Chapter 4- Control Delay Analysis at an Intersection

Intersection control delay

The delay encountered by a traveler at a signalized intersection constitutes the larger part of his and her travel time on an arterial link. Hence, estimating the control delay plays a very important part of the travel time estimation of an arterial link. The methodology used in estimating the control delay is a combination of the shockwave algorithm developed earlier by Hobeika et.al, and adjusted algorithms of HCM2000 that recommends certain methods for computing the delays at a signalized intersection.

The control delay experienced by the observed group has three main components: the uniform stop delay ($d_1$), the over-saturation delay ($d_2$), and the stopped delay caused by the initial queue accumulated behind the intersection from previous cycles ($d_4$):

$$\text{control \_delay} = \text{uniform \_delay} + \text{oversaturation \_delay} + \text{stopped \_delay}.$$

However, in order to compute the stopped delay of the observed vehicle group ($d_4$) which is caused by the initial queue, the initial queue clearance time ($d_3$) needs to be determined first. $d_4$ is the average stopped delay of the observed group during time $d_3$ that certain vehicles in the observed group while the initial queue is being cleared (during $d_3$). Therefore, the intersection control delay in our methodology is computed as follows:

$$\text{control \_delay} = d_1 + d_2 + d_4.$$
4.1 Definition of New Variables Used in Chapter 4

d1: Uniform stop delay of the observed group when dissipating the intersection;

d2: Over-saturation delay of the observed group when dissipating the intersection.

d3: Initial delay encountered by the first vehicle of the observed group to reach the
intersection caused by the queue built up from previous cycles.

d4: Average stopped delay of the observed vehicle group caused by the initial
queue, or during time d3. (See section 4.3.4 for further explanation)

h1: Original uniform headway of the observed group. (sec/veh);

h2: Intersection departure headway.( sec/veh); usually considered to be 2
secs/veh= 3600/1800

h3: Queue departure headway (sec/veh);

\(Q_m\): Number of vehicles from the observed group that joins the queue during the
red phase,(veh);

\(t_q\): The time it takes the queue from the observed group to develop and to
dissipate during a cycle length, (sec)

S1: Average arrival speed of the observed vehicle group at the detector, (mph)

S2: Moving speed of the queue at the intersection (mph);

The following relationships and assumptions are adopted:

a) The control delay is the average control delay of all the vehicles in the
observed group.

b) The headway distribution and the speed of the observed group will change due
to the initial queue. This assumption will be described later on.
c) The first vehicle in the first observed group will arrive at the intersection at the start of the red time phase. The first observed group is shown in Figure 4 - 1 as Group 1.

![Figure 4 - 1 Observed Vehicle Group](image)

Figure 4 - 1 Observed Vehicle Group

d) The observed group will use a full cycle length to pass the intersection irrespective of what time the first vehicle in the group arrives at the intersection. This is consistent with the approach that we are trying to determine the travel time it takes for this observed group to traverse the link. This assumption is used in calculating the uniform delay for this observed group.

e) If the number of vehicles in the last observed group (for example Group 1 in Figure 4 - 1) is greater than the capacity of the intersection, some vehicles are queued at the intersection till the next green time takes place. Based on the aforementioned items c and d, the first vehicle of next observed group (for example Group 2 in Figure 4 - 1) will also arrive at the end of the queue at the start of red time phase.
HCM2000 uses a relatively large observed time period such as 15 minutes, or half an hour to estimate the control delay at a signalized intersection. But in our case, we need to update the travel time of the observed vehicle group for each one or two minutes. Hence, we had to adjust the equations for computing the control delay in HCM2000 in order to apply to our cases.

4.2 Computing the Initial Delay of the First Vehicle in the Observed Group (d3)

In HCM2000, the initial delay \( d_3 \) is computed as follows:

\[
d_3 = \frac{1800Q_b(1+u)t}{CT}
\]

(Eq 4 - 1)

Where

- \( Q_b \) = Initial queue at the start of period \( T \) (veh),
- \( C \) = capacity (v/h);
- \( T \) = duration of analysis period (h);
- \( t \) = duration of unmet demand in \( T \) (h), and
- \( u \) = delay parameter.

