vibration and the peak acceleration due to walking excitation as given in AISC Design Guide 11, *Floor Vibrations Due to Human Activity* (Murray, et al., 1997). For this research several test floors were constructed and tested, and this data was supplemented with test data from actual floors.

A verification of the modeling techniques is presented first. Using classical results, an example from the Design Guide and the results of some previous research, the modeling techniques are shown to accurately predict the necessary results.

Next the techniques were used on a series of floors and the results were compared to measured data and the predictions of the current design standard.

Finally, conclusions are drawn concerning the success of the finite element modeling techniques, and recommendations for future research are discussed. In general, the finite element modeling techniques can reliably predict the fundamental frequency of a floor, but are unable to accurately predict the acceleration response of the floor to a given dynamic load.
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FOR

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CHAPTER I
FLOOR VIBRATIONS BACKGROUND AND LITERATURE REVIEW

1.1 INTRODUCTION

Modern construction techniques make use of lightweight, high-strength materials to create flexible, long-span floors. These floors sometimes result in annoying levels of vibration under ordinary loading situations. While these vibrations do not present any threat to the structural integrity of the floor, in extreme cases they can render the floor unusable by the human occupants of the building.

The wide variety of scales and prediction techniques available to engineers is an indication of the complex nature of floor vibrations. Furthermore, since each method inherently makes numerous assumptions about the structure, not all methods are equally applicable to all situations.

Remedies for annoying floor vibrations are often cumbersome and expensive. It is far better to design the floor properly the first time, rather than retrofit the structure once a problem develops. The easiest way to avoid building a floor that is susceptible to annoying vibrations is to design the floor with an adequate understanding of the physical phenomena and a respect for the consequences of poor design.

For these reasons, the ability of a finite element program to predict both the fundamental frequency and the dynamic response of a multi-bay floor, regardless of the regularity of beam spacing, material properties, damping properties or other limitations, needs to be explored. This study presents the results of a proposed method for predicting the response of complicated floor systems. In this chapter, the scope of the research is presented first, followed by an introduction to the terminology used in the field. Background criteria are presented next, followed by a detailed description of the most
current design standard. Brief mention is given to some other research currently ongoing in the field in the next section, followed by the justification for this particular research and an overview of the successive chapters.

1.2 SCOPE OF RESEARCH

The goal of this study is to examine and evaluate two techniques for determining the fundamental frequency and acceleration response of a complex, multi-bay floor, using finite element computer models. Both three-dimensional finite element models (3D-FEMs) and two-dimensional finite element models (2D-FEMs) are investigated. The floors studied are more complicated than other floors found in the literature, since they are not limited to a single bay, as has been done in most other research. The models provide the natural frequency of the floor as well as the dynamic response of the floor to a given loading, something that is largely absent from the literature. Data from these models are compared to measured data collected in the field and to current design standards recommended by the American Institute of Steel Construction Design Guide 11 Floor Vibrations Due to Human Activity. The floors examined include joist and hot-rolled beam supported floors, and floors with joist-girders. Lightweight and normal weight concrete, and a variety of loads and damping ratios are included in the test data. Some floors were constructed in the laboratory, while others were actual floors currently in use across the United States. Before a discussion of the results is presented, an introduction to relevant terminology used in this study is presented.

1.3 TERMINOLOGY

Definitions of terminology relevant to this study follow.

Vibration – Oscillation of a system about its equilibrium position. Vibration can be free or forced. Free vibration occurs when the system is excited and allowed to vibrate at a natural frequency of the system. Forced vibration occurs when a system is continually excited at a particular frequency and the system is forced to oscillate at that frequency.
Amplitude – Magnitude of the vibration. Depending on what is being measured, the amplitude can be a displacement, a velocity, or an acceleration.

Period – Time for one cycle, usually measured in seconds. See Figure 1.1.

\[ \text{TIME} \]

\[ \begin{array}{c}
\text{RESPONSE} \\
\hline
\text{PERIOD} \\
\hline
\text{AMPLITUDE} \\
\hline
\text{TIME} \\
\end{array} \]

Figure 1.1 – Definition of Amplitude and Period

Cycle – Motion of the system from the time it passes through a given point travelling in a given direction until it returns to that same point and in the same direction.

Fundamental Frequency – The lowest frequency at which a system will tend to undergo free vibration. This frequency is a function of the mass in a system and the system stiffness. Floors that oscillate at frequencies in the range between 4 and 8 Hz are of particular concern.

Damping – Loss of energy per cycle during the vibration of a system, usually due to friction. Viscous damping, damping proportional to velocity, is generally assumed.

Critical Damping – The damping required to prevent oscillation of the system. Damping is usually presented as a ratio of actual damping divided by critical damping. Log decrement damping is determined by taking the natural logarithm of the ratio of successive peaks in the response curve (see Figure 1.2). Modal damping is determined from an analysis of the Fourier spectrum of the response. Modal damping tends to be
smaller than log decrement damping and tends to more accurately match the actual damping present in a structure.

Figure 1.2 – Effect of Modal Viscous Damping on Response

Resonance – When a system is excited at a frequency that is close to any natural frequency of the system the system will undergo resonance which can lead to severe damage to the system.

Fast Fourier Transform (FFT) – A mathematical process by which a time domain response curve is transformed into a frequency domain curve.

Frequency Response Function (FRF) – The resulting frequency domain curve when an FFT is performed on a data curve showing the relative contributions of various frequencies. Peaks represent the various modes of vibration. See Figure 1.3.

Heel Drop – An excitation function used to evaluate floors. The function, shown in Figure 1.4, is defined as the force generated by a 170-lb. man, raised up on his toes approximately two and one half inches, who shifts his weight to his heels as they strike the floor. The actual curve is approximated by the straight-line ramp function shown.

Dynamic Loads – Loads applied over a finite period of time, or loads changing with time, as opposed to static loads. See Figure 1.5.
Figure 1.3 – Typical FRF Plot

Figure 1.4 – Actual and Approximate Heel Drop Functions
Figure 1.5 - Types of Dynamic Loads

(a) Harmonic Load

(b) Periodic Load

(c) Transient Load

(d) Impulsive Load
Mode Shape – The shape a structure assumes when it vibrates at a natural frequency. See Figure 1.6

Figure 1.6 - Typical Beam and Floor System Mode Shapes
1.4 BACKGROUND CRITERIA

Before this particular study is described in detail, it is important to review the floor vibrations criteria historically used. This section briefly highlights the main points of a number of criteria proposed and used over the years, starting with studies on human perception of vibration.

Initial studies showed that human susceptibility to annoying floor vibrations is dependent upon three factors: frequency, initial amplitude, and damping (Lenzen, 1969). The basic element for determining the frequency of a floor system is the T-Beam model shown in Figure 1.7 with the frequency of the T-Beam determined from:

$$ f_n = 1.57 \sqrt[3]{\frac{gEI_t}{WL^3}} $$  \hspace{1cm} (1.1)

Here \( g \) = acceleration of gravity, \( E \) = modulus of elasticity, \( W \) = total weight supported by the T-Beam, \( L \) = beam span.

Reiher and Meister (1931) conducted the first study that examined human perception of vibration. They subjected a group of people to steady state vibrations and asked them to rate the experience. From this data, a chart was created, called the Reiher-Meister Scale, which was eventually applied to floor vibrations. In 1966, Lenzen
discovered that humans are much less sensitive to transient vibrations than they are to steady state vibrations. He modified the Reiher-Meister Scale to reflect this and applied his findings specifically to floor vibrations (Lenzen, 1966). In 1974, Wiss and Parmelee studied the effect of transient vertical vibrations on a group of test subjects, using frequency displacement and damping as parameters (Wiss and Parmelee, 1974). In 1975, Murray introduced a design criterion that utilized a dynamic load factor to more accurately predict the deflection of the system under a given load. Previous methods had used a static analysis for determining deflection and predicted frequency based upon a static load. Murray also introduced Dunkerly’s Equation to account for the fact that the fundamental frequency of a floor system is a function of the fundamental frequency of the beams and the girders, rather than just the beams, as had previously been assumed. Dunkerly’s Equation is:

\[ \frac{1}{f_s^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2} \]  

(1.2)

Here \( f_s \) = the system frequency, Hz; \( f_b \) = the frequency of the beam, Hz; and \( f_g \) = the frequency of the girder, Hz (Murray, 1975). In 1976, Allen and Rainer introduced a design criterion based on a number of walking tests performed on long span floors. This criterion was included in the Canadian Standards Association Standard, CSA S16.1 (CSA, 1989). In 1981, Murray introduced a minimum required damping criterion that was based on a series of tests. Floors were tested and were rated as either acceptable or unacceptable. A line of best fit was then drawn through the data points and was used to determine the minimum damping required in a system for the system to be acceptable (Murray, 1981). In 1985, Wyatt introduced a design criterion for walking similar to the current American Institute of Steel Construction (AISC) Design Guide procedure (Murray, et al., 1997). In 1989, the International Organization for Standardization (ISO) presented the scale shown in Figure 1.8. This scale, which shows the recommended
limits of peak acceleration, due to walking, for human tolerance, later became a core portion of the AISC Design Guide procedure.

![Figure 1.8 – Recommended Peak Acceleration for Human Tolerance](image-url)
In 1990, Allen proposed a simplified model of the dynamic response of a floor due to rhythmic loading. The basic model consisted of a person exerting a sinusoidal force on a lumped mass attached to a spring and a damper. This simplification would later form the basis for the portion of the AISC Design Guide that deals with the acceleration response due to walking excitation. The derivation of the equation used by Allen is included in Section 3.3.

All of the methods described here were based on the assumption that floor vibrations are localized in a single bay. It is also assumed that the vibration characteristics of a single bay are simply a function of the vibration characteristics of a T-beam analysis for the beams and the girders in the bay. Finally, it is assumed that the T-beam behaves as an equivalent single degree of freedom system with a particular stiffness, damping ratio, and a lumped mass oscillating with only one mode of vibration. The most current design criterion, AISC Design Guide 11, *Floor Vibrations Due to Human Activity*, hereafter Design Guide, is also based on these same assumptions.

All of the methods previously surveyed are based on analyses of rectangular bays. Since all bays are not necessarily rectangular, though, there is a need for an analysis technique for evaluating more complex framing schemes. Two techniques were explored for this study, and are presented in the following chapters. Since the Design Guide method is the most current criterion available, it will be used for comparison purposes. Section 1.5 is a detailed overview of the Design Guide procedure for walking excitation.

1.5 AISC DESIGN GUIDE 11

Allen and Murray (1993) proposed a criterion based on limits proposed by the ISO shown previously in Figure 1.8, which later became the basis for the Design Guide procedure. These limits take into account the fact that human tolerance for vibration depends a great deal on the environment. For example, someone trying to study in a library will tolerate far less vibration than someone shopping at a mall for Christmas gifts.
The Design Guide method essentially requires two sets of calculations. First, it is required to estimate the frequencies of the beams, the girders, and then the entire bay. In general, it is assumed that the members are simply supported; however, cantilever sections are considered. The second part is to estimate the peak acceleration due to walking by estimating the effective mass of the system and then calculating the peak acceleration due to a dynamic load of specific magnitude, oscillating at the lowest natural frequency of the system.

**Frequency.** The fundamental frequency of a beam, Equation 1.3, is given in the Design Guide as:

\[ f_j = 0.18 \sqrt{\frac{g}{\Delta_j}} \]

(1.3)

Here \( g \) = acceleration due to gravity and \( \Delta_j \) = deflection of the joist, beam, or girder under the estimated loading. The maximum deflection of the beam occurs at mid-span and is given as:

\[ \Delta_j = \frac{5w_jL^4_j}{384EI_j} \]

(1.4)

Here \( w_j \) = weight per length of beam, \( E_s \) = modulus of elasticity for steel, \( I_j \) = composite moment of inertia, \( L_j \) = length of beam. (Note: Substitution of Equation 1.4 into Equation 1.3 yields Equation 1.1.) For hot-rolled sections, \( I_j \) is the fully composite moment of inertia. However, for joists, \( I_j \) is the effective composite moment of inertia, which is smaller than the fully composite moment of inertia since web shear deformations are significant. For a joist, the effective composite moment of inertia is estimated from:

\[ I_{eff} = \frac{1}{\gamma} \frac{1}{I_{chords}} + \frac{1}{I_{comp}} \]

(1.5)
Here \( I_{\text{chords}} \) = moment of inertia of the bare steel, in.\(^4\); \( I_{\text{comp}} \) = fully composite moment of inertia of the slab and the chords, in.\(^4\); and \( \gamma \) = a reduction factor given as:

\[
\gamma = \frac{1}{C_r} - 1
\]  \hspace{1cm} (1.6)

In Equation 1.6, \( C_r \) is a modification factor proposed by Band and Murray (1996) to estimate the effective moment of inertia of a joist or joist-girder. Their study showed that since the web members underwent significant shear deformation during loading, a reduction in the effective moment of inertia was necessary. Equation 1.7 applies to joists with angle web members, and Equation 1.8 applies to joists with rod webs. In each case, \( L \) = length, in. and \( D \) = nominal depth, in.:

\[
C_r = 0.90(1 - e^{-0.28(L/D)^{2.8}}) \quad \text{for } 6 \leq L/D \leq 24
\]  \hspace{1cm} (1.7)

\[
C_r = 0.721 + 0.00725(L/D) \quad \text{for } 10 \leq L/D \leq 24
\]  \hspace{1cm} (1.8)

For a girder, the calculations for frequency are the same, unless the girder supports joists. To account for the fact that joist seats are not sufficiently stiff to justify the use of the full composite moment of inertia, the following equation is used to calculate the effective moment of inertia of the girder:

\[
I_g = I_{nc} + (I_e - I_{nc})/4
\]  \hspace{1cm} (1.9)

Here \( I_{nc} \) = moment of inertia of the bare steel, in.\(^4\); \( I_e \) = fully composite moment of inertia, in.\(^4\). In the case of a joist-girder, \( I_{nc} \) is:

\[
I_{nc} = C_r I_{\text{chords}}
\]  \hspace{1cm} (1.10)
Walking Acceleration. The second part of the Design Guide method involves predicting the peak acceleration due to a person walking across the floor. To model this, a harmonic forcing function is used to represent a worst case scenario of someone repeatedly stepping at the midpoint of a span with a frequency equal to or a multiple of the lowest natural frequency of the system. This forcing function is given by:

$$F_i = P \alpha_i \cos(2\pi f_{step} t)$$  \hspace{1cm} (1.11)

Here $P = \text{person's weight}$ taken as 157 lbs., $i = \text{harmonic multiple of the step frequency}$, $f_{\text{step}} = \text{step frequency}$, and $\alpha_i = \text{dynamic coefficient from Table 1.1}$.

Table 1.1
Common Forcing Frequencies, $F$, and Dynamic Coefficients, $\alpha_i$ *

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Person Walking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$, Hz</td>
</tr>
<tr>
<td>1</td>
<td>1.6 – 2.2</td>
</tr>
<tr>
<td>2</td>
<td>3.2 – 4.4</td>
</tr>
<tr>
<td>3</td>
<td>4.8 – 6.6</td>
</tr>
<tr>
<td>4</td>
<td>6.4 – 8.8</td>
</tr>
</tbody>
</table>

* Dynamic Coefficient = peak sinusoidal force / weight of person(s)

A resonance response function, Equation 1.12, predicts the peak acceleration that can be used with Figure 1.8.:

$$a = \frac{R \alpha_i P}{\beta W} \cos(2\pi f_{step})$$  \hspace{1cm} (1.12)
Here, \( a/g \) = the ratio of the floor acceleration to the acceleration due to gravity, \( \beta \) = the modal damping ratio, \( W \) = effective weight of the floor, and \( R \) = a reduction factor recommended as 0.7 for footbridges and 0.5 for floor structures with two-way action.

For design, Equation 1.12 simplifies to (Allen and Murray, 1993):

\[
\frac{a_{p}}{g} = \frac{P_{o} \exp(-0.35 f_{n})}{\beta W}
\]  

\( a_{p}/g \) = estimated peak acceleration, \( \beta \) = modal damping ratio, and \( W \) = effective weight supported by the beam panel, girder panel, or combined panel, as applicable, lbs. For entire floor systems, an effective panel weight for the beam is combined with an effective panel weight for the girder to yield an effective panel weight for the bay. In general, the effective weight of a panel is given as:

\[
W = wBL
\]  

Here \( f_{n} \) = natural frequency of the floor, \( P_{o} \) = constant force equal to 65 lb. for floors and 92 lb. for footbridges, \( a_{p}/g \) = estimated peak acceleration, \( \beta \) = modal damping ratio, and \( W \) = effective weight supported by the beam panel, girder panel, or combined panel, as applicable, lbs. For entire floor systems, an effective panel weight for the beam is combined with an effective panel weight for the girder to yield an effective panel weight for the bay. In general, the effective weight of a panel is given as:

\[
W = wBL
\]  

Here \( L \) = member span, \( w \) = weight per unit area, \( B \) = effective width of the panel. For a beam or joist panel, the effective width is given as:

\[
B_{j} = C_{j}(D_{s} / D_{j})^{1/4} L_{j}
\]  

Here, \( D_{s} \) = transformed slab moment of inertia per unit width, \( D_{j} \) = effective moment of inertia of tee-beam per unit width, and \( C_{j} \) = 2 for most joists or beams and 1 for joists or beams parallel to an interior edge. If the edge joist or beam is more than 50% stiffer than the interior beams or joists, \( C_{j} \) should be taken as 2, even though it is an edge member. For a girder, the effective width is:
\[ B_g = C_g \left( \frac{D_j}{D_g} \right)^{1/4} L_g \]  

(1.16)

Now, \( C_g = 1.6 \) for girders supporting joists connected to the girder flange, and 1.8 for girders supporting beams connected to the girder web, and \( D_g = \) effective moment of inertia per unit width. If the girder is an interior edge member, \( B_g \) should be taken as 2/3 of the supported beam or joist span.

