Chapter 4: Single Vertical Arch

4.1 Introduction

This chapter considers a single pressurized arch in a vertical plane. This arch is similar to the arches that will be studied in the following chapter. The single arch has also been considered as a possible support structure for the LANMaS. A parabolic arch, of the same shape that is used for the leaning arches in chapter 5, is studied in order to gain an understanding of the behavior of an inflatable arch as a relatively simple system compared to the leaning arches.

The arch studied in this chapter has the following properties:

- Base span: 25 m (82.0 ft)
- Height: 17 m (55.8 ft)
- Shape: Parabolic \((z(x) = 17 - \frac{17}{12.5^2} x^2)\)
- Cross-sectional radius: 0.4 m (1.31 ft)
- Shell thickness: 2.5 mm (0.098 in.)
- Modulus of elasticity: 7 GPa (1015 ksi)
- Poisson’s ratio: 0.3
- Density: 1440 kg/m\(^3\) (90 lb/ft\(^3\))
- Boundary conditions: Fixed
- Number of elements: 4800 \((m=24, n=200)\)
- Element type: S4R

Three types of loads are considered, full snow, half snow, and wind loads, as discussed in section 3.5. However, six types of pressure distributions for the wind load will be investigated to determine which distribution will be used on the leaning arches.
4.2 Internal pressure

For every analysis, prior to the application of external loads, the self-weight of the structure is applied as well as an internal pressure of 500 kPa (72.5 psi). At the final internal pressure, the arch deflects 5.87 cm (2.31 in.) upwards at the apex. The first three vibration frequencies of the arch are: 4.96 rad/sec, 13.75 rad/sec, and 16.52 rad/sec. These vibration frequencies correspond to mode shapes of side sway, longitudinal sway, and twisting, similar to the modes shown in figures 4.4 through 4.6. For further analysis of external loads, the deflection is reported as the displacement from the equilibrium position of the pressurized arch.

4.3 Full snow load

The full snow load is applied as described in section 3.5. Since the full snow load is vertical and symmetric, only the deflections in the Z-direction at the apex are monitored.

The load is increased on the arch until the first vibration frequency becomes zero, which occurs at a total load of 44.7 kN (10.0 kips). At this bifurcation load, the arch deflects 10.8 cm (4.25 in.) downward from the pressurized equilibrium position. The load-deflection and load-frequency curves are shown in figures 4.1 and 4.2, respectively. The load-deflection curve is slightly softening. In the load-frequency plot, the solid line is the first vibration frequency, the dashed line is the second frequency, and the dash-dot line is the third vibration frequency. The frequencies decrease as the load is increased. A view of the deflected shape at the buckling load in the X-Z plane is shown in figure 4.3, with the initial shape in red. This is the only plane in which deflections are visible. Finally, the first three vibration modes at the buckling load are shown in figures 4.4 through 4.6. The first vibration mode has a frequency of zero and is the buckling mode.
Figure 4.1 Total load vs. deflection for single arch with full snow

Figure 4.2 Total load vs. frequencies for single arch with full snow
Figure 4.3  Deflected shape of single arch with full snow

Figure 4.4  Buckling mode of single arch with full snow
(Side sway)
Figure 4.5  Second vibration mode of single arch with full snow
(Longitudinal sway)

Figure 4.6  Third vibration mode of single arch with full snow
(Twist)

a) Front view

b) Top view
4.4 Half snow load

The half snow load is applied as described in section 3.5. The half snow loading is only symmetric about the x-z plane; therefore, deflections in the x and z directions are monitored and plotted against the total load.

The load is increased until the first frequency becomes zero. This occurs at a total load of 46.1 kN (10 kips). At this bifurcation load, the arch deflects 13.7 cm (5.39 in.) longitudinally and 11.6 cm (4.57 in.) downwards from the pressurized equilibrium position. Load-deflection curves are shown in figure 3.7 where the solid line is the deflection in the x (longitudinal) direction and the dashed line is the deflection in the z (vertical) direction. The load-frequency curve is shown in figure 4.8 and the deflected shape at the buckling load is shown in figure 4.9, where the snow is applied on the right side. The vibration modes are not shown, because they are similar to those shown in figures 4.4 through 4.6; the buckling mode is a side sway mode, the second mode is longitudinal sway, and the third is a twisting mode.