But the initial delay of the first vehicle of the observed group (d3) in our case is different from the one computed in HCM2000. The initial delay of the first vehicle in the observed group in our case, as shown in Figure 4 - 2, is the time it takes the first vehicle in the observed group to travel from the time it arrives at the end of the initial queue to the time it arrives at the intersection. Our method computes the clearing time for the initial queue and not the average delay per vehicle in the queue. In addition,
after the first vehicle in the observed group arrives at the intersection, the character of the observed group will change during the initial delay time. Hence, it will affect the uniform stop delay, as well as the over-saturation delay, which will be discussed later on.

Different from HCM2000, the adopted approach uses the shockwave method to estimate the initial delay of the first vehicle in the observed vehicle group and the initial queue.

If there is no blackout taking place on the loop detector which means that the traffic data observed by the loop detector is reliable, the shockwave method is used to estimate the initial queue length and then estimate the initial delay. The shockwave method utilized here is the same one used in the freeway case.

(Please refer to those notes for complete description of the shockwave method).

There are three cases for estimating the initial delay: a building queue case, a dissipation queue case and no change of queue case.

**Step 1:**

\[ W_u = \frac{V - C}{k_{id} - k_y} \], for the building queue case where \( V > C \) ; .......................... (Eq 4 - 2)
\[ W_d = \frac{C - V}{k_q - k_{id}}, \text{ for the dissipation case where } V < C; \quad \text{………………………… (Eq 4 - 3)} \]

\[ W = 0, \text{ for no change in queue case where } V = C; \quad \text{………………………… (Eq 4 - 4)} \]

Where

\[ k_{id}: \text{ is the vehicular density obtained from the detector;} \]

\[ k_q: \text{ is the jam density.} \]

The queuing rate QR (veh/h) is:

\[ QR = \frac{dn}{dt} = (V - C - W_d \times k_{dr}); \quad \text{………………………………………………... ... (Eq 4 - 5)} \]

**Step 2:**

The number of vehicles in queue which is built during the observed cycle length \( t \) will be:

\[ Q_m = QR \times CL \quad \text{………………………………………………………………………………... (Eq 4 - 6)} \]

The process is repeated for every interval and the total number of vehicles in queue is estimated using the following expression:

\[ Q_t = \max(0, \sum_{m=1}^{t} Q_m) \quad \text{………………………………………………………………………………... (Eq 4 - 7)} \]

\( Q_t \) is the initial queue for next cycle length which is also the queue at the end of cycle \( t \). Thus, if we want to estimate the initial delay for cycle length \( t \), we should consider the queue at the end of last cycle length which is \( Q_{t-1} \).

If \( \frac{Q_{t-1}}{D} < g \), the initial delay is \( \frac{Q_{t-1}}{D} + \text{red} \) (\( D \) is the departure saturation flow in green time veh/h/ln)

Else
If \( \frac{Q_{t-1}}{D_s} < 2g \), the initial delay is \( \frac{Q_{t-1}}{D_s} + 2 \times \text{red\_time} \); (because it should wait for another green time to dissipate the initial queue vehicle for the incoming group of vehicles in time \( t \))

Else

If \( \frac{Q_{t-1}}{D_s} < 3g \), the initial delay is \( \frac{Q_{t-1}}{D_s} + 3 \times \text{red\_time} \);

......

Thus, we came out with the following equation to estimate the initial delay.

When \( (n-1) \times g \leq \frac{Q_{t-1}}{D_s} \leq n \times g \), ...............................................................(Eq 4 - 8)

Therefore, \( d_3 = \frac{Q_{t-1}}{D_s} + (n) \times \text{red\_time} \) .............................................................(Eq 4 - 9)

Step 3:

After we conducted a calibration method using CORSIM, the average length of the vehicle plus the average headway in queue is considered to be 19ft.

Hence the queue length, \( QL = 19 \times Q_t \),

Where

\( Q_t \) is number of the vehicles in the queue at the end of current time interval.

The average speed of vehicles in the observed group is:

\[
\text{Speed} = \frac{V}{N} \cdot \frac{1}{k_{sd}}
\] .........................................................................................(Eq 4 - 10)
4.3 Computing Uniform Stop Delay, \( (d) \)

Uniform delay is caused by cyclic interruption of service caused by the red phase at a signalized intersection. Part of the observed group of vehicles will stop and queue because of the red light, if there is no initial queue. The area of the yellow triangle in Figure 4 - 3 is the total uniform delay time for each cycle length.

