Chapter 5: Leaning Arches with Fixed Bases

5.1 Introduction

This chapter considers leaning arches with fixed bases. The behavior of the leaning arches is investigated for two tilt angles ($\theta$). For each angle, three loading conditions are considered, full snow, half snow, and wind loads, as discussed in section 3.5. The fixed base is described in section 3.4.

For the analyses of this chapter, all vertical deflections are measured from the pressurized equilibrium position with positive deflections being downwards. For the half snow load, positive longitudinal deflections are in the negative x direction, while for wind loads, positive longitudinal deflections are in the positive x direction.

5.2 15 degree tilt angle

The leaning arches considered are the same as in section 4.1 with the following exceptions:

- Height: 16.42 m (53.9 ft)
- Base width: 8.80 m (28.9 ft)

The parabolic shape discussed in sections 3.3 and 4.1 is for the arch in the vertical plane; therefore, as the tilt angle is increased, the height of the leaning arches decreases and the base width increases.

5.2.1 Internal pressure

At the inflation pressure of 500 kPa (72.5 psi) the arches deflect 6.11 cm (2.41 in.) upwards at the apex. The first three vibration frequencies at this pressure are: 13.08 rad/sec, 13.74 rad/sec,
and 16.48 rad/sec. These frequencies correspond to mode shapes of side sway, longitudinal sway, and twisting.

### 5.2.2 Full snow load

The full snow load is applied to the leaning arches as described in section 3.5. Since the load is symmetric, only vertical deflections at the apex are monitored.

The load is increased until the first vibration frequency becomes zero, which occurs at a load of 253 kN (56.9 kips). At the final load, the apex of each arch deflects downward 45.5 cm (17.9 in.) from the pressurized equilibrium position, the second frequency is 9.35 rad/sec, and the third frequency is 10.82 rad/sec. The load-deflection and load-frequency curves are shown in figures 5.1 and 5.2, respectively. The side view of the displaced shape at the buckling load is shown in figure 5.3, with the initial shape in red, and four views of the first three vibration modes are shown in figures 5.4 through 5.6. Since the first frequency is zero, the first vibration mode is also the buckled configuration. Local wrinkling is seen near the top of the structure in figures 5.3 and 5.4.
Figure 5.2 Total load vs. frequencies

Figure 5.3 Deflected shape
Figure 5.4 Buckling mode (side sway)
Figure 5.5  Second vibration mode (longitudinal sway)
Figure 5.6  Third vibration mode (twist)
5.2.3 Half snow load

The half snow load is applied to the leaning arches as described in section 3.5. Both vertical and longitudinal deflections are monitored because the load is only symmetric about the x-z plane.

The load is increased until the analysis fails to converge. Local buckling occurs at a load of 199 kN (44.7 kips). At this load the equilibrium solution does not converge. At the final load the apex of each arch deflects 36.8 cm (14.5 in.) downward from the pressurized equilibrium position and 40.5 cm (15.9 in.) longitudinally. The first vibration frequency is 8.07 rad/sec, the second frequency is 10.1 rad/sec, and the third frequency is 12.2 rad/sec. The load-deflection and load-frequency curves are shown in figures 5.7 and 5.8, respectively. In the load-deflection curve, the solid line gives the longitudinal deflection and the dashed line gives the vertical deflection. The side view of the displaced shape at the buckling load is shown in figure 5.9 with the initial shape in red, and views of the first three vibration modes are shown in figures 5.10 through 5.12. Local wrinkling is seen near the top of the arches in figures 5.10 and 5.11.

![Figure 5.7 Total load vs. deflection](image-url)
Figure 5.8 Total load vs. frequencies

Figure 5.9 Deflected shape
Figure 5.10 First vibration mode (side sway)
Figure 5.11  Second vibration mode (longitudinal sway)
Figure 5.12  Third vibration mode (twist)
5.2.4 Wind load

The rough wind distribution as described in section 4.5 is used as the pressure distribution on the leaning arches. The distribution is applied to the arches as described in section 3.5.