For the combined mode, the effective panel weight is a function of the flexibility of each member type and the effective weight of each panel type, and is given by:

\[ W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_j}{\Delta_j + \Delta_g} W_g \]  

(1.17)

If a beam, joist, or girder is continuous over a support, and the adjacent span is at least seven-tenths of the length of the center span, then the effective weight of that panel may be increased by 50% to account for continuity. Hot-rolled shear connections are acceptable here, but joists that are only connected at the top chord, and girders that frame into a column, are excluded.

If the girder span is less that the effective width of the joist panel \( (L_g < B_j) \), the combined mode is said to be restricted. In this case, the deflection of the girder is multiplied by the factor in Inequality 1.18, subject to the limits shown:

\[ 0.5 \leq \frac{L_g}{B_j} \leq 1.0 \]  

(1.18)

This reduction applies to the calculation of \( W \) in Equation 1.17 and to the frequency of the member, Equation 1.3.

**Cantilevers.** For a cantilever member, the deflection at the tip of the cantilever is most critical. Figure 1.9 defines some of the parameters used to estimate the deflection at the tip of a cantilever.
Since it is rare for a cantilever to be fully fixed at the supported end, the following equations are used to estimate the modal deflections for use with Equation 1.3. Equation 1.19 is used if the cantilever deflection is greater than the deflection of the backspan, while Equation 1.20 is used if the cantilever deflection is less than the deflection of the backspan.

\[
\Delta = \Delta_i = C_m \left[ 1 + \frac{4 L_B}{3 L} \frac{1 + 0.25 \left( \frac{L_B^2}{L^2} \right)}{1 + \frac{n_c k_c}{k_b}} \right] \Delta_f
\]  

(1.19)

\[
\Delta = \Delta_B = C_m \left[ 1 + (2.4) \frac{\left( \frac{L_B^2}{L^2} \right) - 0.5 \frac{k_c}{k_b}}{1 + \frac{n_c k_c}{k_b}} \right] \Delta_{ss}
\]  

(1.20)
Here \( k_b = I_B / L_B \) or the moment of inertia of the backspan divided by the length of the backspan, \( k_c = I_c / L_c \) is for the cantilever, \( C_m = 0.81 \) for a distributed mass and 1.06 for a mass concentrated at the tip, \( n_c = 2 \) for columns above and below, 1 for columns only below, \( \Delta_f \) = deflection of a fully fixed cantilever under the weight supported, \( \Delta_{ss} \) = deflection of the backspan assuming a simply supported member.

**Acceptability Criterion.** Once the peak acceleration response of the system is estimated, it is compared to Inequality 1.21:

\[
\frac{a_p}{g} = \frac{P_o \exp(-0.35f_n)}{\beta W} \leq \frac{a_o}{g}
\]  

(1.21)

Here, \( a_o/g \) = limit of human tolerance as recommended by Murray and shown in Figure 1.8. For the case of office floors, \( a_o/g = 0.5\%g \).

### 1.6 OTHER CURRENT RESEARCH

Many other individuals have done research on the topic of floor vibrations, some of which is directly related to the research presented in this paper. Kitterman (1994) investigated the properties and behavior of steel joists and their impact on floor vibrations. He developed several equations for calculating the effective moment of inertia of a steel joist based on numerous test results and finite element models. Hanagan (1994) explored the use of active control systems to reduce annoying floor vibrations. Her research provided a known input and measured response for a multi-bay floor and is referenced during the verification of the finite element modeling techniques. Band (1996) researched joist and joist-girder supported floors and helped develop the reduction factors used for calculating the effective moment of inertia of a joist. Rottmann (1996) explored the use of tuned mass dampers for controlling floor vibrations. Beavers (1998) used a finite element program to model single-bay joist-supported floors and predict
fundamental frequencies. He also developed the rigid-link elements and joist-seat elements as well as several other modeling conventions used in this study.

1.7 NEED FOR RESEARCH

The wide variety of scales and prediction techniques available to engineers is an indication of the complex nature of floor vibrations. Furthermore, since each method inherently makes numerous assumptions about the structure, not all methods are equally applicable to all situations. This study was done to evaluate the ability of two finite element modeling techniques to predict both the fundamental frequency and the dynamic response of a multi-bay floor, regardless of the regularity of beam spacing, material properties, damping properties or other limitations. Current design standards are based on regularly spaced rectangular bays that are approximated as a series of single degree of freedom systems. Real structures, though, are far more complex. The results of the finite element analyses are compared to current practices and field-measured results. Chapter II discusses the finite element modeling techniques used in this study. The equipment used and testing procedures for the floors are also discussed in Chapter II. In Chapter III the computer models are verified by matching the results to closed form solutions and to the predictions of the Design Guide method. Chapter IV summarizes the results of the analyses performed on a number of in-situ floors. Each floor is described in detail, and the results of both finite element modeling techniques are compared to results from the Design Guide method and to various field measurements for each floor. In Chapter V, conclusions are drawn and the need for more research is suggested. Appendix A contains supporting data for the analyses presented in Chapter IV, while Appendix B contains a detailed example of both finite element modeling techniques.
CHAPTER II
FINITE ELEMENT MODELING AND IN-SITU MEASUREMENT TECHNIQUES

2.1 INTRODUCTION

To examine the ability of a finite element computer model to predict both the fundamental frequency of a floor and the peak acceleration due to walking, two modeling techniques were developed and test results were gathered. This chapter briefly describes the finite element computer program used in this study, the 3D and 2D modeling techniques developed, and the procedure and equipment used to take in-situ measurements.

2.2 FINITE ELEMENT COMPUTER PROGRAM

2.2.1 OVERVIEW

For the purposes of this research, the commercial finite element program, SAP2000 Nonlinear Version 7.1, hereafter SAP2000, from Computers and Structures, Inc. (Wilson and Habibullah, 1998) was used. The program can perform static and dynamic analyses of complex structures, including the effect of modal damping, while running on a typical desktop computer. The program features a graphical interface that makes the input of a structure and the interpretation of results quick and easy.

2.2.2 FINITE ELEMENTS USED

In this study, beam, shell, and plate finite elements were used. The beam element used is referred to in SAP2000 as a three-dimensional frame element. The element is a two-node beam element with three translational and three rotational degrees of freedom at each end. In some cases, pre-defined sections were used, such as hot-rolled sections, but in other instances the properties of the member had to be calculated and input
separately. Typically, each frame element was defined with the material properties of steel, but occasionally, in the case of links and seats especially, another material was used. Once a frame element was created, it could be used to define the properties of elements in the model.

To model the concrete slab, plate or shell elements were used. Shell elements in SAP2000 are four-node elements with six degrees of freedom at each node, and were used for the 3D-FEMs. Plate elements in SAP2000 are also four-node elements, but they allow only three degrees of freedom at each node and were used exclusively in the 2D-FEMs. Both elements are defined in the same way as frame elements, and each element can be assigned a set of material properties, usually concrete.

2.2.3 THREE DIMENSIONAL MODELING TECHNIQUE

Slab. Three-dimensional finite element models (3D-FEMs) were created using frame and shell elements. The shell elements were used to represent the slab and were placed at the centroid of the concrete above the ribs. The thickness of a typical shell element was defined as the thickness of the slab above the ribs of the deck. This assumption is typically made for beams, but is usually not considered to be valid for girders. However, this study assumes that the assumption is valid for floors in general. The concrete was defined as having a modulus of elasticity 1.35 times greater than the modulus generally assumed for structural calculations. This was due to the fact that concrete is stiffer under dynamic loading than it is under static loading (Allen and Murray, 1993). The concrete was defined with a unit weight and unit mass given by:

\[
W = \left(\frac{d_c + d_r}{12}\right)w_c + w_d + w_i\left(\frac{12}{d_c}\right)\left(\frac{1}{1,728,000}\right)
\] (2.1)

\[
M = \frac{W}{386}
\] (2.2)
Here $W =$ weight per unit volume, k/in$^3$; $d_c =$ thickness of concrete above the ribs, in.; $d_r =$ thickness of concrete in ribs, in.; $wc =$ unit weight of concrete, pcf; $w_d =$ estimated additional dead load on the floor, psf; $w_l =$ estimated additional live load on the floor, psf; $M =$ mass per unit volume, k-sec$^2$/in$^4$. This technique allowed for the fact that loading on a real structure will inherently affect the natural frequencies of the structure by increasing the total mass of the system. SAP2000 calculates frequencies based on the mass and stiffness of the system independent of any loading on the structure. Therefore, to include the effect of distributed loads, they must be included in the mass of the system.

**Hot-Rolled Beams.** Floors supported by hot-rolled sections were fairly simple to model. Once the frame elements were defined for each section, they were drawn in the finite element model at the location of their respective centroids. Since the members were long and slender, the frame sections were modified so as to remove the effects of shear deformations by setting the shear area equal to zero.

**Joists and Joist-girders.** Floors supported by joists and joist-girders were more complicated to model. For these members, each element of the joist or joist-girder had to be modeled separately. For angle web members, the end of adjacent members are not coincident at one point, but rather are separated by up to approximately 2 in. To model this, each node in the finite element model was offset 1 in. as shown in Figure 2.1. For rod web members, no offset was used. For the top and bottom chords of the joist, the shear area was set to zero, while it was defined for the shorter web members.

**Rigid Links and Joist Seats.** Once the slab and the supporting members were created in the finite element model, they had to be connected and properly restrained so that the system acted appropriately. For this purpose, two elements were defined, the rigid link and the joist seat. Both elements were defined with a material that had no weight, no mass, and a very high modulus of elasticity (100,000 ksi). The rigid link was defined with a large area (100 in$^2$) and a large moment of inertia in each principal direction (10,000 in$^4$). Essentially, this member was rigid against both axial and bending deformations without adding any mass to the system. This method was much simpler to use than the constraint conditions that SAP2000 allows, since a frame element
The joist seat was defined with a large area (100 in\(^2\)) and a near zero moment of inertia in each principal direction (0.001 in\(^4\)). Essentially, this member was rigid axially, without providing any bending stiffness or additional mass to the system. This method was also simpler than using the constraints provided in SAP2000, and it allowed for the fact that joist seats are not sufficiently stiff to develop the fully composite moment of inertia for the girder. The use of the seat element is illustrated in Figure 2.4.

**Restraints and Releases.** Once all of the elements were connected, the model was restrained, and some joints were altered to accurately represent the actual floors. Restraint was provided by the built-in joint restraints in SAP2000. Figure 2.5 shows the restraints applied to a typical single bay floor assuming that the beams and the girders are located in the same plane. For cases where the girder is deeper than the beams, the restraints are applied at the ends of the girders only. One corner, in this case Corner 1, was restrained in all three translational directions. The corner located at the other end of the girder, in this case Corner 2, was restrained in the vertical direction (Z-direction) and in the out-of plane direction (Y-direction). The diagonally opposite corner, in this case
Figure 2.2 – Rigid Links with Hot-Rolled Beams

Figure 2.3 – Rigid Links with Steel Joists

Figure 2.4 – Joist Seats
Corner 4, was restrained from motion in the vertical direction and in the direction of the girder (X-direction). All other column locations were restrained in the vertical (Z-direction) only. For floors with joists or joists girders, all nodes along the bottom chord were restrained from moving in the out of plane direction. The slab was also restrained from movement in the plane of the slab. A corner not restrained on the girder level, in this case, Corner 3, was restrained in both in-plane directions (X-direction and Y-direction). The diagonally opposite corner, Corner 2, was restrained in one direction only (X-direction).

Coincident joints in SAP2000 are assumed to be fully connected to one another. However, when dealing with joists, continuity over a girder cannot be assumed if only the top chord of the joist is connected to the top flange of the girder. For this reason, moment releases were introduced at these locations. Moment releases were not used when hot-
rolled beams were connected to hot-rolled girders or when hot-rolled girders were connected to columns, and moment releases were not introduced in the slab.

2.2.4 TWO DIMENSIONAL FINITE ELEMENT MODELING TECHNIQUE

**Slab.** Two-dimensional finite element models (2D-FEM) were created using frame and plate elements. The plate elements were used to represent the slab. The thickness of a typical plate element was defined as the thickness of the slab above the ribs of the deck as was done in Section 2.2.3 for the 3D-FEM. The concrete was defined with the dynamic modulus of elasticity and with a unit mass and unit weight which included the concrete above the ribs, the concrete in the ribs, and any additional dead or live load that was uniformly applied to the structure. This technique allowed for the fact that loading on the structure will inherently affect the natural frequencies of the structure. SAP2000 calculates frequencies based on the mass and stiffness of the system, independent of any applied loading. Therefore, the additional loading was included as a distributed mass rather than as a distributed load. The plate element was placed in a plane located at the centroid of the concrete above the ribs.

**Hot-Rolled Beams.** For floors supported by hot-rolled sections, the composite moment of inertia was calculated for each section and then the appropriate frame elements were placed in the finite element model in the same plane as the slab. To avoid counting the moment of inertia of the slab twice, the moment of inertia of the slab about its centroid was subtracted from the composite moment of inertia used to define the frame elements. Again, since the members were long and slender, the frame sections were modified so as to remove the effects of shear deformations by setting the shear area equal to zero.

**Joists and Joist-Girders.** Floors supported by joists and joist-girders were also fairly simple to model in the 2D-FEM. Similar to the hot-rolled sections, the frame members representing joists were defined with the effective moment of inertia from Equation 1.5. For either joist-girders or hot-rolled sections supporting joists, Equation 1.9 was used to calculate the effective moment of inertia. All frame elements were
placed in the same plane as the plate elements representing the slab. Like the hot-rolled sections, the shear area was set equal to zero in\(^2\) since shear deformations were already accounted for in the inertia calculations.

**Rigid Links and Joist Seats.** The 2D-FEM did not require the use of links or seats. The links were not needed since SAP2000 automatically connects coincident joints, and all joints in the 2D-FEM are in the same plane. The seats were not required because their effect was taken into account when the effective moment of inertia was determined for the girders (Equation 1.9).

**Restraints and Releases.** Once all of the elements were connected, the model was restrained, and some joints were altered to represent the actual floors. Restraint was provided by the built-in joint restraints in SAP2000. Typically, 2D-FEMs were restrained in the same manner as the slab in a 3D-FEM with the exception that all column locations also received a vertical support.

Coincident joints in SAP2000 are assumed to be fully connected to one another. However, when dealing with joists, continuity over a girder cannot be assumed if only the top chords of the joist are connected to the top flange of the girder. For this reason, moment releases were introduced at these locations. Moment releases were not used when hot-rolled beams were connected to hot-rolled girders, and moment releases were not introduced in the slab.

### 2.2.5 LOADING PROTOCOL AND TIME-HISTORY ANALYSES.

Once the model was created, three different static loads were applied. The first load applied was simply the dead weight of the structure. This was used to check deflections and the total mass of the system. The second load was a single, concentrated, 600-lb. point load, which is used for the heel drop analysis. The third load was a single, concentrated load with magnitude equal to the magnitude of the walking load used in the Design Guide. Allen (1990) showed that a point load oscillating at a natural frequency of a system would give the same response as predicted by Equation 1.13. The magnitude of the point load is:
Two dynamic analyses were performed for each floor. A heel drop analysis was completed using the static load for the heel drop case and a time history function. The function consisted of a 50ms ramp function that started at 1 and decreased to 0, see Figure 1.4. The analysis used the modal damping ratio that was estimated by the measurement team. A walking analysis was completed using the static load for the walking case and a time history function. Here, the function was a sine function that oscillated at the natural frequency of the system. Again, the analysis used the modal damping ratio estimated by the measurement team.

2.3 IN-SITU MEASUREMENTS

Results from the Design Guide Procedure and the two finite element modeling techniques were compared to one another and to data from the actual floor. A variety of different tests was conducted for each floor, which measured the response of the floor to different types of loading. In addition, a subjective evaluation was made for each floor.

**Testing Equipment.** To measure both the fundamental frequency of a floor and the peak acceleration response of the floor, a series of tests was performed. The data was collected with an Ono Sokki CF-1200 Handheld FFT Analyzer. The analyzer was connected to a seismic accelerometer, manufactured by PCB Piezotronics as model 393C. For each test, the acceleration was measured by the accelerometer and recorded on a data card in the analyzer. The analyzer then performed a Fast Fourier Transform on the data and created an FRF that was also stored as a second record on the data card. Once this was completed, the next test could be run.

**Floor Excitations.** For each floor, as many as six possible excitations were recorded. Not all excitations were recorded for all floors. Generally, the accelerometer was placed as near to the center of the bay as possible since that is the location of
maximum response, and the test was performed close to the accelerometer. Generally, ambient vibration of the floor was measured first. For this test, the measurement team simply recorded the ambient vibration in the building at the time of testing, from which the fundamental frequency of the floor framing was determined. The second type of excitation was the aforementioned heel drop. For this test, a member of the measurement team raised up on his toes approximately 2.5 in. and then shifted all of his weight to his heels and impacted the floor. The third and fourth types of excitation were due to walking. Here, a member of the measurement team walked across the floor parallel to the direction of the beams and then perpendicular to the direction of the beams, each time passing near the accelerometer. The fifth and sixth types of excitation involve resonance with the first or second harmonic of the system, as described by Allen (1990). He found that if the fundamental frequency of a system is the first or second harmonic of the forcing frequency of a periodic input, then the input will adequately excite the fundamental frequency of the floor. The actual input will not be a pure sine function, but will rather be a function that is made up of a number of different frequencies. The forcing input can then be written as a Fourier series, and it will be found that the second or third term in the series will act in resonance with the fundamental frequency of the floor. The fifth type of excitation, therefore, was walking in place. Here, a member of the measurement team walked in place, near the accelerometer, at a pacing frequency that was approximately one-half or one-third of the lowest natural frequency of the floor. The sixth type of excitation was due to bouncing or rhythmic excitation. Here, a member of the measurement team bounced in place at a slowly increasing frequency until harmonic resonance with the lowest floor frequency was achieved.