- **Over-saturation delay:**
  - Area of large triangle
- **Uniform delay:**
  - Area of small triangles

![Figure 4 - 3 Uniform and Over-saturation Delay](image)

A diagram of the yellow triangle is shown in Figure 4 - 4. It shows the build up of the queue during the red phase to \( Q_{\text{max}} \), and then its dissipation during part or whole of the green time phase.

![Figure 4 - 4 Uniform Delay](image)
If there is no initial queue, the uniform delay in HCM2000 is computed as follows, based in Figure 4-4:

\[ d_1 = \frac{0.5 \times Q_{\text{max}} \times t_q}{\min(V, C) \times CL} = \frac{CL}{2} \times \left(1 - \frac{g}{CL}\right)^2 \left(1 - \min(l, \frac{V}{C}) \times \frac{g}{CL}\right) \]  

\text{……..(Eq 4 - 11)}

For a detailed derivation of this equation refer to Appendix A.

We have three main concerns in adopting Equation 4 of HCM2000 in calculating the uniform delay of the observed group. Each of the concerns is discussed separately below:

4.3.1 The Denominator in Equation 2-11

The denominator in Equation 4 reflects the volume approaching the intersection or the capacity of the intersection in calculating the uniform delay. When the volume is greater than the capacity, it uses capacity as the volume to estimate the total uniform delay. In our algorithm, we agree to use the same numerator which is \(0.5 \times Q_{\text{max}} \times t_q\) in equation 4, but since we are considering the average uniform stop delay for all vehicles in the observed group, the total stop delay should be divided by the total number of vehicles in the observed group instead of the value of \(\min(V, C) \times CL\). Hence, the new average uniform stop delay is computed as follows:

\[ \text{Average uniform delay } d_1 = \frac{0.5 \times Q_{\text{max}} \times t_q}{V \times CL} \]  

\text{……………………………………..(Eq 4 - 12)}
Queue Departure Rate $h_3$

HCM2000 assumes that in every second, there will be $\frac{1}{h_1}$ vehicles added to the queue and $\frac{1}{h_2}$ vehicles will be dissipated from the queue. Hence, the queue departure rate is $\frac{1}{h_2} - \frac{1}{h_1}$ (veh/sec) which is $\frac{1}{h_3}$. Therefore, $h_3$ is $\frac{h_1 \times h_2}{h_1 - h_2}$ (sec/veh). This calculation of $h_3$ works well when there is no initial queue at the intersection, but it may not work well when there is an initial queue at the intersection.

To illustrate, consider the problem shown in Figure 4 - 5, where $h_1$ is assumed to be 5 sec/veh, and $h_2$ to be 2 sec/veh. After 10 seconds of green time, vehicle 10 will arrive at the queue according to HCM2000, but in actually it doesn’t, because the queue is moving forward. Vehicle 10 only arrives at a location $L$ where it may starts to reduce its speed in order to enter into the queue. Hence, it doesn’t belong to the queue size yet, because vehicle 9 has moved downstream a little bit. The distance between vehicle 10 and vehicle 9 is greater than queue distance. This example shows that the value of $h_3$ is under-estimated in HCM2000 when there is a long initial queue.
To refigure the new queue dissipating rate (h3) during the green time with an initial queue, the following steps are adopted:

**Step 1- Queue Moving Speed (S2):**

Assuming that the average vehicle length and headway in queue is 19 ft (adopted from CORSIM simulation), and that h2 is 2 sec/veh based on saturation flow of 1800 vehicles in one hour, then the average speed of the moving queue is $19/2=9.5$ (ft/sec)=6.5mph. Hence we assume all vehicles in queue are moving at the speed of 6.5 mph. Therefore, the last vehicle in the queue is also moving at the speed of 6.5 mph.

