FUNDAMENTALS OF THE SIMPLEX COMMUNICATION CHANNEL WITH RETRANSMISSIONS

by

Boris Davidson

Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY in

Electrical Engineering

APPROVED:

_________________________  _________________________
Dr. C. W. Bostian, Chairman  Dr. N. J. Davis IV  Dr. S. F. Midkiff

_________________________  _________________________
Dr. W. L. Stutzman  Dr. W. Kohler

April 14, 1997

Blacksburg, Virginia

Keywords: simplex channel, multiple access, direct-sequence spread spectrum, message collisions

Copyright © 1997, Boris Davidson
The need for multiple access strategies arises whenever a number of users have to share a communication resource, since it is usually either cost prohibitive or impractical to dedicate a communication channel to a particular user. A need for such algorithms arises in many instances, particularly in applications utilizing wireless systems where all users access a common channel or medium. Such random access techniques as ALOHA and slotted ALOHA have been successfully implemented in a number of wireless applications. One of the major drawbacks of these algorithms is the necessity of a return path from the central station to each system user, which makes their use both inefficient and expensive for applications where one-way communication would suffice. For such applications, a need remained for a random access algorithm which can maximize the probability of successful message transmission in a one-way communication environment.

A random access technique that addresses the above-mentioned need is developed. With this technique, each user sends an original message of predetermined length to a central receiver. The user then retransmits the message a specified number of times in a predetermined interval reserved for the retransmission process. The time interval between each successive retransmission of a given message is random. Assuming total annihilation of all colliding messages, the expression for the probability of successful transmission of a given message in terms of the major channel parameters is theoretically formulated. This technique offers a significant improvement, compared to a single transmission, in ensuring that a message is successfully received.

The actual message collision dynamics in this system are experimentally studied using two different types of direct-sequence spread spectrum receivers, one employing a sliding correlator and the other using a matched filter. The spreading code in such systems offers extra protection for messages against possible interferers. The results indicate that it is often possible to properly receive a given message in the presence of co-channel interferers, thus significantly improving the overall system performance. These results are subsequently incorporated with the propagation data for several different types of microcells to arrive at a more precise theory of the link.
They are able because they think they are able.

Vergil (70-19 BC), Roman poet.

ACKNOWLEDGMENTS

The pursuit of knowledge is a driving force in the lives of many individuals. For myself, this work represents a significant milestone in this pursuit. I have been privileged to come in contact with many people who have greatly influenced me and facilitated the completion of this endeavor. I am truly indebted to them all.

I would like to express deepest gratitude to my Ph.D. thesis advisor, Dr. Charles W. Bostian. He has been a mentor and a teacher. His guidance in all matters has been greatly appreciated. During my studies at Virginia Tech, I have truly benefited from his valuable technical advice, which I sincerely hope is reflected in this work.

I wish to thank all of my Ph.D. committee members: Dr. Nathaniel Davis, Dr. Scott Midkiff, Dr. Warren Stutzman and Dr. Werner Kohler, for their time and effort throughout the course of this work. They have contributed to my growth, both academically and as a researcher, which I believe is of primary importance for someone undertaking the pursuit of this degree.

This effort has been greatly aided by Dr. Dennis Sweeney and Andy Harmon who offered invaluable assistance with all aspects of the experimental collision study, as well as Steven Franks and Matt Kurtin who assisted in the writing of the channel simulation program. In addition, I would like to thank my fellow graduate students at the Center for Wireless Telecommunications: Barry Mullins, Aaron Hawes, Todd Fleming, Jeanette Mulligan, Raza Shah, Rusty Baldwin, Andrew Gobien, Gerry Ricciardi, Carl Dietrich and Matt Monkevich, for creating an atmosphere of mutual cooperation and active exchange of ideas.

I would like to acknowledge and thank Interactive Return Service, Inc. and Virginia’s Center for Innovative Technology, whose joint sponsorship made this work possible.

I would also like to thank my family for their constant encouragement and support.
# TABLE OF CONTENTS

**ABSTRACT** ........................................................................................................................................... ii

**ACKNOWLEDGMENTS** .......................................................................................................................... iii

**CHAPTER 1. INTRODUCTION** .............................................................................................................. 1

1.1 MOTIVATION FOR DEVELOPMENT OF THE CHANNEL ................................................................. 2

1.2 OVERVIEW OF EXISTING ACCESS STRATEGIES FOR ONE-WAY CHANNELS ......................... 3

1.3 SPECIFICS OF THE NEW CHANNEL ................................................................................................. 6

**CHAPTER 2. THEORY AND COMPUTER MODELING OF THE CHANNEL** ........................................... 8

2.1 PROBABILISTIC TRAFFIC MODEL OF THE CHANNEL UNDER ASSUMPTION FOR

ANNIHILATION OF COLLIDING MESSAGES ......................................................................................... 8

2.1.1 Generation of initial messages ..................................................................................................... 8

2.1.2 Generation of retransmissions ..................................................................................................... 9

2.1.3 Message collision dynamics ....................................................................................................... 11

2.1.4 Effective collision parameter ..................................................................................................... 16

2.2 OVERVIEW OF THEORETICAL RESULTS AND THEIR APPLICATION ........................................ 17

2.2.1 Effect of initial message arrival rate on system performance ...................................................... 17

2.2.2 Determination of traffic model-based channel parameters ......................................................... 21

2.2.3 Simplification of theoretical results ............................................................................................ 24

2.2.4 Channel utilization ..................................................................................................................... 26

2.3 COMPUTER SIMULATION OF THE CHANNEL ................................................................................. 28

2.4 COMPARISON OF SIMULATED AND THEORETICAL RESULTS .................................................. 30

**CHAPTER 3. EXPERIMENTAL INVESTIGATION ON SURVIVAL OF COLLIDING

MESSAGES** ............................................................................................................................................. 37
CHAPTER 1. INTRODUCTION

The need for multiple access strategies arises whenever a number of users have to share a communication resource, since it is usually either cost prohibitive or impractical to dedicate a communication channel to a particular user. Many such algorithms have been proposed and implemented [1]. Generally, multiple access techniques can be classified in three main categories: deterministic access, controlled access, and random access.

The most common example of deterministic access is time-division multiplexing (TDM), where a portion of an outgoing time frame is allocated to each user. TDM technique has been successfully implemented in, among others, geostationary satellite channels.

With controlled access, the users gain access to the channel either through a central controller (polling) or by passing control from one user to another in a decentralized fashion (token passing) [2]. A microwave channel on which all users transmit on the same frequency is an example.

Random access techniques allow users to transmit at will. Then, these techniques employ various methods to resolve packet collisions that occur whenever two or more users transmit at the same time. One of the more common random access strategies is the ALOHA algorithm, which resolves packet collisions by having a central station recognize a collision and request the user(s) involved to retransmit the message(s) [3]. This process should be repeated until no collision of a given message is detected. The ALOHA transmission algorithm has been implemented in computer communication networks operating in a local-area network (LAN) environment.
1.1 MOTIVATION FOR DEVELOPMENT OF THE CHANNEL

All of the major access strategies discussed in the previous section share a very important feature - they have been implemented primarily in two-way (duplex) channels. In other words, some sort of acknowledgement from the receiving party is required to tell the users about the status of their transmissions. However, many applications exist where unidirectional (simplex) transmission could possibly meet all or most of the system requirements. These applications include, but are not limited to, home-shopping networks, video-on-demand controllers, and various alarm installations. The major problem that exists with any form of simplex transmission is that the transmitting party has no way of knowing whether their message was successfully received. Therefore, it is imperative for a system designer to develop a transmission scheme which would provide for a highly reliable message transfer in a one-way communication environment.

