Chapter 3
Individual Case Study Reports

The following case reports provide a detailed description of the algebra classrooms of three teachers in the study. These three were selected as representative of the six cases. One case, Mrs. Saunders, represents teachers who express some reservations or discomfort toward teaching algebra in the block schedule. Despite that personal opinion, they are using previous training and teaching experience to 'make it fit.' In other words, they believe that they are basically 'teaching like always.' Although the teachers say that they do not prefer the block schedule, they are beginning to enjoy it more, and are using time well. They refer to experience and intuition regarding 'what works' and most of their students are on-task and successful in algebra. A similar attitude and response was discernible in at least one other case (Mrs. Miller) that appears in Appendix A.

The second case, Mr. Reynolds, represents the teachers who find the block schedule somewhat 'a fit to personal style.' These teachers indicate a comfortable attitude or preference toward the block schedule. They appear willing to try new approaches and their teaching behaviors reflect an attempt to use a variety of teaching strategies and materials. They appear ready to adjust when strategies are not working for students. Occasionally one or a few students may be off-task, but generally, they are engaged. Similar attitudes and approaches were reflected in at least two other cases (Mr. Owens and Ms. Nolan) that appear in Appendices B and C respectively. Student achievement varied across these cases; Mr. Reynolds' students earned higher grades than students in the other two classes.

The third case, Mrs. King, represents the teacher whose verbal recognition of the potential of the block schedule is unrealized in the classroom implementation. The teacher experiences some difficulties in deciding which instructional strategies to use and how to manage the schedule in ways that engage students for success in learning algebra. The result is a classroom that is 'consistently inconsistent' and students are often off-task.

The three full case reports follow and the remaining three may be found in the appendices.
Scenarios from Mrs. Saunders' Algebra Classes

At 8:30 a.m. in mid-November the seventeen students in Mrs. Saunders algebra class arrive on time. Mrs. Saunders greets her students at the door, asking them to take a seat and informing them that they will be working with a partner on the practice quiz that she is distributing. Students have permission to refer to their notes or work with others "to jog their memory." They begin promptly and continue working as she monitors them. At 8:50 she begins to call on students to discuss the problems. In three short minutes Mrs. Saunders gathers responses from Matt, Chris, Joseph, Bradley, and Andrea. As they finish checking, she responds to related questions, indicating that the quiz Tuesday will "look like this."

At 9:00 a.m. she tells students to check their homework using the transparency she places on the overhead projector. All students appear to have their homework paper and a calculator. Mrs. Saunders circulates, making notes in her gradebook to credit them with completed homework. "Remember to check the odd problems before you come to class. That helps keep the homework check quick. Now, does anyone see anything that we need to talk about?" Five students ask for help. Mrs. Saunders approaches Andrea to discuss the effect of a slide transformation. Other students observe without talking. She moves to Joseph to talk about his problem. Bradley puts his head on the desk and Saunders remarks "You just told me that you have no idea what's going on. Yet you have your head down. That doesn't make sense." By 9:18 Mrs. Saunders has spoken to each individual who had a question about the homework assignment. Hearing no further questions, she tells the class "we are ready to start section 3-5 so open your book to page 170." Mrs. Saunders stands beside the overhead projector, ready to give students notes on the new material. "Please read to the bottom of the page and study it for a minute." Students get their books and notebooks and begin reading.

"To be honest, I do basically the same thing I did in a 50 minute schedule in 100 minutes. I usually started off with some kind of focus, a quiz or something, and then I went over homework. Then I taught a lesson, wrapped it up, and that was it."

My algebra lesson plan provides the opportunity for students to use calculators daily. (Q27)

I assign homework to my students almost daily. (Q46)

"If I see a student who has [his] head down and I know they need to be with me, then I will address them."

My lesson includes the opportunity for students to read and respond to print material from an algebra textbook. (Q50)
As students finish reading, they look up and Mrs. Saunders makes eye contact with each. She begins to explain the illustration, using a dialogue with students.

"What is happening in step one, Deonte?"
"Subtraction"
"After removing three ounces, you have 4w=8. So what does a box weigh?"
Bradley answers "2 ounces."
Deonte remarks that "you multiply both sides of the equation by the reciprocal."
Mrs. Saunders nods and says "Exactly. That's especially helpful when the problem is not so obvious that we can solve it in our heads. Let's look at two more examples."
After solving two other simple equations with students, Mrs. Saunders writes an equation on the transparency \[2x+1 = 7\] and asks students to assist her with 'getting rid of things' in order to 'have x by itself.' She reminds them that they "keep an equation in balance by doing what you do to one side to the other side as well."

By 9:31 the students have engaged in an exchange with the teacher while solving four simple equations using additive inverses and reciprocals. She writes \[\frac{2}{3}x+19=7\] on the transparency and calls on Matt to suggest what to do first. Matt suggests adding -19 to both sides of the equation. After she transforms the equation to \[\frac{2}{3}x=-12\] Deonte indicates that multiplying by the reciprocal of \[\frac{2}{3}\] is the next step. The teacher models on the transparency the result of multiplying both members of the equation by \[\frac{3}{2}\] to produce \[x = -18\]. At 9:34 she tells the class to try another equation \[-2 = -8x + -6\]. She allows students to work for a minute, then calls on Darnell to direct her as she writes. After reducing the equation to \[4=-8x\] she asks Valerie to take the next step. Mrs. Saunders writes as Valerie describes multiplication by the reciprocal to produce \[\frac{1}{2} = x\] as the solution.

At 9:40 Saunders directs students to a problem in their textbook involving calculation of salary. A worker earns $77.15 from several hours of work at $9.80 per hour, plus $8 for meals and $3 for transportation. Saunders asks students to figure out how many hours the

My lesson demonstrates that learning algebra involves learning a specific set of rules. (Q39)
My algebra lesson includes practical applications. (Q40)
When I was in the 50-minute period [schedule], I don’t know if I ever did group work. Back then, it wasn’t as big. I see myself doing more group work. I put them in pairs and that has worked well.

Bradley uses his calculator to find the product of 66.15 times \( \frac{1}{9.80} \). When Mrs. Saunders asks him how he will find the product, he says "66.15 times 9.80" although the class immediately disagrees and suggests "it's division, not multiplication." Mrs. Saunders agrees as several students begin to call out "6.75." She asks "what does .75 mean?" A boy says "\( \frac{1}{4} \)" and a girl says "45 minutes" as Saunders glances at the clock on the wall.

At 9:45 Mrs. Saunders distributes a practice worksheet, telling the class to "get back with the partner you had at the beginning of class." Students begin to work with partners, talking quietly and using their calculators whenever needed. The teacher circulates to monitor, noticing that students are correct in their work, accurately applying the reciprocal and alternately writing responses as rational fractions or decimals. At 10:00 the teacher writes the homework assignment on the board and tells the class to begin the homework as they finish the practice problems. Most students begin homework by 10:05 and are able to complete a few problems before the bell sounds.

Six days later, another group of eighteen students enter Mrs. Saunders' afternoon algebra class. The observer is covering for Mrs. Saunders while she runs an errand down the hall, so students enter asking "are you our sub?" When told "no, Mrs. Saunders is down the hall" a girl responds "Good! I have my homework and I want her to see. Mrs. Saunders is okay. This class isn't bad." A tall boy with a Buffalo Bills team jacket saunters into the room. When the observer says "I hear you have homework," he says "Why? You a sub?"

"No, Mrs. Saunders is here."

"Good. I don't like to talk business until class starts, but look, [showing his paper] I've got my homework."

A girl with braids says "You've got your homework? Get real! Who are you kidding?" But she smiles as he shows her his paper. There is an easy rapport among the students. A girl enters smiling, carrying a bouquet of roses. She explains to her classmates that they are a gift from her boyfriend.
At 12:55, Mrs. Saunders enters as the bell sounds. She begins to take roll and returns student papers. She then reveals a five-problem quiz on the overhead projector and invites them to "use your notes if you need them." Students begin to work on the five problems. The first two are algebraic expressions to be simplified using the distributive property and rules for combining like terms. The last three are equations to solve after applying the same properties. A short, freckle-faced boy in a Redskins jacket notes that he got two of five problems correct on the paper Mrs. Saunders has returned. His friend in the next row says "That's one more than I got!" while an attractive blond behind him says "Come on! It wasn't that hard."

At 1:03 Mrs. Saunders copies the problems from the overhead onto the whiteboard as students continue working. She clearly plans to discuss the solutions to these problems. A boy in back raises his hand, saying "Can I ask you a question?" She shakes her head and says "Class, you have one more minute." At 1:05 she reminds them to be sure their names are on the papers and asks them to pass their papers to the front of the room. When she has gathered them, she says "I will give you two points extra credit if you come to the board, work a problem, and explain it to the class." Jeremy takes the first problem, explaining that the application of the distributive property results in $6x + 6y + 6z$. Mrs. Saunders asks if he would then have $18xyz$? Jeremy explains that the three terms are not like terms and so they may not be combined.

Chris takes the second problem which also requires the distributive property but needs the extra step of combining like terms. He completes the problem correctly as his classmates observe. Candace explains the solution to the equation $2(x+2)=6$ quickly and accurately. J.D. seems less confident, but he too accurately applies the distributive law before simplifying and solving $3(x+1)+x = 11$.

Lanesha volunteers for the last problem, clearly explaining her work including the last step of applying a multiplicative inverse when she has transformed the equation into $4x=-12$. Her
I assign homework to my algebra students almost daily. (Q46)
"They get their homework out and while they are checking answers, I walk around to see if they’ve done their assignment. Then I go over the homework. I usually go over whatever they need. That’s another thing in the block schedule. I do have a little bit more time."

"Does anybody see anything they want to ask about?" Students have questions about four problems. Saunders calls on Marcus to simplify \(10b(b+c)\) asking him if he "sees any similarity to the problem he simplified earlier-- \(6(x + y +z)\)."

When Carmen is asked to explain her solution to \(7(u+ -3)=0\) she begins by saying "I think I should add 7." When Mrs. Saunders repeats "add?" Carmen says, "no, I mean multiply by 7--to get \(7u + -21 = 0\)." Mrs. Saunders says "Good" and encourages Carmen as she finishes the problem.

Mrs. Saunders calls on another girl for help with problem 26, asking her if she recognizes any need for the distributive property when simplifying \((x^2 +3x+1) + (2x^2 +x+8)\)? The girl says "no, we just have to add."
"Add what?" queries Mrs. Saunders.
"Like terms" and she completes the problem with "\(3x^2+4x+9\)."

Saunders asks Chris to assist with the last problem \([n+.04n+.15n]\) and he explains to others that he "thinks of 1 in front of n and adds to get 1.19n." Saunders nods at his explanation. At 1:25 she informs the students that they have a BIG quiz on Wednesday, "not a little ole quiz." Several students say "Mrs. Saunders, you're supposed to have the holiday spirit!" "I do. I won't give you homework for Thanksgiving (two days away.)"

Mrs. Saunders distributes a paper to students, saying "This is your next assignment. Put it in your notebook. It is the second nine-week project and will
I require students to take notes in class (daily.)

As students begin to read the assignment, Saunders explains that they will design a short survey of personal interest. They will ask 20 students and 20 adults to respond to the questions. They will then use a stacked bar graph to show the results and write a paragraph to describe the findings. "Questions (with four answer choices each) are due the Tuesday after Thanksgiving. A week later (December 11) you must have a tally sheet showing results of the survey. Your stacked bar graph and paragraph are due December 19." She then shows a sample product to the class, saying "This project is worth a C. Why do you think so?"

