

Epicyclic Gear Train Solution Techniques with Application to Tandem Bicycling

Christopher A. Corey

*Thesis submitted to the faculty of Virginia Polytechnic
Institute and State University in partial fulfillment of the
requirements for the degree*

Master of Science
in
Mechanical Engineering

Dr. Charles F. Reinholtz, Chair
Dr. Alfred L. Wicks
Dr. Robert L. West, Jr.

8 December 2003
Blacksburg, VA

Keywords: epicyclic, planetary, gear train, kinematics,
design

Epicyclic Gear Train Solution Techniques with Application to Tandem Bicycling

Christopher A. Corey

(Abstract)

This thesis presents a unification of kinematic and force-based methods for the design and analysis of planetary gear trains along with a discussion of potential applications in tandem biking. Specifically, this thesis will provide a simple solution technique for the general case of a two-degree of freedom (2DOF) planetary gear train along with new graphical design aids. It will also address the use of epicyclic gear trains as a power coupling in a tandem bike.

In the current literature, planetary gear trains are given a clear treatment with regard to the pure kinematics of the system, but little or no literature exists that includes the torques present in the system. By treating both the kinematics and torque balance of the most general case, this thesis attempts to fill a void in the current literature. After developing the solution to the general two-degree of freedom case using the Willis formula, a force analysis will be performed using the conservation of energy principle assuming zero losses. Once the total solution is known, nomographs will be presented as a simple design tool. These graphical aids enable the designer to simultaneously approximate both speeds and torques for the mechanism. After fully developing a satisfactory solution technique and design tools, these will be applied to the problem of coupling the power provided by the riders of a tandem bicycle.

Acknowledgments

Six years ago, if someone had told me I would eventually receive a Master of Science degree in mechanical engineering, I would have laughed. When I entered school here at Virginia Tech, I was treating engineering as a secondary degree to my “real” major, music performance. Many thanks are due to my parents, who were there for me in my transition from a music student to an engineer, even as they watched tuition money fly out the door in a seemingly endless stream. Thanks to my mother, for her constant questions. As frustrating as it was to answer the same question repeatedly, it was a constant reminder of how much she cared about me. I want to thank my father for spending countless hours with me in a workshop as a child, for this is where my appetite for engineering was born. Special thanks to my sister, Tricia, for constantly reminding me that I was smart enough and stubborn enough to accomplish any goal I set for myself.

I want to thank all of my friends here at Tech that helped me through my journey. Each one of you has made it different and enjoyable in your own way. Thank you all for listening to me complain and offering me perspective that you can’t find in a library. Moreover, thanks to my friends that helped me put academics in perspective and helped me to “take the edge off” when plans went haywire and things didn’t work the way I intended. I would like to thank Dave McKee in the music department. Without the time I spent with the Marching Virginians every day, I surely would have gone insane long before completing my degree.

All the thanks in the world to my advisor, Dr Reinholtz, for seeing in me the potential to study at the graduate level and encouraging me every step of the way. Without his honest advice and calming influence, this would not have been possible.

Thank you all for everything.

Contents

Acknowledgements	iii
Table of Contents	iv
List of Figures and Tables	v
Chapter 1: Motivation and Background	1
1.1 Introduction	1
1.2 Motivation	1
1.3 Background	3
1.4 Literature Review	6
Chapter 2: Development of Solution Technique and Design Aids	9
2.1 Motion Analysis	9
2.2 Torque Analysis	10
2.3 Nomography	12
Chapter 3: Case Study: Multi-Rider Human Powered Vehicle	17
Chapter 4: Conclusions and Recommendations	23
4.1 Conclusions	23
4.2 Recommendations	23
References	24
Appendix A: MATLAB Transmission Ratio Selection Code	25
Appendix B: MATLAB Power Contribution Code	26
Vita	28

Figures and Tables

Figure 1: Gear train to be used in the Human Powered Vehicle Team's design effort	2
Figure 2: (a) The elementary epicyclic gear train and (b) its kinematical representation	3
Figure 3: The simple and complex epicyclic gear trains	4
Figure 4: Epicyclic gear train of the lower arrangement of quadrant I in figure 3	5
Figure 5: Epicyclic gear train with a basic transmission ratio of 1.0	10
Figure 6: General layout of nomograph for solution of epicyclic gear trains	12
Figure 7: Completed nomograph, with ranges of basic transmission ratio, R , labeled	14
Figure 8: Nomograph with torques represented as vectors	15
Table 1: Design Constraints For Tandem Bike Design	17
Figure 9: Nomograph With Target Regions for ω_A highlighted	18
Figure 10: Nomographs for (a) $1 < R < \infty$ and (b) $-\infty < R < 0$ with torques drawn as vectors	19
Figure 11: Absolute speeds ω_F and ω_L as a function of ratio R , for $\omega_A=120\text{rpm}$	21
Figure 12: Power required at branches F and L, assuming $R=1.5$, $P_A=1$ hp, and $\omega_A=120$ rpm	22

CHAPTER 1: MOTIVATION AND BACKGROUND

1.1 Introduction

In the current literature available to engineers, planetary gear trains are given a clear treatment as far as a simple kinematic solution. Unfortunately, no publications to date present a simple, concise design and analysis technique that considers both the motion and forces present in a gear train in the general case. This thesis attempts to fill this void by presenting a technique for finding a total speed and force solution to an epicyclic gear train in the most general case possible. After developing this solution, nomographs will be used to create an intuitive design aid, allowing the designer to visualize the performance of a gear train without the need to solve equations repeatedly. Finally, the solution technique and design aids presented will be used to address the practicality of using planetary gear trains as a power coupling element in a new generation of tandem bicycles.

1.2 Motivation

The research contained herein was motivated by a design effort undertaken by the Virginia Tech Human Powered Vehicle Team in 2002. During the early design of the multi-rider entry into the annual ASME competition, it was suggested that the most effective method for coupling the relatively inconsistent inputs of two human riders would be to use a planetary gear train. The concept behind the design attempted by the human powered vehicle team was to use a gear train like the one shown in figure 1 to create a system that would allow both riders to pedal at approximately the same speed and contribute approximately the same percentage of the output power. The planetary system accommodates differences in speed and power input by the two riders. The nature of the system behavior is the focus of this thesis..