![Figure 4.7 Total load vs. deflection for single arch with half snow](image)

Figure 4.7 Total load vs. deflection for single arch with half snow
Figure 4.8 Total load vs. frequencies for single arch with half snow

Figure 4.9 Deflected shape of single arch with half snow
4.5 Wind load

As mentioned in section 2.7, there are many wind pressure distributions available. Six different pressure distributions are applied to the single vertical arch so that a comparison of the effect of each distribution can be made. Each distribution has an equation for the wind coefficient, $k$, in terms of $\theta$, as explained in chapter 2. Six distributions were taken from Soare (1967); these distributions are:

Smooth:  
$$k(\theta) = -0.804 + 0.140 \cos \theta + 1.380 \cos 2\theta + 0.490 \cos 3\theta - 0.318 \cos 4\theta$$  \hspace{1cm} (4.1)

Rough:  
$$k(\theta) = -0.258 + 0.488 \cos \theta + 0.476 \cos 2\theta + 0.328 \cos 3\theta + 0.10 \cos 4\theta$$  \hspace{1cm} (4.2)

Beyer:  
$$k(\theta) = -0.655 + 0.28 \cos \theta + 1.115 \cos 2\theta + 0.40 \cos 3\theta - 0.113 \cos 4\theta - 1.027 \cos 5\theta$$  \hspace{1cm} (4.3)

Dischinger:  
$$k(\theta) = 0.75 \cos \theta + 0.15 \cos 3\theta$$  \hspace{1cm} (4.4)

Girkmann:  
$$k(\theta) = -0.526 + 0.253 \cos \theta + 0.95 \cos 2\theta + 0.462 \cos 3\theta - 0.189 \cos 4\theta$$  \hspace{1cm} (4.5)

Cosine:  
$$k(\theta) = \cos \theta$$  \hspace{1cm} (4.6)

Each equation in terms of $\theta$ for a circular arch, can be converted into an equation in terms of $x$, for any arch shape. The conversion for $0 \leq \theta \leq \frac{\pi}{2}$ is $\theta = \tan^{-1}\left(\frac{1}{z'(x)}\right)$ and the conversion for $\frac{\pi}{2} \leq \theta \leq \pi$ is $\theta = \pi + \tan^{-1}\left(\frac{1}{z'(x)}\right)$. For each distribution, positive values of $k$ signify inward pressure, while negative values of $k$ signify suction. Equations 4.1 through 4.6 are plotted in figure 4.10 as a function of $x$ for the single arch being studied.
For each distribution, the basic wind pressure is increased until approximately 1 kPa (20.9 psf). The pressure-longitudinal deflection curves for the six distributions are shown in figure 4.11, the pressure-vertical deflection curves are shown in figure 4.12, and the load-frequency curves are shown in figure 4.13. In figure 4.11, positive deflections are in the positive x direction, or the direction of wind flow. In figure 4.12, positive deflections are in the positive z direction, or upwards. The apex moves upward for four of the distributions and the first frequency tends to increase as the pressure increases. Finally, the side view of the deflected shapes of a single arch under a wind pressure of approximately 5 kPa (104 psf) are shown in figures 4.14 through 4.19, along with the initial shape (shown in red).
Figure 4.11 Wind pressure vs. longitudinal deflection for single arch with six wind distributions

Figure 4.12 Wind pressure vs. vertical deflection for single arch with six wind distributions
Figure 4.13  Wind pressure vs. first frequency for single arch with six wind distributions

Figure 4.14  Deflected shape of single arch for Beyer wind distribution
Figure 4.15  Deflected shape of single arch for cosine wind distribution

Figure 4.16  Deflected shape of single arch for Dischinger wind distribution

Figure 4.17  Deflected shape of single arch for Girkmann wind distribution
It can be seen from the graphs and pictures above that the cosine and Dischinger distributions give results that are quite different from the other four distributions. These two pressure distributions give small vertical deflections; however, the lateral deflections are very large. The large lateral deflections are due to the fact that both distributions have only pressure on the windward side of the arch and only suction on the leeward face of the arch. These two distributions are very simple models of wind pressure; however, it appears that they are not realistic when compared to the results of pressure distributions which are obtained from wind
tunnel testing. Since the results are different from test results, the Dischinger and cosine distributions will not be considered further.

The remaining four distributions are all derived from wind tunnel tests. The smooth distribution has a very large suction at the apex of the arch (as seen in figure 4.10), gives very large vertical deflections, and actually gives longitudinal deflections into the wind. The smooth distribution also gives the largest increase in natural frequency. Of the remaining three distributions, Beyer, Girkmann, and rough, the rough distribution gives the largest lateral deflections and the smallest increase in natural frequency. All three distributions have approximately the same effect on the arch. The rough distribution is chosen as the wind pressure distribution to be used on the leaning arches of chapter 5 and 6.