Students begin to remark "It's sloppy."
"You didn't use graph paper."
"It's not colorful."
"There are no labels on the sides [axes] to tell what the numbers mean."

Mrs. Saunders seems pleased with the students' remarks. She summarizes by saying "Get your question. Narrow your response choices. Think about it so you can get started. Any questions?"

One girl says "I don't know 20 adults."
"Sure you do! Teachers, parents, neighbors..."

It is 1:35 and the teacher directs students to get ready to take notes on the topic 'Adding Algebraic Fractions.' When the bulb on the overhead blows, Saunders goes to the whiteboard. "Let's see if you can add fractions." She writes \( \frac{1}{2} + \frac{1}{3} \) and \( \frac{1}{5} + \frac{2}{3} \). "Who remembers?" Saunders circulates to see what students are writing in their notes. Candace says "I know it!" but another girl says "I don't like fractions so I copied him."

"Chris, explain to the class how you add these."

He correctly explains the need for common denominators and the addition of numerators. Mrs. Saunders compliments his explanation and says "Now let's do some algebraic fractions," as she writes on the board.

\[
\frac{x + 2y}{3} \quad \text{and} \quad \frac{(-9+3b) + 9}{b} \quad \frac{b}{b}
\]

"Do you notice the common denominators?" Students nod and answer "yes."
"So what do we do?"
"Add the tops."
"The what?"
"The numerators."
Saunders covers the denominators with a piece of paper and says "what is x+2y?" Chorally, the class responds "x+2y."
"And the denominator?"
"3"
"Good job! And the next one?"
Carmen answers "3" and Saunders says "Tell us how." Carmen correctly explains the addition of like terms and the process of reducing the fraction \( \frac{3b}{b} \) to 3.
Mrs. Saunders asks the class to try the following example

\[
\frac{x + 7}{5u} + \frac{2x + -3}{5u}
\]

"Once again you already have a common denominator. Look at the numerators. What is the sum? Can you clean it up? Try it!"

Jeremy responds "x + 2x is 3x; 7 + -3 is 4; so 3x+4 is the numerator and 5u is the denominator."

Another student says "I don't see it."
Saunders urges her to "Be careful. Focus on the numerator first. Then work with the denominator. Try this one."
"Oh, I see. That's easy."
"It looks easy doesn't it" says Mrs. Saunders. "What do you get?"
"I know. It's x."
"I think you may be on the right track, but there's something happening that we might want to investigate. Let's work it out."

She models finding the sum \( \left[ \frac{1}{4} x \right] \) and then simplifying for a final result of x.

"Chris, what would you do with this one?" \( \frac{3x}{4} + \frac{1}{4} \)

Chris explains that he chooses 12 as the common denominator, multiplies as needed to produce \( \frac{9x}{12} + \frac{4x}{12} \), and then finally adds the fractions to get \( \frac{13x}{12} \).

At 1:50 Saunders assigns a problem for students to work

\[
\frac{-3a}{7} + \frac{2a}{7}
\]
as she circulates among the desks. She sees generally accurate responses and goes to the board to write \( x + \frac{1}{2} \) urging students to "try this one."

Desmond says "I've got the first one."

The class is working and there is no off-task chatting although some students are working together on the two problems. Mrs. Saunders walks about the room. Students explain their work and it seems that they are clear in their procedures. She decides to assign the homework and directs them to complete problems 1-17, plus 24, 25, and 26 from the textbook section 3-9. "But right now, please get with your partner" and she gives each pair a worksheet.

Students shift positions to work on the assignment. By 1:56 they are all working, talking softly about the problems on the worksheet. A few students choose to stay as single workers. The teacher (and observer) circulate. Students are discussing the problems and occasionally raising a hand to ask for help.

By 2:10, three boys are off-task. They have finished the worksheet and are talking about other things, sharing silly jokes they have taken from the Internet. Two other groups are finishing. Most groups are more than half-finished with the 24 problems. The teacher continues to circulate. By 2:14 four groups are still working. Mrs. Saunders checks some papers and prompts the pairs to continue working to correct mistakes. She moves to offer one-to-one assistance to Desmond whose hand is raised. "If your worksheet is finished, go on to the homework assignment."

Several students are chatting about boyfriends, girlfriends, flowers, dancing. The teacher ignores the chatting and asks Chris to take the overhead projector to the library for a new bulb. All students (except two) are socializing and talking quietly. At 2:30, one boy begins his homework and asks the observer if his answer is correct. It is. Two girls ask another question about the homework. Four minutes later all are finished with the worksheet and are closing up books and materials as the afternoon announcements on the public address system summarize the activities after school. When the announcements end, the final bell rings and students move quickly to the hall talking about plans that range from athletic practices to jobs and includes just hanging out with friends.

Two months later at 12:55 on January 15, the same group of third-block students are assembling for Mrs. Saunders' algebra
class. They enter talking and one or two students approach Mrs. Saunders to ask for some time after school to catch up on missed assignments. Exams are approaching and the teacher has prepared a review sheet. She welcomes students to class and tells them they will have ten minutes with access to their textbook and notes to work the problems on the review sheet. "When you are finished, check your results with your partner and then turn them in to me. We will then check the problems as a group and I will give you another review sheet for the exam." She distributes calculators to students and takes roll as they begin working. All are working, except one male in the back of the room. He remains quiet. At 1:12 Saunders asks students to work with partners to check and at 1:20 she says "Okay, turn in your papers."

"Let's take out the exam prep sheet. Write correct answers and notes on the [prep] sheet so you can study." She calls on Melinda to answer problem 1 (requiring her to simplify an expression by combining like terms.) She answers correctly as does Ricky who must state an algebraic expression equivalent to a verbal statement. Sharita correctly identifies the commutative property in problem 3 and Rodger responds accurately to problem 4. Sherlene calls out the correct response to problem 5 but another student disagrees with her. The teacher uses a different example to demonstrate the correct solution. Chris is asked to add two algebraic fractions with unlike denominators and he does so. It appears that the class is familiar with the basic properties and processes required to write algebraic expressions and to simplify them by combining terms. By 1:30 the teacher is leading them in a review of solving equations and inequalities. This, too, appears to be familiar content for the class, since they respond correctly when called by name. At 1:35 she tells them to "put that away somewhere you can find it to use in preparing for your exam."

Saunders places a transparency on the overhead showing solutions to last night's homework. As usual, students begin checking as she makes a visual check of their papers and marks in her gradebook. The boy in the back of class is now asleep and one other boy is mumbling to himself absent-mindedly. The teacher tells the students that they will have a quiz on the homework material on Friday. "Are there questions?" When students ask for explanations to six problems, she makes a space on the board for each problem and calls for volunteers who are willing to work a
problem for two points extra credit. Four students go to the board right away. Other students are talking or waiting with their papers in front of them. Melinda has a highlighter and is marking problems as she checks. Her study skills are evident as she notes "Do Over" beside certain problems in her notebook. Two other students finally approach the board to attempt the remaining two extra credit problems.

At 1:50 Mrs. Saunders walks to the board to review each problem. Kevin's solution to the inequality \(5 + -3x + -6 < 10x\) is correct. Another boy's work on problem 14 is wrong. Saunders erases his work and calls on Leanne to explain from her seat. After checking the remaining problems, at 1:55 Mrs. Saunders models the 'set up' for a word problem related to placing an advertisement in a newspaper. "If the Gazette charges $2 plus $.08 per word and the Herald charges $1.50 plus $.10 per word, when is it cheaper to advertise in the Gazette?" She calls on Coriana who solves by stating an inequality \([2+.08x < 1.5+.1x]\) and solving to find \(25<x\). Mrs. Saunders asks "What does this mean? 25 hours? 25 cents a word? Think class. What is this problem about?" She pauses to await their responses.

Finally Leanne answers. "This tells us that when the ad is more than 25 words...."

Her teacher says "Thanks! You're right. If the ad is longer than 25 words, the Gazette is the cheapest option. Does everyone see that?" There are three problems remaining to check and they are completed quickly.

At 2:00 Mrs. Saunders gives students an assignment to help them review for the Friday quiz. Ten problems (from the textbook) are assigned. "If you can't figure it out, raise your hand and I will help you get started." She repeats "Get started" to two girls who are talking.

Leanne raises her hand. "I'm getting confused about when to combine like terms and when to add something to both sides and solve."

"Now stop and think. What has sides? An equation has sides. Look to see if you have an equal sign [=] in there somewhere."

"Oh yeah. I know that. What am I thinking?" and she goes back to work.

Mrs. Saunders moves to another student. "Remember, I need to see your work." Candace asks Mrs. Saunders to "please check this with me. I think I am right." Her teacher reads her work.
and nods. "Yes." Students are working and Saunders circulates as a helpful observer.

At 2:25 Mrs. Saunders assigns homework using an overhead transparency. Eight problems are presented requiring students to graph five linear equations, solve one system of equations, and solve three simple equations. She calls student attention to the eight problems. By 2:30 the class begins to shut down although Saunders reminds them to "keep working. It's not time to stop learning. The more you do now, the less you have to do later."

"But my Mom says I have to bring work home" says one girl. Another girl mentions that she 'needs something to take to her tutor who charges $20 per hour.'

As the afternoon announcements begin, Mrs. Saunders reminds students to copy the homework problems because they are not in the textbook. She collects calculators as they listen to announcements. Students are discussing the Friday basketball game with Wilson Gap. The teams are tied for district honors and students seem excited about the rivalry and the prospects of an exciting game. As the bell rings, they move quickly out of the classroom as the teacher remarks to the observer that she is "surprised that everyone worked so consistently during class. Some of them know that they will fail this semester and will be repeating this material when second semester begins." The observer suggests that perhaps students' continued attention to task in the face of failure is a tribute to her expectation that each must participate while in her class.

Teacher Background
Mrs. Saunders has been teaching for nine years and has taught at least one first-year algebra class during six of the nine years. She has a Masters degree in mathematics education and has spent almost as many years teaching algebra in the alternate day block schedule as she did in the shorter 50-minute daily schedule. She candidly says, when asked her level of preparedness to teach algebra in the block schedule, "[I had] zero [preparation.] I was scared to death. I honestly did not know how to fill that time block. The first year was rough. I just wasn't used to having that amount of time. I knew I had to cover the same amount of material so I knew I would have to double up. Eventually it worked out, but I remember that first year. I sat down and wrote out everything. 'This is going to take 10
minutes, and so on.'" Despite the fact that Mrs. Saunders says she has become accustomed to the longer class periods, she "honestly doesn't like it." She does like having more time to work problems in class so that students are not rushed, but if she had her choice, she would return to the 50-minute daily schedule. "I feel like when they are with me in this room that learning is taking place. My biggest problem is that when they leave me they do not think about this again until they see me. So many students do not crack a book between Friday and Tuesday that when they come back, I almost have to start over."

Distinctive Features of the Case

Variety

Mrs. Saunders' interview comments indicate that she worked from the beginning to ensure that the entire block of time was used efficiently. Although she cites the opportunity for creativity, she dismisses her own creativity by saying that she "does basically the same thing she did in a fifty-minute schedule in the hundred-minute schedule." She doesn't feel pushed for time and considers her style "pretty much the same." Lessons consist of a focus activity using a quiz or series of problems, an opportunity to involve students (individually, in pairs, or in groups) with checking and discussing homework problems, a formal presentation of new material (usually with some lecture and modeling by the teacher as students take notes), guided practice with teacher assistance (including collaborative reading from their text), and a homework assignment.