**Subjective Evaluations.** In addition to the quantitative measurements taken during a test of a floor, qualitative measurements were recorded as well. Members of the measurement team were generally asked to give their impression of the acceptability of the floor under a walking load and were asked to indicate how noticeable the vibrations were. In addition, for floors that were in use at or near the time measurements were taken, the occupants of the building gave a subjective evaluation.
CHAPTER III
VERIFICATION OF FINITE ELEMENT MODELING TECHNIQUES

3.1 INTRODUCTION

This chapter outlines the process of verifying the modeling techniques before applying them to actual floors. Simple structural dynamics of a beam, a continuous beam, a cantilevered section, and a beam-girder system are looked at first. Next, the loading function used to represent walking is verified. Then an example from the Design Guide is modeled to demonstrate that the rigid link elements used are acceptable. Next, a single bay floor, supported by joists and joist-girders, is analyzed. Finally, a floor model is presented representing an actual floor for which experimental data was previously recorded. This is the only multi-bay model used for verification, and the only model for which the actual dynamic response of the structure is known. Multi-bay floor models are largely absent from the literature, as the prevailing assumption is that a single bay accurately represents the behavior of the floor. Dynamic response of a structure due to a known or assumed dynamic load is also largely absent from the literature, and so cannot be used for verification here.

3.2 COMPARISON WITH THE DYNAMICS OF SIMPLE STRUCTURES

This section examines the ability of the proposed 3D and 2D modeling techniques to match the dynamic response of several different members. In all cases, the closed form solution in Biggs (1964) is used for comparison. Results for a simply supported beam, a cantilever, and a continuous beam are presented for both modeling techniques, and the results for a beam-girder system are presented for the 2D technique only.
3.2.1 SIMPLY SUPPORTED BEAM

For a simply supported beam of length, \( L \), constant \( EI \), and a particular mass intensity, with a concentrated dynamic load, Biggs (1964) provides the following closed form solutions for frequency and total deflection:

\[
 f_n = \frac{n^2 \pi}{2l^2} \sqrt{\frac{EI}{m}} \tag{3.1}
\]

\[
y(x,t) = \frac{2F}{ml} \sum_{n} \frac{1}{4f_n^2\pi^2} \sin \left( \frac{n\pi c}{l} \right) (DLF)_n \left( \sin \frac{n\pi x}{l} \right) \tag{3.2}
\]

Here \( n \) = integer mode number, \( F \) = magnitude of the load, \( x \) = location along the member, \( y(x,t) \) = displacement of member at location, \( x \), and time, \( t \), \( c \) = location of load, \( DLF \) = dynamic load factor for the loading function, and \( f_n \) = frequency, Hz.

Choosing the W12X40 beam shown in Figure 3.1, with a 6 ft wide concrete slab, the dynamic load shown in Figure 3.2, and its associated DLF, Equation 3.3, the first fundamental mode of vibration is calculated:

\[
 F = 1000 \text{ lbs.}
\]

\[
 \text{Concrete: } w_c = 145 \text{ pcf, } f'_c = 3 \text{ ksi, } d_c = 3
\]

\[
 I_{\text{comp}} = 807 \text{ in}^4
\]

Figure 3.1 – Simply Supported Beam
Figure 3.2 – Ramp Function for Dynamic Load

\[ DLF_n = 1 - \cos \omega_n t + \frac{\sin \omega_n t}{\omega_n t_d} - \frac{t}{t_d} \quad \text{(for } t < t_d) \] (3.3)

(Note: sometimes, but not here, frequency is presented as an angular frequency, \( \omega_n \), with units of radians per second.)

From the values assigned to the beam, the transformed concrete width is calculated as 10.14 in., yielding a composite moment of inertia of 807 in\(^4\). Using this information with Equations 3.1 and 3.2, and then differentiating Equation 3.2 twice, the frequency, displacement, velocity and acceleration at mid-span are calculated. The values for the closed form solution are compared to the values predicted by the finite element models in Table 3.1. Clearly, the finite element modeling techniques give excellent results for frequency, deflection, velocity, and acceleration in this case. The 3D-FEM gives slightly different results because the effective moment of inertia calculated is slightly less than the full composite moment of inertia which is assumed for the 2D-FEM and the closed form solution.
### Table 3.1
Simply Supported Beam Results

<table>
<thead>
<tr>
<th></th>
<th>Biggs</th>
<th>3D-FEM</th>
<th>2D-FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency ($\omega_n/2\pi$)</td>
<td>31.4 Hz</td>
<td>30.4 Hz</td>
<td>31.4 Hz</td>
</tr>
<tr>
<td>Maximum Deflection</td>
<td>0.014 in</td>
<td>0.015 in</td>
<td>0.014 in</td>
</tr>
<tr>
<td>Minimum Deflection</td>
<td>-0.027 in</td>
<td>-0.028 in</td>
<td>-0.027 in</td>
</tr>
<tr>
<td>Maximum Velocity</td>
<td>3.03 in/s</td>
<td>3.07 in/s</td>
<td>3.02 in/s</td>
</tr>
<tr>
<td>Minimum Velocity</td>
<td>-2.74 in/s</td>
<td>-2.77 in/s</td>
<td>-2.72 in/s</td>
</tr>
<tr>
<td>Maximum Acceleration</td>
<td>570 in/s²</td>
<td>557 in/s²</td>
<td>567 in/s²</td>
</tr>
<tr>
<td>Minimum Acceleration</td>
<td>-570 in/s²</td>
<td>-558 in/s²</td>
<td>-567 in/s²</td>
</tr>
</tbody>
</table>

#### 3.2.2 Cantilevered Section

A similar procedure was used for a cantilever beam, Figure 3.3, of constant EI subject to the same type of dynamic loading as shown in Figure 3.2. The cantilever beam was assigned the following properties:

Load, $F$

\[ F = 1000 \text{ lbs.} \]

Concrete: \( w_c = 145 \text{ pcf}, f'_c = 3 \text{ ksi}, d_c = 3'' \)

\[ I_{\text{comp}} = 807 \text{ in}^4 \]

**Figure 3.3 – Cantilever Beam**
From these values, the transformed concrete width is calculated as 10.14 in., yielding a composite moment of inertia of 807 in\(^4\). For the primary mode of vibration, the fundamental frequency is given by Equation 3.4, and the deflection at the tip is given by Equation 3.5:

\[
\omega_n = \frac{(0.597\pi)^2}{l^2} \sqrt{\frac{EI}{m}} \quad (3.4)
\]

\[
y(t) = \frac{4F}{ml\omega_n^2} \left[ 1 - \cos \omega_n t + \frac{\sin \omega_n t}{\omega_n t_d - \frac{t}{t_d}} \right] \quad (3.5)
\]

Table 3.2 compares the results of the closed form solution (Biggs, 1964) to that obtained from the finite element models. Again, there is excellent correlation.

<table>
<thead>
<tr>
<th></th>
<th>Biggs</th>
<th>3D-FEM</th>
<th>2D-FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency ((\omega_n/2\pi))</td>
<td>17.5 Hz</td>
<td>17.3 Hz</td>
<td>17.5 Hz</td>
</tr>
<tr>
<td>Maximum Deflection</td>
<td>0.071 in</td>
<td>0.073 in</td>
<td>0.070 in</td>
</tr>
<tr>
<td>Minimum Deflection</td>
<td>-0.210 in</td>
<td>-0.220 in</td>
<td>-0.210 in</td>
</tr>
<tr>
<td>Maximum Velocity</td>
<td>14.9 in/s</td>
<td>14.9 in/s</td>
<td>14.9 in/s</td>
</tr>
<tr>
<td>Minimum Velocity</td>
<td>-12.4 in/s</td>
<td>-12.4 in/s</td>
<td>-12.4 in/s</td>
</tr>
<tr>
<td>Maximum Acceleration</td>
<td>1504 in/s(^2)</td>
<td>1481 in/s(^2)</td>
<td>1500 in/s(^2)</td>
</tr>
<tr>
<td>Minimum Acceleration</td>
<td>-1504 in/s(^2)</td>
<td>-1481 in/s(^2)</td>
<td>-1499 in/s(^2)</td>
</tr>
</tbody>
</table>
3.2.3 CONTINUOUS BEAM

A continuous beam, shown in Figure 3.4, was analyzed next. As before, the load was applied dynamically according to Figure 3.2, and the displacement, velocity and acceleration were obtained at the point of the load for the first two fundamental modes.

Load, \( F \) = 1000 lbs.

Concrete: \( w_c = 145 \text{ pcf}, f'_c = 3 \text{ ksi}, d_c = 3 '' \)

\( l = 10 ' \)

\( I_{\text{comp}} = 807 \text{ in}^4 \)

From these values, the transformed concrete width is calculated as 10.14 in., yielding a composite moment of inertia of 807 in\(^4\). For a continuous beam, the first two modal frequencies are given by Equations 3.6a and 3.6b and the displacement at the location of the load is given by Equation 3.7a for the first mode and Equation 3.7b for the second mode.

\[
\omega_1 = \frac{\pi^2}{l^2} \sqrt{\frac{EI}{m}} \quad (3.6a)
\]

\[
\omega_2 = \frac{3.92^2}{l^2} \sqrt{\frac{EI}{m}} \quad (3.6b)
\]
Using Equations 3.7a and 3.7b, and differentiating each twice, the displacement, velocity and acceleration at the point of load are determined. The values are compared with values obtained from the finite element analyses in Table 3.3. There is good agreement between all three in terms of frequency, displacement, velocity and acceleration for the first mode, while the 3D-FEM predicts slightly different values for the second mode. The 3D-FEM gives slightly different values because the effective moment of inertia is lower than the full composite moment of inertia assumed for the 2D-FEM and the closed form solution.

### Table 3.3
**Continuous Beam Results**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequency (ω₀/2π)</th>
<th>Biggs</th>
<th>3D-FEM</th>
<th>2D-FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.5 Hz</td>
<td>17.3 Hz</td>
<td>17.5 Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Maximum Deflection</td>
<td>0.00075 in</td>
<td>0.00068 in</td>
<td>0.00073 in</td>
</tr>
<tr>
<td></td>
<td>Maximum Velocity</td>
<td>0.344 in/s</td>
<td>0.321 in/s</td>
<td>0.337 in/s</td>
</tr>
<tr>
<td></td>
<td>Maximum Acceleration</td>
<td>150 in/s²</td>
<td>129 in/s²</td>
<td>146 in/s²</td>
</tr>
<tr>
<td>2</td>
<td>110.2 Hz</td>
<td>96.2 Hz</td>
<td>108.7 Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Maximum Deflection</td>
<td>0.00031 in</td>
<td>0.00041 in</td>
<td>0.00029 in</td>
</tr>
<tr>
<td></td>
<td>Maximum Velocity</td>
<td>0.220 in/s</td>
<td>0.268 in/s</td>
<td>0.217 in/s</td>
</tr>
<tr>
<td></td>
<td>Maximum Acceleration</td>
<td>150 in/s²</td>
<td>159 in/s²</td>
<td>146 in/s²</td>
</tr>
</tbody>
</table>
3.2.4 BEAM-GIRDER SYSTEM

The last system used for verification, Figure 3.5, was a beam-girder system with the properties shown, where $\Omega$ is the forcing frequency of the sinusoidal load. Only a 2D-FEM and the closed form solution were compared for this case.

F = 5000 lbs.
$\Omega = 45 \text{ rad/sec (7.16 Hz)}$
$EI_b = EI_g = 2 \times 10^{10} \text{ lb-in}^2$
$m_b = m_g = 0.1 \text{ lb-sec}^2/\text{in}^2$

Figure 3.5 – Beam-Girder System
Using LaGrange’s equations and the energy method, the kinetic energy and the strain energy of the system are calculated, assuming that each member vibrates in the fundamental bending mode of a simply supported beam, yielding a 2 degree of freedom system. From these results, the equations of motion are found, as is the determinant of the 2X2 coefficient matrix, which leads to the fundamental frequencies of the system. From these, the maximum deflection is found for the beam, the girders, and the system in total (Biggs, 1964). The results are shown in Table 3.4, along with the results from the finite element analysis. Clearly, the results from the finite element model are in good agreement with the closed form solution.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequency ($\omega_n/2\pi$)</th>
<th>Biggs</th>
<th>2D-FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.1 Hz</td>
<td>11.0 Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max. Deflection of Girder</td>
<td>0.048 in.</td>
<td>0.048 in.</td>
</tr>
<tr>
<td></td>
<td>Max. Deflection of Beam</td>
<td>0.088 in.</td>
<td>0.088 in.</td>
</tr>
<tr>
<td>2</td>
<td>25.6 Hz</td>
<td>25.5 Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max. Deflection of Girder</td>
<td>-0.00803 in.</td>
<td>-0.00803 in.</td>
</tr>
<tr>
<td></td>
<td>Max. Deflection of Beam</td>
<td>0.0112 in.</td>
<td>0.0115 in.</td>
</tr>
<tr>
<td>Combined</td>
<td>Max. Deflection of Girder</td>
<td>0.04 in.</td>
<td>0.04 in.</td>
</tr>
<tr>
<td></td>
<td>Max. Deflection of Beam</td>
<td>0.099 in.</td>
<td>0.010 in.</td>
</tr>
</tbody>
</table>

### 3.3 DESIGN GUIDE WALKING LOAD

According to the Design Guide, the peak acceleration of a floor due to a person of average weight walking across the middle of the floor in harmony with the fundamental frequency of the floor is given as:

$$\frac{a_p}{g} = \frac{P_o \exp(-0.35f_n)}{\beta W}$$  \hspace{1cm} (3.8)
To use the finite element techniques, it was necessary to determine the load that would need to be applied to the model to match this prediction. The standard solution for a single degree of freedom system under a sinusoidal load is given by Allen (1990) as:

\[
x = \frac{P}{k} \frac{\sin(2\pi f t - \phi)}{\sqrt{[1 - (f / f_n)^2]^2 + (2\beta f / f_n)^2}}
\]  

(3.9)

Here \( P \) = magnitude of load, \( k \) = stiffness, \( f \) = forcing frequency, \( f_n \) = natural frequency, \( \beta \) = damping ratio, \( \phi \) = phase angle between the load and the response of the structure. Taking two derivatives and setting the forcing frequency equal to the natural frequency, the acceleration response is equal to:

\[
a_p = \frac{P \sin(2\pi f t)}{2M\beta}
\]  

(3.10)

Here \( M \) = mass of the system. Dividing by the acceleration due to gravity and noting that the equivalent mass of a simply supported beam represented as a single degree of freedom system is equal to 50% of the actual mass, the previous equation reduces to:

\[
a_p / g = \frac{P \sin(2\pi f t)}{W\beta}
\]  

(3.11)

It follows then that the maximum acceleration, if \( f = f_n \), is:

\[
a_p / g = \frac{P}{W\beta}
\]  

(3.12)

Therefore, to obtain the same response in the finite element model as required in the Design Guide procedure, the static load applied, \( P \), must have a magnitude equal to
$P_0 \exp(-0.35f_n)$ and be applied as a harmonic load with a forcing frequency equal to the natural frequency of the system.

### 3.4 DESIGN GUIDE FOOTBRIDGE EXAMPLE

The next step in the verification of the finite element modeling technique is to model a simple footbridge and compare the results obtained from both 3D-FEMs and the 2D-FEM to those provided in Chapter IV of the Design Guide. For this purpose, Example 4.2 of the AISC Design Guide is used.

**Footbridge Description.** The footbridge is shown in Figure 3.6:

![Footbridge Cross Section](image)

Concrete: $w_c = 145$ pcf, $f'_{c} = 4000$ psi
Slab + deck weight = 75 psf
Beams: W 21 X 44, $A = 13.0$ in.$^2$, $I = 843$ in.$^4$, $d = 20.66$ in.
Span: $L = 40$ ft
$I_{\text{comp}} = 5,818$ in.$^4$

**Figure 3.6 – Footbridge Cross Section**

**Design Guide Procedure.** Using the dynamic modulus of elasticity for the concrete, the information provided, and the equations given in Chapter I, the natural frequency of the footbridge is found to be 6.6 Hz. Assuming a damping ratio of 0.01 for
an outdoor footbridge, the peak acceleration due to walking is found to be 0.027g or 2.7%g.

**Finite Element Modeling Techniques.** Three finite element models were created for this example. The first model was a 3D-FEM, as described in Section 2.2.3. The second model was also a 3D-FEM, as described in Section 2.2.3, except that constraints connected the beam to the slab rather than the rigid links previously used. The third model was a 2D-FEM, as described in Section 2.2.4. The results for each model are shown in Table 3.5 and are compared with the values obtained from the Design Guide procedure.

<table>
<thead>
<tr>
<th>Table 3.5</th>
<th>Footbridge Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide</td>
</tr>
<tr>
<td>Self Weight Deflection</td>
<td>0.286 in.</td>
</tr>
<tr>
<td>Frequency</td>
<td>6.6 Hz</td>
</tr>
<tr>
<td>Total Weight</td>
<td>33.5 Kips</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>2.7% g</td>
</tr>
</tbody>
</table>

From Table 3.5, it is obvious that there is excellent agreement between the 3D-FEMs, the 2D-FEM and the Design Guide procedure. Furthermore, the 3D-FEM that used constraints matched the 3D-FEM that used links. As before, though, the 3D-FEMs predict a slightly lower effective moment of inertia than the full composite moment of inertia that is generally assumed. For all other analyses, links are used, since they are much easier and faster to model and they give good results.