In particular, Interactive Return Service, Inc. (hereafter referred to as IRS, Inc.) contracted with the Center for Wireless Telecommunications (CWT) at Virginia Tech to develop a one-way channel that would be able to support some of the above-mentioned services. This channel would operate in a microcellular wireless environment in a 900 MHz ISM (Industrial, Scientific, and Medical) band. It would also employ direct-sequence spread spectrum for interference rejection. IRS, Inc., envisioned each user having a hand-held transmitter the size of a TV remote control enabling them to transmit data (such as product and service requests) to a central station. Sufficient reliability must be achieved with just a simplex transmission since there would be no return path from the central station. This would be significantly simpler and less expensive than incorporating
receiving capability into each transmitter. Thus, the need arose in a multiple access strategy for a simplex channel that would meet all of the requirements mentioned above.

1.2 OVERVIEW OF EXISTING ACCESS STRATEGIES FOR ONE-WAY CHANNELS

The problem of developing a traffic model for a unidirectional channel has interested researchers for a number of years. Several models for a collision channel without feedback have been considered previously.

One such model was investigated by Massey and Mathys [4]. This model was developed for a situation where all users must share a common communication resource but, because of their inability to synchronize their clocks, cannot transmit their data packets in a time-sharing mode and, due to the lack of a feedback link, can never be sure of the outcomes of their individual packet transmissions. This inability of users to synchronize their transmissions forces them to employ random accessing. Thus, a channel model had to be developed that would demonstrate the possibility of reliable random access communication without a feedback link.

The basic channel model for the collision channel without feedback looked at a case where each of the channel users would occasionally send a packet of some fixed duration, but otherwise be silent. In addition, there is no common time reference between any of the users or the receiver. This was modelled by introducing time offsets for each user. These offsets governed the relative packet transmission times for each of the channel users. Two cases for the possible values of the unknown time offsets were
considered: the slot-synchronized case in which the time offsets are arbitrary integer multiples of packet duration, and the unsynchronized case in which the time offsets are arbitrary real numbers. In the slot-synchronized case, if all users align their packet transmissions within time slots, collisions will result only when received packets completely overlap. In the unsynchronized case, however, the users have no way to avoid collisions that result from only partial overlapping of packets.

In real random access systems, information is transmitted only via the contents of packets. In other words, the randomness of the information is not used in the selection of transmission times. Therefore, a constraint on channel usage was needed for the channel model. This constraint would eliminate the dependence of starting times on information to be transmitted. Additionally, it had the desirable effect that system performance would not vary with the statistical nature of the information transmitted. The constraint was realized by requiring that each user have a protocol signal generator whose output is a predetermined periodic waveform that completely specifies the transmission times for that user. This protocol signal \( s_i(t) \) for user \( i \) had period \( \tau_i \), took on value either zero or one for all \( t \), and obtained value one only over semi-open intervals whose lengths were integer multiples of the packet duration. Each user could emit packets only when its protocol signal took on value one. Otherwise, that user was required to be silent. Thus, predetermined periodic protocol signals were used to control access in this channel model, with each user having no knowledge of the protocol signals for other users.

For the above channel model, Massey and Mathys were able to determine the capacity region and the zero-error capacity region for both the unsynchronized and slot-synchronized cases. The capacity region is defined as the set of all joint user rates such
that it is possible to communicate with some arbitrarily small error probability at any joint rate inside the set, but it is impossible to do so at any joint rate outside this set. By the zero-error capacity region one means the joint rate region where zero-error probability is possible. It was shown that these four regions coincide and that throughput approaches $1/e$ packets/slot. In other words, the maximum throughput of this channel is limited to 36 percent of the line capacity. This value is notable in that it is also the maximum throughput of a slotted ALOHA duplex system [5].

Another model for a collision channel without feedback was considered over twenty years ago by Huber and Shah [6], who were interested in applications to alarm and telemetry systems. They looked at a system consisting of many peripheral transmitters and a single central receiver with unidirectional information flow. The transmitters would send short messages consisting of their own addresses and a small number of additional information bits. The transmitters had no way of recognizing whether the channel is busy or not and were totally independent from each other. The information flow would be carried over a single binary channel consisting of a radio link.

Huber and Shah addressed several important questions regarding the above-described channel. They investigated the transmitter repetition rate that should be selected for ensuring a maximum of correctly received messages, the effect of the average transmission rate on the behavior of the system, and the optimal strategy that each individual station should use when transmitting the message. They determined the number of transmissions for maximum data flow, and pointed out that a message will be received correctly with a probability of $1/e = 0.368$ when the system is optimized. Finally, they
postulated that some form of stochastic message distribution by each individual station is necessary in order to improve system performance.

1.3 SPECIFICS OF THE NEW CHANNEL

The model proposed by Massey and Mathys in [4] for a channel without feedback required each user to have a protocol signal generator allowing message transmission only during a time period determined by a generator. This feature of the model made it unusable for the application specified by IRS, Inc., since each user had to be able to initiate a transmission at any time. IRS, Inc. also required a higher probability of successful message transmission than would be possible with a single transmission in cases of anything other than an extremely lightly loaded channel (that is, a channel with low message arrival rate). One of the ways to achieve this is to introduce stochastically distributed message retransmissions into the channel protocol. This also rendered the channel model in [4] inapplicable and necessitated a random access technique with retransmissions that would meet all of the specified requirements. It is also worth noting that in [6] Huber and Shah were primarily concerned with determining the optimal average transmission rate which would maximize the (expected) total number of correctly received messages from all users during some observation period (which they considered to be the time interval during which all system users executed their transmission attempts). They made no attempt to quantitatively develop a retransmission strategy which would improve the chances of each user having at least one message received correctly for a situation where the observation period for that user does not perfectly coincide with observation
periods of all other users. Therefore, the development of a new random access protocol with retransmissions for one-way channels where observation (retransmission) periods for all users do not necessarily overlap, and incorporation of that protocol into an actual propagation environment became the key questions to be addressed by this dissertation.

With respect to the actual spread spectrum system implementation, the question of capturing a signal in the presence of simultaneous transmissions has recently received considerable attention from researchers investigating mobile radio systems [7]-[8]. However, the issue of what happens to a spread-spectrum signal that is being demodulated while another signal with an identical spreading code arrives at the receiver has not been thoroughly researched. This question is of central interest in the development of the channel model, and it served as a motivation for the collision study of the direct-sequence spread spectrum system utilizing the developed transmission strategy.

With respect to characterizing the propagation environment, path loss measurements have recently been made for signals around 900 MHz for several different wireless environments [9]-[10]. What had to be accomplished was tying in these results with the developed traffic model and studied receiver collision dynamics to arrive at a more precise theory of the channel.

CHAPTER 2. THEORY AND COMPUTER MODELING OF THE CHANNEL

As was stated in Chapter 1, the problem addressed here required the development of a random access strategy to be utilized in a home-shopping network application. This system would require users to transmit product requests over a wireless channel in response to TV advertisements. Thus, the focal point of this dissertation became the development of a probabilistic traffic model and strategy for the proposed configuration where a number of users can access the same physical channel (in this case, the same transmission frequency band) in a purely random manner in order to transmit messages to a common central receiving station. The goal was to achieve the required probability of successful message transmission in a situation in which the transmitting user will not get any status information from the receiving party. This was accomplished by introducing message retransmissions and subsequently randomizing their origination times.