**Step 2- Vehicle Arrival Rate to the Queue during Green Time Phase**

As the vehicles in the observed group reach the end of the queue, developed from
the red time of the previous cycle length, they will change their speeds and correspondingly their headways in order to enter the queue. When the queue is moving, the arrival rate of the observed group to enter the queue will change as stated in section 2.3.2. The following discussion is centered on how to determine the arrival rate of the observed group to the queue during the green time phase. Assume that a vehicle in the observed group will join the moving queue in t seconds. Let vehicle n be the last vehicle in the observed group that arrives at the queue in the red time phase of the current cycle length. After t seconds, the next vehicle n+1 will join the queue. To compute the value of t or headway, the following relationships and assumptions are adopted:

A) Only two speeds and consequently two headways are adopted to occur in this movement of the observed group of vehicles. We already have shown that the saturated departure headway (h2) in green time at the intersection is 2sec/veh and the queue moving speed is 6.5 mph (S2). The incoming speed (S1) of observed group is obtained from the detector, and so its uniform headway (h1).

B) The difference in the distance traveled by the nth vehicle and its follower (n+1) in time t during the green time phase is computed based on the difference in speeds. The distance between vehicle n and (n+1) before n enters the queue is S1*h1. At time t0, vehicle n enters into the queue. At time t0+t, vehicle n+1 arrives at the end of queue behind vehicle n with 19 ft headway. Hence, in time interval t, the distance change between
vehicle n and n+1 is \((S1*h1-19)\).

Let the difference in distance traveled be represented as \((S1 - S2) \times t\).

Hence,

\[
(S1 - S2) \times t = S1 \times h1 - 19
\]

\[
\Rightarrow t = \frac{S1 \times h1 - 19}{(S1 - S2)}
\]

Equation 4 - 13

And the arrival rate to the queue is \(t\) (sec/veh) which means one new vehicle will join the moving queue in every \(t\) seconds.

**Step 3- Actual Queue Dissipating Rate (h3)**

Now we know that the new arrival rate to the queue is \(t\) (sec/veh). Hence, \(\frac{1}{t}\) vehicles will come to the queue as well as \(\frac{1}{h2}\) vehicles will dissipate from the queue in every second. Hence, the actual dissipating rate of the queue in vehicle per second is \(\frac{1}{h2} - \frac{1}{t}\) (veh/sec) which is \(\frac{1}{h3}\). Therefore, the new \(h3\) (different from HCM2000) is \(\frac{t \times h2}{t - h2}\) (sec/veh). It is represented by the angle shown in Figure 4-6.

![Figure 4-6 Queue Dissipating Rate (h3)](image)

From Figure 4-6, we can compute the \(tq\) based on the new angle of \(h3\) and the
Step 4- Computing the Uniform Delay with no Initial Queue

When there is no initial queue, the average uniform stop delay for the observed group is computed based on the above discussions, as follows:

\[
d_1 = \frac{0.5 \times Q_{\text{max}} \times t_q}{\text{Total vehicles in Observed Group}}\]

(Eq 4 - 14)

Where

\[
Q_{\text{max}} = \frac{\text{red time}(r)}{h_1}
\]

\[
t_q = h_3 \times Q_{\text{max}} + r
\]

Total vehicles in the observed group = \(V \times CL\)

4.3.2 Computing the Uniform Stopped Delay with an Initial Queue

The headway and speed of the observed group will also change when they arrive at the intersection if there is an initial queue. For example, at time \(t\) in Figure 4 - 7, the first vehicle of the observed group arrives at the end of the initial queue as explained earlier and reduces its speed and consequently joins the queue at the intersection. When the green phase starts, it will take \(d_3\) seconds to dissipate the initial queue till vehicle 1 in the observed group arrives at the intersection. As shown in Figure 4 - 8, vehicle 1 arrives at the intersection, and other \(n\) vehicles following it will join the queue during the time interval of \(d_3\). But there are some vehicles in the observed group that are still moving with original \(h_1\) headway which is 5sec/veh.
The queue in the observed group will attempt to leave the intersection in the remaining green time period $g_1=(g-d_3)$ after $d_3$ seconds has expired to dissipate the initial queue. Figure 4-9 shows the queue build up and dissipation for the observed group during a full cycle length period. The area under the curve in Figure 4-9 represents the uniform stopped delay with an initial queue, which is different from the area under the curve in Figure 4-6 when there is no initial queue. It is immediately realized that $g_1$ is dependent on $d_3$, which is influenced by the size of the initial queue. So, the following relationships are established to encompass the various initial queue size situations.