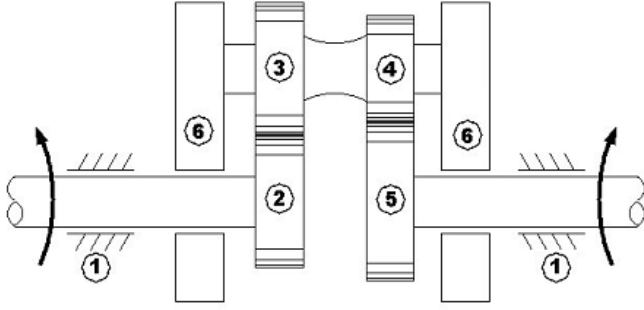


Figure 1: Gear train to be used in the Human Powered Vehicle Team's design effort

Using Willis's [1] method for finding the kinematic solution of the gear train, it was found that the mechanism was governed by

$$\omega_6 = \frac{R\omega_5 - \omega_2}{R - 1} \quad (1)$$

where the ω 's represent rotational speeds of each element in the gear train, and R is the basic transmission ratio of the gear train. Performing a static analysis, the torques were found to be controlled by

$$T_6 = (N_2 + N_3) \left(\frac{T_2}{N_2} + \frac{T_5}{N_5} \right) \quad (2)$$

$$T_5 = -RT_2 \quad (3)$$

where the T 's represent torques on each element in the train, and the N 's represent number of teeth in each gear in the train. Using these equations, it became apparent that the goal of achieving power balance at equal and opposite input speeds was impossible. If ω_2 and ω_5 are assumed to be equal and opposite, then to achieve a power balance, T_2 and T_5 must also be equal and opposite. According to equation 3, this means R must be 1. Unfortunately, this takes the denominator of equation 1 to zero, which drives ω_6 to infinity. What had seemed intuitively a simple problem to solve had led to a singularity in the solution space. With deadlines for competition closing in, the design effort was abandoned in favor of a simpler solution. However,

the research done in attempting to design a specific gear train became the foundation of a much broader research project.

The drive of this project, rather than the design of a gear train for a specific purpose, is to create a concise design method that will allow development of planetary gear trains for any number of possible applications. By dealing with the planetary in the most general case possible, this project explores the reasons for the failure of the HPV team's design as well as allowing engineers to define the kinematic relationships between the three branches of the planetary gear train without first selecting a physical arrangement of gears.

1.3 Background

A planetary gear train is defined as any gear train containing at least one gear that orbits by rotating about its own axis and also about the axis of an arm, or carrier. The elementary planetary, or epicyclic, gear train is shown in figure 2, along with the simplified representation to be used for the remainder of this thesis. The elementary train consists of two gears, the sun (1) and planet (2) gears, and a third member, hereafter referred to as the planet carrier or arm (3).

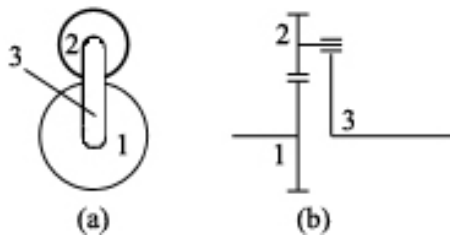


Figure 2: (a) The elementary epicyclic gear train and (b) its kinematical representation

Since it is difficult to directly transmit motion to or from the planet gear, the elementary epicyclic gear train is somewhat limited in practical application. More useful, however, are the epicyclic trains referred to as the simple and complex planetary gear trains, where a second sun gear is used. These gear trains can be realized in any of the twelve arrangements set forth in

figure 3, as originally presented by Lévai. The trains in quadrants *I* and *III* are classified as simple epicyclic trains, since the planet gears are in mesh with both sun gears. Those in quadrants *II* and *IV* represent the complex trains, where the planet gears are partially in mesh with each other and partially in mesh with the two sun gears. Notice that, regardless of arrangement, only one planet carrier may be used.

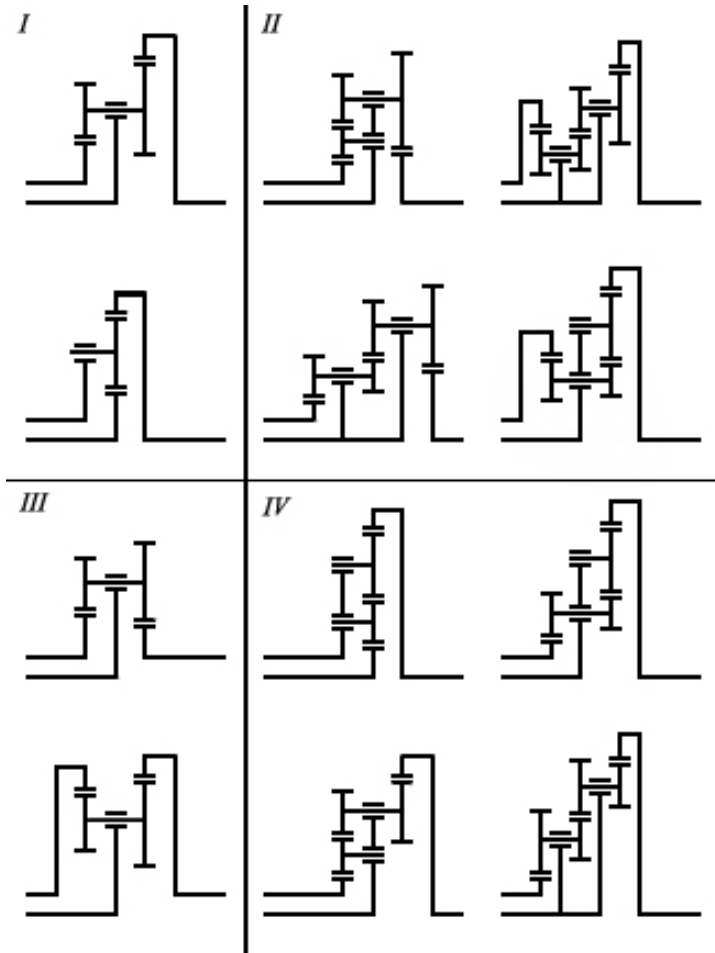


Figure 3: The simple and complex epicyclic gear trains

While this figure clearly shows the twelve possible arrangements of the epicyclic gear train, the notation used is difficult to grasp. To aid in the visualization of the actual trains represented, figure 4 shows a gear train of the lower arrangement in quadrant *I*.