2.1 PROBABILISTIC TRAFFIC MODEL OF THE CHANNEL UNDER ASSUMPTION FOR ANNIHILATION OF COLLIDING MESSAGES

2.1.1 Generation of initial messages

It is clear that one cannot predict when any given system user will originate a message transmission on a channel under consideration. This is because this message could be in response to any TV program or advertisement that is broadcast during a single day. However, once that message is transmitted, its subsequent retransmissions could be
structured in any given manner. Therefore, the statistics for the generation times of initial
messages and subsequent retransmissions will obviously differ. The arrival of initial
messages at the central station is a Poisson process which serves as a good approximation
in modeling the arrival of a large number of messages from totally uncorrelated sources
[11]. The total initial message arrival rate is then described by the parameter \( \lambda N \) (in
messages/sec), where \( \lambda \) is the average rate of initial messages transmitted per user and \( N \)
is the total number of users on the system. Thus, the probability \( p(k) \) of \( k \) original
(Poisson) arrivals in time interval \( t \) is given by [12]:

\[
p(k) = e^{-\lambda N t} \frac{(\lambda N t)^k}{k!}.
\]  

Expression (2-1) is the probability density function (pdf) of a Poisson distribution. It is
important to note that the arrival of initial messages at the central station with the
proposed channel will in fact satisfy the major condition of a Poisson process, namely that
the arrivals are memoryless: an arrival in one time interval of length \( t \) is independent of
arrivals in previous or future intervals. In other words, if a user has responded to a given
TV advertisement, he/she is no more or less likely to respond to future such
advertisements.

2.1.2 Generation of retransmissions

Upon completion of their initial message transmission, a user will enter their
retransmission period. It is during this time interval that all retransmissions of the initial
message from that user will take place. As was stated in [6], periodic repetition of the
same message cannot be employed since using identical periods for all transmitters would
mean that any coincidence which might fortuitously occur would be repeated again and
again. This would cause all transmitted messages to suffer collisions. Using different
periods for different transmitters would give an advantage to the transmitters with the
shortest periods, which is generally not desired. Therefore, some sort of stochastic
distribution of message retransmissions by each user is necessary. At this point, the
statistics of retransmissions can be addressed.

The time interval $T$ from the end of the original transmission of a message to the
end of its final retransmission is broken into $E$ identical sections, since there are $E$ such
retransmissions. Each retransmission can occur at any point within this time period.
However, if a retransmission is completed prior to the end of this time interval, a new
retransmission will not be allowed to occur before the start of the next time interval.
Since each message is $\tau$ seconds in duration, the maximum time interval between
successive retransmissions is given by $\frac{T}{E} - \tau$. This time interval between successive
retransmissions for any user can be modeled as a uniformly distributed random variable on
the interval $[0, \frac{T}{E} - \tau]$. Therefore, each retransmission delay has a probability density
function $f(t)$ given by:

$$f(t) = \begin{cases} 
\frac{1}{\frac{E}{T} - \tau} & \text{for } 0 < t < \frac{T}{E} - \tau \\
0 & \text{otherwise}
\end{cases}$$

Uniform distribution of retransmissions was selected for a number of reasons. First of all,
uniformly distributed events can be easily implemented in practice using random number
generator circuits. Secondly, as was pointed out in [6], uniform message distribution
produces significantly better results than the other common technique, namely, exponential distribution of the time stretches between the messages.

Having outlined the criteria for the transmission of initial messages and retransmissions by each user with the proposed channel, the message collision process can now be addressed.

### 2.1.3 Message collision dynamics

The probability of a successful transmission of a given message, denoted $P$, is the probability that at least one of $(E + 1)$ transmissions of that message was successful (that is, transmitted without collision with any other messages), where $E$ is again the number of retransmissions for each message. This can be expressed as follows:

$$P = 1 - P_{\text{all (E + 1) transmissions of a given message suffered a collision}}$$

$$= 1 - P_1$$

(2-3)

with

$$P_1 = P_{\text{original transmission failed}}\times P_{\text{all retransmissions failed}}.$$  

(2-4)

A transmission by a given user can collide with either an original transmission or a retransmission from any other user. As was stated in Section 2.1.1, the statistics for the generation of original transmissions and subsequent retransmissions differ. However, every transmission encounters identical conditions on the channel. In other words, the message generation processes for all users do not vary with time. Therefore, we have:

$$P_{\text{original transmission failed}} = P_{\text{any one of E retransmissions failed}}.$$  

(2-5)
and

\[ P\{\text{all retransmissions failed}\} = \left[P\{\text{any one of } E \text{ retransmissions failed}\}\right]^E. \quad (2-6) \]

Thus, the term \( P_1 \) (the probability that all \( E + 1 \) transmissions of a given message suffered a collision) can be expressed as:

\[ P_1 = P_2^{E+1}. \quad (2-7) \]

where \( P_2 \) represents the probability that a collision occurred. In the subsequent analysis, only the worst-case scenario (one that will maximize \( P_2 \)) will be considered.

Every message, sent by the user of interest, creates a ‘collision window’ of fixed duration equal to \( 2\tau \). If any other message arrives at the receiver during this period, a collision will take place. The probability that initial messages are present on the channel during \( 2\tau \) is just the probability that at least one Poisson arrival occurred in \( 2\tau \), and it is given by:

\[ P\{\text{at least one arrival in } 2\tau\} = 1 - e^{-2\lambda N \tau}. \quad (2-8) \]

From the standpoint of collisions, all Poisson messages have equal probability of suffering a collision with some given message. This probability depends only on the message length and on the number of messages generated per unit of time. This implies that any Poisson message is equally likely to undergo a collision with a given message. Hence, in this case, Poisson message distribution reduces to a uniform one from the collisional viewpoint.

The following important point should be noted. Even though the arrival of initial messages obeys Poisson statistics, we do not have a pure Poisson process. Instead, Poisson initial messages trigger retransmissions by each user. In order to represent the probability of collision with original messages in this compound system, an effective
collision parameter $\tau_{\text{eff}}$ is introduced and will be discussed later in Section 2.1.4. This allows us to treat, from collisional dynamics point of view, the initial message generation and retransmission processes independently, although the retransmission process for each user is obviously conditioned on an original message having been transmitted. With this modification, the expression (2-8) for the probability of at least one initial message having been generated during the collision window is given by:

$$P\{\text{at least one initial message generated during the collision window}\} = 1 - e^{-2\lambda N \tau_{\text{eff}}}.$$  \hspace{1cm} (2-9)

When a given user transmits a message, there are three possible combinations of messages from all other users that can be present on the channel during the interval $2\tau$ (and therefore cause a collision):

1. There can be only initial messages from other users and no retransmissions on the channel during $2\tau$ (the probability of which will be denoted $P_3$).

2. There can be only retransmissions from other users and no initial messages on the channel during $2\tau$ (the probability of which will be denoted $P_4$).

3. The channel can contain both initial messages and retransmissions during this interval of time (the probability of which will be denoted $P_5$).

Therefore, the probability of collision $P_2$ can be expressed as the sum of the above probabilities as follows:

$$P_2 = P_3 + P_4 + P_5.$$  \hspace{1cm} (2-10)
In order to determine the expression for $P_3$ in (2-10), the problem of collision with retransmitted messages must be considered. There are two situations that will prevent retransmissions from being on the channel during $2\tau$:

a. No users sent an initial message during a specified interval of time ($T$) before $2\tau$ (and therefore could not possibly send a retransmission during $2\tau$) or

b. None of the users eligible to send a retransmission during $2\tau$ (that is, those who sent an initial message during $T$ before $2\tau$) did so.