When $d_3$ is smaller than a cycle length:

$$g_1 = Initial\_queue \times h2,$$

where $h2$ is 2 sec/veh.
\[ g_2 = green\_time - g_1; \]

When: \( \text{cycle length} \leq d_3 \leq 2 \times \text{cycle length} \)

\[ g_1 = Initial\_queue \times h_2 - green\_time \]

\[ g_2 = green\_time - g_1 \]

When: \( 2 \times \text{cycle length} \leq d_3 \leq 3 \times \text{cycle length} \)

\[ g_1 = Initial\_queue \times h_2 - 2 \times green\_time \]

\[ g_2 = green\_time - g_1 \]

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Figure 4 - 9 Uniform Stopped Delay

Therefore, it is necessary to estimate the new characteristics of the observed group to compute the uniform stopped delay of the queued vehicles in the observed group while attempting to dissipate through the intersection. The uniform stopped delay will vary, as stated above with the size of the initial queue, and the volume of the observed group.

We have four cases that describe these variations in queue size and volume in determining the stopped delay. They are:

Case 1- where the volume of the observed group is smaller than the intersection
capacity and where \(d_3\) is smaller than the cycle length

Case 2- where the volume of the observed group is greater than the capacity and \(d_3\) is smaller than a cycle length

Case 3- where \(d_3\) is greater than a Cycle Length

Case 4- is a special case where vehicle 1 in the observed group does not reach the initial queue at the beginning of the red phase.

**Case 1- the volume of the observed group is smaller than the intersection capacity and \(d_3\) is smaller than the cycle length**

Figure 4-10 shows the development of the queue size in vehicles over time at a signalized intersection. As we have stated earlier the queue is made up of two components: 1) the initial queue from previous cycles, and 2) the queue of the observed group in this time interval or cycle length.

Let \(I_q\) be the initial queue at the intersection from previous cycles. As shown in Figure 4-10 the queue will grow during the red phase to include the incoming vehicles from the observed group. At the end of the red time, the queue is now \(I_q + m\). It will take a time of \(g_1\) from the green phase to clear the initial queue. At this time the queue size is \(k\) because some additional vehicles have joined the queue from the observed group. The queue will still decrease till the end of the green phase \((g_1+g_2)\) and now reaches the level of \(L\). The queue will start to build up again during the red phase. However, we need to separate the status of the vehicles in the observed group to the status of what is happening at the intersection as a whole. In that regard, we separated the queue development into a) the queue of the observed group and b) the queue at the
intersection. The queue of the observed group, which is our main interest, now increases for a short period of time (t4-t3) to i, as shown in Figure 4-10, and then remains constant during the red phase, and then dissipates again during the next green phase using a time (t6-t5).

After t3 in the Figure 4-10, the broken upward line represents the queue at the intersection which composes of current queued vehicles and the anticipated queues from the next observed group. The dark line in Figure 4-10 represents the queue size of the current observed group. The area under this line, areas A, B, and C will determine the uniform stopped delay of the observed group.

To better illustrate the progression of the queue development in Figure 4-10, the following Figures are provided by key time periods.

**At time t0**

Figure 4 - 11 shows the beginning situation at time t0, where the first vehicle in the group arrives at the end of the initial queue (Iq) at the start of the red phase.
At time t1

During the red time phase from t0 to t1, the queue size at the intersection will increase at a queue building rate of $h_1$ (as explained earlier), reaching the value of $m$. Hence, $m$ is equal to $\frac{\text{red time}}{h_1}$. Therefore, at the end of red time phase, the queue size at the intersection is $m + Lq$ as shown in Figure 4-10.

From t1 to t2

During the green time phase $g_1$ from time t1 to t2, the queue size at intersection will be reduced at the rate of $h_3$ to the level $k$. The estimation of $h_3$ is stated earlier in
section 2.3.2.