This case is limited in several important ways, though. First, this example is only a single bay. The effect of continuity with adjacent spans as well as the increased mass of the system, both of which are factors in a multi-bay analysis, are not factors here. Second, the entire weight of the structure is assumed to be effective. Again, if other bays were involved in the analysis, the effective weight may be different than the total actual
weight, which would have an effect on the predicted peak acceleration. Third, the primary bending mode of the structure is a simple beam mode; there are no girders to complicate the analysis. Fourth, the system consists solely of hot-rolled sections. The ability of the finite element modeling techniques to accurately model joists and joist-girders is not tested with this case.

3.5 JOIST AND JOIST-GIRDER MODEL

The next step in the verification of the finite element modeling technique is to analyze a single bay floor, shown in Figure 3.7, supported by joists and joist-girders. Joist details are shown in Figures 3.8 and 3.9. Joist-Girder details are shown in Figure 3.10. The floor was analyzed using the Design Guide procedure, a 3D-FEM and a 2D-FEM.

**Floor Description.** The bay is 30 ft X 44 ft, with the following properties:
Concrete: 3 in. total thickness on 0.6 in. deck, $f_c = 3.5$ ksi, $w_c = 145$ pcf
Loading: Self Weight + 4 psf dead + 11 psf live
Damping: 3% Assumed

![Figure 3.7 – Plan View of Joist-Supported Floor](image-url)
d = 36”
Overhang = 6”
Top Chord = 2L3X3X0.236
Bottom Chord = 2L2.5X2.5X0.22
Web 1 = 2L1.5X1.5X0.15
Web 2 = 1L1.5X1.5X0.129
Web 3 = 2L1.5X1.5X0.129
Web 4 = 1L1.25X1.25X0.118
Web 5 = 1L1.75X1.75X0.15
Web 6 = 1L1.5X1.5X0.15
Cr = 0.8627
I_{chords} = 1412 in^4

Figure 3.8 – 36LH450/300 Joist Details
It is noted that the exterior joists are different than the interior joists. In the Design Guide procedure, if the exterior member on a free edge is more than 50% stiffer than the interior members, then the floor is not classified as a mezzanine, and $C_j = 2.0$ in Equation 1.15. In this case, the edge joists are not 50% stiffer than the interior joists, and the effect that this has on the finite element models will be examined.

**Design Guide Procedure.** Using the Design Guide procedure, outlined in Chapter I, the moments of inertia for the interior joists and the joist-girders were computed. The edge joists were not included in this procedure, since it was assumed that the vibrations are a function of the interior joists and the joist-girders only. As noted previously, the edges of the floor are free and the exterior joist is not more than 50% stiffer than the interior joist, so the bay is classified as a mezzanine and $C_j = 1.0$ in Equation 1.15. The results are presented in Table 3.6.

**Finite Element Technique.** Two finite element models were analyzed. A 3D-FEM was analyzed using the procedure outlined in Section 2.2.3. Joist seats and rigid
links were used, as were joint releases, and joint offsets. A 2D-FEM was analyzed using the calculated composite moments of inertia and the procedure outline in Section 2.2.4. The results for each model are compared with the AISC results in Table 3.7.

Table 3.6
Design Guide Results for Joist Supported Floor

<table>
<thead>
<tr>
<th></th>
<th>Joist</th>
<th>Joist-Girder</th>
<th>Bay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Moment of Inertia</td>
<td>1960 in$^4$</td>
<td>2988 in$^4$</td>
<td>N/A</td>
</tr>
<tr>
<td>Dead Load + Live Load Deflection</td>
<td>0.24 in.</td>
<td>0.26 in.</td>
<td>0.50 in.</td>
</tr>
<tr>
<td>Frequency</td>
<td>7.2 Hz</td>
<td>7.5 Hz</td>
<td>5.0 Hz</td>
</tr>
</tbody>
</table>
| $a_p/g$                | N/A   | N/A | 0.98 %g if $C_j = 1.0$
|                        |       |         | 0.78 %g if $C_j = 2.0$

Table 3.7
Joist Supported Floor Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Design Guide Procedure</th>
<th>3D-FEM</th>
<th>2D-FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load + Live Load Deflection</td>
<td>0.50 in.</td>
<td>0.55 in</td>
<td>0.46 in.</td>
</tr>
<tr>
<td>Frequency</td>
<td>5.0 Hz</td>
<td>4.9 Hz</td>
<td>5.4 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>0.98 %g if $C_j = 1.0$</td>
<td>0.82 %g</td>
<td>0.70 %g</td>
</tr>
<tr>
<td></td>
<td>0.78 %g if $C_j = 2.0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As seen in Table 3.7, the results for each analysis are very similar. The 3D-FEM and the Design Guide procedures differed mainly in the calculation for $a_p/g$. This is attributed to the fact that in the Design Guide, this floor is treated as a mezzanine. If that assumption is ignored, the frequency remains the same, while $a_p/g$ drops to 0.78%g, nearly matching the 3D-FEM. The 2D-FEM is slightly stiffer than the other two
analyses, as evidenced by the higher frequency and smaller deflection of the static analysis.

3.6 MULTI-BAY FLOOR WITH DYNAMIC LOAD

The final step in the verification of the finite element modeling technique is a comparison with a multi-bay floor where the frequency response and the dynamic response to a specific load are both known. For this floor, the Design Guide procedure was used only to predict a fundamental frequency, since the procedure can not predict the dynamic response of the structure to the given load. A 3D-FEM was also constructed and the results were compared to the measured response.

Description of Floor. This building is a multi-story laboratory located in Vermont. The building footprint is an area 57 ft X 95 ft and consists of three bays in the short direction and six bays in the long direction. Of the bays in the short direction, the first is a laboratory spanning 21 ft 7 in., the second is a corridor spanning 7 ft and the third is another laboratory spanning 28 ft 7 in. The larger of the two laboratories is where testing occurred. In the long direction, the tested bay spans 18 ft, while the bays on either side span 15 ft 1 in. The floor slab consists of 3 in. (total depth) of 4-ksi normal-weight concrete supported by a 0.6-in. deck. The slab is supported on 16K7 and 16K2 joists, spaced at 24 in. on center, with cross-sectional areas of 1.955 in$^2$ and 1.069 in$^2$ and moments of inertia of the bare chords of 109 in$^4$ and 59 in$^4$ respectively. The joists rest on hot-rolled beams, and the exterior edges of the slab are integral with the exterior wall. A framing plan for a typical area of this building is shown in Figure 3.11.

The modal damping for this structure was measured as 4.5%. The loading on the structure at the time of testing was estimated to consist of 3 psf of dead load and 5 psf of live load, which includes island type workbenches where the intended function has been impaired due to the annoying levels of vibration.

Design Guide Procedure. Using the given moments of inertia for the joists and the estimated loading, the fundamental frequency of the floor was calculated as 7.13 Hz.
Figure 3.11 – Chemistry Laboratory Floor Plan

**Measured Test Results.** Hanagan (1994) presents the results of her measurements of the floor. The fundamental natural frequency of the floor was measured as 7.1 Hz. In addition to this, a series of heel drops was performed at the center of the bay tested. The force input was recorded, as was the velocity response of the floor. The input and output are shown in Figure 3.12.

**Finite Element Results.** A 3D-FEM was constructed according to the method described in Section 2.2.3. As mentioned, the exterior girders were restrained to model the fact that the slab was integral with the exterior walls at the edges. The frequency
predicted by the model is 7.25 Hz. Additionally, a series of theoretical heel drops were applied at the center of the tested bay to represent the loading applied to the actual structure. The dynamic response of the 3D-FEM is shown in Figure 3.13. Clearly, both
the predicted frequency and dynamic response of the structure closely match the measured results reported by Hanagan (1994).

3.7 CONCLUSION

In general, the finite element modeling technique provides excellent results for this portion of the study. Natural frequencies and dynamic responses were in good agreement between closed form solutions, the Design Guide method, and the finite element technique. However, due to limitations in the literature, only rather simple systems could be verified. Even the multi-bay floor, for which test results existed, was fairly simple, since the narrow corridor in the middle acts to stiffen the system and to isolate the exterior bays. In this way, even a multi-bay floor behaves as a single bay. In Chapter IV the results for more complicated framing schemes of multi-bay floors are given.
CHAPTER IV
DESCRIPTION OF TEST CASES AND COMPARISON OF RESULTS

4.1 INTRODUCTION

Once the finite element modeling techniques were verified, they were used to analyze a number of floors. The floors studied are more complicated than other floors found in the literature, since they are not limited to a single bay, as most other research is. The floors examined included joist and hot-rolled beam-supported floors, as well as floors with joist-girders. Both light weight and normal weight concrete was used, as were a variety of loads and damping ratios. Some floors were constructed in the laboratory, while others were actual floors currently in use across the United States. This chapter presents a brief description of each floor, summarizes the results of the Design Guide predictions and the finite element analyses, both 3D-FEMs and 2D-FEMs, and compares these results to the field measurements taken at each site. Before discussing each floor in detail, some general comments are presented here.

Frequency Response Function. For each floor, several FRFs are presented in Appendix A. A sample experimental FRF is shown in Figure 4.1. The first and second peaks are labeled as the first and second vibration frequencies, respectively. Generally, at least three FRFs are shown in Appendix A, one for the 3D-FEM, one for the 2D-FEM, and one for the measured experimental response, for each floor. In some cases, a fourth FRF is shown, corresponding to a special case. The FRFs from the 3D-FEM and 2D-FEM are generated using a spreadsheet program. The acceleration response from a heel drop load is saved as a text file, imported into a spreadsheet, and a Fourier Analysis is performed. The resolution of the FRF is a function of the number of data points stored and the length of the time history analysis. For these cases, 4096 data points were stored
over a 4 second time period, corresponding to a frequency resolution of 0.25 Hz. This was chosen because it matches the resolution of the handheld analyzer used for all experimental measurements. It is noted that the absolute value of each peak has very little significance; only the relative magnitude of each peak is important.

![Graph showing amplitude vs. frequency with peaks at 0 (Fundamental Frequency), 2nd Lowest Frequency, and various other frequencies.](image)

**Figure 4.1 – Sample FRF**

**Acceleration.** For each FEM analysis, the peak acceleration due to walking was also recorded. A sample acceleration trace predicted by a 3D-FEM is shown in Figure 4.2. The acceleration traces for each FEM are essentially identical, except for their magnitude, because in each case the system reaches a steady-state condition. Therefore, the acceleration traces are not included in Appendix A, rather the peak value is simply reported for each floor.

Measured acceleration responses are shown in Appendix A for each floor tested. Each acceleration response corresponds to one of the six excitations described in Section 2.3.
A detailed description of each floor and a summary of the results are presented in the following sections.

4.2 BUILDING 1

Building 1, constructed in 1998, is a three-story office building. The building footprint is an area 120 ft X 182 ft and consists of three bays in the short direction and six bays in the long direction. The outside bays in the long direction are relatively open with no permanent partitions, but they do contain a large number of closely spaced demountable partitions. The interior bays contain full height partitions; the business environment is categorized as a “paper” office, with about 3% modal damping. The floor slab consists of 5-5/8 in. (total depth) of 4-ksi lightweight concrete supported by a 3 in. deck. A detailed framing plan for a typical area of this building is shown in Figure 4.3.
This floor system was analyzed using the Design Guide procedure, a 3D-FEM, and a 2D-FEM. The results of these three analyses are compared with experimental test results.

**Design Guide Procedure.** The loading on the structure was assumed to include 4 psf dead load and 11 psf live load, in addition to the weight of the floor slab and supporting members, since the building was fully occupied at the time of testing. The modal damping in the structure was assumed to be 0.03 because of the presence of partitions, mechanical equipment, and filing cabinets, hence the categorization of “paper” office. Since the exterior girder was integral with the exterior wall, this member was assumed to have a very large stiffness. Because of the full-height partitions present in the interior bay, this bay was significantly stiffer than the exterior bays. The fundamental frequency of the floor, therefore, was taken to be the fundamental frequency of the beams. Also, as a result of this increased stiffness, the allowed 50% mass increase was
ignored when calculating the peak acceleration. (The Design Guide procedure allows for a 50% increase in mass if the adjacent span length is at least 70% of the span being studied, and the beams are shear connected to the girders.) The net composite moments of inertia for the beams and the girders were 2856 in\(^4\) and 4420 in\(^4\), respectively.

**3D-FEM.** The 3D-FEM used the same loading assumptions as the Design Guide procedure. The exterior girder was restrained in the model because the exterior cladding greatly increased the stiffness of the section. All other sections were modeled as described in Section 2.2.3.

**2D-FEM.** The 2D-FEM was created with the same assumptions as the 3D-FEM. All other sections in the 2D-FEM were created as described in Section 2.2.4.

**Experimental Measurements.** The floor system was tested for vibrations using a series of techniques on several different bays. A heel drop was performed at the center of each bay tested. The response of the system was measured, but the excitation was not since the main purpose of this particular test was to excite all frequencies in the floor and determine the first fundamental frequency. Two walking tests were performed with a member of the measurement team first walking parallel to the beams and then walking in place at a pace that was a harmonic of the fundamental frequency of the floor; 40 steps in 15 seconds. In some bays a rhythmic excitation test was performed where a member of the measurement team bounced on the floor at an ever-increasing frequency until a harmonic of the natural frequency of the floor was felt. Again, the excitation was not measured, only the response. Finally, in some locations, the ambient vibration of the system was measured.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for the 3D-FEM, the 2D-FEM, and the experimental results. The Design Guide procedure predicts a fundamental frequency of 5.06 Hz. The 3D-FEM and 2D-FEM predict fundamental frequencies at 5.10 Hz and 4.65 Hz respectively. The corresponding measured frequency is 5.5 Hz.

In addition to frequency, the peak acceleration due to walking was also calculated using each of the three analytical methods. As seen in Table 4.1, the Design Guide
procedure predicts a peak acceleration of 0.30%\(g\), the 3D-FEM predicts a peak acceleration of 0.41%\(g\), and the 2D-FEM predicts a peak acceleration due to walking of 0.73%\(g\). The measurement team recorded a peak acceleration of 0.98%\(g\) when walking parallel to the beams, a peak acceleration of 0.95%\(g\) when walking in place, and a peak acceleration of 2%\(g\) when rhythmically exciting the floor.

The predicted peak walking accelerations for the Design Guide Procedure and the 3D-FEM are both below the Design Guide recommended limit for human tolerance in an office environment of 0.50%\(g\). The 2D-FEM and the measured peak accelerations are all greater than this same criterion. The subjective evaluation of the floor, by both the occupants of the building and the measurement team, is that the floor exhibits annoying vibrations caused by normal office-type activities.

**Table 4.1**

**Summary of Results for Building 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design Guide</th>
<th>3D Finite Element Model</th>
<th>2D Finite Element Model</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5.06 Hz</td>
<td>5.10 Hz</td>
<td>4.65 Hz</td>
<td>5.5 Hz</td>
</tr>
<tr>
<td>(a_p/g)</td>
<td>0.30 %(g)</td>
<td>0.41 %(g)</td>
<td>0.73 %(g)</td>
<td>// 0.98%(g)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>In Place 0.95%(g)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rhythmic 2 %(g)</td>
</tr>
<tr>
<td>Acceptability</td>
<td>Acceptable</td>
<td>Acceptable</td>
<td>Not Acceptable</td>
<td>Not Acceptable</td>
</tr>
</tbody>
</table>

In this case, the frequency predictions consistently underestimated the actual response of the floor, but not excessively. In addition, the predicted accelerations underestimated the actual response of the floor to an even greater degree. There was fair correlation between the Design Guide procedure and the 3D-FEM, while the 2D-FEM differed somewhat.
4.3 BUILDING 2

Building 2, constructed in 1998, is an open-air addition to an existing office building. The building footprint is an area 49 ft X 249 ft and consists of one normal bay and one cantilevered bay in the short direction and twelve bays in the long direction. The addition was connected to an existing building at the exterior edge of the cantilevered section. The entire structure is open, with no permanent partitions and a hung ceiling. The floor slab consists of 4 in. (total depth) of 3-ksi normal weight concrete supported by a 1-1/2 in. deck resting on 20KSP1 joists (I<sub>eff</sub> = 555 in<sup>4</sup>). A detailed framing plan for a typical area of this building is shown in Figure 4.4. The floor system was analyzed using two slightly different procedures, one from the Design Guide and one from a paper by Murray (1991). A 3D-FEM, and two 2D-FEMs, one with normal support conditions and one examining the possibility of an increased stiffness due to the adjoining building, were constructed. The results of the analyses are compared with experimental test results.

**Design Guide Procedure.** The loading on the structure was assumed to include 4 psf of dead load and no live load, in addition to the weight of the floor slab and supporting members, since the building was not occupied at the time of testing. The modal damping in the structure was assumed to be 0.015 because of the presence of a hung ceiling and mechanical equipment only. Since the exterior girder was integral with the exterior wall, this member was assumed to have a very large stiffness. It was assumed at this time that the cantilevered bay was a true cantilever and received no support from the adjoining building. A 2D-FEM later verified this assumption. The Design Guide procedure was outlined in Section 1.5 for the case where a cantilever is involved.

**Murray Procedure.** The Murray procedure used the same assumptions as the Design Guide procedure except that the calculated frequency is for a cantilever beam with a backspan.

**3D-FEM.** The 3D-FEM used the same loading assumptions as the Design Guide procedure. The exterior girder was restrained in the model because the exterior cladding
greatly increased the stiffness of the section. All other sections were modeled as described in Section 2.2.3.

Figure 4.4 – Partial Floor Plan for Building 2

2D-FEM. The first 2D-FEM was created with the same assumptions as the 3D-FEM. All sections in the 2D-FEM were created as described in Section 2.2.4. The second 2D-FEM restrained the cantilevered end from vertical movement to explore the possibility of additional stiffness contributed by the adjoining building. All other sections of this model remained the same.