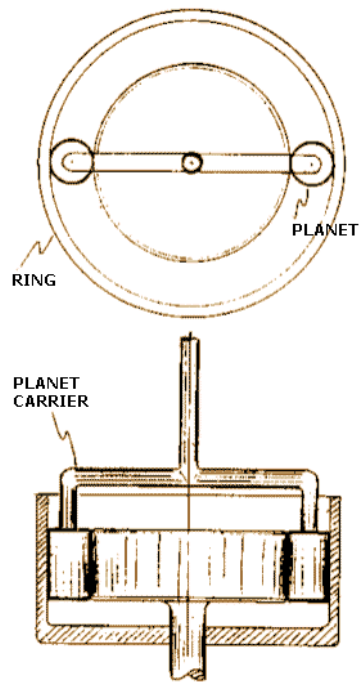


Figure 4: Epicyclic gear train of the lower arrangement of quadrant I in figure 3

The planetary gear train first appeared in ancient China, around 2600 BC, in a device referred to as the south pointing chariot. At a time when the magnetic compass was still centuries away from its birth, the Chinese faced the difficult task of navigating across the relatively featureless Gobi Desert. To surmount this difficulty, the south pointing chariot was developed. This device used a relatively complex planetary gear train attached to the two wheels of a cart to maintain a figure atop the cart pointing in the same direction, regardless of the path taken by the cart. The complexity of this device seems to indicate that the Chinese had been using differential drives for quite some time before the birth of the south pointing chariot.

At this point, the planetary gear train disappears from history for quite some time. This is more likely due to a lack of writing on the subject, rather than the actual disuse of the principle. After the south pointing chariot, the next appearance of the planetary is in what has been named the Antikythera machine. Discovered by sponge divers off the coast of the Greek island of

Antikythera in 1901, it has been identified by scholars as a type of calculator used for predicting eclipses and other astrological events. This particular device has been dated back to approximately 82 BC, leaving a gap of roughly 2500 years during which the planetary gear train passed relatively unnoticed through human history [8].

The principle of the planetary gear survived Europe's dark ages in the Far East, evidenced by the discovery of a device similar to the Antikythera machine by an Iranian savant named Al-Bizûna in the late first century AD. During the Great Renaissance, the planetary garnered wide use in astrolabes and clocks. The use and development of the mechanism continued throughout the Renaissance and on until present day. It is interesting to note at this point that, while the planetary has been successfully used since 2600 BC, it was not until the 1841 publication of Willis's Principles of Mechanism [1] that any attempt was made to create an analytical model of the device.

1.4 Literature Review

Robert Willis's 1857 publication, Principles of Mechanism, is widely regarded as the first publication dedicated solely to the field now called kinematics. In his work, Willis discusses for the first time in published literature the analytical modeling of an epicyclic gear train. As this work is a study purely in mechanism, Willis presents only a solution for the rotational speeds in the gear train. After developing this solution, the author spends the remainder of the work dedicated to epicyclic gear trains in discussing applications of the mechanism. While this discussion is well conceived, it covers four remarkably obscure applications of the epicyclic gear train, owing to the age of the work. As stated previously, this work studied only the pure kinematics of the gear train, without any discussion of the torques present in the mechanism.

In his doctoral dissertation for The Technical University of Building, Civil and Transport Engineering in Hungary, Theory of Epicyclic Gears and Epicyclic Change-Speed Gears, Dr Z. Lévai attempts to unify all of the previously written literature on epicyclic trains and what he calls “epicyclic change speed gears”, which appear to simply be multiple speed transmissions. In explaining to the reader exactly what constitutes an epicyclic train Lévai identifies, for the first time, the twelve possible variations on the epicyclic train. It is also stated that these twelve variations can be neatly divided into those with and without auxiliary planets or planet pairs. This is the first publication where any attempt was made to clearly and concisely define all possible arrangements of the planetary train.

After defining the epicyclic train, Lévai turns his attention to its solution. After briefly discussing the solution method laid out by Willis, and the graphical method of Kutzbach [5] as it applies to trains without auxiliary planets, he discusses at length two different modifications that can be performed to apply the Kutzbach method to a train with auxiliary planets. Again, he offers no treatment of the torques present in the system.

Deane Lent, professor of Mechanical Engineering at Massachusetts Institute of Technology, published his work, Analysis and Design of Mechanisms, in 1961. In this work Lent again presents in detail the methodology of Willis for finding the rotational speeds of each branch of the epicyclic gear train, along with specific methods for the design of three and four gear trains. While these techniques are well written and simple to follow, there is again no discussion of torques present in the system. Also included in this publication are several applications of the planetary gear train, all significantly more relevant than those discussed by Willis.

Joseph Shigley and John Uicker published their kinematics text, Theory of Machines and Mechanisms, in 1980. Within this work are not only a treatment of Willis's methodology, but also a more complete definition of the epicyclic gear train. Not only do they dedicate a significant amount of discussion to this definition, but they also reproduce Lévai's figure demonstrating the twelve possible variations of the planetary gear train. Most importantly, however, they present a solution technique for the torques present in the gear train. Unfortunately they do not approach the static force analysis for the general case; rather they present the solution in terms of free body diagrams for a specific arrangement of the planetary. While this method is relatively simple, it limits the designer to a single arrangement early in the design process.

Mechanisms and the Dynamics of Machinery, the publication of Hamilton Mabie and Charles Reinholtz, presents largely the same information as Shigley and Uicker. While the treatment of the kinematics and static forces of the mechanism are nearly identical, Mabie and Reinholtz also present a brief section considering circulating power flow in controlled planetary gear systems. While this discussion has no direct application to this thesis, it does hint at the methods used herein to solve for the static forces in the gear train for the general case.

John Molnar published his Nomographs in 1981. This work presents an excellent introduction to nomographs, as well as discussing at length their use and construction. This work was instrumental in the construction of the nomographs presented herein. While the bulk of this publication is dedicated to the reproduction of nomographs covering the broad general category of problems dealing with air, water, and related mechanical devices, the introduction provides more than enough information for a novice to completely understand the construction and use of nomographs for the solution of nearly any problem.

CHAPTER 2: DEVELOPMENT OF SOLUTION TECHNIQUE AND DESIGN AIDS

2.1 Motion Analysis

Willis suggests the use of a generic “transmission ratio” in defining the kinematic motion of a planetary gear train. This transmission ratio, hereafter called R , is defined as the speed ratio between the first and last gear in the train when the arm is held stationary. For the purposes of analysis, this transmission ratio can be found using knowledge of the arrangement of the particular gear train. In design, R can be selected arbitrarily by the designer as any value other than 1. Once R is known, one can write

$$\frac{\omega_{FA}}{\omega_{LA}} = R \quad (4)$$

where ω_{FA} is the rotational speed of the first sun gear in the train and ω_{LA} is the rotational speed of the last sun gear, both relative to the arm. Mathematically, this can be written

$$\frac{\omega_F - \omega_A}{\omega_L - \omega_A} = R \quad (5)$$

It is convenient to represent this in terms of the rotational speed of the arm, ω_A , as

$$\omega_A = \frac{R\omega_L - \omega_F}{R - 1} \quad (6)$$

Equation 6 makes it mathematically apparent why 1 is an invalid selection for the transmission ratio, R . Physically, a transmission ratio of 1 represents the indeterminate case where the arm can rotate at any speed regardless of the speeds of the first and last gears of the train. This singularity is perhaps best illustrated by the picture in figure 5, which shows an epicyclic gear train constructed using the upper arrangement of quadrant *II* in figure 3 with a basic transmission ratio of 1. Upon inspection, it should be obvious to the reader that even if the two sun gears are held stationary, the arm can rotate freely about its axis.