At time t2

At time t2, the first vehicle of the observed group arrives at the intersection as shown in Figure 4-14. Let $k$ be the queue size at the intersection at time $t2$. From Figure 4-10, $k$ is derived to be equal to $k = m + Iq - \frac{gI}{h3}$. At time $t2$, all the vehicles in the initial queue have cleared the intersection, because $d3$ is less than the cycle length. Meanwhile, all vehicles in the queue at the intersection at time $t2$ are the vehicles of the observed group. Hence, $k$ is also equal to the queue of the observed group.
At time t3

The red time phase starts again at time t3. As shown in Figure 4-10, t3 is also the time for the last vehicle in this observed group to attempt to arrive at the intersection (t3-t0=100seconds=the assumed cycle length). Figure 4 - 15 shows the distribution of the vehicles in the current observed group at this time. As shown in Figure 4-10, from time t2 to t3 which represents the remaining green time g2, some vehicles in the observed group have already cleared the intersection, while some others have joined the queue, and some others are still attempting to join the queue since the approaching volume V is smaller than the capacity. For instance, as shown in Figure 4 - 15, vehicle 20, which is the last vehicle in the observed group, doesn’t join the stopped vehicles at the intersection till t4 in the red time phase. The value of L in Figure 4 - 15 and in Figure 4-10 represents the number of vehicles in the observed group that are part of the stopped vehicles at the intersection at the start of the red phase.

Based on Figure 4-10, L is computed as follows:

\[ L = m + Iq - \frac{g_1 + g_2}{h_3} \]  

….(Eq 4 - 15)

And

When L>0

The area of A, which represents part of the uniform stopped delay, is a trapezoid which can be calculated as follows:

The area of A is  

\[ a = 0.5 \times (k + L) \times g_2 \cdots \]  

….(Eq 4 - 16)

When L=0, the area of A is a triangle,
From t3 to t5

The time from t3 to t5 represents the red time phase, as shown in Figure 4-10. During this time some of the remaining vehicles in the observed group that did not already join the queue are still attempting to join the stopped queue at an arrival rate of h1. From time t4 to t5, all the remaining vehicles in the observed group would have joined the stopped queue as shown in Figure 4-16. The number of vehicles in this queue would remain constant for the rest of the red time phase. The number of vehicles in queue from t4 to t5 is equal to the number of vehicles in the observed group which can not dissipate the intersection during time t2 to t3 (g2). The dissipating rate of the observed group in the queue from time t2 is h2 which is always 2 sec/veh. Hence, the number of vehicles in queue from t4 to t5, represented as (i), is computed by subtracting the number of vehicles in the observed group (V.CL) from the number of vehicles that already left the intersection (g2/h2):

\[ i = V \times CL - \frac{g^2}{h2} \]  

(Eq 4 - 18)

The vehicles that would be still approaching the queue from this group during this period are the difference between the two sizes of queue i and L as shown in Figure
\[ t_4 - t_3 = (i - L) \times h_1 \]

Therefore, the area B under the curve, which represents the stopped delay can now be calculated as follows:

\[ b = 0.5 \times (L + i) \times (t_4 - t_3) + i \times (\text{redtime} - (t_4 - t_3)) \] .................................(Eq 4 - 19)

From Figure 4-10, the C area under the curve that represents the uniform stopped delay is computed as follows:

\[ c = 0.5 \times i \times i \times h_2 = 0.5 \times h_2 \times i^2 \] .................................(Eq 4 - 20)

Hence, the average stopped delay of case 1 is computed by adding areas A, B, and C and divide by the number of vehicles in the observed group (VxCL):
\[ d_1 = \frac{a + b + c}{V \times CL} \]

**Case 2, Volume of the observed group is greater than the intersection capacity and d3 is smaller than the cycle length**

This case 2 is very similar to case 1 except that intersection capacity is now used instead of the observed volume in calculating the dissipating volume at the intersection. This logically would result that all vehicles in the observed group can dissipate the intersection during the whole green time, and that the queue at the start of the next cycle is the same as the starting initial queue. Therefore, from Figure 4-17 the value of i in case 2 is equal to the initial queue size (Iq) which is equal to

\[ C \times CL - \frac{g_2}{h_2} = \frac{g_1}{h_2} \]

![Figure 4-17 Uniform Stopped Delay Case 2](image)

**Case 3- d3 is Greater Than the Cycle Length**

Since d3 is greater than the cycle length, the observed group will experience at
least two red times and a green time before it arrives at the intersection. All vehicles in the observed group are assumed to be in the queue before they begin to depart the intersection.