Experimental Measurements. The floor system was tested for vibrations using a series of techniques on several different bays. Each set of techniques was performed on both the interior bays and the cantilevered bays. A heel drop was performed at the center of each bay tested. The response of the system was measured, but the excitation was not
since the main purpose of this particular test was to excite all frequencies in the floor and
determine the first fundamental frequency. Two walking tests were performed with a
member of the measurement team first walking parallel to the beams and then walking
perpendicular to the beams. In some bays a rhythmic excitation test was performed
where a member of the measurement team bounced on the floor at an ever-increasing
frequency until a harmonic of the natural frequency of the floor was felt. Again, the
excitation was not measured, only the response.

Comparison of Results. Typical FRF plots are shown in Appendix A for the 3D-
FEM, both 2D-FEMs, and the experimental results. The Design Guide procedure
predicts a fundamental frequency of 5.58 Hz, while the Murray procedure predicts a
frequency of 5.64 Hz. The 3D-FEM predicts a fundamental frequency of 5.73 Hz, the
first 2D-FEM predicts a fundamental frequency of 5.56 Hz, while the second 2D-FEM
predicts a fundamental frequency of 7.56 Hz. The corresponding measured frequency is
5.75 Hz. Clearly, modeling the cantilevered end as a supported end is inaccurate.

In addition to frequency, peak acceleration due to walking was also calculated
using all procedures. As seen in Table 4.2, the Design Guide procedure and the Murray
procedure predict similar peak accelerations of 1.24%g and 1.21%g respectively. The
3D-FEM predicts a peak acceleration due to walking of 0.67%g, while the 2D-FEM with
normal support conditions predicts a slightly higher acceleration of 0.75%g. The
measurement team recorded a peak acceleration of 2.3%g when walking parallel to the
joists, 1.9%g when walking perpendicular to the joists, and 1.5%g when rhythmically
exciting the floor.

The predicted peak walking accelerations are all above the Design Guide
recommended limit for human tolerance in an office environment of 0.50%g. The
measured accelerations are also greater than this criterion. The subjective evaluation of
the floor, by both the occupants of the building and the measurement team, is that the
floor exhibits annoying vibrations caused by normal office-type activities.

In this case there was excellent agreement between the predicted fundamental
frequency and the measured frequency, for all analytical models. While the predicted
peak accelerations varied between procedures, they all exceeded the recommended limits, thus categorizing the floor as unacceptable. One possible explanation for the discrepancy between analytical models is that the finite element models might be over-estimating the effective mass in the system as compared to the Design Guide method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Murray Procedure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3-D Finite Element Model</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-D Finite Element Model (Normal)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-D Finite Element Model (Fixed)</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>5.58 Hz</td>
<td>5.75 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>1.24%g</td>
<td>// 2.3%g</td>
</tr>
<tr>
<td></td>
<td>1.21%g</td>
<td>1.9%g</td>
</tr>
<tr>
<td>Acceptability</td>
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</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>Not Acceptable</td>
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</tr>
</tbody>
</table>

**4.4 BUILDING 3**

Building 3, constructed in 1996, is a two-story office building. The building footprint is an area 180 ft X 190 ft and consists of five bays in the short direction and six bays in the long direction. The interior of the building is free of full-height partitions and consists mainly of open spaces and computer workstations. The business environment is categorized as an “electronic” office, with 2% modal damping. The floor slab consists of 3 in. (total depth) of 3-ksi normal weight concrete supported by a 9/16 in. deck. The floor
slab is supported by joists and joist-girders. A detailed framing plan for a typical area of this building is shown in Figure 4.5. The 24K7 joists (I_{eff} = 414 \text{ in}^4) are shown in Figure 4.6 and the 24G13N9.0K joist-girders (I_{eff} = 2195 \text{ in}^4) are shown in Figure 4.7.

Figure 4.5 – Partial Floor Plan for Building 3
d = 24”
Overhang = 1.25”
Seat = 2.5”
Top Chord = 2L1.75X1.75X0.155
Bottom Chord = 2L1.5X1.5X0.155
Web 1 = 7/8” Rod

Web 2 = 1L1X1X0.109
Web 3 = 1L1.75X1.75X0.155
Web 4 = 1L1.5X1.5X0.123
Web 5 = 1L1.25X1.25X0.109
Cr = 0.8838
I_{comp} = 527 in^4

Figure 4.6 – Typical 24K7 Joist for Building 3

d = 24”
Top Chord = 2L5X5X0.4375
Bottom Chord = 2L5X5X0.4375
Web 1 = 2L3.5X3.5X0.375
Web 2 = 2L1.5X1.5X0.155
Web 3 = 2L2.5X2.5X0.25
Cr = 0.8704
I_{comp} = 5,336 in^4

Figure 4.7 – Typical Joist-Girder for Building 3
This floor system was analyzed using the Design Guide procedure, a 3D-FEM, and a 2D-FEM. The results of these three analyses are compared with experimental test results.

**Design Guide Procedure.** The loading on the structure was assumed to include 2 psf of dead load in addition to the weight of the floor slab and supporting members, and 5 psf of live load, since the building was sparsely occupied at the time of testing. The modal damping in the structure was assumed to be 0.02 because no partitions were present and the office was categorized as an “electronic” office. A typical interior bay was analyzed, so the edge conditions for the floor did not affect the results.

The fundamental frequency of the floor was taken to be a function of the frequency of the joists and the frequency of the joist-girders, related by Dunkerly’s Equation, given as Equation 1.3. Since the joists are only connected at the top chord, the 50% mass increase allowed for continuity was ignored when calculating the peak acceleration.

**3D-FEM.** The 3D-FEM used the same loading assumptions as the Design Guide procedure. All sections were modeled as described in Section 2.2.3.

**2D-FEM.** The 2D-FEM was created with the same assumptions as the 3D-FEM. All sections in the 2D-FEM were created as described in Section 2.2.4.

**Experimental Measurements.** The floor system was tested for vibrations using a series of techniques on several different bays. A heel drop was performed at the center of several bays. The response of the system was measured, but the excitation was not since the main purpose of this particular test was to excite all frequencies in the floor and determine the fundamental frequency. Three walking tests were performed with a member of the measurement team first walking parallel to the joists, then walking perpendicular to the joists, and finally walking in place taking 48 steps in 15 seconds.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for the 3D-FEM, the 2D-FEM, and the experimental results. The Design Guide procedure predicts a natural frequency of 3.70 Hz, while the 3D-FEM and the 2D-FEM predict natural
frequencies of 3.71 Hz and 3.80 Hz respectively. The corresponding measured frequency is 3.75 Hz.

In addition to frequency, acceleration due to walking was also calculated using each of the three analytical methods. As seen in Table 4.3, the Design Guide procedure predicts a peak acceleration due to walking of 1.02%g, which is above the Design Guide recommended limit for human tolerance in an office environment, while the 3D-FEM and the 2D-FEM predict essentially the same acceleration due to walking, 0.46%g – 0.49%g, which is below the limit of 0.50%g. The measurement team recorded a peak acceleration of 7.7%g when walking parallel to the joists, 6.4%g when walking perpendicular to the joists, and 27%g when walking in place. The subjective evaluation of the floor, by both the occupants of the building and the measurement team, is that the floor exhibits annoying vibrations caused by normal office-type activities.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide</td>
<td>3.70 Hz</td>
</tr>
<tr>
<td>Frequency</td>
<td>3.71 Hz</td>
<td>3.82 Hz</td>
</tr>
<tr>
<td>Frequency</td>
<td>3.75 Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3D Finite Element Model</td>
<td>0.46 %g</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>0.49 %g</td>
<td></td>
</tr>
<tr>
<td>Acceptability</td>
<td>Not Acceptable</td>
<td>Acceptable</td>
</tr>
<tr>
<td>Acceptability</td>
<td>Acceptable</td>
<td></td>
</tr>
<tr>
<td>Acceptability</td>
<td>Not Acceptable</td>
<td></td>
</tr>
</tbody>
</table>

In this case there was excellent agreement between the measured fundamental frequency and the predicted fundamental frequency for all analysis procedures. However, there was a great discrepancy between the peak acceleration values predicted.
While all three methods gave values significantly less than the measured values, only the Design Guide procedure correctly identified the floor as being unacceptable. One possible explanation for this is that the finite element models predict a greater effective mass than the Design Guide does. Mass is inversely proportional to the peak acceleration response, so a larger mass would lead to a smaller peak acceleration, similar to what happens in this analysis.

4.5 BUILDING 4

Building 4 was constructed in 1997. The ballroom, where vibrations are a problem, is an area 180 ft X 220 ft and consists of two bays in the short direction and eight bays in the long direction. The ballroom is free of columns and partitions. Some full height partitions are in use below the ballroom in some meeting rooms on the lower level. The floor slab consists of 5-1/2 in. (total depth) of 4-ksi normal weight concrete supported by a 3 in. deck. A detailed framing plan for a typical area of this building is shown in Figure 4.8. The floor slab is supported by hot rolled sections and a series of built-up plate girders. The effective composite moments of inertia of the beams and girders are 3050 in$^4$ and 131,000 in$^4$, respectively.

This floor system was analyzed using the Design Guide procedure, and two different 3D-FEMs. No 2D-FEM was created for this floor. The results of these three analyses are compared with experimental test results.

**Design Guide Procedure.** The loading on the structure was assumed to include 8 psf of dead load, in addition to the weight of the floor slab and supporting members, for a walking analysis and an additional 12.5 psf of live load for a rhythmic analysis. The modal damping in the structure was assumed to be 0.03 because of the large quantities of structural steel, flooring, mechanical and electrical equipment present.

An interior portion of the ballroom floor, two bays wide and three bays long, was analyzed, so the edge conditions for the floor were unimportant. The fundamental frequency of the floor was taken to be a function of the fundamental frequencies of the beams and the girders, as related by Equation 1.3. Also, since the slab is supported by
hot-rolled beams shear connected to the girders, and the adjacent spans are at least 7/10 of the span in question, the allowed 50% mass increase was included when calculating the peak acceleration.

**Figure 4.8 – Partial Floor Plan for Building 4**

**3D-FEM.** Two 3D-FEMs were created for this analysis. The first model was created in the same manner as described in Section 2.2.3. The second model was created with the assumption that each support should be restrained in all three translational directions. Each model was analyzed both for a walking load and for a dancing load. The dancing load was applied as a uniform sinusoidal load over one bay of magnitude 12.5 psf and a frequency that is a fraction of the fundamental frequency, but is limited to the range of one to three hertz.

**Experimental Measurements.** The floor system was tested for vibrations differently than other floors in the study. The mass of the floor is so great that a single individual performing a heel drop is unable to effectively excite the floor. Therefore, two
groups of ROTC cadets, one consisting of 26 Cadets and one of 17 Cadets, were asked to assist in this study. Using an electronic drum, the Cadets marched in time at a variety of frequencies. Accelerations were recorded for each frequency. A third test was performed with roughly seventy students actively dancing in order to excite the floor. Again, accelerations were recorded.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for both 3D-FEMs, and the experimental results. The Design Guide predicts a fundamental frequency of 3.82 Hz. The original 3D-FEM predicts a fundamental frequency of 3.67 Hz, while the 3D-FEM with the additional restraints predicts a fundamental frequency of 4.50 Hz. The measured response shows a fundamental frequency of 4.75 Hz. This was achieved when the Cadets marched at a pacing frequency of 2.4 Hz, or roughly half of the fundamental frequency of the floor.

In addition to frequency, peak accelerations due to walking and dancing were also calculated using both analytical methods. As seen in Table 4.4, the Design Guide procedure predicts a walking acceleration of 0.12%g and a dancing acceleration of 28.6%g. The original 3D-FEM predicts a walking acceleration of 0.1%g, and a dancing acceleration of 19.6%g. The restrained 3D-FEM predicts a walking acceleration of 0.09%g and a dancing acceleration of 18.7%g. All three walking accelerations are below the Design Guide recommended limit for human tolerance in an office environment of 0.50%g, but all three accelerations for dancing are above the Design Guide recommended limit of 2%g for rhythmic activities combined with dining. The measurement team recorded an acceleration of about 10%g when the Cadets were marching, and an acceleration of 7%g was recorded under the dancing load. The subjective evaluation of the floor, by both the occupants of the building and the measurement team, is that the floor is acceptable under a walking load, but is unacceptable under the conditions it is likely to be used for, namely, dancing.

In this case, the Design Guide procedure and the original 3D-FEM are in good agreement with one another in terms of both fundamental frequency and peak walking acceleration. On the other hand, the restrained 3D-FEM is in good agreement with the
measured results in terms of fundamental frequency. Clearly, the column supports are adding more rigidity to the system than is generally assumed. A strict comparison of the marching and dancing loads is invalid here, but is useful in order to determine the acceptability of the floor under the loading that it is likely to see.

**Table 4.4**
**Summary of Results for Building 4**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure</td>
<td>3D-FEM Original</td>
</tr>
<tr>
<td>Frequency</td>
<td>3.82 Hz</td>
<td>3.67 Hz</td>
</tr>
<tr>
<td>(a_p/g) (Walking)</td>
<td>0.12 %g</td>
<td>0.10 %g</td>
</tr>
<tr>
<td>(a_p/g) (Marching)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Acceptability</td>
<td>Acceptable</td>
<td>Acceptable</td>
</tr>
<tr>
<td>(a_p/g) (Dancing)</td>
<td>28.6%g</td>
<td>19.6%g</td>
</tr>
<tr>
<td>Acceptability</td>
<td>Not Acceptable</td>
<td>Not Acceptable</td>
</tr>
</tbody>
</table>

**4.6 BUILDING 5**

Building 5, constructed in 1996, is a two-story office. The building footprint is an area 110 ft X 225 ft and consists of three bays in the short direction and six bays in the long direction. The building contains numerous demountable partitions and other office equipment. The building also contains several large openings that affect the vibration characteristics of the floor. The business environment is categorized as a “paper” office, with 2.5% modal damping. The floor slab consists of 4 in. (total depth) of 4-ksi normal weight concrete supported by a 2 in. deck. A detailed framing plan for a typical area of this building is shown in Figure 4.9. The effective composite moments of inertia of the
beams and girders are 1254 in$^4$ and 6256 in$^4$, respectively. This floor system was analyzed using the Design Guide procedure, a 3D-FEM, and a 2D-FEM. The results of these three analyses are compared with experimental test results.

**Figure 4.9 – Partial Floor Plan for Building 5**

**Design Guide Procedure.** The loading on the structure was assumed to include 8 psf of dead load and no live load, in addition to the weight of the floor slab and supporting members, since the building was not occupied at the time of testing. The modal damping in the structure was assumed to be 0.025 because of the presence of partitions, mechanical equipment, and filing cabinets, hence the categorization of “paper”
office. Since the exterior girder was integral with the exterior wall, this member was assumed to have a very large stiffness. The fundamental frequency of the floor was taken to be a function of the frequency of the beams and the girders as related by Equation 1.3.

3D-FEM. The 3D-FEM used the same loading assumptions as the Design Guide procedure. The exterior girder was restrained in the model because the exterior cladding greatly increased the stiffness of the section. All other sections were modeled as described in Section 2.2.3.

2D-FEM. The 2D-FEM was created with the same assumptions as the 3D-FEM. All other sections in the 2D-FEM were created as described in Section 2.2.4.

Experimental Measurements. The floor system was tested for vibrations using a series of techniques on several different bays. A heel drop was performed at the center of each bay tested. The response of the system was measured, but the excitation was not since the main purpose of this particular test was to excite all frequencies in the floor and determine the first fundamental frequency. Three walking tests were performed with a member of the measurement team first walking parallel to the beams, then walking perpendicular to the beams, and finally walking in place, taking 34 steps in 15 seconds.

Comparison of Results. Typical FRF plots are shown in Appendix A for the 3D-FEM, the 2D-FEM, and the experimental results. The Design Guide procedure predicts a natural frequency of 3.33 Hz. The 3D-FEM predicts a fundamental frequency of 3.90 Hz, and an even greater response at a frequency of 4.50 Hz. The mode shape, however, indicates that the smaller frequency is associated with the primary bending mode for this floor. The 2D-FEM predicts a fundamental frequency of 3.68 Hz. The corresponding measured frequency is between 3.75 Hz and 4 Hz.

In addition to frequency, acceleration due to walking was also calculated using each of the three analytical methods. As seen in Table 4.5, there is a large discrepancy between the values predicted. The Design Guide procedure predicts a peak acceleration of 0.77%g, while the 3D-FEM predicts a peak walking acceleration of 0.37%g, and the 2D-FEM predicts a peak walking acceleration of 0.85%g. The acceleration predicted by the Design Guide procedure and the 2D-FEM exceeds the Design Guide recommended
limit of 0.50%g for office floors. However, the acceleration predicted by the 3D-FEM does not. The measurement team recorded an acceleration of 2%g when walking parallel to the beams, 3%g when walking perpendicular to the beams, and 8.75%g when walking in place. The subjective evaluation of the floor, by both the occupants of the building and the measurement team, is that the floor exhibits annoying vibrations caused by normal office-type activities.

In this case there is relatively good agreement between the predicted and measured frequencies, with the Design Guide prediction being the furthest away. However, in terms of peak acceleration, there is very little agreement, except between the Design Guide procedure and the 2D-FEM. All predictions greatly underestimated the actual response of the floor, and the 3D-FEM even predicted an acceleration well below the recommended limit.

### Table 4.5
**Summary of Results for Building 5**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure</td>
<td>3D Finite Element Model</td>
</tr>
<tr>
<td>Frequency</td>
<td>3.33 Hz</td>
<td>3.90 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>0.77 %g</td>
<td>0.37 %g</td>
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<tr>
<td>Acceptability</td>
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<td>Acceptable</td>
</tr>
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</table>

**4.7 BUILDING 6**

Building 6, constructed in 1997, is a three-story office building. The building footprint occupies an area 56 ft X 112 ft and consists of one bay in the short direction and five bays in the long direction. At the time of testing, the building was under
construction, with only the structural steel and concrete deck in place. The floor slab consists of 5 in. (total depth) of 4-ksi normal weight concrete supported by a 1-1/2 in. deck. Both the second and third floors of this building were analyzed. The only difference between the floors was the size of the joists. For this analysis, the web members were unavailable, so the effective moment of inertia was calculated based on the chord sizes and spacing. A detailed framing plan for a typical area of this building is shown in Figure 4.10.