Homework

Mrs. Saunders routinely assigns homework and considers homework an essential part of learning algebra. "I'm giving more homework than I would if I saw them everyday. Some students are overwhelmed. They don't realize this is for two days. They see the amount, it's a turn off, and they do nothing. [Even in my upper level classes,] kids will tell me that if today is Thursday and I gave homework, 99% of them would wait until Sunday night to do their homework. They open their notebooks and they cannot remember. It was clear when they were in class, but by the time days have gone by, they have no clue about what they are to do." Mrs. Saunders advises them that they "should do homework the night it is assigned and the [next] night before they come [to class] they should review it. Just look over the notes for 10-15 minutes and refresh it in their minds." She believes students would be more successful if they met (and had homework) on a daily schedule.
Engagement
Mrs. Saunders’ students tend to be highly attentive in class and she agrees that the majority of students pay attention for the entire period (Q8.) However, she is sensitive to eye contact and body language. "Body language, especially in the afternoon, tells me that students start getting tired. Anybody can sit through anything for 50 minutes..." She thinks it is hard for students to sit through longer classes and recalls her own experience as a college student with longer classes on Tuesday and Thursday. "I personally could not sit for an hour and a half (or did not want to) and listen to the teacher. So I chose Monday-Wednesday-Friday classes. I see the same thing for these kids."

Individual Perspective
Mrs. Saunders does not believe that algebra students have realized any benefits from the extended block schedule. However, as a teacher, she is not dissatisfied. "As far as I go, if you take the kids out of it and just look at me, I think the block schedule is easier for a teacher. I am less stressed; I have a longer amount of time to get through what I need to get through; I have that nice planning period every other day. Having a planning time that is long enough to get something done is an opportunity of it [the block schedule.]" She does admit "it is hard the day you teach all day. I am exhausted at the end of the day. If you teach the way you should, you don't sit down and you're on the go the whole time."

Advice to Algebra Teachers Beginning a Block Assignment
"I would probably tell them first that they are going to have to teach two days worth of material in one to get through the curriculum. Just because you teach two lessons doesn't necessarily mean you are going to give the same amount of homework. I've made that mistake. If it's too much, it is overwhelming. You have to be careful in that respect. And at first, be careful with your time and make sure you plan out your activities. I was worried I would have thirty minutes time left at the end [of class] and not know what to do." She is reassuring as she predicts that everything "eventually works out." Although she honestly does not like the block schedule and would prefer to modify or eliminate it, she grades her algebra lessons as "B" and believes that her students are generally successful in algebra. "I feel like I am doing the best I can do to get through to the kids." She agrees that most of them do what is expected in her class.
**Student Achievement**

At the end of the year, twenty-seven of Mrs. Saunders' students earned an A, B, or C in algebra and another nine received a D. Seventeen students failed algebra (32%).
Scenarios from Mr. Reynolds' Algebra Classes

It is the first week of December and Mr. Reynolds' 8:30 algebra class is gathering for the first time since Thanksgiving break. Mr. Reynolds is checking the work turned in to the substitute who met with the class the last day before break. He addresses the class. "Look at the questions in lesson 10. the 'cracking the code' exercise. Remember the relationship between the frequency of letters and the code?" An example problem is on the board. Message: Meet me on Friday.

\[
\begin{bmatrix}
13 & 5 & 5 & 20 & 27 & 13 \\
5 & 27 & 15 & 14 & 27 & 6 \\
18 & 9 & 4 & 1 & 25 & 27 \\
\end{bmatrix} + \begin{bmatrix}
3 & 3 & 3 & 3 & 3 \\
3 & 3 & 3 & 3 & 3 \\
3 & 3 & 3 & 3 & 3 \\
\end{bmatrix}
\]

"What kind of encrypting process is this?" Students' responses vary. "Add 3." "Add C's." "No, not C, just 3 letters past... What's this called?" "Shifting."

"Suppose we did something more complicated. Perhaps add a more complex matrix?" Mr. Reynolds attempts to get the class to suggest what they would need to know in order to discover the message. It is 8:45 and students are beginning to recall the need to know the coding matrix. When a student suggests that need, Mr. Reynolds responds.

"Does everyone see this? Suppose we use a coding matrix like ANIMAL." The nineteen students appear to understand Mr. Reynolds' intention as he adds the coding matrix ANIMAL to the message matrix to produce the encoded message matrix

\[
\begin{bmatrix}
14 & 19 & 14 & 33 & 28 & 25 \\
6 & 41 & 24 & 27 & 28 & 18 \\
19 & 23 & 13 & 14 & 26 & 39 \\
\end{bmatrix}
\]

At 8:50 he engages students in the process of coding his name (Reynolds converted into a 2 x 4 matrix) using another 2 x 4 matrix (lamb) as the coding matrix. A girl develops her own example using her name and points out that in the final encoded
"If I can get them organized and started, it is pretty successful. When I am a little disorganized at the beginning and don’t have everything ready to go, I lose some students and the rest of the period is spent getting them back on track. Getting them started is the biggest thing."

"If I try to develop the problem and work it through, it takes away from their discovery. Some of the applications are complex stuff. With this material, I wouldn’t even want to try to teach in 50 minutes. The class work is based on discovery and group work."

matrix, a repeated numeral may or may not represent the same letter. Students are variably engaged at 8:55. Four students put their heads down, one writes a note, and another stares out the window toward the parking lot. Mr. Reynolds asks everyone to complete lesson 10. "I'll collect it next class." He begins to distribute slips of paper printed with the sequential letters of the alphabet. One student observes that there is a row of uppercase letters and a row below it of lowercase letters. The teacher, joking, remarks on the student's astute powers of observation. He asks one girl to distribute scissors as he makes tape available to use later. As they prepare their models, he sets up the overhead projector. At 9:05 he speaks to the class.

"Have you got them cut? Together you have a pair. (He holds up the two strips side by side, aligning A with a, etc.) Suppose we did a shift cipher of nine? What would happen?" (Again demonstrating holding one strip fixed and shifting the second one by nine letters.)

One boy says, "a goes to J."
"Right. And b goes to..."
"K"
"And p shifts to..."
"Y."

"Now, let's bring the strips into circles and tape them." Students form bands or 'bracelets', tape them together, and chat among themselves as they finish the task. Mr. Reynolds rolls a sheet of notebook paper and then loops the paper bands over the paper to produce a moveable model that will spin. As he does this, he says "Use your magic decoder bracelets to code zebra with a shift 9. Be sure you know which is your first or original reference and which is not."

A boy says, "It won't matter."
"We'll experiment and see," replies Reynolds. As he prepares to illustrate, Zack suggests taping one circle in place so that only one spins.
"Good idea."
Mr. Reynolds models spinning the lower case bracelet forward to encode 'zebra' as INKAJ (reading the letters on the uppercase band that match the lower case letters for zebra.) He then describes the same spin but switches the translation (to illustrate 'forgetting which band he moved') and reads from the stationary band (uppercase ZEBRA) to produce 'qvsir' as the code. "It isn't the same," he concludes.

At that moment (9:20), a student rises from his seat and walks out of the classroom, seeming angry in response to words between him and a girl seated next to him. Mr. Reynolds leaves the room briefly to follow the boy, but quickly returns.

"Let's look at the graphical representation of this process. Look at section 11-2." Mr. Reynolds is standing by the overhead projector, using a coordinate graph showing uppercase letters and integers on the x-axis and lowercase letters and integers on the y-axis. "Normally, if A pairs with a, B pairs with b, and so on, we'd plot (1,1), (2,2),...right through here," says Reynolds, placing points on the coordinate plane. (There is a brief interruption as a student delivers a note from a counselor requiring a boy to leave class.) "Let's plot the 9 shift." He plots (1,10),(2, 11), ...(17, 26). "We have a ceiling here since there are only 26 letters of the alphabet. But we could bring the 'extras' back" indicating the point (18,1) "...we can't pass the ceiling so we bring the circle back. This is another type of arithmetic known as modulo. The days of the week are modulo. Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, then Monday again. The ceiling. . .then repeat. Think of another kind of modulo."

"Time," says one student.
"Months of the year," says another.

The time is 9:30 and most students appear engaged with the models used during the lesson. However, a trio of boys near the window are talking, another three boys and one girl at the front of the room have their heads down.
"Suppose we added 54 hours to the current time. What time would it be?" asks Mr. Reynolds. He works with two girls in front of the overhead projector. Students discuss the repetition of 24 hours and 48 hours bringing them back to the same time. There would be seven more hours to add.

"So, 55 hours is the same as 7 hours, if all we care about is the time of day, not the day of the week. Is that right?" asks Reynolds. Two students chuckle. "I am saying that 55 hours equates to 7 hours. They are the same."

"I don't understand" says one boy.
"If all you see is the clock. If you don't have a calendar. You can't see outside."
Another boy asks "What if the clock says a.m. or p.m.?"
Yet another asks "What if it is military time?"
"What difference does it make?" asks a girl.
"I don't see the point" says a blonde girl in the back of the room as she opens a cosmetic catalog on her desk and glances at her manicured nails.
R. J. says "My birthday is tomorrow."
Reynolds responds. "Your birthday is tomorrow? Really?"
"Yeah. December fourth."
Mr. Reynolds writes "Wednesday, December 4, 1996" on the board. "What day will R. J.'s birthday be next year?"
A girl asks "Is it a leap year?"
A boy says "My birthday was Saturday last year. January 27. Where is January 27 this year (1997)?"
Mr. Reynolds glances at a calendar and says "It's Monday."
The boy decides "So skip a day and it's the next day."
"No" disagrees a girl. "My birthday was June 2. Last year it was Friday. This year its Saturday. Why didn't that work?"
"But it's (1996) a leap year," says another girl.
Their teacher replies, "Oh! So his birthday passed a leap year. Hers doesn't. How many days do we have to wait for our birthday to return?"
"365" is the immediate response from several students.
"How many weeks is that?" asks Reynolds.
"52" replies a boy in front.
"Are we sure? Exactly? Let's divide. 365 divided by 7 is 52, with a remainder of 1. Is that our one day? So, R. J. What day is your birthday in 1997?"
R. J. answers "Thursday."
"Right! And in leap year situations (366 days) how many days do you advance?"
Almost daily, the algebra lesson includes practical applications. (Q40)

A girl quickly responds "Two!"

"Suppose I asked you what day of the week your twentieth birthday will be? You'll have to consider leap years...Let's go in another direction." Mr. Reynolds goes to the board and writes \textbf{What day of the week were you born?} "Work it out. On paper. Tell me that you were born on 'Tuesday, March\underline{\hspace{2cm}}, 198\underline{\hspace{2cm}'}" It is 9:54 and students seem interested as they begin to scribble on papers.

The blonde with the manicure says "I don't know what I'm doing." Two other girls near the door ask "How do you do it?" Three boys are apparently sleeping. The two girls in front are talking about the problem. One of the trio of boys near the window continues to watch outside; another is looking at the question on the board; the third says "I already know I was born on Wednesday, January 27, 1980 at 10:52 a.m." Another boy in the back of the room says, "I was born on June 22. Where do I start?" Mr. Reynolds gets his calendar and helps the student to determine that last June 22 was Saturday. He then moves to the trio and says "Do this or take a zero. This is an assignment." He moves to respond to another student's questions. Just after 10:00 Mr. Reynolds writes on the board \textbf{What day of the week was I born? January 18, 1957}

Students are engaged with the exception of two who still appear to be sleeping. One girl asks "What day was it this year?" When he answers, they begin to work. At 10:06 a girl calls out "You were born on a Saturday." Mr. Reynolds grins good-naturedly. "I don't know. I was too young to remember."