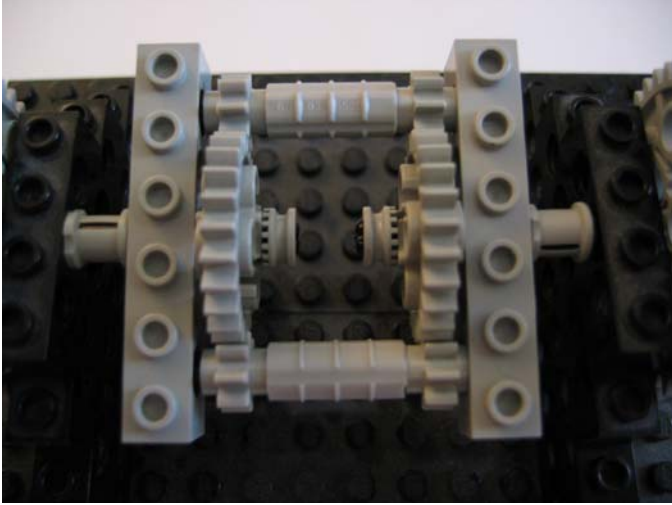


Figure 5: Epicyclic gear train with a basic transmission ratio of 1.0

2.2 Torque Analysis

With the motion of the planetary gear train fully defined, it becomes important to understand the torque requirements of the system. This has traditionally been achieved by a static force analysis of a specific gear train. While this is a valid technique, it requires the selection of a specific planetary arrangement, and it is a long computational process, introducing many opportunities for error on the part of the designer. It is simpler to use the principle of energy conservation. The energy balance equation for the general planetary gear train can be written as

$$T_A \omega_A + T_F \omega_F + T_L \omega_L = 0 \quad (7)$$

where T 's represent the torques applied to each branch of the gear train. While this at first does not appear to yield a great amount of information, the careful selection of two specific cases for ω_F and ω_L will quickly yield equations completely defining the torques on the three branches of the gear train.

The first case to examine is the instance when the gear train is moving as a solid axle, that is $\omega_F = \omega_L = \omega_A = \omega$. In this case, equation 7 can be rewritten as

$$T_A\omega + T_F\omega + T_L\omega = 0 \quad (8)$$

Collecting terms and assuming ω is non-zero, it can be quickly deduced that

$$T_A + T_F + T_L = 0 \quad (9)$$

This is the first of two equations that will define the torque requirements of the planetary gear train.

The second case of interest is that of zero speed at the arm. Using equation 6 and substituting zero for ω_A , it becomes readily apparent that the numerator on the right side must be zero. Hence,

$$\omega_F = R\omega_L \quad (10)$$

Making this substitution and substituting zero for ω_A in equation 7,

$$T_F(R\omega_L) + T_L\omega_L = 0 \quad (11)$$

Again, collecting terms and rearranging into a more convenient form, the second torque governing equation is found to be

$$T_L = -RT_F \quad (12)$$

Using this equation, along with equation 9, one can fully characterize the torque requirements at any two branches of the gear train, given the torque at the remaining branch.

With this analysis complete, the system has been reduced to the solution of equations 6, 9, and 12, involving a total of seven variables. While this seems to imply that the designer has free choice of any four variables, the torque equations are independent of rotational speed. This means that the designer must select one torque, along with either three speeds or two speeds and a gear ratio. With so many variables being selected by the engineer, it becomes important to be able to visualize the response of the design to changes in any of these variables. A convenient graphical aid to the design effort is the nomograph.

2.3 Nomography

A nomograph is defined as three or more axes, or scales, arranged such that problems of three or more variables can be solved using a straightedge. The origins of nomographs date back to professor Maurice d'Ocagne in late nineteenth century France. With the advent of the computer, nomography has become somewhat outmoded, but is still valuable in allowing quick and simple visualization of complex problems.

In the particular case of epicyclic gear trains, a nomograph can be quickly constructed using three parallel axes. Figure 6 shows the basic form of the graph to be created, where the axes labeled ω_F , ω_L , and ω_A represent the rotational speed of the first gear, last gear, and the arm from Willis's solution technique discussed previously. The location of the ω_A axis may actually be to the left of the ω_F axis or between the ω_F and the ω_A axes, depending upon the value of R (and hence d).

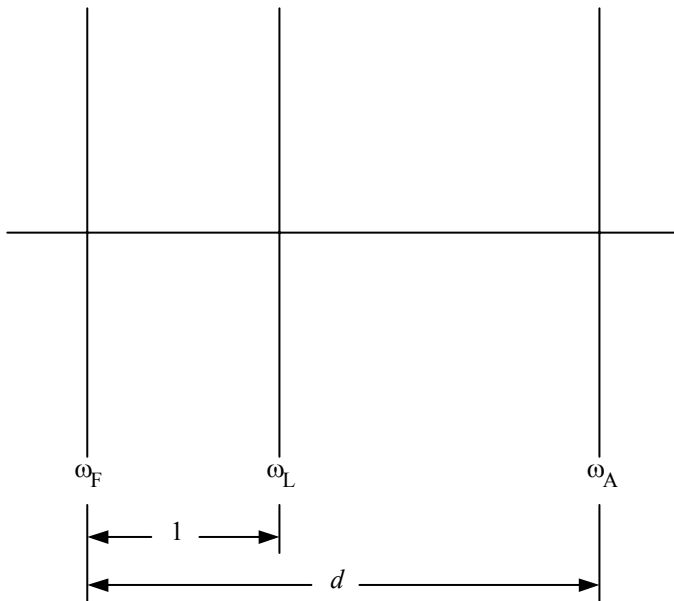


Figure 6: General layout of nomograph for solution of epicyclic gear trains

In order to make the construction of a nomograph as simple as possible, the ω_F and ω_L axes are chosen to be 1 unit apart. It is also important to note that the scales of all three axes must be identical and aligned with one another. By using the equation of a line through 2 points and some knowledge of the behavior of epicyclic gear trains, the distance d can be readily defined in terms of R , the general transmission ratio of the gear train. From equation 6 of the kinematic solution, it is readily apparent that for any R , ω_A is zero when $\omega_F = R\omega_L$. Using this fact, d can be found by considering the ω_F axis to be the y -axis and the line connecting the zeros of the three axes to be the x -axis. As illustrated in Figure 6, the distance d will simply be the x -intercept of the line through the points $(0, R\omega_L)$ and $(1, \omega_L)$. The slope-intercept form of this equation can be quickly found to be