The probability of the first case taking place is given by $e^{-\lambda NT}$ (the probability of no Poisson arrivals in $T$). Since the retransmission interval for each user is uniformly distributed, the probability that any one user who is eligible to retransmit did not do so during $2\tau$ is expressed as:

$$\int_{2\tau}^{T-E\tau} \frac{1}{E - \tau} dt = 1 - \frac{2E\tau}{T - E\tau}.$$  \hspace{1cm} (2-11)

The expression above is just the probability that the retransmission was originated somewhere else in the interval $\frac{T}{E} - \tau$ reserved for each retransmission, and not during $2\tau$. Thus, the probability of the second scenario (that is, the case where none of the users who sent an initial message during $T$ retransmitted their messages during $2\tau$) is given by

$$(1 - e^{-\lambda NT})(1 - \frac{2E\tau}{T - E\tau})^E.$$ The exponent $E$ in the previous expression arises because each user has $E$ possible retransmissions for any given message. Putting together all of the above yields the expression for $P_3$ in (2-10):
\[ P_3 = (1 - e^{-2\lambda NT_{\text{eff}}})[e^{-\lambda NT}(1 - \frac{2E\tau}{T - E\tau})^E]. \] (2-12)

Next, the expression for probability \( P_4 \) in (2-10) must be derived. That is the probability that only retransmissions from other users and no initial messages are present on the channel during the interval \( 2\tau \), whereby causing collisions with the message of interest. In order for retransmissions to be present on the channel during the collision window, at least one user must send an original message during \( T \) and retransmit that message during \( 2\tau \). The probability of that scenario taking place is given by \((1 - e^{-\lambda NT})(1 - (1 - \frac{2E\tau}{T - E\tau})^E)\). Multiplying this expression by the probability \( e^{-2\lambda NT_{\text{eff}}} \) that no initial messages are present on the channel during the collision window, we arrive at the final expression for \( P_4 \):

\[ P_4 = (1 - e^{-\lambda NT})(1 - (1 - \frac{2E\tau}{T - E\tau})^E) e^{-2\lambda NT_{\text{eff}}}. \] (2-13)

Finally, the expression for \( P_5 \) in (2-10) - the probability of the channel containing both original messages and retransmissions during the collision window - must be determined. This expression is similar to expression (2-13) and is given by:

\[ P_5 = (1 - e^{-\lambda NT})(1 - (1 - \frac{2E\tau}{T - E\tau})^E)(1 - e^{-2\lambda NT_{\text{eff}}}). \] (2-14)

Using (2-12), (2-13), and (2-14) in (2-10) and simplifying yields the final expression for the probability of collision \( P_2 \):

\[ P_2 = (1 - e^{-2\lambda NT_{\text{eff}}}) + (1 - e^{-\lambda NT})(1 - (1 - \frac{2E\tau}{T - E\tau})^E) e^{-2\lambda NT_{\text{eff}}}. \] (2-15)

Using (2-15) in (2-7) leads to the expression for the probability \( P_1 \) that a given message was not transmitted successfully:
Finally, using (2-16) in (2-3) gives the expression for the probability of successful transmission of a message using the proposed algorithm:

\[
P_1 = \left[ (1 - e^{-2\lambda N t_{eff}}) + (1 - e^{-\lambda NT})(1 - (1 - \frac{2E\tau}{T - E\tau})^E)e^{-2\lambda N t_{eff}} \right]^{E+1}.
\]  

(2-16)

At this point, it is worth reemphasizing that expression (2-17) represents the probability that a user has successfully transmitted at least one of the \((E + 1)\) transmissions of a given message. This expression is valid for all system users since the proposed algorithm gives no preference to any transmitter.

2.1.4 Effective collision parameter

As was pointed out in the previous section, the retransmission process for each user with the proposed algorithm is conditioned on the origination of an initial message by that user. At any given instant in time, there may be users who are in their retransmission periods as well as users who are sending their initial transmissions. Thus, the channel has concurrent interdependent processes taking place: Poisson distributed initial message arrivals and uniformly distributed message retransmissions. The introduction of an effective collision parameter \(t_{eff}\) serves the purpose of describing this situation. It also allows us properly to treat the initial message generation and retransmission processes in this compound system.

In order to determine the expression for \(t_{eff}\), we assume that each message and all of its retransmissions are Poisson distributed and compare the results to the actual
transmission model. In other words, if all \((E+1)\) arrivals of every message followed Poisson statistics, the probability that no such arrivals would take place during the collision window is expressed by \(e^{-2\lambda N (E+1)\tau}\). In actuality, the probability that no arrivals of a initial message and any retransmission take place during the collision interval is given by \(e^{-2\lambda N \tau_{eff}} \left[ e^{-\lambda N T} + (1 - e^{-\lambda N T}) \left(1 - \frac{2E\tau}{T - E\tau}\right) \right]\). Equating the last two expressions yields the formula for \(\tau_{eff}\):

\[
\tau_{eff} = (E + 1)\tau + \frac{1}{2\lambda N} \ln \left[ e^{-\lambda N T} + (1 - e^{-\lambda N T}) \left(1 - \frac{2E\tau}{T - E\tau}\right) \right].
\]

(2-18)

Therefore, in order to evaluate the expression (2-17) for the probability of successful transmission of a message, one must first determine the effective collision parameter \(\tau_{eff}\) using (2-18). Subsequently, that value should be substituted into (2-17).

### 2.2 OVERVIEW OF THEORETICAL RESULTS AND THEIR APPLICATION

#### 2.2.1 Effect of initial message arrival rate on system performance

Upon closer examination of expressions (2-17) and (2-18), it is clear that the probability of successful message transmission is a function of four major parameters: the initial message arrival rate \(\lambda N\), the message length \(\tau\), the number of retransmissions \(E\) for each message, and the total retransmission interval \(T\). A system designer developing a channel using the proposed traffic model would be able to select all of the above parameters with the exception of the initial message arrival rate. It is therefore imperative
to study the channel behavior (namely, message success probability) for various arrival rates. Using some fixed values for the message length and the retransmission interval, the optimal number of retransmissions can be determined for any given arrival rate. This is accomplished by first calculating, using (2-18), the effective collision parameter $\tau_{\text{eff}}$ for a given rate and the number of retransmissions. Then, this value is substituted into (2-17) to determine the probability of successful message transmission for a specified number of retransmissions and a given rate of initial message arrivals on that channel.

The channel behavior is analyzed for the case of $\tau = 4.6$ msec (corresponding to a 184-bit message transmitted at 40 Kbits/sec) and $T = 30$ sec. These numbers are the parameters of a system proposed for IRS, Inc., which will utilize the above mentioned retransmission strategy over a wireless direct sequence spread spectrum channel. The system will incorporate 4 independent channels, serving approximately 100 users per channel, in a cell with radius of roughly 0.5 mile. The above-mentioned retransmission interval is selected due to the fact that this system will be designed for users responding to TV advertisements lasting an average of 30 seconds. This corresponds to a message arrival rate of 4 messages per second or less. The system development is currently underway in the Center for Wireless Telecommunications at Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA.

Figure 2.1 shows the plots of the calculated message error rate (MER) versus the number of retransmissions on a semi-log scale for initial message arrival rates of 1, 2, and 4 messages/sec, respectively. MER is equal to $P_1$, where $P_1$ is defined in (2-16). These rates are selected because they fall in the range of expected arrival rates for the specified application of the channel. Thus, they represent the most adverse conditions that can be
Figure 2.1 Message error rate (MER) versus number of retransmissions (E) for message rates equal to 4 messages/sec, 2 messages/sec, and 1 message/sec.
expected to occur on the channel. It is obvious that the primary goal here is to achieve a required reliability of transmitting uncorrupted messages using minimum number of retransmissions.

The plots in Figure 2.1 illustrate several important points regarding the proposed channel model. First of all, the retransmission process significantly improves channel reliability compared to employing just a single transmission of each message. For example, given the initial message arrival rate of 4 messages/sec, the value of MER is approximately $2 \cdot 10^{-2}$ (or about 2 completely lost messages out of every 100 messages transmitted) when the retransmission process is not employed. This value decreases to about $1 \cdot 10^{-3}$ (corresponding to 1 out of every 100,000 transmitted messages being completely wiped out) when the retransmission process of the proposed channel model is utilized generating 10 retransmissions in 30 seconds. The improvement is even more dramatic when the loading on the channel is lighter (that is, when the initial message arrival rate is lower).