"Well, you were."

Mr. Reynolds glances at the clock and says "Lesson 10. Complete and correct, next class. Kent, thanks for collecting the scissors." Students gather their jackets, books, and back packs and begin to saunter toward the door. The bell rings and they enter the hallway.

It is five weeks later when the same class assembles at 8:30. Mr. Reynolds opens the lesson with a whole-group activity. On the overhead projector is a transparency indicating what is planned.

1. Writing equations for algebraic expressions
   Trust Fund
   Speed of Bowling Ball
Cost of Rental Car
Price Comparisons between Kroger and Food Lion

2. Graph of linear equations $y = mx + b$

Twenty-two students are present and Mr. Reynolds is calling on them to respond to questions. A tardy student enters and hands Reynolds a pass. Before going on with the lesson, he prompts a student whose head is on the desk to "Wake up. Try to hang with us." He also speaks to five students near the window. "I want you guys to help with this graph." He calls on a girl to go to the board to graph $y = 3 + x$. As she begins to graph, he asks "How are you plotting this line?"

"I say first, suppose $x$ doesn't exist. Then $y=3$. So I marked [a point where] $y=3$ on this line [the y-axis]."

"You're not really saying it [$x$ doesn't exist]. You're saying it is what?" asks Mr. Reynolds.

"Zero?"

"Right."

The girl then plots a point (1,0).

"And what does that point represent?" asks Reynolds.

"$x$ is one."

"Okay. But what should the $y$ value be?"

"What?" She looks puzzled.

Reynolds prompts, "Look at your equation. What should $y$ be if $x$ is one?"

Another student answers "four."

"She's right" compliments Reynolds. "How did she know it was 4?"

The first girl says "$3+1=4$", plots (1,4) and then connects the segment between (0,3) and (1,4). Her teacher suggests that she "show all the points on that line." At first there is no reaction; then she draws arrows on each end of the line segment. Reynolds urges her to "extend that line a bit" and she takes the ruler he offers and extends the line into the second quadrant.

"Try the next one ($y = -3 + x$)" suggests Mr. Reynolds. "Class, can you help her? Does anyone remember another way to graph? Maybe easier?"

Several classmates suggest she locate (0, -3) and then use 'up one, over one' [slope]. When she is finished with her sketch, Mr. Reynolds asks a boy to take the next problem. He refuses and
Reynolds calls on another boy (Greg) to graph \( y = 3-x \). Greg plots \((0,3)\) immediately, then says "-x tells me to go down x, over one."

A girl says "up one and left one."

Their teacher suggests plotting a point in each described direction and examining how [the points] look.

The girl says "they all line up."

Mr. Reynolds calls on the five students at the window, but they decline to graph the next problem. Another boy and another girl who are called upon also decline. "I'll make a deal. I'll leave you alone if you'll stop talking and pay attention. Maybe then, you'll volunteer to do one..." He then asks Vanessa to try \( y = -1 + 2x \).

She goes to the overhead projector and locates -1 on the y-axis. She then plots a second point \((2,-2)\).

Mr. Reynolds inquires "Why that particular point?"

"I don't know. There's a two here (in the equation as the coefficient of x.)"

He approaches the board. "Let's think a minute." He writes \( y = -1 + 2x \) on the board. "When x is zero, the y-value was what?"

"-1"

"Now, Vanessa, if you chose to use 2 [if \( x=2 \)], then y would be what?" At the same time, he begins to complete a table of values.

As he writes \( y = -1 + 2(2) \) another girl responds "five."

"No," a student corrects, "three."

"Three is correct," agrees Mr. Reynolds. "So let's plot the point." Vanessa goes to 2 on the x-axis, moves up 3, and says, "I think this is right."

Mr. Reynolds returns to the equation. "Look at what we know. As \( x \) goes to the right one unit (from 0 to 1 or from 1 to 2) what happens to \( y \)? How does \( y \) respond? In the past, we have called \( x \) the explanatory variable and \( y \) is [the] responding. Who can make a chart to show what happens?" He uses the table of values to focus on incremental change.

"How much change occurs here?" he points to the changes in \( x \).

"One," answers a girl.

"And here?" he points to the table.

"One," replies a boy.

"How much change here?" [in \( y \) values.]

"Two."

"And here?"
"Two."
"Let's think of this... As x goes over 1, what does y do?"
A girl says "I see! It goes up 2."
"Right. So we have an idea about the relationship. Over 1 and up 2 (or up 2 and over 1.) That [change of] 2 is making the graph steeper." Mr. Reynolds locates the points from the table on the graph, connects them, and extends the line. "Look at the next one. [y = -1 - 2x] Do I have a volunteer?"

Charlene volunteers and plots -1 on the y-axis. She moves up 2 units and back (left) 1 unit to mark a second point. She repeats the process again to locate another point on the graph and looks to her teacher.

"Good. Does anyone have questions?"
Another girl says "I don't see that."
"Look at the equation and think about the one before. As x goes over one (gets one larger), what happens because of this?"
"Y goes down 2. Is that right? -2?"
"Yes" says Mr. Reynolds. "There is a negative relationship here. Go ahead and connect your points," he tells Charlene.

Mr. Reynolds asks Kent to work problem f. [y = 2 - 2x]
"I'm not sure."
"Go ahead and try."
Kent goes to the overhead projector. Mr. Reynolds also approaches the overhead and covers (-2x), asking "what if you start here [with the 2?]
Kent locates (0,2) and marks a point.
"Now, let's see the effect of this part." Mr. Reynolds uncovers (-2x). "What happens if you have 1 for x? As x increases by 1, what is the change in y? If x goes over one, y will do what?"
Kent: "Go down 2."
"Right. Let's see that on the graph."
Kent sketches another point by moving over one unit and down two units. He then connects the points and sketches the graph of the line.

"Good," says Reynolds. The time is 9:20 and a boy arrives late. Except for one boy, all students have been attentive during the graphing activity. He asks Mr. Reynolds for permission to go to the restroom, which Reynolds denies. "I hoped you would work. I'm not pleased. Maybe later." The student threatens to walk out, even if it means being suspended.
"My Mom will come cuss you out."
"I'll just have to deal with it."
"Then my Dad will come..."
Mr. Reynolds ignores the comment and continues to examine the graphs as students work. He returns to the overhead projector. "Anything noticeable about these graphs?" he asks as he points to the graphs of \( y = -1 - 2x \) and \( y = 2 - 2x \). "How do they look?"

Cary says "Like they are parallel."

"They do look that way," agrees Mr. Reynolds. "Look at their equations. [Do you] notice anything?"

A girl reports that "They both have \(-2x\)."

"Right, and we talked about the \(-2\) and what it does to the graph. . . how a change in \(x\) changes \(y\); how, as we move over 1 for \(x\), the value of \(y\) changes. How?"

The group answers "Down 2."

It is 9:30 and Mr. Reynolds seems satisfied with the conclusions drawn from the graphing thus far. "Let's look at the next group." He sends a boy (who has been extremely attentive) to the overhead to graph \( y = -3 - 2x \). The boy plots \((0, -3)\) and then says "my next point will go off the graph."

Mr. Reynolds asks him to explain why. "What are you thinking? Now that you have your [first] point, where do you want to go?"

"\(-2x\) means down 2 and over 1, but it [the point] is off the graph."

Mr. Reynolds nods and says "I did this on purpose so that Danika could suggest another way to go."

Before she can respond, another boy says "Go up 2 and back to the left 1 unit."

"Great!" says Reynolds. "Do that again and connect the points. Now, the next one. Who'll volunteer? Come on." He calls on three students.

"I'm not sure."

"I don't get it."

"I'm not ready to do one yet."

"Let me look at some of your papers," responds Mr. Reynolds. As he glances at their work, he says "Doreen, go ahead and put yours on the graph."

A young lady with long wavy red hair, tie-dyed shirt and jeans quietly goes to the board. She plots \((0, 2)\) and moves down one unit, over one unit to plot a second point. She then connects the points, extends the segment in both directions and puts an arrow on either end to indicate a line. She returns to her seat when she is finished.

"Good job, Doreen. Let me do the next problem." It is 9:36 and Mr. Reynolds returns to the overhead projector.
The next equation is \( y=2+(\frac{1}{x}) \). "Let me go back to one we've done. Vanessa's problem was \( y = -1 + 2x \). I'll write it as \( y = -1 + (\frac{1}{2})x \). That may seem silly because \( \frac{1}{2} \) is 2. However, look at the graph. We have a point at \((0,-1)\)" he says, pointing to -1 in the equation, "and I move up 2 and over 1 to the next point." (indicating the connection to the ratio \( \frac{1}{2} \).) "That may help me look at this one. It may not be so bad. The fraction may not be such a big deal." Mr. Reynolds plots a point \((0,2)\) and then continues. "I'll use this (pointing to \( \frac{1}{2} \)) to guide me. . . up 1, over 2." He marks a second point, connects the two, and extends. "So these fractions may be actually very helpful to me. Roslyn, let's try d."

Roslyn goes to the overhead and plots \((0,-1)\), the first step in graphing the linear equation \( y = -1 - \frac{x}{2} \). Then she says "I'll go down 2." Mr. Reynolds interrupts with "why?"

"Because of the negative sign here [in front of the fraction]."

"Okay. And then?"

"Over three."

"Okay."

Roslyn moves left three units and Mr. Reynolds again interrupts to ask "why?" Roslyn pauses to think. "Oh! It should be down 2 and right 3 units." She plots that point. "Then up 2 and left 3 units." She plots another point, connects all three, and extends her sketch.

"Good!" exclaims Mr. Reynolds.

Another girl seated near Roslyn says "I want to do the next one." Her teacher nods and says "Go ahead." She correctly graphs the equation \( y = 0 + x \). It is 9:46 and the majority (fourteen) of the class apparently understand the process of producing a graph from these types of equations. They are working on the assigned problems and raising a hand for Mr. Reynolds' attention and verification. The five students near the window are passing notes, talking, and looking out the windows. Two students on the far right have heads down on the desks. One boy in the center has his head propped on his hand, eyes staring blankly, disengaged from the lesson activities.

Mr. Reynolds suggests that they examine some other lines, looking at perpendiculares. "Let's look at page two. I see you need to work some more on this assignment. We'll discuss these on Friday."

One of the boys seated nearby says "I think I'm just getting this. Will we do more?"
"Yes. But now, let's shift gears." Mr. Reynolds begins to distribute some other pages to the class. "Do you recall stem-and-leaf plots? [We took] heights of boys and girls and made a double stem-and-leaf. Remember?" He illustrates by showing a sketch on the board.