The floor slab is supported by composite joists resting on hot-rolled sections. The second floor joists are 40 in. deep with a top chord of $2L2.5X2.5X0.23$ and a bottom chord of $2L3X3X0.30$. The third floor joists are 36 in. deep with a top chord of $2L2.5X2.5X0.23$ and a bottom chord of $2L3X3X0.25$. The effective moments of inertia for the joists of the second and third floors were 4073 in$^4$ and 2979 in$^4$, respectively. Both the second and third floors were analyzed using the Design Guide procedure, a 3D-FEM and a 2D-FEM. The results of these three analyses are compared with experimental test results.

Figure 4.10 – Partial Floor Plan for Building 6
**Design Guide Procedure.** Since only the structural steel and concrete slab were in place at the time of testing, no additional loading was assumed. The modal damping in the structure was assumed to be 0.01 for the same reasons. Since no exterior wall was present at the time of testing, the edges of the slab were not stiffened. Because of the spacing of members and the long span of the joists, the frequency of the floor was taken as the frequency of the joists.

**3D-FEM.** A 3D-FEM was created for each floor that used the same assumptions as the Design Guide procedure. Since the web members of the joists were not available, the effective moment of inertia was calculated based on the chord sizes and spacing. A frame element with a cross sectional area equal to that of the joists was placed at an elevation such that the composite moment of inertia of the section was equal to the effective moment of inertia calculated by hand for the joists. All other sections were created as described in Section 2.2.3.

**2D-FEM.** A 2D-FEM was created for each floor using the same assumptions as the 3D-FEM. All sections were created as described in Section 2.2.4.

**Experimental Measurements.** The second floor of the building was tested for vibrations using a heel drop and a walking load. The heel drop was applied at the center of the floor, while the walking load consisted of a member of the measurement team walking parallel to the joists at the center of the floor. In each case, the response of the system was measured, but the excitation was not. The third floor of the building was tested under a heel drop loading only.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for the 3D-FEM, 2D-FEM and the experimental results associated with each floor. For the second floor, the Design Guide procedure predicts a fundamental frequency of 4.47 Hz, the 3D-FEM predicts a fundamental frequency of 4.40 Hz, while the 2D-FEM predicts a fundamental frequency of 4.27 Hz. The measurement team recorded a fundamental frequency of 4.25 Hz. For the third floor, the Design Guide procedure predicts a fundamental frequency of 3.84 Hz, the 3D-FEM predicts a fundamental frequency of
3.90 Hz, while the 2D-FEM predicts a fundamental frequency of 3.71 Hz. The measurement team recorded a fundamental frequency at 3.75 Hz.

In addition to frequency, acceleration due to walking was also calculated using all three analytical methods. As seen in Table 4.6 concerning the second floor, the Design Guide procedure predicts a peak walking acceleration of 1.12%g, the 3D-FEM predicts a peak walking acceleration of 0.80%g, while the 2D-FEM predicts 0.66%g. The measurement team recorded an acceleration of 0.8%g for walking on the second floor. As seen in Table 4.7 concerning the third floor, the Design Guide procedure predicts 1.28%g for the peak walking acceleration, while the 3D-FEM and the 2D-FEM each predict peak walking accelerations of 0.92%g. In all cases, the accelerations are above the Design Guide recommended limit for human tolerance in an office environment of 0.50%g. It is noted, though, that the structure was under construction at the time of testing. It is estimated that with the increased mass and damping when the building is occupied the floor will be acceptable. The measurement team evaluated the floor as unacceptable at the time of testing.

### Table 4.6
Summary of Results for Second Floor of Building 6

<table>
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<tr>
<th>Parameter</th>
<th>Design Guide Procedure</th>
<th>3D Finite Element Model</th>
<th>2D Finite Element Model</th>
<th>Measured Data</th>
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<tr>
<td>Frequency</td>
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<td>4.40 Hz</td>
<td>4.27 Hz</td>
<td>4.25 Hz</td>
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Table 4.7  
Summary of Results for Third Floor of Building 6

<table>
<thead>
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<td>3.90 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>1.28 %g</td>
<td>0.92 %g</td>
</tr>
<tr>
<td>Acceptability</td>
<td>Not Acceptable</td>
<td>Not Acceptable</td>
</tr>
</tbody>
</table>

In both cases, there was good correlation between the predicted fundamental frequency and the measured frequency. There was also good correlation between the predicted and measured peak walking accelerations. This was expected since the floor is only one bay wide, thus making its behavior much more predictable.

4.8 BUILDING 7

Building 7, constructed in 1996, is a three-story office building. The building footprint occupies an area 78 ft X 120 ft and consists of three bays in the short direction and seven bays in the long direction. The footprint for this building is not perfectly rectangular, and not all bays are supported by columns at their corners. At the time of testing, the building was unoccupied, with only a minimal amount of office equipment present. The floor slab consists of 4 1/2 in. (total depth) of 3-ksi normal weight concrete supported by a 2 in. deck. Two bays, one on the second floor and one on the third, were analyzed. Furthermore, the effects of a stiffening truss were also analyzed, and data was recorded after the truss was installed. The second and third floor bays were identical before the trusses were added. The bay on the second floor received two trusses, while the bay on the third floor received three. A detailed framing plan for a typical area of this
building is shown in Figure 4.11. The area studied, framed by column lines 1 and 5 and column lines E and F, is shaded. The effective composite moments of inertia for the W12X19 beams, and the W14X22 girders were 559 in\(^4\) and 798 in\(^4\), respectively.

![Figure 4.11 – Partial Floor Plan for Building 7](image)

The floor slab is supported entirely by hot-rolled sections. The stiffening truss that was added after initial measurements were taken is shown in Figure 4.12. In all cases, the trusses were added to the middle 10 ft of the section that was stiffened. On the
second floor, trusses were added to the W12X19 along column line E and to the W12X19 located 14 ft – 9 in. to the right, yielding an effective composite moment of inertia of 1394 in$^4$. On the third floor, trusses were added at the same location as the floor below as well as the W14X22 girder along column line 5, yielding an effective composite moment of inertia of 1739 in$^4$. Both the second and third floors were analyzed using the Design Guide procedure, and a 3D-FEM for the initial case and for the case with the additional trusses. The results of these analyses are compared with experimental test results.

![Figure 4.12 – Stiffening Truss for Building 7](image)

**Design Guide Procedure.** The loading on the structure was assumed to include the self weight of the structure and an additional 2 psf of dead load because there was minimal equipment located on the floor at the time of testing. The modal damping in the structure was assumed to be 0.03 because of the ceiling and mechanical equipment present. Since the edge of the floor slab was integral with the exterior wall, the exterior girder was stiffened. The fundamental frequency of the bay was taken to be a function of the fundamental frequency of the beams and the girders. For the cases where a stiffening truss was added, the fundamental frequency of the bay was taken to be a function of the fundamental frequency of the stiffened beams and the girder.

**3D-FEM.** Three 3D-FEMs were created for this building which used the same loading and damping assumptions as the Design Guide procedure. The first 3D-FEM
modeled both the second and third floors before the trusses were added. The other two 3D-FEMs modeled the effect of the stiffening trusses by including the actual members used. All sections were created as described in Section 2.2.3.

**Experimental Measurements.** Both floors of the building were tested in the same manner, both before and after the trusses were added. For each case, a heel drop and two walking loads were used. The heel drop was applied at the center of the bay, while the walking loads consisted of a member of the measurement team first walking parallel to the beams at the center of the bay, and then walking perpendicular to the beams. In all cases, the response of the system was measured, but the excitation was not.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for all 3D-FEM analyses and all experimental results. The FRF from the second floor 3D-FEM without the trusses is shown first, followed by the experimental FRF for the same case. Then the FRF from the second floor 3D-FEM with the trusses is shown, followed by the experimental FRF. Results for the third floor follow the same pattern. The Design Guide procedure predicts a fundamental frequency of 4.18 Hz, while the 3D-FEM predicts a fundamental frequency of 5.57 Hz before the trusses were added. The corresponding measured frequency is 5.75 Hz for the second floor and 5.50 Hz for the third. The Design Guide procedure predicts a fundamental frequency of 5.16 Hz after the trusses were added. The 3D-FEM for the same area predicts a fundamental frequency of 5.71 Hz., while the measured frequency is 6.25 Hz. The Design Guide procedure predicts a fundamental frequency of 6.17 Hz for the third floor after the trusses were added. The 3D-FEM for the same area predicts a fundamental frequency of 5.83 Hz., while the measured frequency is 6.5 Hz.

In addition to frequency, acceleration due to walking was also calculated using both analytical methods. As seen in Tables 4.8 and 4.9, the Design Guide procedure predicts a peak acceleration due to walking of 0.95%g before the trusses were added, while the 3D-FEM predicts a peak acceleration of 1.50%g. After the trusses were added on the second floor, the Design Guide procedure predicts a peak acceleration of 0.70%g, while the 3D-FEM predicts 1.27%g. For the third floor, the Design Guide predicts
0.52\%g as compared to 1.10\%g for the 3D-FEM. The measurement team recorded a number of peak accelerations due to walking on the floor both before and after the trusses were installed. In all cases, both the predicted values of peak acceleration and the measured response of the system are above the Design Guide limit of 0.50\%g for an office environment. The subjective evaluation of the occupants of the building and the measurement team is that both bays exhibited annoying levels of vibration before the trusses were added, but do not exhibit annoying levels of vibration now that the trusses are in place.

Before the trusses were added, the Design Guide procedure did not accurately predict the fundamental frequency of the bays, while the 3D-FEM predicted values that closely matched the experimental results. Neither model accurately predicted the peak accelerations. After the trusses were added, both models predicted higher fundamental frequencies and lower peak accelerations, as expected. However, none of the predicted values were comparable to the measured values with the exception of the third floor where the Design Guide predicted a fundamental frequency that was close to the measured frequency. The irregular column spacing and complex framing scheme for this floor made accurate predictions exceedingly difficult to make.

### Table 4.8
**Summary of Results for Second Floor of Building 7**

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<td>No Truss With Truss</td>
<td>No Truss With Truss</td>
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<td>Frequency</td>
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<td>1.50%g</td>
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<td>1.27%g</td>
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<td># 2.2%g</td>
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Table 4.9
Summary of Results for Third Floor of Building 7

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<thead>
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<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure No Truss</td>
<td>Design Guide</td>
<td>Measured</td>
</tr>
<tr>
<td></td>
<td>3D-FEM No Truss</td>
<td>Guide Procedure With Truss</td>
<td>Data No Truss</td>
</tr>
<tr>
<td>Frequency</td>
<td>4.18 Hz</td>
<td>6.17 Hz</td>
<td>5.83 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>0.95%g</td>
<td>0.52 %g</td>
<td>1.10%g</td>
</tr>
<tr>
<td></td>
<td>⊥ 2.7%g</td>
<td>⊥ 1.4%g</td>
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<td>Not Acceptable</td>
<td>Not Acceptable</td>
</tr>
</tbody>
</table>

4.9 BUILDING 8

Building 8, constructed in 1999, is a four-story academic building. The building has several wings and a non-rectangular footprint. In general, the building is three bays wide and more than five bays long. The exterior bays are designed as classrooms, while the interior bay consists of two cantilevered hallways and a number of large openings. It is assumed, therefore, that each side of the building will vibrate independently, and so only half of the entire floor will be modeled. At the time of testing, the building was open to the air, with only the structural steel and concrete deck in place. The floor slab consists of 6 in. (total depth) of 4-ksi normal weight concrete supported by a 3 in. deck. Two different areas of the second floor were analyzed. A detailed framing plan for the first area tested is shown in Figure 4.13, while the second area is shown in Figure 4.14.

The floor slab is supported by a combination of composite joists and hot-rolled sections resting on hot-rolled girders. The joists in the first area are 26 in. deep with a top and bottom chord of 2L4X4X0.44. The joist seats are 5 in. and the joists are cantilevered.
over the W27 girders, with the bottom chord attached to the girder. The effective moment of inertia for the composite joist is 3852 in$^4$.

![Diagram](image)

**Figure 4.13 – Area 1 for Building 8**

The joists for the second area are the same as the joists for the first area with the exception of their length. The effective moment of inertia for the composite joist is 3586 in$^4$.

Both areas were analyzed using the Design Guide procedure, a 3D-FEM and a 2D-FEM. The results of these three analyses are compared with experimental test results.

**Design Guide Procedure.** The loading on the structure was assumed to be zero, since only the structural steel and concrete slab were in place at the time of testing. The modal damping in the structure was assumed to be 0.01 for the same reasons. Since no exterior wall was present at the time of testing, the edges of the slab were not stiffened.

**3D-FEM.** A 3D-FEM was created for each area, which used the same assumptions as the Design Guide procedure. Since the web members of the joists were not available, the effective moment of inertia was calculated based on the chord sizes and
spacing. A frame element with a cross-sectional area equal to that of the joists was placed at an elevation such that the composite moment of inertia of the section was equal to the effective moment of inertia calculated by hand for the joists. The girder over which the joist is cantilevered was assumed to act non-compositely since the only attachment between the girder and the slab was through the entire depth of the joist. All other sections were created as described in Section 2.2.3.

![Figure 4.14 – Area 2 for Building 8](image)

**2D-FEM.** A 2D-FEM was created for each floor using the same assumptions as in the 3D-FEM. All sections were created as described in Section 2.2.4.

**Experimental Measurements.** Both areas were tested using a variety of techniques. A heel drop was performed at the center of the central bay for each area. Three walking loads, one with a member of the measurement team walking parallel to the joists, a second with a member of the team walking perpendicular to the joists, and a third with a member of the team walking in place at a pacing frequency equal to a multiple of
the fundamental frequency of the floor, were recorded. Additionally, rhythmic excitation was applied at the center of the central bay in each area. In all cases, the response of the system was measured, but the excitation was not.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for the 3D-FEM, 2D-FEM and the experimental results associated with each area. For the first area, the Design Guide procedure predicts a fundamental frequency of 7.41 Hz, the 3D-FEM predicts a fundamental frequency of 8.59 Hz, while the 2D-FEM predicts a fundamental frequency of 8.63 Hz. The measurement team recorded a fundamental frequency of 9.0 Hz. For the second area, the Design Guide procedure predicts a fundamental frequency of 6.12 Hz, 3D-FEM predicts a fundamental frequency of 6.04 Hz, while the 2D-FEM predicts a fundamental frequency of 6.14 Hz. The measurement team recorded a fundamental frequency of 6.75 Hz.

In addition to frequency, acceleration due to walking was also calculated using all three analytical methods. As seen in Table 4.10 for the first area, the Design Guide procedure predicts a peak acceleration due to walking of 0.97%g, the 3D-FEM predicts 0.75%g, while the 2D-FEM predicts 0.88%g. The measurement team recorded a peak walking acceleration of 1.0%g when walking parallel to the joists and 0.9%g when walking perpendicular to the joists in the first area. When stepping 42 times in 15 seconds, the team recorded a peak acceleration of 10.5%g. When the first area was rhythmically excited, a peak acceleration of 2.5%g was recorded.

As seen in Table 4.11 for the second area, the Design Guide procedure predicts a peak acceleration due to walking of 0.89%g, while the 3D-FEM predicts 0.41%g and the 2D-FEM predicts 1.28%g. The measurement team recorded a peak walking acceleration of 1.1%g when walking parallel to the joists, and 0.7%g when walking perpendicular to the joists. When stepping 45 times in 15 seconds, the team recorded a peak acceleration of 6.3%g. When the second area was rhythmically excited, the measured acceleration peaked at 1.2%. The team evaluated the floor as unacceptable at the time of measurement.
### Table 4.10
Summary of Results for Area One of Building 8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide</td>
<td>3D Finite Element Model</td>
</tr>
<tr>
<td>Frequency</td>
<td>7.41 Hz</td>
<td>8.59 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>0.97 %g</td>
<td>0.75 %g</td>
</tr>
<tr>
<td></td>
<td>// 1.0%g</td>
<td>(\perp 0.9)</td>
</tr>
<tr>
<td>Acceptability</td>
<td>Not Acceptable</td>
<td>Not Acceptable</td>
</tr>
</tbody>
</table>

In all cases but the 3D-FEM for the second area, the accelerations are above the Design Guide recommended limit for human tolerance in an office environment of...
0.50%g,. It is noted, though, that the structure was still under construction at the time of testing. It is estimated that with the increased mass and damping when the building is occupied the floor will be acceptable.

For the first area, there was good correlation between both the predicted frequencies and accelerations and the measured frequencies and accelerations, with the exception of the Design Guide procedure which significantly underestimated the fundamental frequency of the floor. For the second area, the predicted frequencies were a bit lower than the frequency measured, and the peak walking accelerations were widely dispersed. The 3D-FEM predicted a much lower acceleration than was measured or predicted by the other analyses, while the 2D-FEM predicted an acceleration that was higher than either the measured or predicted values from other analyses.

4.10 BUILDING 9

Building 9, constructed in 1998, is an auditorium. The area of concern in this building is a balcony that is supported by a plate girder spanning 99 ft. The balcony is sloped, and each row of seats rests one step higher than the row in front, on its own concrete pad. A plan view of the structural steel framing is shown in Figure 4.15. The elevations for the endpoints of members are shown in parentheses, taking the centroid of the plate girder as elevation zero.