Secondly, if the heaviest expected initial message arrival rate is known, a system designer using the proposed traffic model can determine if the required channel reliability can be achieved for a given message length and retransmission interval. If this is possible, the minimum number of retransmissions necessary to achieve this reliability can also be determined. A maximum acceptable MER value of $1 \cdot 10^{-5}$ is often quoted for a communication channel. Then, as can be seen from Figure 2.1, the message success probability with this scheme will exceed the minimum required value ($1 - 10^{-5}$) by using:

1. 3 repetitions, if the initial message rate is equal to 1 message/sec,
2. 4 repetitions, if the initial message rate is equal to 2 messages/sec, or

3. 10 repetitions, if the initial message rate is equal to 4 messages/sec.

2.2.2 Determination of traffic model-based channel parameters

In the analysis of the previous section, it was assumed that the retransmission interval is given and the initial message arrival rate is known. Often, this is not the case, as a major question facing a system designer would be the trade-offs that exist between the length of the retransmission interval, the minimum number of retransmissions, and the maximum initial message arrival rate that the channel will be able to support and still meet the required reliability. A MATHCAD module, based on expressions (2-16) and (2-18), was developed to calculate the maximum initial message arrival rate that a channel with given retransmission interval and number of retransmissions could support (see Appendix A). Table 2.1 and Figure 2.2 illustrate these results for message length of $\tau = 4.6$ msec (same as used previously) and MER of no greater than $1 \cdot 10^{-5}$.

Several important points should be noted. As the retransmission period is increased, the maximum throughput that can be supported by the channel also increases. However, lengthening the retransmission period has progressively smaller and smaller effect on the throughput. For example, as the retransmission interval is doubled from 5 to 10 seconds, the maximum supportable initial message arrival rate increases from 2.68 messages/sec to 3.45 messages/sec, an increase of 28 percent. Further doubling the retransmission period to 20 seconds increases the maximum throughput to 3.94 messages/sec, an increase of only about 14.5 percent. In addition, the number of
<table>
<thead>
<tr>
<th>NUMBER OF RETRANSMISSIONS</th>
<th>RETRANSMISSION INTERVAL (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>0.67</td>
</tr>
<tr>
<td>3</td>
<td>1.28</td>
</tr>
<tr>
<td>4</td>
<td>1.81</td>
</tr>
<tr>
<td>5</td>
<td>2.21</td>
</tr>
<tr>
<td>6</td>
<td>2.47</td>
</tr>
<tr>
<td>7</td>
<td>2.62</td>
</tr>
<tr>
<td>8</td>
<td>2.68</td>
</tr>
<tr>
<td>9</td>
<td>2.67</td>
</tr>
<tr>
<td>10</td>
<td>2.61</td>
</tr>
<tr>
<td>11</td>
<td>2.51</td>
</tr>
<tr>
<td>12</td>
<td>2.37</td>
</tr>
<tr>
<td>13</td>
<td>2.22</td>
</tr>
<tr>
<td>14</td>
<td>2.04</td>
</tr>
<tr>
<td>15</td>
<td>1.84</td>
</tr>
</tbody>
</table>

Table 2.1 Maximum supportable arrival rate (in messages/sec) with message length $\tau = 4.6$ msec for message error rate of less than $1 \times 10^{-5}$ (depending on retransmission interval and number of retransmissions).
Figure 2.2 Maximum throughput versus number of retransmissions for message error rate of less than $1 \times 10^{-5}$.
retransmissions required to achieve the maximum throughput for any given retransmission interval becomes progressively larger with the increased retransmission period. This is because for longer retransmission intervals, significantly more retransmissions from all users are needed to saturate the channel. This effect is particularly pronounced when the retransmission period is 5 seconds. In this case, introducing more than the optimal number of retransmissions for each user quickly saturates the channel, thus degrading its performance.

2.2.3 Simplification of theoretical results

As was shown in Section 2.1.4, the effective collision parameter \( \tau_{\text{eff}} \) is given by expression (2-18). Thus, it contains two major terms: the first term, \( (E + 1)\tau \), arising from the introduction of retransmissions into the channel model, and the second term, 
\[
\frac{1}{2\lambda N} \ln \left[ e^{-\lambda NT} + (1 - e^{-\lambda NT})(1 - \frac{2E\tau}{T - E\tau}) \right],
\]
which is the correction factor used to account for interdependence of initial message generation and message retransmission processes.

The contributions of each of these two major terms in \( \tau_{\text{eff}} \) are analyzed for the case of \( \tau = 4.6 \) msec and \( T = 30 \) sec (the same parameters as were used to generate Figure 2.1). Figure 2.3 shows both \( \tau_{\text{eff}} \) from (2-18) and the first term of (2-18), \( (E + 1)\tau \equiv \tau_{\text{app}} \), as functions of the number of retransmissions \( E \) for original message arrival rate of 1 message/sec. It is evident from Figure 2.3 that the second term in (2-18) has a very small contribution given any practical number of retransmissions and expected arrival
Figure 2.3  Effective message lengths versus number of retransmissions for arrival rate of 1 message/sec.
rates. Thus, for any practically realizable, small number of retransmissions, the effective message length $\tau_{\text{eff}}$ can be approximated by:

$$\tau_{\text{eff}} = (E + 1)\tau.$$  \hspace{1cm} (2-19)

Using (2-19) in (2-17) leads to the simplified expression for the probability of successful transmission of a given message:

$$P = 1 - \left[ (1 - e^{-2\lambda N(E+1)\tau}) + (1 - e^{-\lambda NT})(1 - \left(1 - \frac{2E\tau}{T-E\tau}\right)^E) e^{-2\lambda N(E+1)\tau} \right]^{E+1}. \hspace{1cm} (2-20)$$

which is applicable to channels with small number of retransmissions.

Finally, it should be noted that if no retransmissions take place (that is, $E = 0$), $\tau_{\text{eff}}$ in (2-18) becomes simply $\tau$, and expression (2-17) for the probability of successful transmission of a message becomes $e^{-2\lambda N\tau}$, as expected, which is the probability of no Poisson arrivals during the collision window $2\tau$.

2.2.4 Channel utilization

In order to describe the performance of a given communication channel, it is often desirable to know the extent of channel utilization that is achieved by a certain message arrival rate. A parameter that is used for this purpose is the normalized channel throughput $S$, the fraction of time (fraction of an Erlang) a channel is utilized [2]. One Erlang represents the amount of traffic carried by a channel that is completely occupied. For example, a radio channel that is occupied for thirty minutes during an hour carries 0.5 Erlangs of traffic. The normalized throughput is given as the total offered load $G$ times the probability of successful transmission, i.e.:
\[ S = G \times P\{\text{no collision}\}. \] 

(2-21)

In determining the utilization of the proposed channel corresponding to a given message arrival rate, it is important to remember that the total traffic on the channel will consist of newly transmitted Poisson messages plus uniformly distributed retransmissions. Under these conditions, the total offered load is given by:

\[ G = \lambda N(E + 1)\tau. \] 

(2-22)

with all the terms retaining their previous meanings. Expression (2-22) makes use of (2-19) which is valid for practically implementable, small number of retransmissions. It is worth noting that \( 1/\tau \) represents the channel capacity in units of messages/sec transmitted. Thus, the total offered load on the channel is simply \( \lambda N(E + 1) \), the aggregate rate of messages attempting transmission over the channel (newly generated plus retransmitted ones), divided by the maximum channel capacity \( 1/\tau \). The probability that a message transmission does not suffer a collision in the proposed channel is expressed as:

\[ P\{\text{no collision}\} = 1 - P_2. \] 

(2-23)

where \( P_2 \) is the probability of collision from expression (2-15) with the effective collision parameter \( \tau_{\text{eff}} \) given by (2-19). Putting together all of the above leads to the following expression for the normalized channel throughput:

\[
S = \lambda N(E + 1)\tau \\
\times \left[ 1 - \left( (1 - e^{-2\lambda N(E+1)\tau}) + (1 - e^{-\lambda NT})(1 - \frac{2E\tau}{T - E\tau})e^{-2\lambda N(E+1)\tau} \right) \right] \] 

(2-24)

Expression (2-24) can be used to analyze channel utilization corresponding to initial message arrival rates and other channel parameters considered in Figure 2.1. It was seen that, for example, 10 retransmissions were necessary to achieve MER of \( 1 \cdot 10^{-5} \) for
initial message arrival rate of $\lambda N = 4$ messages/sec, message duration $\tau = 4.6$ msec, and retransmission period $T = 30$ sec. Under these conditions, the normalized channel throughput will be approximately 13.1 percent. Thus, when the initial message arrival rate is 4 messages/sec and 10 retransmissions are employed, the channel is utilized at 13.1 percent of its maximum capacity of 40 Kbits/sec.