"This represents a couple of girls with height 160 cm, one who is 161 cm tall, and another whose height is 163 cm. Now, let's look at a box-and-whisker plot. Let's order the planets in the solar system by the number of moons each has." The whole class is engaged as he lists the planets and the count of moons on the board.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Moons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0</td>
</tr>
<tr>
<td>Venus</td>
<td>0</td>
</tr>
<tr>
<td>Earth</td>
<td>1</td>
</tr>
<tr>
<td>Pluto</td>
<td>1</td>
</tr>
<tr>
<td>Neptune</td>
<td>2</td>
</tr>
<tr>
<td>Mars</td>
<td>2</td>
</tr>
<tr>
<td>Uranus</td>
<td>15</td>
</tr>
<tr>
<td>Jupiter</td>
<td>16</td>
</tr>
<tr>
<td>Saturn</td>
<td>23</td>
</tr>
</tbody>
</table>

"The median splits the data. What is the middle value?" Students answer "2" as he points to the data set. "Now, step two. Take the four planets with the fewest moons. Find the median of these."

A girl says ".5" and he smiles as he says "Good, you remembered! It is 0.5—the first quartile value. Now, find the third quartile value."

Another girl says "15."

"Not quite. Who can help?"

"15.5" suggests a different girl.

"So the third quartile is 15.5 What is the least value?"

"Zero" is the choral response.

"What is the greatest value?"

"Twenty-three."

At this point, several students are disengaged. Tony in front appears to be sleeping. A couple are talking. The rest of the class are awake and writing as they listen to Mr. Reynolds. It is slightly before 10 as he models the five steps just described and uses those values to create a box-and-whisker plot of the data set.

"What's inside the box?" asks Mr. Reynolds.

"The middle quarters of the data."
"We can see some important things here. The spread of the data. . .The extremes. . .Box-and-whisker gives a good overall view of what's going on with the data. The median being near the end, not in the middle of the graph, tell us there are some extremes. Look at this item (taken from yesterday's newspaper.) These are names of companies planning to hire in this area in the near future as well as the number of new jobs. Make a box-and-whisker. Do it on the back of this paper."

Mr. Reynolds circulates to assist any students needing help. A few ask brief questions, but most begin to collect their materials in anticipation of the bell that is moments away. As the bell rings, all twenty-two teenagers move out of their algebra class and on to second block.

Once again, it is almost five weeks later when the first-block algebra class enters Mr. Reynolds' room. Eighteen students are present today and as they enter they take out notes and materials, settling in as their teacher says "Let's look at lesson 3-4."

"Was that homework?"
"Yes. Let's see the homework." They begin to check their homework questions orally. The problems relate to images on a video monitor produced by frames moving at various rates (frames per second.) Mr. Reynolds asks several questions in quick succession.

"How many frames per second? How did you set it up? What do you do with that? How did you decide what to do? Tell us what you did."
A student consults his notes and asks "At second per frame, how many frames would there be in ten seconds?"
Mr. Reynolds draws a sketch on the board.

"In one second, how many frames?" asks Reynolds.
The class responds "Thirty."
"And in ten seconds?"
"Three hundred."
A girl continues, "There are 300 frames every 10 seconds. So if there are 720 frames, could we use 300 + 720?"

Mr. Reynolds pauses. "Based on what we learned here [from the example and our model] how can we use that in this problem to help us?" He writes on the board: \[ \frac{1}{30} + \frac{1}{30} + \frac{1}{30} + \ldots = \frac{7}{30} \]
"How many changes occur in one second? How many are needed to total \( \frac{1}{3} \)?

A boy responds "thirty changes."

"That's right," says Mr. Reynolds. "So let's think about it. How long in seconds would 720 changes take?"

Girl: "Let's divide 30 into 720."

Another girl interrupts excitedly. "I know! I remember! I used \( 720 \times \frac{1}{30} \)"

"Oh," says Reynolds, "that's different from what you said earlier. So both of you are saying the same thing in a different way." He writes

\[ 720 \times \frac{1}{30} = \frac{720}{30} \]

"This would require 24 seconds. And how many frames would be needed in a 60-second commercial?"

Another girl replies "1800."

"Can you give me the math for that answer?"

"Sixty seconds times 30 frames per second."

"I said 300 x 6" says another student.

"Why?" asks Reynolds.

"Because there are 300 changes in 10 seconds so 60 seconds would be six of them."

"So 1800 what's are our answer?"

"Frames," she says.

Mitchell interrupts. "I'm lost."

Reynolds notes "You've been absent. Let's see if you can catch up." He moves to the next problem. "How about pixels? Andy?"

"I don't know."

Mr. Reynolds sketches a 'video screen display.'

\[ \begin{array}{|c|}
\hline
250 \\
\hline
| 0 | 200 |
\hline
\end{array} \]

"Roslyn?" She shakes her head; no response. Another boy volunteers. "You start at 90 on the screen."

"Where is 90?" asks Mr. Reynolds.
"At (90,0)."

Mr. Reynolds starts at (90,0) and moves 8 pixels to the right per second. "When does it fall off the screen?"

The same boy says \( \frac{200-90}{8} = \frac{110}{8} = 13.75 \)"

"13.75 what?"

"Seconds."

Mr. Reynolds redirects his question to the girl. "Do you want to chunk by 8's on the grid?"

"No."

"How fast does it move to leave the screen in 12 seconds? How does this relate to the previous problem? How do we get velocity or pixels per second?"

Students make no response.

"How many pixels do we want to cover?" prompts Mr. Reynolds.

A girl says "I don't know."

"Think. Start at 90 and go to....."

"Two hundred? Oh! 110 pixels."

Mr. Reynolds writes 110 pixels on the board. "And how many seconds do we have?"

"Twelve seconds."

"Kent, what do we need to get our answer?"

\( \frac{110}{12} \) or 110 pixels per 12 seconds."

"How could we say this?"

"9.16 pixels per second."

Students are attentive to the dialogue. It is 9:05 and Mr. Reynolds says, "Next question. Suppose there is a balloon at (0,192) on the screen and moving down at 16 pixels per second. How long will it take to reach the ground?"

One girl suggests "Divide 192 by -16."

"What seconds do we get from \( \frac{192}{-16} \)?"

"Twelve seconds."

"Really? Are we forgetting something?"

"The negative sign?"

"How does that figure in? Does it take -12 seconds?"

"No."

"How can we explain that? Did we account for everything?"

"We should use -192."

"That's a good idea. Why?"

"Because it's moving down."

"Good. And \( \frac{-192}{-16} \) gives us what?"

"Twelve seconds" says Robby.
Mr. Reynolds distributes three problems on a worksheet. "You can work with a partner or alone." One student asks to be excused, telling his teacher that he will be back right away. Mr. Reynolds begins to monitor student work and to distribute calculators. For three minutes, Reynolds works with a student one-on-one, then he moves to a pair of students. "You two work on this. You can do it. Look at page five and your notes."

At 9:17, Mitchell says "I'm lost." Mr. Reynolds suggests that he draw a screen to represent this problem.

Mitchell: "A pixel is like a dot, right?"
Reynolds: "Yes. Note there are 250 pixels across. What will the screen look like? Some kind of rectangle, right?"
M: "Right."
R: "But where's it start [starting position?]"
M: "At 50."
R: "So show me a sketch of what's happening."
M: "It moves like this (drawing a sketch.)"
R: "Where are you after one change?"
M: "At 65 [(65,0)]."
R: "Two changes?"
M: "80"
R: "You could continue this. But is there a math process you could use?" Mitchell nods and Mr. Reynolds moves to a pair of girls at 9:20. They ask several questions and he tells them to "go ahead with the idea." At 9:22 he assists two other girls with their questions and again moves to work with others. At 9:30 he writes a problem on the board.

Assume the computer screen is 250 pixels wide and 200 pixels high. Suppose a balloon is at the top of the screen and falls 5 pixels from one frame to the next. How many changes are needed to animate the balloon to the bottom of the screen? How many frames are needed?

"Let's all focus. Listen," and Mr. Reynolds reads the problem aloud. What is the velocity?"
Someone answers "5 pixels per frame."
"Not quite." Mr. Reynolds calls on a boy he overhears. "What did you just say you learned? You told your partner...
"Negative five [pixels per frame.]
"Okay! So it's going to fall how far?"
The choral response from the class is "-200 pixels."
"And at what velocity?"
In unison, "-5 pixels per frame."
Mr. Reynolds writes $\frac{200}{3} = 40$ changes. "There are 40 changes required to animate the balloon to the bottom of the screen. So how many frames are needed?" Hearing no response, Mr. Reynolds says "forty-one...the first plus forty more."

At 9:35 Mr. Reynolds directs their attention to problem #9. Write a closed form equation. "How can we write it? Remember your recursive formulas?"
He writes: new $V_{\text{vertical}} = \text{old } V_{\text{vertical}} + \underline{\text{______}}$.
A student says "plus five (+5)."
"Is it really?" Mr. Reynolds points to the model. "Here's the new vertical position [on the video screen.] Is it the old plus 5?"
"No, it's down 5 so -5."
Reynolds writes: new $V = \text{old } V - 5$. "How many '-5s' do we have here? Let's write an equation to match the problem." He continues writing on the board.
$\underline{?} = \text{initial position} + (40)(-5)$
"Is this right?"
Students nod and say "yes."
Again, Mr. Reynolds writes on the board.

$\underline{?} = 200 + \underline{-5}$

"How do I know how many '-5s'? Remember how we knew? At any one time, $V_{\text{vertical}} = 200 + ?(-5)$. Do I always move -5 pixels?"
"Yes."
"What is variable in this? How many changes occur?"
"That isn't the same always."
"Right. After 40 changes, you're here." He points to the bottom of the screen. "After 20 changes, you're here, about half way. So $V = 200+x(-5)$" and Mr. Reynolds writes the equation as he speaks. "Let's use another variable to help us remember what this [x] represents. What are we talking about? What is x?"
A boy says "changes."
"Okay," agrees Reynolds, "or another word?"
One of the girls suggests "frames."
"Okay!" And Mr. Reynolds writes a final equation, $V = 200+F(-5)$. "At any time, the vertical position is 200 plus the number of frames you move down." He pauses. It is 9:45 and Mr. Reynolds goes to answer Mitchell's question, speaking to four students who are off-task and talking. "You are in pairs. Let's focus on the problem."
A girl retorts "It's not just me."
"Right. But you need to take care of just you." He turns back to Mitchell and works with him for several minutes. Roger begins collecting the rulers that students have been using during the lesson. Mr. Reynolds goes to assist a girl who is working alone. She says "Everything is fine." He addresses the class.

"Pass your papers forward, please. Now we are going to change the object to a helicopter. Get a sheet of paper to follow along." He collects their papers and then sketches a rectangular screen on the board. "Let's move across and up the screen at the same time. Not a balloon moving up or down. Not a car moving forward. But a helicopter." He sketches:

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"If we look at the horizontal position 3, what is the vertical position?"
"Two," says one girl.
"At the same time that it is moving horizontally, it's moving up. One object will have two equations, both related to time. This unit is diagonal motion." Mr. Reynolds distributes a new page to the class at 9:55. Students chatter as he hands them a paper.
"Gosh, girl. You need help." says one student to another.
"This class is too much," says another student.

Mr. Reynolds calls on Crystal. "Let's get to 1A." He writes the two equations:
\[ H = 2 + 3(t) \]
\[ V = 3 + 6(t) \]
"Let's use a three-column variable table." Again, he writes on the board:

<table>
<thead>
<tr>
<th>T (sec)</th>
<th>Horiz (feet)</th>
<th>Vert (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

"What are our variables?"
One boy says "time"; another says "height."
"Let's be sure we don't get confused. Does H equal height?"
"No," answers a girl. "H is horizontal."
"Right. Okay." It is 10:00 and an announcement on the intercom calls for seniors to leave for a class activity. One student leaves the room.
"Let's look at 0 seconds of time. If no time has elapsed, the position is not at 0. Let's look." Mr. Reynolds models substituting zero for time in each equation. "H=2+3(0) and V=3+6(0) or H=3, V=3. So at zero seconds, where is the helicopter on screen?"
"At (2, 3)."
"Let's fill in the data table for the first six seconds in time."