![Diagram illustrating traffic flow and delay](image)

**Figure 4 - 18 Uniform Stopped Delay Case 3**

According to this assumption, the number of vehicles in the queue ($m$) in Figure 4-18 is equal to $V \times CL$ when $V$ is smaller than the capacity, otherwise, $m$ is equal to $C \times CL$. Because as stated earlier the uniform stopped delay only deals only with observed group volumes of less or equal to capacity. If the observed group volume is greater than intersection capacity, the delay of the excess volume beyond the intersection capacity will become part of the oversaturation delay which is discussed later. The dissipation rate of the observed group is $h_2$ (2 sec/veh). From Figure 4-18,

$$i = m - \frac{g_2}{h_2}$$  \hspace{1cm} (Eq 4 - 21)

Area A is computed as follows:

$$a = 0.5 \times (i + m) \times g_2$$  \hspace{1cm} (Eq 4 - 22)

Areas B and C are computed as follows:

$$b = r \times i$$  \hspace{1cm} (Eq 4 - 23)
\[ c = 0.5 \times (i \times h2) \times i = 0.5 \times h2 \times i^2 \] ...........................................................................................................(Eq 4 - 24)

The average stopped delay for the observed group for this case is \[ d_1 = \frac{a + b + c}{V \times CL} \]

Case 4 - the volume of the previous observed group is smaller than the capacity, which means that the first vehicle in this observed group doesn’t arrive at the stopped queue at the start of the red time phase.

The observed group arriving in this cycle will arrive at the end of the current group at the beginning of the red phase at time \( t_3 \). However it will take a time of \( (t_4-t_3) \) for the first vehicle in the current observed group to join the stopped queue after the start of red time phase as shown in Figure 4-19. Let \( dt \) be the time of \( t_4-t_3 \) from the last observed group. This \( dt \) is shown in the Figure 4 - 20 with the mark of ‘change’ to reflect the new queue development of the current observed group.

Therefore, the value of \( m \) (the queue size of the current observed group at the end of the red phase) is revised to:

\[ m = \frac{(r - dt)}{h1} \] ...........................................................................................................(Eq 4 - 25)

The other equations remain the same as the other cases.
4.4 Over-saturation Delay (d2):

Over-saturation delay is caused by the accumulation of queues when the incoming volume of the observed group exceeds capacity. So, during the time of a cycle length part of the vehicles in the observed group has not cleared the intersection.
The over-saturation delay computes the time it takes for these vehicles to reach and clear the intersection.

The area of the blue triangle in Figure 4-21 is the total over-saturation delay, which can be computed as follows:

\[
\frac{0.5 \times Q \times T}{V \times CL} = \frac{0.5 \times Q_m \times CL}{V \times CL} = \frac{0.5 \times Q_m}{V}
\]

(Eq 4 - 26)

where:

\(Q_m\) : Queue established by the observed group in a whole cycle length. (veh) It can be obtained directly from the Shockwave method. (Refer to 2.1.1 initial delay)

- Over-saturation delay:
  - Area of large triangle
- Uniform delay:
  - Area of small triangles

Figure 4 - 21 Uniform and Over-Saturation Delay

This over-saturation delay equation does not account for the time it takes to build the queue \(Q_m\) and to dissipate it as shown in Figure 4-22. It only estimates the area of triangle B and does not consider the area of triangle C. Both these triangles constitute the over saturation delay encountered by the observed group when the volume exceeds capacity.
Hence, the equation is revised to:

\[ d_2 = \frac{\text{area}(B) + \text{area}(C)}{V \times CL} = \frac{0.5 \times Q_m \times CL}{V \times CL} + \frac{0.5 \times Q_m \times Q_m \times h_2}{V \times CL} \] ........................... (Eq 4 - 27)

It is possible that the dissipating time for \( Q_m \) is greater than \( g_2 \). Hence, some vehicles of this group will experience additional red time delay as shown in Figure 4-23. In this figure, the value of queue level \( i \) is equal to \( Q_m - \frac{g_2}{h_2} \).