This value of channel throughput is relatively low, which may seem alarming at first. However, it should be noted that, based on the maximum projected number of 250 users per channel per cell, this arrival rate would correspond to approximately 50 percent of the user population transmitting during any given 30-second TV advertisement. In other words, it would represent some of the heaviest loading that can be expected on the proposed channel. It is also worth mentioning that the goal of this transmission strategy is to ensure that a user will successfully complete at least one transmission attempt, and not necessarily every single transmission. In addition, the primary application of this channel is to carry highly bursty traffic. Under these conditions, the requirement of low message error rates leads to relatively low channel utilization.

2.3 COMPUTER SIMULATION OF THE CHANNEL

In order to validate the probabilistic traffic model presented above, a program to simulate channel behavior was jointly written with CWT graduate students Steven Franks and Matt Kurtin. The simulator generates messages for the channel, and, for each message, its retransmissions. All of the traffic is analyzed for collisions and errors, and the results are compared to the analytical model.
The simulator has three main parts: message generation, retransmission generation, and channel analysis. Message arrival times are generated using a Poisson distribution based on two parameters: the duration of simulation (its run time) and average message density. The origination time of each message is determined by exponentially distributing interarrival times based on the average original message density. This approach has been shown to produce a Poisson arrival process [13]. In addition, all original messages are assigned a unique integer to identify them.

The simulator’s algorithm for generating retransmissions is as follows. First, the total retransmission period is divided by the number of retransmissions, yielding equal subintervals. Each retransmission occurs within its respective subinterval at a time determined using a random number generator. This yields a uniform distribution of message retransmission times. Each retransmission of a given message has the same integer identification number as the original message.

A collision occurs anytime a new message or retransmission begins while another message is still being transmitted. When this happens, both messages are lost. An error occurs if a message and all of its subsequent retransmissions experience collisions. The error probability is the number of errors divided by the number of original messages. If an error occurs, the origination time of a message is considered to be the time of the error. These times can then be examined to verify that errors occur with a reasonable distribution within the simulation duration.

Due to the nature of the simulation process - start generating messages, do so for a specified time period, and then stop - messages at the beginning and the end of the simulation period do not experience the same amount of traffic as those messages in the
middle. In order to avoid simulation results being skewed by this, all messages originating during these periods are ignored when computing collisions and errors.

The simulator places all generated messages into a list sorted by message generation time. Once all messages have been inserted in this list, the simulator analyzes message transmissions and retransmissions to find error and collision probabilities as well as the complete message loss (the probability that all $E+1$ transmissions of a given message suffered collisions) as a function of time.

### 2.4 COMPARISON OF SIMULATED AND THEORETICAL RESULTS

In order to verify the validity of the closed-form solution for the MER derived in Section 2.1, computer simulation of the channel was performed in accordance with the stipulations outlined in the previous section. The parameters used for the simulation were identical to those used in Figure 2.1 with the initial message arrival rate being 4 messages/sec.

Upon closer examination of the theoretical curve for $\lambda N = 4$ messages/sec in Figure 2.1, it is obvious that MER of less than 1 in 100,000 messages can be expected when the number of retransmissions is in the range of 10 to 15. Therefore, the total number of simulated messages has to be at least one order of magnitude greater than the minimum number required, or approximately 1,000,000 messages. Towards that end, the simulation was repeated 17 times as the number of retransmissions was varied from 1 to 15. Each time, approximately 60,000 new messages were generated for a total of about 1,020,000 such messages. The simulation file containing all of the simulation parameters
can be found in Appendix B. For each number of retransmissions, any given simulation run resulted in a proportion of a number of lost messages to total number of messages generated. Subsequently, the mean of all 17 proportions was calculated for each number of retransmissions. Figure 2.4 shows the comparison between the theoretically calculated MER values and the means of simulated results. An important point to be noted from Figure 2.4 is that there is good agreement between theoretical and simulated results. In addition, the simulated MER is never greater than its corresponding theoretical value. In other words, the theoretical model never underestimates the MER value for any given number of retransmissions, which is of particular importance to a system designer implementing such a traffic model.

At this juncture, one major point needs to be addressed with this or any other simulation - it is the issue of just how believable are these simulation results. The concept of a confidence interval, particularly the 95% confidence interval, is often used to gain a measure of accuracy of a given simulation [14]. In this case, we are after an interval that is 95% certain, based on simulation results, to contain the true MER. However, as was stated previously, we have a relatively small sample of 17 observations for each number of retransmissions. In order to increase the accuracy of the calculated 95% confidence interval, the well-known statistical technique of bootstrapping can be employed [15].

The basic idea behind the bootstrap is rather simple. Suppose that a random sample of \( n \) values \( X_1, X_2, \ldots, X_n \) is taken from a population and used to estimate a parameter \( \Theta \), which could be, for example, the variance or a measure of skewness. Then the \( n \) observed values are regarded as the best indication of the population distribution of \( X \). The empirical cumulative distribution function (ecdf) from this set of observations is
Figure 2.4  Calculated (solid line) and simulated (points) MER versus number of retransmissions for arrival rate equal to 4 messages/sec.
used to represent the underlying distribution of the entire population. As the sampling
distribution of the estimator $\hat{\Theta}$ is unknown, the ecdf is employed to approximate this
distribution. The sampling variation in the estimator $\hat{\Theta}$ of $\Theta$ is obtained by taking
random samples of size $n$ from this approximate distribution. In order to induce
variability, the random sampling must be performed with replacement. Samples taken
from this approximated distribution of $X$ are called bootstrap samples, and each sample
provides a bootstrap estimate of $\Theta$. The simplest way to determine the distribution that
these estimates have is to take a large number of random samples from the distribution of
equally likely $X$-values. Bootstrap theory states that as the number of random samples
increases, the distribution of the bootstrap estimates tends toward the underlying
population distribution.

In this case, we have a sample of 17 MER values and would like to estimate the
mean MER. An SAS (Statistical Analysis Software) module was written (see Appendix
C) that generates 1000 bootstrap samples (each having 17 values) for any given number of
retransmissions. It then calculates a bootstrap estimate of mean MER for each of 1000
samples. Finally, it determines the smallest interval containing 950 consecutively ordered
bootstrapped mean MER estimates, which then becomes the 95% confidence interval.
Table 2.2 and Figure 2.5 summarize the results. They contain, for each number of
retransmissions from 1 to 15, the theoretical MER value based on expression (2-16), the
mean simulated MER value, as well as the upper and lower limits of the bootstrapped 95%
confidence interval.