As individual students complete the data table on their own, one girl signals for Mr. Reynolds' help to get started. He assists her and then returns to the board to complete the table with their help. "Emma, give me the next line in the table."
"[H is] four [and V is] six," is her response.
"Oh, let's see. What do we get when we substitute one second for time."
"Five and nine."
"Let's verify this."
Robby says "The horizontal goes up by 3s and the vertical goes up by 6s."
"Right. Your homework is page 3."

Students begin to move toward the door, anticipating the bell. Mr. Reynolds asks them to return to their seats. He repeats the homework assignment and the bell rings. Reynolds nods to dismiss them and students hurry out of the classroom, talking and laughing together.

Teacher Background
Mr. Reynolds has been teaching at this high school for ten years. In that time, he has taught algebra many times, although not in the past four years. His last algebra teaching assignment was in a 50-minute daily schedule. This year is his first experience with teaching algebra in the alternate day block schedule. In terms of advance preparation, Reynolds says his was "probably not very good. I am preparing as I go along. With material that is application based, I am more flexible. I really don't always know how much time students need. As for training....I wouldn't call it training. We had small workshops, but nothing to prepare you about writing lesson plans for the block. Getting students started is the biggest thing. If I can get them organized and started, it is pretty successful. When I
am a little disorganized at the beginning, I lose some students and the rest of the period is spent getting some back on track.

Distinctive Features of the Case

Student engagement through applications of algebra
Mr. Reynolds prefers to offer students activities that demonstrate the applications of algebra in the real world because he believes that approach interests his students more than a lecture. He believes that students get a "better sense of what is going on" when they are engaged in application-based activities rather than when they are memorizing rules. He admits that some of the applications he has used have been "complex stuff" but he attempts to balance the complexity with the support that comes from working with a small group or a partner. He also offers more one-to-one assistance during the extended class time. The block allows students time to accomplish application-based activities in one sitting rather than splitting the lesson over two class periods.

Student commitment
According to Mr. Reynolds, student attendance has not improved under the block schedule. Even when students are present, some of them do not engage in the algebra lesson. "For a lot of students, the time is too long for them to stay on task. And some won't do anything after they walk out the door." Although Reynolds is an advocate of activity-oriented lessons based on practical applications, it was clear that a few students (one to half-a-dozen in any given lesson) were reluctant to participate and were chronically off-task. "Students sometimes want you to lecture and work examples, but the student who learns the concept [by doing an application] is better off than one who memorizes rules. It takes a bit for them to get used to. . ."

Individual Perspective
Teaching in the alternate day block instead of the daily 50-minute schedule "took getting used to," according to Mr. Reynolds. "You have to curb the easy way of just cramming two lessons into one day. That's not easy to do. It actually helped that we got new textbooks at the same time [that we changed to a block schedule.] You didn't already have a plan in mind. New book in a new time frame. You had to figure out how to fit it together."

Advice to Algebra Teachers Beginning a Block Assignment
Mr. Reynolds advises any teacher who will teach algebra in an extended block for the first time to prepare lessons in the
summer. "Adapt to the extended time without trying to squeeze a two-[lessons]-for-one deal." He does not recommend having an outsider come in to do a workshop. "You've got your material, you know your curriculum, you've got professionals around your department. Just like here. We need time to sit down and share ideas. We need unencumbered time to think about it. If planning were to take place unencumbered and before students arrive, I think it would be powerful."

**Student achievement**
Sixteen of Mr. Reynolds' twenty-four students earned A, B, or C in algebra. Two others received a D and six failed (25%).
Scenarios from Mrs. King's Algebra Classes

It is 12:45 p.m. in mid-November and Mrs. King is greeting students as they wander into class from the hallway. "Katrina, how are you doing today?"

"Fine."
"Did you study for your test?"
"I didn't know nothin' about it."

Mrs. King tells her that the class has spent the last two class periods while she was absent preparing for a test which is to be given today. Katrina looks concerned and goes to her desk.

"Matt! Welcome back! We missed you last time. Were you sick?"
He nods and takes a seat as the teacher continues to circulate and greet students. One girl who enters says "I've studied for my test a little bit." Mrs. King smiles appreciatively and reminds students to take a seat before the bell rings. Two girls ask permission to go to the restroom but Mrs. King says "No, not now. I'll let you go later. I promise." By the time the bell rings, twenty-one algebra students have arrived and taken their seats.

"Ladies and gentlemen...you were supposed to have your test today. Instead, I have 51 problems here" (waving a handout) "and they are just like your test will be. You have 25 minutes. Check yourself out! Use your notebooks."
"Will you go over the problems with us?" asks one girl.
"Yes, if we need it," says Mrs. King. The students are very quiet. "Is anybody upset?"
One girl nods her head. "Me! I was ready."
A boy agrees. "Your test is the only reason I came to school today."

Mrs. King urges everyone to begin working. "If this were a test, what would you do? I'm giving you time now. At 1:25 we will go over these problems."

It is 12:58 and a pretty girl with braids finishes a bag of popcorn. Three students go to sharpen their pencils. Mrs. King begins to circulate to assist students who indicate they have questions. To one boy she says "It is a good thing we have this pretest. You need to review union." She reviews the concept with him, using two finite sets as an example as well as a sketch of two highways intersecting as an illustration. Mrs. King explains "It seems you people find it hard to find time to study. Here is some time to study in class. If you are stuck, pass it [the problem] by so you can know what questions you
have." She is moving about as she speaks, looking at individual work. "[Lack of] attendance hurts you. It steals your success when you don't come." She walks to the windows and opens them slightly. Although it is cold outside, the afternoon sun is shining on the windows and the classroom is comfortably warm. A young blond male in the back of the room rests his head in his hand and outlines the shape of his pencil on the desk. Another boy is scribbling on his pretest. The girl with braids adjusts her barrette and another girl raises her hand. Mrs. King walks over to glance at her paper, nods, and says "You have the right idea." The same disinterested fellow stretches his arms over his head, picks up the pretest, taps the edge of it on the desk as he looks down at his own foot tapping on the floor.

Mrs. King is standing beside a student when she asks "Class, can you buy \( \frac{1}{2} \) cards when you buy a card?" Several students indicate that is not possible and Mrs. King moves on. Sixteen students are working. Two are browsing through the chapter in their textbook. One boy has his head down. The blond remains disengaged. One girl leaves to go to the restroom. The blond goes to the pencil sharpener and then saunters slowly back to his seat. It is 1:15 when Mrs. King asks "Are you ready for some help?" Hearing "yes" in chorus, she tells students to "mark those you want me to work out."

As the girl returns from the restroom, she slams the door, and walks to the front of the room to return the wooden hall pass to its place in the chalk tray. She signs in and returns to her seat. One boy raises his hand with a question as the blond yawns loudly. In response to questions from two students, Mrs. King works the first problem involving union of sets, then does an example with intersection. Six students sit back in their seats, arms folded, unresponsive. Three others have their heads down on their desks.

"Sandra, do you know how to do #2? Katrina, how about #2 on the worksheet?" Mrs. King calls to two of her "sleepers."

"Raise your hand if you can work #4."
Nine hands go up. She calls on a young man whose hand is not raised. "I understand it."

"Then raise your hand! Marcus, come show us problem 4."
The blond boy goes to get the hall pass, signs out, and leaves the room as Marcus correctly works the problem involving Venn diagrams.

"How many of you like Venn diagrams?" asks King.
"You don't have to use them on the test do you?" asks a girl. After reviewing union and intersection using Venn diagrams to illustrate, Mrs. King begins to call on students to
define terms such as integer. A student asks "Is zero a whole number?" The blond is returning and answers "Yes." Mrs. King asks the whole class "Is 3.1 an integer?" The choral response is "No."

Mrs. King reacts to the general air of disinterest. "What's wrong people? The room is stuffy?" "Everybody is sick," explains a girl.

The clock reads 1:35 and Mrs. King says "I'll tell you what. I'll put you together and each group will have someone come to the board. I can't have a bunch of sleepy heads here! Awake! Awake! Awake!" she claps her hands. As she moves about the room, Mrs. King gathers two or three students together and numbers the groups from one to eight. "Everybody do problems 10-14. Then I'll tell someone from the group to come to the board."

The general energy level rises as the students begin to chat about the problems. They appear to be working at 1:38. Mrs. King circulates, saying "Teach each other." She glances up and sees Keith is not working. "I want to see you working Keith."

"We're finished."

"You couldn't be! Then do the next five..."

For the next ten minutes, most students are engaged. The blond is back at the pencil sharpener. At 1:48, Mrs. King calls on students to put problems 10, 12, and 13 on the board. "I'm marking a grade for each group. If you get it wrong, everyone gets an F. That's right." Keith asks if he may work #17. Two boys are laughing and talking as Mrs. King checks the work on the board. One group gets a zero for graphing the solution to the inequality \(-4 < x \leq 1\) as the set of six integers from -4 to 1 inclusive. The teacher quickly comments about each problem and then assigns four more problems at 1:55. The group in back talks among themselves. They received the zero for their earlier work. Mrs. King speaks to Keith, complimenting his work and then she asks Julia to "help these three girls."

Keith and his friend are discussing in animated voices the possibility of having \(\frac{1}{4}\) of a book as the solution to a word problem. Mrs. King is helping one group. Two girls are helping another group. All students appear to be on task at 2:01. Students are discussing the meaning of the word "or" as it relates to the graph of a compound inequality. Mrs. King interrupts the discussions to say "Put your pencils and pens
down and look at the screen. I want your complete attention on #15." She reads the inequality statements aloud as she writes

\[ M = \{x > 3\} \]
\[ T = \{x < -1\} \]

"Keith, turn around and look at the screen. Let's graph the union of M and T." Mrs. King pauses briefly, then continues to speak. "If it is in M, I shade it. If it is in T, I shade it."

The blond boy asks "Suppose you wanted the intersection of M and T?"

Mrs. King says "There's no number greater than 3 and also less than -1, so you couldn't have an intersection. Now, for #18. . ."

"Sometimes the temperature goes over 77 degrees." comments Toni.

"We're not talking about prediction. We are talking about a record of what happened with temperature over several days. The temperatures were between 60 and 77 degrees." Mrs. King writes \[ 60 < t < 77 \] and tells students to make a graph.

One girl says "We got our graph right, but not the statement."

One boy shows Mrs. King his graph. "That's incorrect."

The group of girls near the door are chattering and Mrs. King says "I'll wait until you finish your entertainment. . . ." Ashleigh has the hiccups and the others in her group are giggling. The blond is tapping his foot. Mrs. King continues, "Let's discuss the cost of the cards." She writes on the transparency as she reads the guidelines for calculating the prices of several cards.

\[
\begin{align*}
[\text{If}] \quad n < 10 & \quad [\text{then } c =] \quad 0.70 \\
10 < n < 18 & \quad 0.60 \\
n > 18 & \quad 0.50
\end{align*}
\]

Most students are watching her write and seem to understand the various inequalities. One boy says "I have a question" as he shows Mrs. King his paper. She privately responds to his question. It is 2:18 and the teacher speaks to one student about another problem. She then addresses the entire class regarding #21. "This is a question to see if students know what is a whole number."