The W10X22’s, Figure 4.16, are stiffened from the free end to a point 5 ft 1½ in. on the opposite side of the plate girder by a kicker. Since the slab is not continuous over the steel, it is assumed to have no effect on the stiffness of the floor, but the mass of the concrete is included in the analysis. The plate girder consists of two 16 in. by 2 in. flanges connected by a 44 in. by 3/8 in. web. The calculated moment of inertia for this section is approximately 36,500 in$^4$.

**FEM Analysis.** This structure was analyzed differently than other floors in this study. A 3D-FEM was the only analysis performed. The concrete slab was not included in the model for stiffness, but the mass of the concrete was included. In addition, it was assumed that 40 psf of dead load, which includes the concrete, and 25 psf of live load
were present at the time the floor was analyzed. The modal damping for this floor was not estimated since the acceleration of the floor was not required. Only the fundamental frequency of the floor and some suggested remedies for the vibration problem were explored.

![Steel Framing Plan for Building 9](image)

**Figure 4.15 – Steel Framing Plan for Building 9**

The structural steel members in this frame were created as described in Section 2.2.3, with one exception. Instead of using the unit weight and unit mass of steel for each member, a unit weight and unit mass were calculated separately for each section based on the tributary loading of that section.

In addition to the as-built analysis described above, several proposed remedies for the control of the vibrations induced by a lively crowd were also analyzed. The first solution included the addition of two columns at 8 ft 0 in. from the support along the W10X100s and the addition of two more columns at 13 ft 6 in. from either end of the plate girder. The second solution was identical to the first except that the columns along the plate girder were moved to 21 ft 5 ½ in. from the end of the plate girder. The third
proposed remedy consisted of two 16 in. by 2 in. plates welded to the bottom flange of the plate girder separated by an 18 in. by 3/8 in. vertical plate.

\[ \text{Figure 4.16 – Kicker For Building 9} \]

**Results.** The results of the various analyses for this building are shown in Table 4.12. The frequency predicted by the 3D-FEM for the as-built case was 2.65 Hz. The frequency of the system under the given loading is actually 2.7 Hz. This frequency was estimated from an incident that occurred shortly after the building was opened. Audience members attending a rock concert bounced in rhythm with a song being played and caused disturbing vibrations in the balcony by driving it to resonance. The frequency of the song is known to be 2.7 Hz; therefore, the frequency of the system under the loading present at the time of the concert was estimated to also be 2.7 Hz.

The first retrofit solution proposed increased the predicted frequency of the system to 5.40 Hz. The second proposed retrofit solution increased the predicted frequency to 6.60 Hz. The third proposed retrofit solution only increased the frequency
to 2.95 Hz. None of the proposed retrofit solutions have been completed as of yet, so no new test data is available for comparison.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>3D-FEM As Built</th>
<th>3D-FEM Retrofit #1</th>
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<tr>
<td>Frequency</td>
<td>2.65 Hz</td>
<td>5.40 Hz</td>
<td>6.60 Hz</td>
<td>2.95 Hz</td>
</tr>
</tbody>
</table>

**Table 4.12**  
**Summary of Results for Building 9**

**4.11 LABORATORY FLOOR 1**

Laboratory Floor 1, constructed in 1998, is a single bay, joist-supported floor. The floor is approximately 31 ft X 31 ft, and the floor slab consists of 3 in. (total depth) of 3.9-ksi normal weight concrete supported by a 0.6 in. deck. A detailed framing plan for the floor is shown Figure 4.17.

This floor system was analyzed using the Design Guide procedure, two 2D-FEMs, and one 3D-FEM. Two analyses were needed to account for some discrepancy in the effective moment of inertia of the joists. The calculated moment of inertia for the joists and the measured moment of inertia for the joists were different. Therefore, two sets of analyses were run. The theoretical moment of inertia of the bare joist is 1074 in$^4$, but the measured moment of inertia is only 860 in$^4$. The joist used is shown in Figure 4.18. The results of all analyses are compared with experimental test results.

**Design Guide Procedure.** The additional loading on the structure was assumed to be zero, since the floor was bare at the time of testing. The modal damping in the structure was assumed to be 0.01 for the same reason. The joists rest on a concrete masonry wall, so the fundamental frequency of the floor was taken to be the fundamental frequency of the joists. Since the edges of the floor are free, the structure was assumed to
behave as a mezzanine. One analysis was performed using the predicted moment of inertia of the bare joist, which yielded an effective moment of inertia of $1546 \text{ in}^4$. A second analysis was performed using the measured moment of inertia of the bare joist, which yielded an effective moment of inertia of $1314 \text{ in}^4$.

Figure 4.17 – Floor Plan for Laboratory Floor 1
d = 36”  
Top Chord = 2L3X3X0.25  
Bottom Chord = 2L2.5X2.5X0.212  
Web 1 = 2L2X2X0.232  
Web 2 = 1L1.5X1.5X0.151  
Web 3 = 2L2.5X2.5X0.151  
Web 4 = 2L1.25X1.25X0.13  
Web 5 = 1L2X2X0.187  
Web 6 = 1L1.25X1.25X0.13  
Web 7 = 1L1.75X1.75X0.155  
Cr = 0.7582  
I\text{chords − theory} = 1074 \text{ in}^4  
I\text{chords − measured} = 860 \text{ in}^4

Figure 4.18– Typical Joist for Laboratory Floor 1

**3D-FEM.** The 3D-FEM used the same loading assumptions as the Design Guide procedure. The ends of the joists were simply supported to model the fact that the masonry wall will not have any effect on the vibration characteristics of the floor. The predicted moment of inertia for the joist matched the measured moment of inertia in the 3D-FEM. All other sections were modeled as described in Section 2.2.3.

**2D-FEM.** Two 2D-FEMs were created with the same assumptions as the Design Guide procedure. All other sections in the 2D-FEM were created as described in Section 2.2.4.

**Experimental Measurements.** The floor system was tested for vibrations using a series of techniques. A heel drop was performed at the center of the floor. The response of the system was measured, but the excitation was not since the main purpose of this particular test was to excite all frequencies in the floor and determine the first fundamental frequency. Three walking tests were performed with a member of the
measurement team first walking parallel to the joists, secondly walking perpendicular to the joists, and finally walking in place at a pace that was a harmonic of the fundamental frequency of the floor, 43 steps in 15 seconds.

**Comparison of Results.** Typical FRF plots are shown in Appendix A for the 3D-FEM, both 2D-FEMs, and the experimental results. The Design Guide procedure predicts a fundamental frequency of 17.6 Hz for the theoretical moment of inertia and 16.2 Hz for the measured moment of inertia. The 3D-FEM predicts a fundamental frequency of 16.92 Hz. The 2D-FEM corresponding to the theoretical moment of inertia of the joists shows a fundamental frequency of 18 Hz, while the 2D-FEM corresponding to the measured moment of inertia of the joists shows a fundamental frequency of 16.58 Hz. The corresponding measured frequency is 16.75 Hz.

In addition to frequency, acceleration due to walking was calculated using each of the analytical methods. As seen in Table 4.13, the Design Guide procedure predicts a peak walking acceleration of 0.12%g for the theoretical case and 0.19%g for the measured case. The 3D-FEM predicts a peak walking acceleration of 0.06%g. The 2D-FEM predicts a peak walking acceleration of 0.06%g for the theoretical case and 0.1%g for the measured case. The measurement team record peak walking accelerations equal to 0.044%g, 0.05%g, and 0.23%g for walking perpendicular to the joists, walking parallel to the joists, and walking in place, respectively. The predicted walking accelerations are all below the Design Guide recommended limit for human tolerance in an office environment of 0.5%g. The measured accelerations are also below this same limit. The subjective evaluation of the floor, by the measurement team, is that the floor does not exhibit annoying vibrations caused by walking even in the bare state.

In this case, the predicted fundamental frequencies closely matched the measured value. This is of particular note since the 3D-FEM correctly predicted the moment of inertia of the joists based on the actual member properties. The calculations for the moment of inertia of the joist yielded results that were quite different from the measured results which required that two analyses be run for both the Design Guide procedure and the 2D-FEM, for comparison. In terms of walking acceleration, the Design Guide
procedure overestimated the response, while the 3D-FEM and the 2D-FEM for the measured case closely matched the measured response.

Table 4.13
Summary of Results for Laboratory Floor 1

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<tr>
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<th>3D-FEM</th>
<th>2D-FEM</th>
<th>Measured Data</th>
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4.12 LABORATORY FLOOR 2

Laboratory Floor 2, constructed in 1996, is a single bay, joist-supported floor. The floor is approximately 31 ft X 32 ft, and the floor slab consists of 2-1/2 in. (total depth) of 4.2-ksi light weight concrete supported by a 1 in. deck. A detailed framing plan for the floor is shown in Figure 4.19.

This floor system was analyzed using the Design Guide procedure, a 2D-FEM, and a 3D-FEM. A typical joist used for this floor is shown in Figure 4.20, with the web members following. A typical joist-girder is shown in Figure 4.21, with the web
members following. The effective moments of inertia for the joists and joist-girders were 522 in\(^4\) and 2960 in\(^4\), respectively. The results of all three analyses are compared with experimental test results.

**Figure 4.19 – Floor Plan for Laboratory Floor 2**

**Design Guide Procedure.** The additional loading on the structure was assumed to be zero, since the floor was bare at the time of testing. The modal damping in the structure was assumed to be 0.01 for the same reason. The joists rest on joist-girders that
are supported at the ends by steel tubes. Therefore, the frequency of the floor was taken to be a function of the frequency of the joists and the frequency of the joist-girders as related by Equation 1.3. Since the edges of the floor are free, the structure was assumed to behave as a mezzanine. The Design Guide procedure assumes that column supports are rigid. Upon closer analysis of this floor, though, it was determined that the columns acted as springs rather than rigid supports. While the results of the analysis using this assumption turned out to be good, that may have simply been by chance.

**3D-FEM.** The 3D-FEM used the same loading assumptions as the Design Guide procedure. The axial stiffness of the supports was measured as 2630 kips/in. In the 3D-FEM, the vertical restraints at the ends of the joist-girders were modeled as springs of the same stiffness. All other sections were modeled as described in Section 2.2.3.

**2D-FEM.** A 2D-FEM was created with the same assumptions as the Design Guide procedure and included the springs described for the 3D-FEM. All other sections in the 2D-FEM were created as described in Section 2.2.4.

![Figure 4.20 – Typical Joist for Laboratory Floor 2](image)

- d = 28”
- Top Chord = 2L1.75X1.75X0.162
- Bottom Chord = 2L1.5X1.5X0.163
- Web 1 = 0.935” φ Rod
- Web 2 = 1L1.25X1.25X0.125
- Web 3 = 1L2X2X0.163
- Web 4 = 1L1.75X1.75X0.155
- Web 5 = 1L1.5X1.5X0.135
- Cr = 0.8354
- I_{comp} = 522 in^4
d = 36”
Seat Depth = 3”
Top Chord = 2L4X4X0.39
Bottom Chord = 2L4X4X0.39
Web 1 = 2L3.5X3.5X0.37
Web 2 = 2L1.75X1.75X0.155
Web 3 = 2L2.5X2.5X0.26
Web 4 = 1L1.5X1.5X0.153
Web 5 = 2L2X2X0.19
Cr = 0.7550
I\text{comp} = 2960 \text{ in}^4

Figure 4.21 – Typical Joist-Girder for Laboratory Floor 2

Experimental Measurements. The floor system was tested for vibrations using a series of techniques. A heel drop was performed at the center of the floor. The response of the system was measured, but the excitation was not since the main purpose of this particular test was to excite all frequencies in the floor and determine the first fundamental frequency. Two walking tests were performed with a member of the measurement team first walking parallel to the joists and then walking perpendicular to the joists.

Comparison of Results. Typical FRF plots are shown in Appendix A for the 3D-FEM, 2D-FEM, and the experimental results. The Design Guide predicts a fundamental frequency of 7.11 Hz. The 3D-FEM predicts a fundamental frequency of 7.66 Hz, while the 2D-FEM predicts a fundamental frequency of 8.10 Hz. The corresponding measured frequency is 6.75 Hz.

In addition to frequency, acceleration due to walking was calculated using each of the analytical methods. As seen in Table 4.14, the Design Guide procedure predicts a
peak walking acceleration of 4.83\%g, the 3D-FEM predicts a peak walking acceleration of 2.75\%g, and the 2D-FEM predicts a peak walking acceleration of 2.86\%g. The measurement team record walking accelerations equal to 5\%g for walking perpendicular to the joists, and 4.9\%g for walking parallel to the joists. The predicted walking accelerations are all above the Design Guide recommended limit for human tolerance in an office environment of 0.50\%g. The measured accelerations are also above this same limit. The subjective evaluation of the floor, by the measurement team, is that the floor exhibits significant vibrations due to walking in its bare state.

In this case, both FEMs overestimated the fundamental frequency while the Design Guide procedure came much closer. In terms of peak accelerations, again, the Design Guide procedure was fairly accurate, while the 3D-FEM and the 2D-FEM were well below the measured values.

Table 4.14
Summary of Results for Laboratory Floor 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analysis Method</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide</td>
<td>3D Finite Element Model</td>
</tr>
<tr>
<td>Frequency</td>
<td>7.11 Hz</td>
<td>7.66 Hz</td>
</tr>
</tbody>
</table>
| $a_{p}/g$ | 4.83 \%g        | 2.75 \%g      | 2.86 \%g      | // 5\%g  \\
|           |                 |               |               | \perp 4.9\%g |
| Acceptability | Not Acceptable | Not Acceptable | Not Acceptable | Not Acceptable |
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION
This study explored the ability of two finite element modeling techniques to predict the fundamental frequency and peak acceleration of a variety of floors. Some of these floors were constructed with lightweight concrete, while others were constructed with normal weight concrete. Some floors were supported by joists, while others were supported by hot-rolled beams. Each floor was loaded differently, and had a unique damping ratio. The results from the finite element analyses were compared to predictions from the Design Guide procedure and to experimental measurements.

5.2 CONCLUSIONS
Frequency. In general, as seen in Table 5.1, the prediction of the fundamental frequency of a floor using either of the finite element techniques described earlier was reliable. The 3D-FEM was used to predict the fundamental frequency of a floor seventeen times, and correctly matched the measured frequency to within 0.5 Hz twelve times. The 3D-FEM did not predict the correct frequency for Building 4 due to the fact that the ballroom supports were more rigid than is generally assumed. The 3D-FEM did not predict the correct frequency for Building 7 after the stiffening trusses were added, due to the fact that the trusses stiffened the floor more than the model calculated. The 3D-FEM did not predict the correct frequency for Building 8, Area 2, for an unknown reason. It is noted, though, that all three analyses for this particular floor were in error, but were within 0.1 Hz of one another. The conclusion for this area is that the floor was not constructed exactly as the plans indicated. The 3D-FEM did not predict the correct
frequency for Laboratory Floor 2 because of possible errors in the measurement of the axial stiffness of the columns.

### Table 5.1
Summary of Predicted and Measured Frequencies

<table>
<thead>
<tr>
<th>Building</th>
<th>Analysis Procedure</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure</td>
<td></td>
</tr>
<tr>
<td>Building 1</td>
<td>5.06 Hz</td>
<td>5.10 Hz</td>
</tr>
<tr>
<td>Building 2</td>
<td>5.58 Hz</td>
<td>5.73 Hz</td>
</tr>
<tr>
<td>Building 3</td>
<td>3.70 Hz</td>
<td>3.71 Hz</td>
</tr>
<tr>
<td>Building 4</td>
<td>3.82 Hz</td>
<td>Original 3.67 Hz</td>
</tr>
<tr>
<td>Building 4</td>
<td>Restrained 4.50 Hz</td>
<td></td>
</tr>
<tr>
<td>Building 5</td>
<td>3.33 Hz</td>
<td>3.90 Hz</td>
</tr>
<tr>
<td>Building 6 2nd Floor</td>
<td>4.47 Hz</td>
<td>4.40 Hz</td>
</tr>
<tr>
<td>Building 6 3rd Floor</td>
<td>3.84 Hz</td>
<td>3.90 Hz</td>
</tr>
<tr>
<td>Building 7 2nd Floor, No Truss</td>
<td>4.18 Hz</td>
<td>5.57 Hz</td>
</tr>
<tr>
<td>Building 7 2nd Floor, Truss</td>
<td>5.16 Hz</td>
<td>5.71 Hz</td>
</tr>
<tr>
<td>Building 7 3rd Floor, No Truss</td>
<td>4.18 Hz</td>
<td>5.57 Hz</td>
</tr>
<tr>
<td>Building 7 3rd Floor, Truss</td>
<td>6.17 Hz</td>
<td>5.83 Hz</td>
</tr>
</tbody>
</table>
Table 5.1
Summary of Predicted and Measured Frequencies (Continued)

<table>
<thead>
<tr>
<th>Building</th>
<th>Analysis Procedure</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure</td>
<td>3D-FEM</td>
</tr>
<tr>
<td>Building 8 Area 1</td>
<td>7.41 Hz</td>
<td>8.59 Hz</td>
</tr>
<tr>
<td>Building 8 Area 2</td>
<td>6.12 Hz</td>
<td>6.04 Hz</td>
</tr>
<tr>
<td>Building 9</td>
<td>N/A</td>
<td>2.65 Hz</td>
</tr>
<tr>
<td>Laboratory Floor 1</td>
<td>16.22 Hz</td>
<td>16.92 Hz</td>
</tr>
<tr>
<td>Laboratory Floor 2</td>
<td>7.11 Hz</td>
<td>7.66 Hz</td>
</tr>
</tbody>
</table>

The 2D-FEM was used to predict the fundamental frequency of a floor ten times, and correctly matched the measured frequency to within 0.5 Hz seven times. Two of the analyses that were incorrect, for Building 8 and Laboratory Floor 2, were incorrect for the same reasons as the 3D-FEM. The 2D-FEM did not predict the correct frequency for Building 1 for an unknown reason.