$$y = \omega_L(1 - R)x + R\omega_L \quad (13)$$

Substituting zero for y and d for x , then solving for d yields

$$d = \frac{R}{R - 1} \quad (14)$$

The nomograph is now totally defined. Referring to the nomograph in figure 7, the graph can be divided into the three sections labeled in the figure. The sections represent the locations of the ω_A axis for the cases of R listed. For example, if $R = 0.5$, $d = -1$ from equation 14, and the ω_A axis must lie to the left of the ω_F axis. This nomograph gives the designer the ability to quickly select a broad range for R , depending on how the gear train is intended to react to typical input speeds.

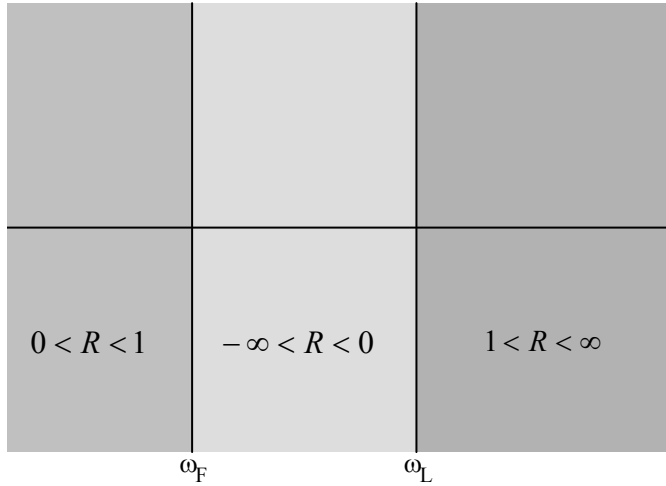


Figure 7: Completed nomograph, with ranges of basic transmission ratio, R , labeled

Aside from selection of a broad range for R , the nomograph also has the distinct advantage of providing a clear visual representation of both the torque response of the gear train and the power flow through the planetary train. Figure 8 shows the torques at F, L, and A represented as arrows along the three axes. Upward vectors represent positive torques. After some examination, one finds that the torques shown in figure 8 can be solved for in the same way as forces on a rigid beam may be solved for in elementary static analysis. Summing the forces in the y-direction yields

$$T_F + T_L + T_A = 0 \quad (15)$$

which is exactly equation 9 from the conservation of energy solution previously presented.

Similarly, summing moments about the point on the ω_A axis generates the equation

$$T_F \cdot d + T_L \cdot (d - 1) = 0 \quad (16)$$

Substituting equation 14 into equation 16,

$$T_F \frac{R}{R-1} + T_L \left(\frac{R}{R-1} - 1 \right) = 0 \quad (17)$$

solving for T_L and simplifying, one arrives at the equation

$$T_L = -R \cdot T_F \quad (18)$$

which is exactly equation 12 derived using conservation of energy. Now, if power flowing out of the system is assumed to have a negative sign, one can immediately see that any torque vector pointing toward zero is representative of power flow out of the system.

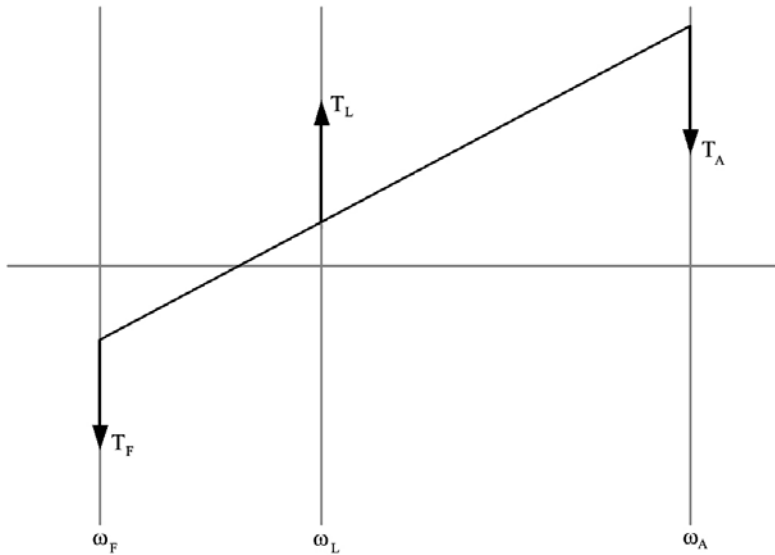


Figure 8: *Nomograph with torques represented as vectors*

Nomographs, like the one in figure 8, allow designers to visualize critical details of a design's response. As previously mentioned, the designer can freely select one torque and either all three rotational speeds or two rotational speeds and the general transmission ratio, R . Nomographs provide some insight into the fundamental differences between these two approaches. In the first approach, one can visualize drawing a straight line between the selected speeds on the ω_F and ω_L axes and placing the ω_A axis such that it intersects this line at the selected rotational speed at the arm. In the latter approach, the designer would simply place the ω_A axis according to equation 14, draw a straight line between the two selected speeds, and read off of the third axis the unspecified speed.

Obviously, there are advantages and disadvantages to each approach, depending on the specific requirements of a particular design. In cases when the torque at each branch of the mechanism is of paramount importance, the second approach has the advantage of immediately specifying the relationships between torques by selection of a gear ratio. Conversely, when the speeds of the three branches of the mechanism are to be closely controlled, it becomes advantageous to simply select all three speeds at their operating points and find the corresponding gear ratio and torques. Using these two approaches, along with a nomograph to aid visualization, a designer can conveniently examine the trade-offs inherent in any design. This is perhaps best illustrated through a case study.

CHAPTER 3: CASE STUDY: MULTI-RIDER HUMAN POWERED VEHICLE

As previously stated, the Human Powered Vehicle team at Virginia Tech has in the past considered planetary gear trains as a viable means of coupling the inputs of two riders. What follows is the design of such a gear train using the methodology developed in this thesis.