"Is it on the test?" asks one boy.

"It's in the chapter" replies Mrs. King. "Let's all try #24." Keith pulls the hood of his sweatshirt over his head.
The girls in Ashleigh's group turn to look at the clock. Mrs. King says quietly "Ashleigh."

"I'm not talking!"

Another girl approaches Mrs. King with a question and Ashleigh begins to chuckle. Her group is laughing and talking. Mrs. King looks up. "Keith, take off that hood. Pay attention... immediately!"

Two girls in a trio have their heads down. Toni's head is down. Ashleigh's group continues to talk. The blond is tapping his foot on the floor. Another girl puts her head on the desk at 2:25. Keith and his partner are working. He quietly directs his partner to "do the like terms, man."

Mrs. King assists Jackie with a problem. "Add the x's first..."

The blond takes the wooden pass and leaves the room. Mrs. King asks Ashleigh to simplify -8h + 1h, but Ashleigh does not respond. "Suppose you are short $8 and someone gives you $1?" asks Mrs. King.

Ashleigh answers "-7h" as the blond returns. "I want to be sure everyone can do #35. Sandra, come to the board."

Toni says "I want to do it."
The blond says "I'll do it."

"No," says Mrs. King, "I want Sandra."

Toni speaks (to no one in particular), "I'm gonna go home, fix me a sandwich, then go to the bus stop and go to work. I can't wait until my birthday. Then I'll be takin' myself to work."

Mrs. King is asking Sandra to solve the equation -p + 13 = 19. What do you need to do? Add -13 to both sides, right?" She models the transformation and simplifies to write -p = 6. "Now, -p = 6 means that p equals how much, Sandra?"

"-6."

"Right! Let's see #37." It is 2:35 and the intercom interrupts with afternoon announcements. Students begin to stand, moving desks back into rows. As the voice on the intercom continues, Mrs. King asks students to put away calculators carefully. "Write this in your notes. Homework. Study for test on chapter three."

Students are talking, gathering books and backpacks, putting on sunglasses, combing hair, getting out baseball caps, and generally preparing to exit as soon as the bell sounds. At the tone, they exit in a rustling, rumbling wave. Mrs. King approaches the observer with her own thoughts about the lesson.
On January 7 at 8:30 a.m., the bell rings in Mrs. King's algebra class as she explains the need for students to fill out the Federal Impact Aid forms she is distributing. A note on the board indicates that the class will answer homework questions, review section 6-5, and take a quiz on section 6-4. When students each have a copy of the questionnaire, Mrs. King reminds them of the project that is due on Friday. "Your project is 10% of your grade and this is your last week to turn it in to me. Who hasn't completed your project? (Three students raise their hands.) Write yourself a note!"

She selects a problem from their homework and uses it to remind students that they are working with arithmetic sequences. She reviews the important terms in arithmetic sequences and assigns a problem. All students are working and at 8:50 Mrs. King reminds them that they may use calculators if they wish. Several students leave their seats immediately to borrow a calculator as they attempt to find the number of terms in a specific arithmetic sequence. At 8:55 Mrs. King begins to collect papers from students who are finished. One girl stops her to ask a question and they talk quietly for a moment. "When you finish, get your homework papers and check to see what questions you wish to ask." At 8:57 Mrs. King uses a question and answer strategy with several students as a means of checking the results they achieved to the assigned problem. All students watch her model the problem but none volunteer to respond until called by name. At 9:05 she asks if they have any questions about the problem or about arithmetic sequences. "Suppose your first term was 1000, the common difference was still -3, and you wanted to find the fifty-sixth term? How would you simplify this?” as she writes 1000-3(56-1) on the board. Several students raise a hand and she calls on Sherry to respond. After her correct response, Jacob says "I have a better way. I use a
Students need to take breaks. Stretch or move. Go to the restroom. I tell them, ‘go wash your face if you’re sleepy.’ They like that little break.”

“We start the lesson by putting our objective in the notes. Then we take questions about the homework. Then I explain new material and then I do one-to-one work. I say ‘Let’s do this problem. Everybody try it.’ Then we go over it to check answers. If there are questions, I’ll explain. Then do another one. Drill and practice. I give them problems and then help individuals. I give a quiz every 3 sections. They know that. Sometimes, when we have a quiz, they come and they haven’t done their homework. They still have questions. So I’ll put the quiz off. That happens a lot.”

Vella S. Wright  Chapter 3: Individual Case Study Reports  64

calculator and I don't need the nth term formula." He uses repeated subtraction to find the designated term.

At 9:12 Mrs. King says "Class, let's open the windows. Stretch. Stand up. Turn around. Wake up. Let's come back together. I need you and your brains. Now, let's take the homework questions." Students return to their seats after the two-minute break. Katelyn asks Mrs. King to work problem #19. King models the process of taking notes before working the problem. "Let's see. There is 6% tax. The cost of the item with tax is $720.80"

She writes on the overhead:

\[
\begin{align*}
6\% \hspace{1cm} \text{tax} \\
\text{cost: $720.80 with tax}
\end{align*}
\]

and then asks students how to write six percent as part of her equation. One girl says "write .06x" so Mrs. King writes \[x + .06x = 720.80 \] "Who can solve this? Gena?"

"Subtract the tax."

"Can we do that yet?"
Jacob says "Solve 1.06x = 720.80"
"Right. Let's do it. Now what"
Rita suggests dividing by 1.06
A boy with a calculator says "x is $680."
"This is a good 'real-life' problem, isn't it?" comments Mrs. King.

[Mrs. King notices a cut on her finger and asks the observer to watch the class while she leaves briefly to attend to the cut. Individual students ask questions about two other problems and the observer works with them until their teacher returns.]

At 9:34 Mrs. King draws the entire class to attention with a problem from their homework. She sets up the equation and asks students to assist her with the solution. As they complete the problem, a girl near the window asks a question about problem #26 (involving salary earned for regular and
overtime work.) The problem requires students to determine how many hours an employee must work in order to earn $250.

"Try it in your notebook. You may help each other." Mrs. King notes that the regular rate of pay is $5.25 per hour.

"How much do we earn for overtime?" she asks.

"Time and a half" says one boy.

"And what is 1\frac{1}{2} times $5.25?"

A girl uses her calculator to multiply and says "$7.87"

"So what would our equation be?" asks Mrs. King.

A boy suggests writing "40($5.25) + $7.87h = $250"

Mrs. King directs the class to "Work it out. I want to see your answer." She allows students to work the problem before asking Monique the next question.

"If he wants to earn $250, will working 45 hours be enough?"

"No, not quite; but 46 would be enough. He would earn more than $250."

Mrs. King glances at the clock. It is 10:00. "We have not gotten to the third part of our lesson. For your homework, you will read pages 284 to 286 and answer questions 1-9 [about the reading.] I will give you a homework quiz on your reading. Now, stand and stretch." She pauses to let the students move around. "Come back now. Let's look at #29 which uses repeated addition. An airport parking lot charges $0.30 for the first hour and $0.20 for each additional hour. How long would the car be parked if the driver owed $1.90?"

Matt says "Eight hours. I used my calculator."

Rita suggests "First subtract $0.30 from $1.90."

"Yes, and then divide by $0.20" says Chris.

Mrs. King writes on the board:

\[
\begin{align*}
N &= \text{hours parked} \\
A + (N-1)D &= $1.90 \\
.30 + (N-1)(.20) &= 1.90 \\
(N-1)(.20) &= 1.60 \\
N-1 &= 8
\end{align*}
\]

Students finish taking notes on the problem and begin to close their books and gather materials, anticipating the bell. It rings at 10:10 and the students hurry from class.

One week later at 12:15 on a Tuesday in mid-January, Mrs. King asks the twenty-one students in her afternoon algebra block to take out their homework papers. "We will check these before we begin the new section. Remember, your exam is one week from Thursday. You have four classes in between, counting this one."
Our lesson today will involve solving inequalities using our knowledge of solving equations. Let's look at this equation.

She writes on the board

$$\frac{1}{2}a + 3 = \frac{1}{4}a + 4$$

"Let's add \(-\frac{1}{4}a\) to both sides." She shows the transformation leading to the following

$$\frac{1}{4}a - \frac{1}{3}a + 3 = 4$$

$$\left(\frac{1}{4} - \frac{1}{3}\right)a + 3 = 4$$

"Keith, can you simplify this?" A girl who had been talking says \(\frac{5}{6}\).

"Is it? What did you do?"
Keith suggests converting one-half to three-sixths.
"Right. And what then?"
He explains simplifying the fractions to produce \(\frac{1}{6}a + 3 = 4\).
A boy suggests subtracting 3 from both members.
"Now what? Toni?"
"Multiply by 6 or divide by one-sixth."
After this suggestion, Mrs. King writes the final solution as \(a = 6\). "Let's review this at home. Learn to do problems like this one."

"Will we have a quiz?" asks a girl near the front of the room.
"Yes. Thursday."

It is 12:11 and Mrs. King writes on the board: \(5x + 2 = 3x + 6\)
Toni asks "Can I come [to the board] and solve it?" Mrs. King nods, asking students to pay attention. As her classmates look on, Toni solves the equation. Mrs. King interrupts her work to ask "What do you think so far?" (Students nod.) "She's right" says Mrs. King as Toni finds the solution to be \(x = 2\). "I am so proud of you" says Mrs. King. "You are a smart girl. [To the class] You can all make A or B. Let's look at this." She writes \(5x + 2 < 3x + 6\). "Is this an equation?"

In unison the class says "No."
"They are unequal, right?"
A boy in back says "We call this an inequality."

Mrs. King smiles at his statement. She tapes four pennies on the board, writes the symbol ">" and then tapes two more pennies on the other side of the inequality sign. Beneath the pennies she writes \(4p > 2p\).
"If I add one penny to each side, what do I have?" she asks. As she tapes one penny to each side, she says "Is the statement still true?" Students nod as she writes $5p > 3p$. "This demonstrates that our true statement using greater than is still true if we add the same amount to both sides. Suppose we subtract two pennies from each side. What is the statement? Is it still true?"

A boy suggests "3p > 1p; but an inequality won't ever be equal." Mrs. King agrees, saying "To hold a statement true, what you do to one member, you must do to both."

[There is a brief interruption on the intercom. It is 1:18 and students are reminded that exams will be held on schedule, regardless of weather. They should take books home to study. If it snows, exams will be held the day school reopens. Students groan quietly.]

At 1:20 Mrs. King asks the class to solve $5x + 2 < 3x + 6$. She monitors their work and asks for someone to explain. Toni describes the process she uses to find $x < 2$. "Did all of you get it?" asks Mrs. King. "Copy this problem. Try it." She writes $3x + 1 < 2x - 7$. "Try it. Ashleigh. Rita. Everybody. Try it. Please. Everybody." Mrs. King circulates to see their work. "Keith, don't you have a bigger piece of paper? Get it out. Don't be lazy. You're a smart guy!" It is 1:25 and students are working as she offers suggestions. "Be careful. You used an equals sign. (pause) This is not an equation. (pause) Good. (pause) Good. (pause) You made a careless mistake. Look again. (pause) Let's work this together."

At 1:28 Mrs. King models the procedure for solving the inequality. As she writes $x = -8$, Toni and Jasmine put their heads down on the desk. Keith was bouncing in his seat, humming a popular song. "Keith, stop talking. This is a warning. Next time you're talking, I'm going to make you write some statements. Not here. At home. Raise your hand if you know what to do. Matt, what would you do to find the solution to $3x < 15$?" Matt explains he would find $x < 5$ by dividing both members by three. Another boy in class says "Where do you change signs?"