The most important result to be noted from Table 2.2 and Figure 2.5 is that the
theoretically calculated MER falls within the 95% confidence interval of the corresponding
<table>
<thead>
<tr>
<th>NUMBER OF RETRANSMISSIONS</th>
<th>THEORETICAL MER</th>
<th>MEAN SIMULATED MER</th>
<th>BOOTSTRAPPED 95% CONFIDENCE INTERVAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower bound</td>
</tr>
<tr>
<td>1</td>
<td>5.035E-3</td>
<td>4.806E-3</td>
<td>4.716E-3</td>
</tr>
<tr>
<td>2</td>
<td>1.16E-3</td>
<td>1.152E-3</td>
<td>1.086E-3</td>
</tr>
<tr>
<td>3</td>
<td>3.676E-4</td>
<td>3.352E-4</td>
<td>2.997E-4</td>
</tr>
<tr>
<td>4</td>
<td>1.467E-4</td>
<td>1.417E-4</td>
<td>1.279E-4</td>
</tr>
<tr>
<td>5</td>
<td>7.005E-5</td>
<td>6.606E-5</td>
<td>5.415E-5</td>
</tr>
<tr>
<td>6</td>
<td>3.863E-5</td>
<td>2.758E-5</td>
<td>2.16E-5</td>
</tr>
<tr>
<td>7</td>
<td>2.401E-5</td>
<td>1.674E-5</td>
<td>9.827E-6</td>
</tr>
<tr>
<td>8</td>
<td>1.651E-5</td>
<td>8.867E-6</td>
<td>4.913E-6</td>
</tr>
<tr>
<td>9</td>
<td>1.237E-5</td>
<td>1.083E-5</td>
<td>6.883E-6</td>
</tr>
<tr>
<td>10</td>
<td>9.996E-6</td>
<td>6.902E-6</td>
<td>2.947E-6</td>
</tr>
<tr>
<td>11</td>
<td>8.616E-6</td>
<td>4.942E-6</td>
<td>9.91E-7</td>
</tr>
<tr>
<td>12</td>
<td>7.862E-6</td>
<td>4.923E-6</td>
<td>9.827E-7</td>
</tr>
<tr>
<td>13</td>
<td>7.542E-6</td>
<td>5.904E-6</td>
<td>1.965E-6</td>
</tr>
<tr>
<td>14</td>
<td>7.562E-6</td>
<td>5.93E-6</td>
<td>1.964E-6</td>
</tr>
<tr>
<td>15</td>
<td>7.882E-6</td>
<td>4.924E-6</td>
<td>2.087E-6</td>
</tr>
</tbody>
</table>

Table 2.2  Summary of theoretical and simulated results ($\lambda N = 4$ messages/sec, $\tau = 4.6$ msec, $T = 30$ sec).
Figure 2.5  Calculated (solid line) and simulated (points) MER versus number of retransmissions for arrival rate equal to 4 messages/sec, with error bars showing 95% confidence intervals for simulated results.
simulation in almost all cases, thus confirming the validity of the theoretically developed traffic model. For the three cases that this is not true (namely for 1, 6, and 8 retransmissions), it is just outside the upper limit of the corresponding confidence interval. In other words, the theoretical model slightly overestimates the MER in these instances (the deviation from the upper limit of the confidence interval is never greater than about 12%). This is not of a particular concern, and likely occurs because the number of simulations is only one order of magnitude larger than the minimum required.
CHAPTER 3. EXPERIMENTAL INVESTIGATION ON SURVIVAL OF COLLIDING MESSAGES

As was stated previously, the system utilizing the access strategy proposed in Chapter 2 will operate in a direct-sequence spread spectrum (DS-SS) environment. Employing spread spectrum communication has many advantages: selective addressing capability, multiple access by code division multiplexing, low-density power spectra for signal hiding, message screening from eavesdroppers, high-resolution ranging, and interference rejection. These properties come about as a consequence of the coded signal format and the broad bandwidth that results.

The property of spread spectrum systems that is of particular interest for this dissertation is their interference rejection capability. It leads us to believe that all DS-SS messages colliding at the receiver need not be annihilated (as was assumed in the traffic model developed in the previous chapter). Therefore, it is possible that system performance may even surpass theoretical predictions. The desire to quantify this improvement served as a major motivation for the collision study which will subsequently be described in full detail. However, before we can proceed with this task, the basic operating principles of spread spectrum systems and their features that allow interference rejection need to be addressed.
3.1 SPREAD SPECTRUM TECHNIQUES

3.1.1 Why are spread spectrum communications used?

A spread spectrum system is one that transmits messages using bandwidth much wider than the minimum required by the information being sent. Spreading messages into wider bandwidths is accomplished by modulating them with a wideband encoding signal. The basis of spread spectrum technology is expressed by C. E. Shannon in the form of channel capacity [16]:

\[ C = W \log_2 \left(1 + \frac{S}{N}\right). \]  \hspace{1cm} (3-1)

where \( C \) is the capacity in bits per second, \( W \) is the bandwidth in hertz, \( N \) is the noise power, and \( S \) is the signal power. Equation (3-1) shows the relationship between the ability of a channel to transfer error-free information, the signal-to-noise ratio existing in the channel, and the bandwidth used to transmit the information. For small signal-to-noise ratios, say \( \leq 0.1 \) (as would be desired in an antijam system, for example), expression (3-1) can be approximated by:

\[ W = \frac{NC}{1.44S}. \]  \hspace{1cm} (3-2)

by employing logarithmic expansion. Thus, it is obvious that information-error rate, which is inversely proportional to the signal-to-noise ratio, can be lowered by increasing the bandwidth used to transfer the information. Therefore, using spread spectrum is particularly desirable for applications requiring low information-error rates.
3.1.2 Direct sequence systems

Direct sequence (or, to be more exact, directly carrier-modulated, code sequence modulation) systems are the best known and most widely used spread spectrum systems. Direct sequence modulation is simply modulation of a carrier by a code sequence. This code sequence possesses a bandwidth much larger than that of a message signal and is called the spreading signal. It is worth noting that the information itself may be embedded in the spreading signal by several methods. The most common is that of adding the information to the spectrum-spreading code before its use for spreading modulation.

Typically, code sequence modulation can assume a number of different modulation formats. It may be AM (amplitude modulation), FM (frequency modulation), or any other amplitude- or angle-modulation form. Very common, however, is binary phase shift keying (BPSK), where the phase of a constant amplitude carrier signal is switched between two values according to the two possible signals \( m_1 \) and \( m_2 \) corresponding to binary 1 and 0, respectively. Normally, the two phases are separated by 180°. BPSK uses coherent or synchronous demodulation, which requires that information about the phase and frequency of the carrier be available at the receiver. A power spectrum typical of this signal format takes the form of a \( \text{sinc} \left( \frac{\sin x}{x} \right) \) squared function in the frequency domain. The main lobe of the sinc squared function defines the transmission bandwidth of the spread signal. The sinc function in the frequency domain appears as a rectangular pulse in the time domain, and the sinc squared function appears as a triangular pulse.

Another modulation technique commonly used in DS-SS systems is differential phase shift keying (DPSK). DPSK is a noncoherent form of phase shift keying which
avoids the need for a coherent reference signal at the receiver. Noncoherent receivers are easy and cheap to build, and hence are widely used in wireless communications. Unlike BPSK, DPSK uses the change in state of the modulating voltage, and not the state itself, to determine the transmitted phase. In DPSK systems, the input binary sequence is first differentially encoded and then modulated using a BPSK modulator. The differentially encoded sequence \( \{d_k\} \) is generated from the input binary sequence \( \{m_k\} \) by complementing the modulo-2 sum of \( m_k \) and \( d_{k-1} \). The effect is to leave the symbol \( d_k \) unchanged from the previous symbol if the incoming binary symbol \( m_k \) is 1, and to toggle \( d_k \) if \( m_k \) is 0.