The first step in the design process is to set forth any constraints on the design. In this case, the HPV team has several requirements that must be met for a design to be considered useable. The riders are to be seated with their backs to one another. This means that at the system's operating point, ω_F and ω_L must have opposite signs. The team has stated that they would like the output speed, assumed to be the speed at the arm, to be slightly higher than the 80-rpm pedal speed usually associated with efficient riding. Also important to the team is that each rider is contributing exactly half of the total system power. Based on research done by the team, one can only realistically expect a total power output of about 1 horsepower from two riders, assuming no losses. Finally, assuming that power out of the system has a negative sign, the power at the F and L branches must be positive. Table 1 shows these constraints in notation consistent with that used throughout this paper.

Table 1: Design Constraints For Tandem Bike Design

Constraint	Explanation
$P_F = P_L > 0$	The input powers are equal and positive.
$P_A = -1hp$	The total system power is 1 horsepower.
$\omega_F / \omega_L < 0$	The two inputs rotate in opposite directions.
$80 \leq \omega_A \leq 120$	The output moves at or slightly above nominal pedal cadence.

To make the design of this system simpler, nomographs can be used to quickly and simply remove ranges of R from consideration. Figure 9 shows the first of these nomographs. The two horizontal bands highlighted represent the target regions for ω_A . As can be seen in the figure, any line through arbitrary points on the ω_F and ω_L axes which intersects the target regions

to the right of the ω_L axis also intersects the regions to the left of the ω_F axis. To narrow the search for an appropriate R , one of these regions can be ignored, and thus the region to the left of the ω_F axis, corresponding to R -values between zero and one, can be eliminated from consideration.

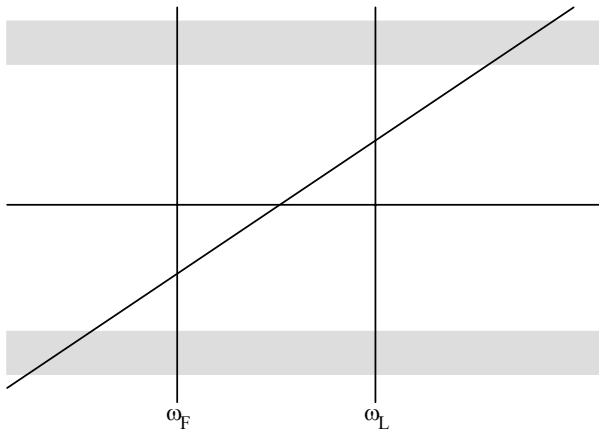


Figure 9: *Nomograph With Target Regions for ω_A highlighted*

In order to eliminate another range of R , one can examine the second design constraint in the context of nomographs. Figure 10 shows two nomographs for the system with the torques drawn as vectors. Notice that when R is (a) between one and infinity, the vectors at ω_F and ω_L point away from zero. This implies that the power at each of these two branches is positive. Conversely, when R is (b) between negative infinity and zero, the vectors both point upward, implying that the power at branches F and L have opposite signs, and therefore cannot be equal, as required by the design constraints. With this range of R -values eliminated, the only range that needs to be considered during the design effort from here forward is between one and infinity.

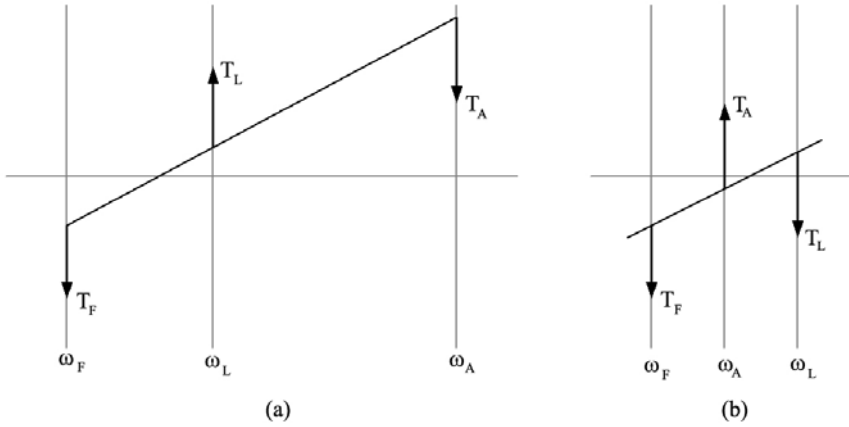


Figure 10: Nomographs for (a) $1 < R < \infty$ and (b) $-\infty < R < 0$ with torques drawn as vectors

After establishing a broad range of R -values to be considered, the next task is to attempt to apply the remaining design constraints to the governing equations of the system. Examining the first design constraint, it is convenient to rewrite it in the form

$$T_F \omega_F = T_L \omega_L \quad (19)$$

Substituting equation 12 from the energy balance solution into equation 19 and dividing through by T_F yields

$$\omega_F = -R \omega_L \quad (20)$$

By virtue of this design constraint, the designer has removed a degree of freedom from the design effort. The system now stands at four equations (namely eqs 6, 9, 12 and 20) and seven variables (the three torques, the three angular velocities and R). With three free choices, it is still difficult to generate any sort of two-dimensional plot to assist in the selection of a gear ratio. Thus, the designer's attention turns to the second design constraint, which defines the total power in the system. Rewriting the design constraint in terms of the variables used in the governing equations,

$$T_A \omega_A = -1hp \quad (21)$$

Adding this equation to the system gives the designer a total of five equations and seven variables. Now, with a free choice of only two variables, ω_A and R , it becomes relatively simple to create plots to assist in the final selection of a gear ratio. Figure 11 shows a plot of the absolute speeds ω_F and ω_L in rpm required to generate a speed of 120 rpm at the arm for a range of R values between 1 and 5. This plot was created using the MATLAB file ‘ratiosselect.m’, included as appendix A. Obviously, since equation 20 must hold for the power contribution from each rider to be equal, the speeds at branches F and L can never be equal. Since the human body provides most efficient power between speeds of 80 and 100 rpm, it would be advantageous to select a gear ratio that would place the absolute values of ω_F and ω_L in this range. Unfortunately, as can be seen in figure 11, the value of ω_L asymptotically approaches one-half of the arm speed, ω_A , while ω_F increases linearly with R . Obviously, the designer could select a very high arm speed along with an appropriate gear ratio to force both speeds into the ideal region, but this would fail the third design constraint, requiring that the output turn at only slightly higher than a nominal cadence. This seems to imply that, in order to allow riders to pedal at their most efficient pace, a gear reduction will be required between their pedals and the planetary itself.