The Design Guide procedure was used to predict the fundamental frequency of a floor fifteen times, and correctly matched the measured frequency to within 0.5 Hz only seven times. The analyses for Building 4 and Building 8, Area 2, were in error for the same reasons as the 3D-FEM. The Design Guide procedure did not predict the correct frequencies for Buildings 5 and 7, Building 8, Area 1, and Laboratory Floor 1 for unknown reasons, since the FEMs used the same basic information and correctly predicted the frequencies.

In general, the finite element modeling techniques did a better job of predicting the fundamental frequency of a floor than did the most current design standard.
Peak Acceleration. In general, as shown in Table 5.2, the prediction of the peak acceleration due to walking was unreliable. In almost all cases, the measured values for walking were quite different from those predicted by any of the three analytical methods. This discrepancy, though, was most likely due to the fact that the loads applied to the models were not measured values, but instead were estimations of the force applied. All three models, though, used the same loading, so it is disturbing that the 3D-FEM, 2D-FEM and Design Guide procedure frequently predicted different walking accelerations even when they predicted very similar frequencies. Clearly, the prediction of peak accelerations is an area requiring more attention.

Table 5.2
Summary of Predicted and Measured Peak Accelerations Due to Walking

<table>
<thead>
<tr>
<th>Building</th>
<th>Analysis Procedure</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure</td>
<td>3D-FEM</td>
</tr>
<tr>
<td>Building 1</td>
<td>0.30 %g</td>
<td>0.41 %g</td>
</tr>
<tr>
<td>Building 2</td>
<td>1.24 %g</td>
<td>0.67 %g</td>
</tr>
<tr>
<td>Building 3</td>
<td>1.11 %g</td>
<td>0.46 %g</td>
</tr>
<tr>
<td>Building 4</td>
<td>0.12 %g</td>
<td>Original 0.10 %g</td>
</tr>
<tr>
<td>Building 5</td>
<td>0.77 %g</td>
<td>0.37 %g</td>
</tr>
<tr>
<td>Building 6 2nd Floor</td>
<td>1.12 %g</td>
<td>0.80 %g</td>
</tr>
<tr>
<td>Building 6 3rd Floor</td>
<td>1.28 %g</td>
<td>0.92 %g</td>
</tr>
<tr>
<td>Building 7 2nd Floor, No Truss</td>
<td>0.95 %g</td>
<td>1.50 %g</td>
</tr>
</tbody>
</table>
### Table 5.2

**Summary of Predicted and Measured Peak Accelerations Due to Walking**

(Continued)

<table>
<thead>
<tr>
<th>Building</th>
<th>Analysis Procedure</th>
<th>Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design Guide Procedure</td>
<td>3D-FEM</td>
</tr>
<tr>
<td>Building 7 2nd Floor, Truss</td>
<td>0.70 %g</td>
<td>1.27 %g</td>
</tr>
<tr>
<td>Building 7 3rd Floor, No Truss</td>
<td>0.95 %g</td>
<td>1.50 %g</td>
</tr>
<tr>
<td>Building 7 3rd Floor, Truss</td>
<td>0.52 %g</td>
<td>1.10 %g</td>
</tr>
<tr>
<td>Building 8 Area 1</td>
<td>0.97 %g</td>
<td>0.75 %g</td>
</tr>
<tr>
<td>Building 8 Area 2</td>
<td>0.89 %g</td>
<td>0.41 %g</td>
</tr>
<tr>
<td>Building 9</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Laboratory Floor 1</td>
<td>0.19 %g</td>
<td>0.06 %g</td>
</tr>
<tr>
<td>Laboratory Floor 2</td>
<td>4.83 %g</td>
<td>2.75 %g</td>
</tr>
</tbody>
</table>

### 5.3 RECOMMENDATIONS FOR FUTURE RESEARCH

This study has shown that the finite element modeling techniques described are capable of accurately predicting the first fundamental frequency of a floor, but are generally not capable of accurately predicting the actual peak acceleration response of a floor due to a dynamic load. Some possible areas of future research are presented here, along with the recommendations of this study.
**Non-Structural Components.** The effect of non-structural components on the vibration characteristics of a real floor needs to be studied. Non-structural components can serve to both dampen and stiffen a floor. Their contribution is generally neglected in both the equations presented in the Design Guide and in the finite element modeling techniques.

**Effective Mass.** The effective mass of a real floor needs to be determined experimentally. The effective mass is inversely proportional to the acceleration response of the system. Clearly, from the results of this study, the effective mass calculated for a system in the Design Guide procedure is very different from the equivalent effective mass in a finite element model since different peak accelerations are predicted for the same floor even when the same frequency is predicted. A study needs to be undertaken to determine the actual percentage of mass that is generally effective in a real building.

**Continuity.** The effect of continuity needs to be examined. An underlying assumption in the Design Guide is that a series of bays can be modeled as a single bay. In the finite element models, though, adjacent bays can serve to stiffen a bay and thus change its behavior. How accurate these assumptions are for real buildings needs to be determined.

**Acceleration Response.** It is recommended that future research concentrate on measuring both the force input and response of a structure for a dynamic load. Since reality is the true measuring stick here, comparing the finite element results to the Design Guide results is of little value if the Design Guide results are in error.

**Transient Nature of Walking.** It is also recommended that the transient nature of walking be examined and included as a parameter in future studies. It is a simple matter to apply a transient load to a finite element model, and it may, perhaps, yield more accurate results.
LIST OF REFERENCES


LIST OF REFERENCES (CONTINUED)


Appendix A

FRF Plots and Acceleration Traces
Figure A.1 – FRF Plots for Building 1
Figure A.2 – Measured Accelerations for Building 1
c) Ambient Vibration

d) Rhythmic Excitation

Figure A.2 – Measured Accelerations for Building 1 (Continued)
Figure A.3 – FRF Plots for Building 2
c) Measured Data From Heel Drop

d) 2D-FEM with Supported Cantilever

Figure A.3 – FRF Plots for Building 2 (Continued)
Figure A.4 – Measured Accelerations for Building 2
Figure A.4 – Measured Accelerations for Building 2 (Continued)
Figure A.5 – FRF Plots for Building 3
Figure A.6 – Measured Accelerations for Building 3
c) Walking in Place (48 Steps in 15 Sec.)
Figure A.7 – FRF Plots for Building 4
Figure A.8 – Measured Accelerations for Building 4
Figure A.8 – Measured Accelerations for Building 4 (Continued)
Figure A.9 – FRF Plots for Building 5
Figure A.10 – Measured Accelerations for Building 5
Figure A.10 – Measured Accelerations for Building 5 (Continued)
Figure A.11 – FRF Plots for Building 6
Figure A.11 – FRF Plots for Building 6 (Continued)
Figure A.11 – FRF Plots for Building 6 (Continued)
Figure A.12 – Measured Accelerations for Building 6
Figure A.13 – FRF Plots for Building 7
Figure A.13 – FRF Plots for Building 7 (Continued)
Figure A.13 – FRF Plots for Building 7 (Continued)
Figure A.13 – FRF Plots for Building 7 (Continued)
Figure A.14 – Measured Accelerations for Building 7
c) Walking Parallel to the Beams
With Truss - 2nd Floor

Figure A.14 – Measured Accelerations for Building 7 (Continued)

d) Walking Perpendicular to the Beams
With Truss - 2nd Floor

Figure A.14 – Measured Accelerations for Building 7 (Continued)
Figure A.14 – Measured Accelerations for Building 7 (Continued)
g) Walking Parallel to the Beams
   With Truss - 3rd Floor

h) Walking Perpendicular to the Beams
   With Truss - 3rd Floor

Figure A.14 – Measured Accelerations for Building 7 (Continued)
Figure A.15 – FRF Plots for Building 8
Figure A.15 – FRF Plots for Building 8 (Continued)
Figure A.16 – Measured Accelerations for Building 8
c) Rhythmic Excitation - 1st Area

d) Walking Parallel to the Joists - 2nd Area

Figure A.16 – Measured Accelerations for Building 8 (Continued)
Figure A.16 – Measured Accelerations for Building 8 (Continued)
Figure A.17 – FRF Plots for Laboratory Floor 1
c) Measured Data From Heel Drop

d) 2D-FEM - I Theory

Figure A.17 – FRF Plots for Laboratory Floor 1 (Continued)
Figure A.18 – Measured Accelerations for Laboratory Floor 1
Figure A.18 – Measured Accelerations for Laboratory Floor 1 (Continued)
Figure A.19 – FRF Plots for Laboratory Floor 2
Figure A.20 – Measured Accelerations for Laboratory Floor 2
Appendix B

Finite Element Modeling Techniques
A detailed example of both finite element modeling techniques follows. The floor used for this example is the same one bay floor used in Section 3.5. Figure B.1 is a plan view of the floor, and the joist details are shown in Figures 3.8, 3.9 and 3.10. The floor is 30 ft X 44 ft, supported on joists and joist-girders, with the following properties:

Concrete: 3 in. total thickness on 0.6 in. deck, $f'_c = 3.5$ ksi, $w_c = 145$ pcf

Loading: Self Weight + 4 psf dead + 11 psf live

Modal Damping: 3%

All edges are free

![Plan View of Example Floor](image)

**Figure B.1 – Plan View of Example Floor**

In general, the 3D-FEM and 2D-FEM techniques use many of the same conventions. For this reason, a 3D-FEM will be described in detail here. Where applicable, the 2D-FEM will also be described; otherwise it is assumed that the technique for the 2D-FEM is identical to the technique for the 3D-FEM. The 3D-FEM procedure is:
1) **Establish a Coordinate System.** The X-Y plane is the plane of the slab. Draw a grid using the centerlines of the members in plan. For 2D-FEMs, all vertical (Z-Direction) coordinates are zero. For the 3D-FEM add a vertical gridline at the centroid of the beams, the girders, the top and bottom chords of the joists, and at the centroid of the slab above the ribs.

2) **Define Materials.** Define the unit weight, unit mass, and modulus of elasticity for the “Conc” material and the “Other” material. The unit weight and unit mass of concrete are given in Equations 2.1 and 2.2. The modulus of elasticity is the dynamic modulus for concrete. The unit weight and unit mass of the “Other” material are set to zero, and the modulus is set to 100,000 ksi. For the 2D-FEM, the “Other” material is unnecessary.

   In this case, the concrete is defined with the following properties:
   \[ M = 3.363 \times 10^{-7} \text{ kip-sec}^2/\text{in}^4, \quad W = 1.403 \times 10^{-4} \text{ k/in}^3, \quad E_c = 4410 \text{ ksi} \]

3) **Define Frame Elements.**

   **3D-FEM.**

   Define hot-rolled sections and all members of the joists using the “Steel” material by either selecting them from the SAP2000 database or inputting their properties individually. Modify all members except the web members of a joist by setting the shear area multiplier equal to zero.

   Define rigid links using the “Other” material, a cross sectional area of 100 in², a moment of inertia in each principal direction of 10,000 in⁴, and with all other properties set to zero.

   Define joist seats using the “Other” material, a cross sectional area of 100 in², a moment of inertia in each principal direction of 0.001 in⁴, and with all other properties set to zero.

   **2D-FEM**

   Define all sections using the “Steel” material. The cross sectional area is equal to the area of the chords, in the case of a joist, or the actual area of the hot-rolled section. The moment of inertia is equal to the effective composite moment of inertia of the T-Section, less the moment
of inertia of the slab about its own centroid. The shear area is set to zero. The torsional constant is set to one. All other properties are set to zero.

In this case, the joists are defined in the 2D-FEM with the following properties using the Equations presented in Section 1.5:

Area = $\Sigma$ Chord Area = 4.824 in$^2$

$I_{\text{chords}} = I_{\text{TopChord}} + I_{\text{BottomChord}} + \Sigma A d^2 = 1412$ in$^4$

$I_{\text{composite}} = 2519$ in$^4$ using $b_{\text{tr}} = 5.47$ in

$C_r = 0.90(1 - e^{-0.28(L/D)})^{2.8} = 0.8627$

$\gamma = \frac{1}{C_r} - 1 = 0.1591$

$I_{\text{slab}} = (1/12) bh^3 = 6.3$ in$^4$

$I_{\text{eff}} = \frac{1}{\gamma} + \frac{1}{I_{\text{chords}}} - I_{\text{slab}} = 1955$ in$^4$

$A_{xx} = A_{sy} = 0, J = I_y = 1$

The joist-girders are similarly defined in the 2D-FEM with the following properties:

Area = $\Sigma$ Chord Area = 11.44 in$^2$

$I_{\text{nc}} = I_{\text{TopChord}} + I_{\text{BottomChord}} + \Sigma A d^2 = 3270$ in$^4$

$b_{\text{tr}} = 21.9$ in

Seat Depth = 5 in

$I_{\text{comp}} = 8288$ in$^4$

$C_r = 0.90(1 - e^{-0.28(L/D)})^{2.8} = 0.7550$

$\gamma = \frac{1}{C_r} - 1 = 0.3245$

$I_c = \frac{1}{\gamma} + \frac{1}{I_{\text{nc}}} = 4547.35$ in$^4$

$I_{\text{slab}} = (1/12) bh^3 = 25.3$ in$^4$

$I_g = I_{nc} + (I_c - I_{nc})/4 - I_{\text{slab}} = 2970$ in$^4$

$A_{xx} = A_{sy} = 0, J = I_y = 1$
4) **Define the Slab.** For the 3D-FEM, define a shell element using the “Conc” material and a thickness equal to the thickness of concrete above the ribs, in this case 2.4 in. For the 2D-FEM, define a plate element with the same properties.

5) **Define Load Cases.** Define three static load cases, one for dead weight with a self-weight multiplier equal to one, one for a heel drop and one for walking, each with the self-weight multiplier equal to zero.

6) **Define Time-History Function.** Define a function called “Heeldrop” as a decreasing ramp function with a value of one at time zero and a value of zero at a time 50ms.

7) **Define Time-History Case.** Define a time-history case called “Heeldrop” using the “Heeldrop” time-history function and the “Heeldrop” load case. Use 4096 time steps of 9.766e-4 seconds each for a total trace of 4 seconds. These values were chosen to match the resolution of the data recorded by the measurement team. Define the modal damping for all modes, in this case 0.01, a linear analysis, a scale factor of 1 and an arrival time of 0.5 seconds.

8) **Draw Frame Elements.** Draw one beam and one girder. For the 2D-FEM simply replicate the elements to form a grid. For the 3D-FEM draw each element at its respective centroid. Also draw joist seats from the centroid of the top chord to the centroid of the supporting member where necessary. Draw rigid links from the centroids of the beams to the centroids of the supporting girders and from the centroid of the slab down to the centroid of the supporting members. The links should be spaced such that when they are used to mesh the slab they provide an aspect ratio as close to 1:1 as possible.

9) **Draw Slab.** Draw the entire slab as one shell or plate element at the centroid of the concrete above the ribs.

10) **Mesh.** Mesh each frame element into at least 10 sections. Use the intersection with the rigid links for convenience. Be sure to mesh the joist chords at least once between the joint with each web member. For the 3D-FEM, mesh the shell by using the intersection with the ends of the rigid links. For the 2D-FEM, simply mesh the plate with the intersection of the joints that
represent the ends of each section that the frame elements were previously meshed into.

11) **Restraints.** Restrain the model as described in Section 2.2. Release the ends of joists and joist-girders.

12) **Heel Drop.** Apply a 600-lb. point load, named “Heeldrop”, at the center of the bay to be analyzed.

13) **Analyze.** Set the active degrees of freedom and the number of modes sought and then run the model.

14) **Walking.** Using the results of the analysis, calculate a ten-second sine function of the same frequency as the fundamental mode and save as a text file. Calculate the magnitude of the load from Equation 2.3, in this case 11.57 lbs. Define a time-history function from the text file saved earlier. Define a time-history case called “Walking” using the walking time-history function and the walking load case. Use 10,000 time steps of 0.01 seconds each. Define the modal damping for all modes, a periodic analysis, a scale factor of 1 and an arrival time of 0.

15) **Walking Load.** Apply the point load called “Walking” to the center of the bay analyzed.

16) **Analyze.** Analyze the model a second time. Record the maximum output of the acceleration response to the walking load at the point of application.

17) **Response Spectrum.** Examine the response spectrum predicted by SAP2000 for the “Heeldrop” time-history case. Record the frequencies of the peaks and examine the corresponding mode shapes predicted by SAP2000.

The results of the 3D-FEM and 2D-FEM analyses are provided in Table B.1. For comparison, the results of the Design Guide Procedure are also provided.
Table B.1
Example Floor Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Design Guide Procedure</th>
<th>3D-FEM</th>
<th>2D-FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection</td>
<td>0.50 in.</td>
<td>0.55 in</td>
<td>0.46 in</td>
</tr>
<tr>
<td>Frequency</td>
<td>5.0 Hz</td>
<td>4.9 Hz</td>
<td>5.4 Hz</td>
</tr>
<tr>
<td>$a_p/g$</td>
<td>0.98 %g</td>
<td>0.82 %g</td>
<td>0.70 %g</td>
</tr>
</tbody>
</table>
VITA

Michael Joseph Sladki was born March 5, 1976 in Olney, Maryland, to Joseph and Ann Sladki. He spent his childhood growing up in Derwood, Maryland, before moving to Laytonsville, Maryland during high school. He attended Richard Montgomery High School and graduated from the International Baccalaureate program in 1993. He enrolled in the engineering program at Virginia Polytechnic Institute and State University in the fall. After taking a semester off to battle cancer, a semester to study in Australia, and working several summers for Michael Baker Jr., Inc., and Dewberry and Davis Engineering, he received a Bachelor of Science Degree in Civil Engineering and a minor in history in 1998. He then entered the graduate program in the Structural Engineering and Materials Division of Civil Engineering at the same university, working under Dr. Thomas M. Murray.

Michael J. Sladki