The wind load is increased until the pressure is 11.31 kPa (236.2 psf). The analysis is stopped at this load because the wind pressure is excessive and the behavior of the structure can be determined from the loading sequence; therefore, larger loads do not need to be applied. At this pressure the longitudinal deflection of the arches is 56.0 cm (22.0 in.) and the vertical deflection is 4.13 cm (1.6 in.). The pressure-deflection curves are shown in figures 5.13 and 5.14. At the final load, the first vibration frequency is 14.0 rad/sec, the second frequency is 14.4 rad/sec, and the third frequency is 18.6 rad/sec. The pressure-frequency curves are shown in figure 5.15. The side view of the deflected shape at the final load is shown in figure 5.16, with the initial shape in red, and the first three vibration modes of the arches are shown in figures 5.17 through 5.19.
Figure 5.14 Wind pressure vs. vertical deflection

Figure 5.15 Wind pressure vs. frequencies
Figure 5.16  Deflected shape

Figure 5.17  First vibration mode (side sway)
Figure 5.18  Second vibration mode (longitudinal sway)

Figure 5.19  Third vibration mode (twist)
5.3 30 degree tilt angle

The properties of the leaning arches considered are the same as in section 4.1 with the following exceptions:

- Height: 14.72 m (48.3 ft)
- Base width: 17.0 m (55.8 ft)

5.3.1 Internal pressure

At the inflation pressure of 500 kPa (72.5 psi), the leaning arches tilted at 30° with fixed bases deflect 6.7 cm (2.6 in.) upwards at the apex. The first three vibration frequencies at this pressure are: 13.7 rad/sec, 16.5 rad/sec, and 18.6 rad/sec. These frequencies correspond to mode shapes of longitudinal sway, twisting, and side sway, respectively.

5.3.2 Full snow load

The distribution of the full snow load on the leaning arches tilted at 30° is the same as described in section 3.5. The load is increased until the total load is 204 kN (45.9 kips). At this load the analysis stops. At the final load, the arch deflects 48 cm (18.9 in.) downwards at the apex, the first vibration frequency is 9.98 rad/sec, the second frequency is 11.4 rad/sec, and the third vibration frequency is 13.7 rad/sec. The load-deflection curve is shown in figure 5.20, the load-frequency curve is shown in figure 5.21, the side view of the deflected shape at the final load is shown in figure 5.22 with the initial shape in red, and the first three vibration modes are shown in figures 5.23 through 5.25. Local wrinkling is seen near the top of the arches in figures 5.23 and 5.25.
Figure 5.20  Total load vs. deflection

Figure 5.21  Total load vs. frequencies
Figure 5.22  Deflected shape

Figure 5.23  First vibration mode (longitudinal sway)

a) Side view

b) Top view
Figure 5.24  Second vibration mode (twist)

a) Front view  b) Top view

Figure 5.25  Third vibration mode (side sway)

a) Front view  b) Side view  c) Top view
5.3.3 Half snow load

The distribution of the half snow load on the leaning arches tilted at $30^\circ$ is the same as described in section 3.5. The load is increased until the total load is 173 kN (38.9 kips). At this load the analysis stops. At the final load, the arch deflects 40.7 cm (16.0 in.) downwards at the apex, and 32.6 cm (12.8 in.) longitudinally at the apex. At the final load, the first vibration frequency is 10.4 rad/sec, the second frequency is 12.3 rad/sec, and the third vibration frequency is 14.6 rad/sec. The load-deflection curve is shown in figure 5.26, where the solid line is the deflection in the x direction at the apex and the dashed line is the downward deflection at the apex. The load-frequency curve is shown in figure 5.27, the side view of the deflected shape at the final load is shown in figure 5.28, where the initial shape is in red, and the first three vibration modes are shown in figures 5.29 through 5.30. Local wrinkling is seen near the top of the arches in figure 5.31.

![Figure 5.26 Total load vs. deflection](image-url)
Figure 5.27  Total load vs. frequencies

Figure 5.28  Deflected shape
Figure 5.29  First vibration mode (longitudinal sway)

Figure 5.30  Second vibration mode (twist)

Figure 5.31  Third vibration mode (side sway)
5.3.4 Wind load

The wind loads are applied as discussed in sections 3.5 and 5.2.4. The wind load is increased until a pressure of 10.81 kPa (255.7 psf) is reached. The analysis is stopped at this load because the pressure is very large. At this load the longitudinal deflection is 47.1 cm (18.5 in.) and the vertical deflection at the apex is negligible; however, the arches do deflect vertically at points other than the apex, as shown in figure 5.34. Figure 5.32 shows the pressure-longitudinal deflection curve. At the final load, the first vibration frequency is 14.71 rad/sec, the second frequency is 18.28 rad/sec, and the third frequency is 19.9 rad/sec. Figure 5.33 shows the pressure-frequency curves, figure 5.34 shows the side view of the displaced shape of the arches at the final load where the initial shape is in red, and figures 5.35 through 5.37 show the first three vibration frequencies of the arches.

![Figure 5.32 Wind pressure vs. longitudinal deflection](image)
Figure 5.33  Wind pressure vs. frequencies

Figure 5.34  Deflected shape
Figure 5.35  First vibration mode (longitudinal sway)

Figure 5.36  Second vibration mode (twist)

Figure 5.37  Third vibration mode (side sway)