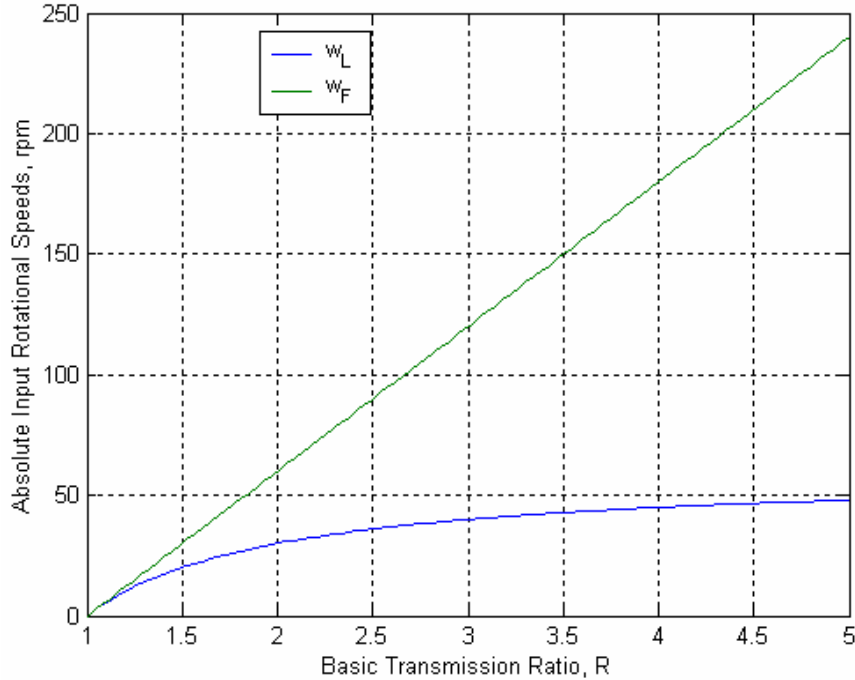


Figure 11: Absolute speeds ω_F and ω_L as a function of ratio R , for $\omega_A=120rpm$

With the realization that a speed change will be required between the riders and the planetary gear train, one might at first assume that the selection of an R -value and an arm speed have become completely arbitrary. While this may be partially true for the arm speed, intelligent selection of R can still have a large impact on the effectiveness of the design. Because the torque at each input is fixed by defining the power and rotational speed of the output, the power at each input becomes solely a function of the rotational speed at that branch. This fact can be used to make the design adaptable to riders of different fitness levels. Figure 12 shows the power contribution from each input for a range of speeds, assuming an R -value of 1.5, an output power of 1 horsepower, and a constant output speed of 120 rpm. This figure was generated using MATLAB file ‘power_response.m’, included as appendix B. Reviewing this figure, one quickly observes that by changing ω_F from -30 to -24 rpm and changing ω_L from 20 to 24 rpm, the power balance can be shifted by ten percent in favor of branch L. Obviously, this relationship can be established for any value of R . The advantage of using a value of 1.5 is that for a ten

percent swing in the power balance between inputs, the rotational speed of the two input branches becomes equal, eliminating the need to mechanically modify the input speed from one of the riders.

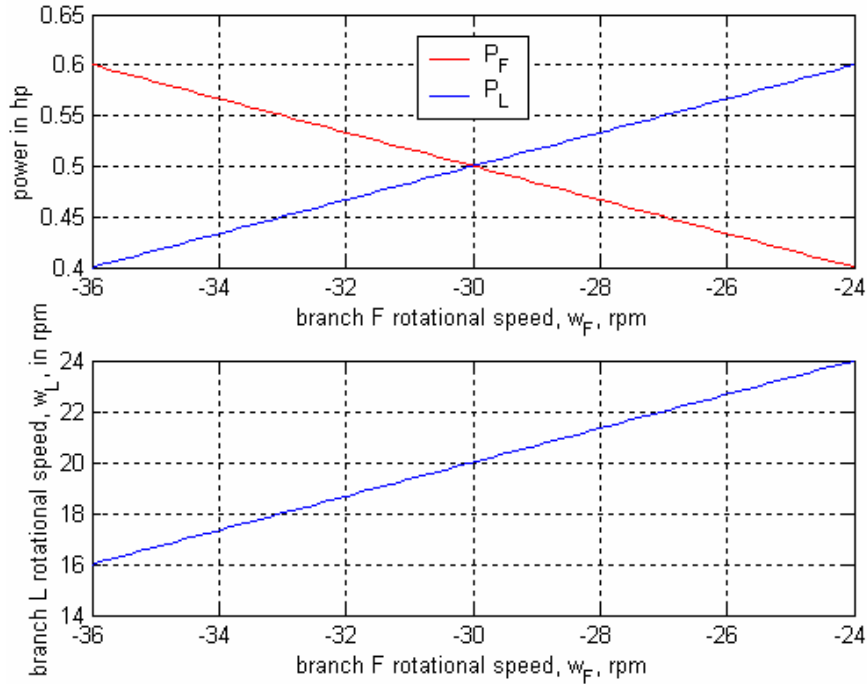


Figure 12: Power required at branches *F* and *L*, assuming $R=1.5$, $P_A=1$ hp, and $\omega_A=120$ rpm

After selecting a gear ratio, it becomes important to decide upon one of the twelve possible arrangements of the gear train. In order to simplify the actual construction of the device, only the simple trains in quadrants *I* and *III* will be considered. Taking the arm to be fixed, one can quickly deduce by examination that only the two trains in quadrant *III* yield a positive ratio between the two concentric gears. In order to avoid the expense of an internal gear, the topmost train in quadrant *III* is selected to complete this design. Selection of actual gear sizes and materials is beyond the scope of this thesis.

CHAPTER 4: CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

This thesis has presented for the first time a complete and thorough design and analysis technique for the class of mechanisms referred to as planetary or epicyclic gear trains. Based largely on the work of Robert Willis, techniques have been provided to find the motion of and forces present in an epicyclic gear train in the most general case. Nomographs have also been developed as a design aid, allowing the designer to quickly visualize the response of the system to a broad range of speed and torque inputs. These tools have been applied to the problem of using an epicyclic gear train as a power coupling in a tandem bicycle. As seen in the previous section, the design of a relatively complex system was simplified through intelligent application of the tools presented in this thesis.