Mrs. King illustrates her answer with the inequality $-2x < 10$. "If you had this inequality, you would divide both sides by the coefficient of x, right? Divide by $-2$. When you do, the direction is going to switch." She writes $x > -5$.

Mark asks "Why?"
"It's the rule. Let me explain to you the 'why.'" Mrs. King writes the simple inequality $2 < 5$ on the board. Suppose I divide both sides by negative one. Would $-2 < -5$ be a true statement?"

Two or three students say (softly) "yes."

"Look at this graph." Mrs. King quickly sketches a number line showing the positions of $-5$, $-2$, $0$, and $+2$.

Others in class say "No" and "-2 is greater than -5."

"So that's why you change the less than sign," Mark says.

"That's right," says Mrs. King as she erases the less than (<) sign in the statement, replacing it with a greater than (>) sign. "This is a fact. It is a mathematical property. Now work this one. You may talk to each other, but softly." She writes $3w -8 +5w < 16$ on the board. A few moments later (1:35) she asks "What do we do first?"

Students direct Mrs. King to add like terms. She simplifies to transform the inequality into $8w -8 < 16$. Another student suggests adding $+8$ to both members and still another student completes the solution by dividing both members by $8$. Everyone is satisfied with the solution $w < 3$. Mrs. King writes another problem on the board. "Please put this example in your notes." $-10x + 18 > 12 - 13x$

As students work, Mrs. King moves to Jasmine's desk and speaks with her about sleeping in class. At 1:42 King calls on Jake and Mark to explain their solution to the problem. Toni assists them in finding $x > -2$ with no errors in their work. Mrs. King speaks to Sharon (who is toying with her calculator absent-mindedly.) "Sharon, are you trying?" Sharon nods, looking down at her desk.

"Let's divide into two groups." Students noisily move desks, sliding them across the floor to form two circles. Julia from the larger group is asked to join the smaller group for balance. "Open your books to page 307. Try #10. I will pick one from each group to come [to the board] and work it. Be sure everyone knows what to do. Help each other." Students work for two or three minutes and Mrs. King circulates to observe their work. "If you have questions, you may ask me."

Toni tries to show the students in her group how to solve $13<7-x$. Keith resists her help, but Sharon takes her advice and changes her work. Mrs. King pauses to say "Be sure he (Keith) understands."
"He does; he just got his sign wrong," Toni says.

The other group is talking and laughing. Mrs. King moves to see their work. At 1:53 she says "Are you ready, groups? Malaina and Rafiq, let's see what you have."

Mrs. King examines each problem and announces "This one (Malaina's) is right. I'll give this group 5 points. This other group gets 4 points." She points out the 'lost variable' midway through Rafiq's problem and cautions students to be careful in their work. She assigns the next problem to the groups and reminds them "It's your responsibility to teach each other. Don't lose points because somebody doesn't get it. Josh, please help Tynisha." Mrs. King allows the groups to work briefly on the inequality \( v - 3v > 2 \). She asks Kim and Tynisha to come to the board to represent their respective groups.

Mrs. King models the solution to the problem for both groups. "Kim, what should have happened when you divided by \(-2\)?"

"Change the signs?"

"Yes. That's an important point. I'll give Kim's group 3 points." After asking about the apparent 'early' reversing of signs in Tynisha's problem, Mrs. King accepts the group explanation that they reversed the signs knowing there was division by a negative number. "This group may have 5 points then. Now, try problem #25 on page 308."

Most students in the groups begin working. However, Julia is not working, nor are Keith and Jasmine. In the other group, three girls are talking, not contributing. Mrs. King records grades in her gradebook while students work and then she moves to the groups to check their efforts. "Jasmine, do you know what you're doing?"

She nods.

"Honest?"

Another nod.

"Trisha and Amy, please go to the board. Tell me groups, are we learning how to do this or just wasting time? Do I need to go over anything with you? Tell me honestly." She looks at both problems and announces "These are both right." Each group earns another 5 points. It is 2:10 when Mrs. King asks them to work #26 in their groups. "We will do two more problems and then I will explain some new material." She watches the groups work and then says "Isolate the variable. Get the variables by themselves." She checks papers and speaks softly to Sandra who just entered the
room. A moment later, she walks over to see Keith's work. "You did it right!"
"I know it."
"You did make a mistake here (pointing) where you subtracted instead of adding. Lissa and Mark, please go to the board."

\[
\begin{array}{ccc}
11h + 71 & > & 13h - 219 \\
-11h & -11h & -11h \\
71 & > & 2h - 219 \\
+219 & +219 & +219 \\
290 & > & 2h \\
\frac{2}{2} & \frac{2}{2} & \frac{2}{2} \\
h & > & 145 \\
\text{Lissa's group} & \text{Mark's group}
\end{array}
\]

"Let's work with Lissa's problem first." Mrs. King reviews Lissa's problem to the point where she divides by 2.
"I would write 145 > h. Is that the same as saying h > 145?" Several students say "No."
"I agree. So you should write h < 145."
"I will give your group 4 points, Lissa. Mark's group gets 5 points."
"Mrs. King, can I go to the board this time?" asks one girl.
"Please pay attention so you can do a different type of problem. Does anyone need to get up and stretch for a minute?"
"No. We're cool."

"Ladies and gentlemen, listen and take notes." Mrs. King writes 'distributive property' on the board and asks "Who remembers how it works?" Hearing no response, she continues. "It says if you have any number, say a, multiplying two other numbers in parentheses, say (b+c) then your product will be equal to a times b plus a times c." For emphasis she writes \[a(b+c)=ab+ac\]. "Mandy, use the property to multiply \[2(3 + x)\]."
Mandy says nothing.
Mrs. King writes \[2(3)+2(x)=6+2x\] beneath the original expression.

At 2:25 an intercom announcement asks all freshmen basketball players to report to the gum. Mrs. King writes another algebraic expression on the board. \[3(x-1) + 2(x+5)\]
"Remember to multiply signs, coefficients and variables. Pay attention. This will give us +3 times the x, then +3 times the -1, and so on." She writes \[3x + -3 + 2x + 10\]
"Now what? Toni, turn around. Lissa. Now. I'm cutting your daily grade."
Another student responds. "We get 5x + 7."
"Good. Jake, simplify -(x - 3)"
When he does not respond immediately, Mrs. King points to the negative sign in front of the parentheses. "What is the number here? Ladies and gentlemen, talk to me. I've lost some of you. Here, solve this problem on your paper." 3-(x+1)=5
"Get to work everybody." One girl asks to go the restroom. "You may. Sign out." At 2:29 Mrs. King asks Matt to "come do this one." He writes 3 - x = 5 beneath the original.
"Class, what did he do wrong?"
"Used the distributive property," a girl replies.
"But" says Matt, "I thought -1 + 1 would be zero, so I left it out."
Mrs. King shows Matt and the class how to solve the problem using the distributive property. "I'll give you problems to prepare for the quiz. Copy this [assignment] in your notebook. Chapter review, pages 308-309, problems 1-22 and 25-32. If you can do these, you can do the quiz alright." It is near bell time and three heads are down. Three other girls are chatting. "Do not resign, ladies and gentleman. Please put your desks back." There is a noisy transition as students talk while dragging desks across the floor. Mrs. King ignores the noise to say "You're helpful!" The intercom interrupts with afternoon announcements. Students collect their materials and prepare to end the day as the bell sounds at 2:40.

Teacher Background
Mrs. King has been teaching since 1971 with a break for graduate school in the late 1970s. She has worked here since 1985 and has taught algebra nine or ten years, the last three of them in the alternate day hundred-minute block schedule. Upon reflection, Mrs. King indicates that she likes the longer block of time but does not feel successful with it. "I don't think I was given enough time to get ready to teach the block. There aren't enough hours in the day [to prepare.]

Distinctive Features of the Case

Preparation
Mrs. King spends a great deal of time planning her lessons, but often changes them. The students' attention wanders and there is no consistent procedure for student accountability. "I find
it very hard to teach them in a traditional method. I feel like I have to come up with more ideas or physical or visual activities. I find I'm not having a life after school any more." She believes she would be more effective if she had more equipment and hands-on materials as well as more software. "I believe the block needs a more hands-on classroom. I need more time to prepare lessons, to get the kids interested. I don't have access to the materials I need. I don't think we have access to software that we could use to make it [algebra] more interesting."

**Individual Perspective**

Mrs. King finds fault with not seeing algebra students on a daily basis. "They have often forgotten what we did in class." In fact, she has talked with other teachers of ninth graders (who make up the majority of the algebra enrollment) and believes that ninth graders have trouble adjusting to the block schedule in all subjects. "Maybe we need a different schedule for ninth graders. Give them a transition. Or group only ninth graders in the algebra class. When they are mixed with juniors and seniors, they tend to imitate. I had a class of mostly ninth graders and it was my most successful [algebra] class." One positive point about the longer class is the additional time to reinforce material. Mrs. King says that she would prefer a class longer than 50 minutes but shorter than 100 minutes. "Neither 50 nor 100 are really very effective."

**Advice to Algebra Teachers Beginning a Block Assignment**

Mrs. King advises teachers to "get ready! Have your plans. Be prepared. Planning! Planning! Planning! It [the block] is a lot of time." In her experience, students are not always interested in the subject and the majority of them cannot pay attention. Teacher planning is essential to ensure that time is well spent. Mrs. King tries to offer some variety by occasionally taking a walk outside of the classroom to discuss the mathematics displays (such as the Pythagorean theorem model in the hall) or simply to work problems outside. She believes that the block schedule challenges her to try new instructional strategies. "I would like to have a field trip, maybe to see how math is used in an engineering job or by an architect. Or perhaps find internships for students. We could publish a mathematics newsletter." However, she expresses the difficulty with putting these ideas into actions. "Locating materials and preparing for class... That's what I'm having trouble with..."
Student Achievement
Seven of Mrs. King's twenty-two students earned an A, B, or C for their final algebra grade. Eight received a D and seven students failed algebra (32%).

Complete case reports for the remaining three cases appear in the appendices. Mrs. Miller is an experienced algebra teacher who does not find the block compatible with her teaching style. She has been trying new strategies but primarily continues to teach 'the way she has always taught.' Her focus is on a quick start to each lesson and use of a variety of strategies to maintain student attention. She is relaxed but firm with her students and is concerned about poor homework habits and absenteeism. She worries about the maturity of ninth graders and their readiness for algebra in the long block configuration.

Mr. Owens is another experienced algebra teacher who has taught in a variety of settings and who feels comfortable teaching in the block schedule. He cites the need for variety to maintain student interest and so he attempts to change activity or focus every 20 to 30 minutes. Although he does not use many hands-on activities, he values finding applications to illustrate concepts. He uses coaching strategies and frequent (brief) assessments. Owens is most concerned about the alternate-day aspect of the schedule and its connection to continuity of learning.

Ms. Nolan has fewer years of teaching experience than other teachers in the study. Her focus is on developing good learning habits and attitudes in her students (for example, engaging in dialogue with the teacher and other students, doing homework, taking notes, taking responsibility for studying for tests.) She tries to vary activities, seeking application problems, projects, and computer connections for students. She acknowledges the importance of teacher planning and enjoys the longer block, hoping that the school will 'stick with it.'