Therefore, the equation of over-saturation delay is revised to:
\[
d_2 = \frac{\text{area}(A) + \text{area}(B) + \text{area}(C) + \text{area}(D)}{V \times CL} \\
= \frac{0.5 \times Q_m \times CL}{V \times CL} + \frac{0.5 \times (i + Q_m) \times g^2}{V \times CL} + \frac{r \times i}{V \times CL} + \frac{0.5 \times i \times i \times h^2}{V \times CL} \\
= \frac{0.5 \times Q_m \times CL + 0.5 \times (i + Q_m) \times g^2 + r \times i + 0.5 \times h^2 \times i^2}{V \times CL} \\
\]

Where \[i = Q_m - \frac{g^2}{h^2}\]

4.5 Average Stopped Delay (d4) of the Observed Vehicle Group Caused by the Initial Queue.

The average stopped delay of the observed vehicle group caused by the initial queue is the cumulative stopped delay experienced by the vehicles in the observed group when the first vehicle in the observed vehicle group reaches the stop-line of the intersection.

The cumulative stopped delay is influenced by the variations in initial queue size and by the incoming volume. There are three cases that cover these variations. They are:

Case 1 - where the volume of the observed group is smaller than the intersection capacity and where d3 is smaller than the cycle length
Case 2 - where the volume of the observed group is greater than the capacity and d3 is smaller than a cycle length
Case 3 - where d3 is greater than a Cycle Length

4.5.1-Case 1

In this case, the queue size of the observed group vs. time is shown in Figure
4.24. As computed earlier, let \( k \) represents the number of vehicles in the queue of the observed vehicle group at time \( t_2 \) when the first vehicle in the observed group arrives at the intersection. The total stopped delay encountered by the observed group during time \( d_3 \) is shown as area \( D \) in Figure 4-24. In addition, used variables such as \( h_1, h_2, h_3, \) and \( m \) are the same ones used in computing the case 1 of uniform delay. Therefore, the area \( D \), which represents the stopped delay by initial queue, can now be calculated as follows:

\[
D = 0.5 \times r \times m + 0.5 \times (m + k) \times g_1 \hspace{1cm} \text{(Eq 4 - 29)}
\]

The average stopped delay of the observed vehicle group is then computed by dividing \( D \) by the total number of vehicles in the observed group:

\[
d_4 = \frac{D}{\text{Total\_vehicle\_in\_the\_Observed\_Group}} \hspace{1cm} \text{(Eq 4 - 30)}
\]

![Diagram showing initial stopped delay](image)

**Figure 4 - 24 Initial Stopped Delay**

**4.5.2-Case 2**

In this case, the volume of the observed group is greater than the intersection capacity and \( d_3 \) is smaller than the cycle length. The only difference from case 1
above is that we need to recompute the variables \( h_1, h_2, h_3, k \) and \( m \) based on the detected volume instead of intersection capacity using the same algorithms as stated before. The use of intersection capacity as the incoming volume will under-estimate the stopped delay.

After recomputing \( k \) and \( m \), area \( D \), which represents the stopped delay caused by the initial queue, is similarly calculated as follows.

\[
D = 0.5 \times r \times m + 0.5 \times (m + k) \times g_1 \tag{Eq 4 - 31}
\]

Consequently, the average stopped delay of the observed vehicle group is computed as follows:

\[
d_4 = \frac{D}{\text{Total_vehicle_in_the_Observed_Group}} \tag{Eq 4 - 32}
\]

4.5.3-Case 3

In this case, \( d_3 \) is greater than the Cycle Length (CL), which means that none of the vehicles in the observed group will dissipate during this time interval as shown in Figure 4-25. Let us assume that all vehicles in the observed group will join the queue with a uniform distribution during the cycle length. Therefore, the total stopped delay during the first cycle length time period when the first vehicle of the observed group arrives at the end of initial queue can be computed as follows:

\[
0.5 \times V \times CL \times CL = 0.5 \times V \times CL^2 \tag{Eq 4 - 33}
\]

In addition, all vehicles in the observed group will experience additional stopped delay which is \( d_3 - CL \).

Therefore, the average stopped delay by initial queue in this case can be
computed as follows:

$$d4 = \frac{0.5 \times V \times CL^2}{V \times CL} + d3 - CL = d3 - 0.5 \times CL \quad \ldots \quad (Eq \ 4 \ - \ 34)$$

Figure 4-25 Initial Delay Case 3