4.2 Recommendations

The research contained in this thesis provides an excellent starting point for more in-depth research in the area of epicyclic gear trains. Specifically, this research does not address the issue of mechanical design of the mechanism. Sizing of gears, for even a simple train, can be a difficult process. In this case, the process of gear selection has been further complicated by epicyclic motion of the planet gears. The mechanical design of these epicyclic gear trains promises to be a challenging task, worthy of in-depth study. Also of interest are multiple stage systems. By combining epicyclic gear trains in series, gear trains can be created with many degrees of kinematic freedom and a single degree of freedom in torque. These gear trains promise to be very interesting from a theoretical standpoint and to be applicable to power sharing devices where multiple power sinks of highly varied requirements are to be supplied from a single power source.

References

1. Willis, Robert. Principles of Mechanism, Second Edition. London: Longmans, Green and Co, 1870.
2. Lévai, Zoltan. Theory of Epicyclic Gears and Epicyclic Change Speed Gears. Budapest: Technical University of Building, Civil, and Transport Engineering, 1966
3. Lent, Deane. Analysis and Design of Mechanisms, Second Edition. Englewood Cliffs, NJ: Prentice Hall, 1970
4. Shigley, Joseph Edward and John Joseph Uicker, Jr. Theory of Machines and Mechanisms. New York: McGraw-Hill Book Company, 1980
5. Mabie, Hamilton H. and Charles F. Reinholtz. Mechanism and Dynamics of Machinery, Fourth Edition. New York: John Wiley and Sons, 1987
6. Molnar, John. Nomographs. Ann Arbor: Ann Arbor Science Publishers, Inc, 1981
7. Price, Derek J. de Solla. On the Origin of Clockwork Perpetual Motion Devices and the Compass. Washington, D.C.: Smithsonian Institution, 1959
8. Dudley, Darle W. The Evolution of the Gear Art. Washington D.C.: American Gear Manufacturers Association, 1969

Appendix A: MATLAB Transmission Ratio Selection Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% ratiosselect.m
%% Programmer: Chris Corey
%% Version: 1.0
%% Date: 10/28/03
%% Description:
%% This program will output a plot of wF and wL for a
%% range of R and wA values selected by the user. Also
%% given by the user is the expected power at the arm.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all

%% Input user values
pA=input('input pA (Defaults to -1hp) ==> ');
if isempty(pA) pA=-1; end;
pA=pA*550; %convert from hp to lb-ft/s
disp(' ');
wA=input('input wA (Defaults to 80rpm) ==> ');
if isempty(wA) wA=80; end;
wA=wA*2*pi/60; % convert from rpm to rad/s
disp(' ');
Rlow=input('input the lowest R to be considered (defaults to 1) ==> ');
if isempty(Rlow) Rlow=1; end;
disp(' ');
Rhigh=input('input the highest R to be considered (defaults to 10) ==> ');
if isempty(Rhigh) Rhigh=10; end;
disp(' ');

R=linspace(Rlow,Rhigh);

%% Calculations
wL=wA.*((R-1)./(2.*R));
wF=-1.*R.*wL;
wL=abs(wL.*30./pi);
wF=abs(wF.*30./pi);
figure;
plot(R,wL,R,wF)
xlabel('Basic Transmission Ratio, R');
ylabel('Absolute Input Rotational Speeds, rpm');
legend('w_L','w_F',0);
grid on;
```

Appendix B: MATLAB Power Contribution Code

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% power_response3.m
%% Programmer: Chris Corey
%% Version: 1.0
%% Date: 10/28/03
%% Description:
%% This program will output a plot of the percent power
%% at wF and wL based on a given R and a range of speeds,
%% assuming wA and pA are constant.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all

%% Input user values
R=input('input the desired gear ratio (defaults to 1.5) ==> ');
if isempty(R) R=1.5; end;
disp(' ');
pA=input('input pA (Defaults to 2hp) ==> ');
if isempty(pA) pA=2; end;
pA=pA*550; %convert from hp to lb-ft/s
disp(' ');
wA=input('input wA (Defaults to 100rpm) ==> ');
if isempty(wA) wA=100; end;
wA=wA*2*pi/60; % convert from rpm to rad/s
disp(' ');
wFlow=input('input the lowest wF to consider (defaults to -150rpm) ==> ');
if isempty(wFlow) wFlow=-150; end;
wFlow=wFlow*2*pi/60; % convert from rpm to rad/s
disp(' ');
wFhigh=input('input the highest wF to consider (defaults to 150rpm) ==> ');
if isempty(wFhigh) wFhigh=150; end;
wFhigh=wFhigh*2*pi/60; % convert from rpm to rad/s
disp(' ');

%% Calculations
wF=linspace(wFlow,wFhigh);

wL=wF./R+wA*(R-1)/R;

tA=pA/wA; % Torque at the arm assuming power is normalized to Pa

tF=tA/(R-1); % Solution to torque equations
tL=-R*tF; % tA + tF + tL = 0
% tL=-R*tF

pF=tF.*wF; % Power calculations
pL=tL.*wL;
pA=pA.*ones(1,length(pF));

pF=pF./550; % convert from ft-lb/s to hp
pL=pL./550;
pA=pA./550;

wA=wA.*ones(1,length(wL));

wA=wA.*60./(2*pi); % convert from rad/s to rpm
wF=wF.*60./(2*pi);
wL=wL.*60./(2*pi);

%% Plot Results

```

```
figure;
subplot(2,1,1)
plot(wF,pF,'r',wF,pL,'b');
grid on;
%ylim([-5,5])
xlabel('branch F rotational speed, w_F, rpm');
ylabel('power in hp');
legend('P_F','P_L',0);

subplot(2,1,2)
plot(wF,wL,'b');
grid on;
xlabel('branch F rotational speed, w_F, rpm');
ylabel('branch L rotational speed, w_L, in rpm');
```


Vita

Christopher A. Corey was born June Fifth, 1979 to Joan and Craig Corey in Baltimore, MD. The majority of his childhood was spent in Lugoff, South Carolina. In 1997, Chris graduated valedictorian from Lugoff-Elgin High School, and came to Blacksburg, Virginia to attend Virginia Polytechnic Institute and State University. Five years later, in May of 2002, he received his Bachelor's degree in mechanical engineering, graduating *cum laude*. During his undergraduate education, Chris was fortunate enough to work for Westinghouse Commercial Nuclear Fuels Division on two separate summer internship assignments, and Honeywell Engines and Systems on a three-term cooperative education assignment. After completing five years of undergraduate education, his motivation to learn was still strong, and thus he continued on his academic journey as a graduate student at the institution that had treated him so well throughout his undergraduate years. As a graduate student, Chris spent many hours working with undergraduates, taking TA positions for two different classes, and serving as an informal advisor on many of the capstone design projects. In December of 2003, Chris will receive his Master of Science degree in mechanical engineering from Virginia Tech.