The purposes for using codes in spread spectrum communication systems are to offer greater protection against interference, secure messages for privacy, and reduce the effect of interference and noise in a communication channel. The code “spreads” a message into a wider transmission bandwidth which, as previously described, improves the signal-to-interference characteristics of the channel. Pseudonoise (PN) codes or pseudorandom codes are most often used in spread spectrum systems. Their name is derived from the fact that these codes appear random but can be reproduced in a deterministic manner by intended receivers. The PN sequence controls a spread spectrum communication system by acting as a multiplier for the baseband data. Pulses of the PN waveform are called chips. The code clock determines the chipping rate or the rate at which these chips are generated. The multiplication by the PN waveform spreads the baseband data into a wider baseband data stream at the chipping rate. Modulation of the RF carrier by this stream produces the double sideband spectrum. At the receiver,
multiplication by the locally generated PN code effectively despreads the message. The received spread spectrum signal for a single user can be expressed as [17]:

\[ S_{ss}(t) = \sqrt{\frac{2E_s}{T_s}} m(t) p(t) \cos(2\pi f_c t + \theta). \quad (3-3) \]

where \( m(t) \) is the data sequence, \( p(t) \) is the PN spreading sequence, \( f_c \) is the carrier frequency, and \( \theta \) is the carrier phase angle at \( t = 0 \). The data waveform is a time sequence of nonoverlapping rectangular pulses, each of which has an amplitude equal to +1 or -1. Each symbol in \( m(t) \) represents a data symbol with duration \( T_s \) and energy \( E_s \). Each pulse in \( p(t) \) represents a chip, and is usually rectangular with an amplitude equal to +1 or -1 and a duration of \( T_c \). The transitions of the data symbols and chips coincide such that the ratio \( T_s \) to \( T_c \) is an integer. If \( B_{ss} \) is the bandwidth of \( S_{ss}(t) \), and \( B \) is the bandwidth of \( m(t)\cos(2\pi f_c t) \), the spreading due to \( p(t) \) gives \( B_{ss} >> B \).

Assuming that code synchronization has been achieved at the receiver (code synchronization is of particular importance in DS-SS systems and will be discussed in greater detail in Section 3.2), the received signal passes through the wideband filter and is multiplied by the local replica of the PN code sequence \( p(t) \). If \( p(t) = \pm 1 \), then \( p^2(t) = 1 \), and this multiplication yields the despread signal \( s(t) \) given by:

\[ s(t) = \sqrt{\frac{2E_s}{T_s}} m(t) \cos(2\pi f_c t + \theta). \quad (3-4) \]

at the demodulator input. Because \( s(t) \) has the form of a BPSK signal, the corresponding demodulation extracts \( m(t) \). Figure 3.1 shows a simple block diagram of DS-SS system with binary phase modulation.
Figure 3.1 DS-SS transmitter [a] and receiver [b] block diagrams.
An important feature of a pseudonoise or pseudorandom sequence is that it has autocorrelation that resembles, over a period, the autocorrelation of a random binary sequence. Its autocorrelation also roughly resembles the autocorrelation of band-limited white noise. Autocorrelation, in general, is defined as the integral:

$$\psi(\tau) = \int_{-\infty}^{\infty} f(t) f(t - \tau) dt.$$  (3-5)

which is a measure of the similarity between a signal and a phase-shifted replica of itself. An autocorrelation function is a plot of autocorrelation over all phase shifts $(t - \tau)$ of the signal.

Another important parameter of PN sequences that one needs to consider is their cross-correlation. Cross-correlation is of interest in several areas such as code division multiple access (CDMA) systems in which receiver response to any signal other than the proper addressing sequence is not allowable, and in antijamming systems that must employ codes with extremely low cross-correlation as well as unambiguous autocorrelation. Cross-correlation is the measure of similarity between two different code sequences. The only difference between autocorrelation and cross-correlation is that in the general convolution integral for cross-correlation a different term is substituted:

$$\psi_{(cross)} = \int_{-\infty}^{\infty} f(t) g(t - \tau) dt.$$  (3-6)

Cross-correlation for different code sequences can be tabulated by generating a comparison table and curve of agreements minus disagreements when the codes are compared chip by chip. This is identical to the procedure used to determine
autocorrelation for a code sequence, except that here the code is compared with a phase-shifted version of itself.

Most DS-SS communication systems employ maximal length PN codes. A maximal length code is the longest code that can be generated by the $m$-stage feedback shift register which is diagrammed in Figure 3.2. It consists of consecutive stages of two-state memory devices and feedback logic. Binary sequences are shifted through the shift registers in response to clock pulses, and the output of the various stages are logically combined and fed back as the input to the first stage. Typically, the feedback logic consists of exclusive-OR gates.

The initial contents of the memory stages and the feedback logic circuit determine the successive contents of the memory. If a shift register reaches zero state at some time, it would always remain in the zero state, and the output would subsequently be all 0’s. Since there are exactly $2^m - 1$ nonzero states for an $m$-stage feedback shift register, the period of a PN sequence produced by such a shift register cannot exceed $2^m - 1$ symbols. Therefore, the number of chips in a maximal length PN code is given by $2^m - 1$ [19]. These PN sequences have the total number of ones and zeros that differs by only one chip.

Maximal length PN sequences (also called m-sequences) possess certain characteristics that make their use in DS-SS systems extremely desirable. Their autocorrelation is $-1$ except in the region between the zero and plus or minus one chip shifts where it increases linearly. Thus, the autocorrelation function for an m-sequence is triangular in shape. This characteristic autocorrelation is used to great advantage in communication and ranging systems. Two communicators may operate simultaneously, for instance, if their codes are phase shifted more than one chip. By comparison, a typical
Figure 3.2 Block diagram of a feedback shift register with $m$ stages [17].
autocorrelation function for a nonmaximal code exhibits additional minor correlation peaks. They are dependent on the actual code used and are caused by partial correlations of the code with a phase-shifted replica of itself. When such minor correlations occur, a receiving system’s ability to synchronize may be impaired because it must discriminate between the major (±1 chip) and minor correlation peaks, and the margin of discrimination is reduced.

3.1.3 Interference rejection

As mentioned previously, the ability of DS-SS systems (and spread spectrum systems, in general) to reject interference is critically important to most communication systems. Several parameters are essential if one is to gain a measure of interference rejection capability of a DS-SS system. They include near-far effect, process gain, and jamming margin.

The question of capturing a signal in the presence of multiple transmissions is of great interest in most DS-SS systems. With spread spectrum communication, it is often possible for the strongest transmitter to successfully capture the intended receiver, even when many other users are also transmitting. Often, the closest unintended transmitter is able to capture a receiver because of the small propagation loss. This is called the near-far effect [17]. This effect offers both advantages and disadvantages in practical systems. Because a particular transmitter may capture an intended receiver, many transmissions may survive despite collision on the channel. However, a strong transmitter may make it
impossible for the receiver to detect a much weaker transmitter which is attempting to communicate to the same receiver. This is known as the hidden transmitter problem.

The most commonly used quantity in describing or specifying spread spectrum systems is that of process gain. Process gain is easy to calculate, if the bandwidth employed in a system is known and the information rate is available. Expression (3-1) (Shannon’s information-rate theorem) showed that one can send information without error, if some method can be devised that employs a wide enough bandwidth to transmit the information. The process gain is an embodiment of that theorem. In spread spectrum processors, the process gain can be estimated by the following rule of thumb:

\[
\text{process gain} = G_p = \frac{BW_{RF}}{R_{inf.o}}.
\]  

(3-7)

where the RF bandwidth \((BW_{RF})\) is the bandwidth of the transmitted spread spectrum signal and the information rate \((R_{inf.o})\) is the data rate in the information baseband channel. As will be explained later, the process gain is an important parameter in describing the ability of a spread spectrum system to reject narrowband interference.

A spread spectrum system develops its process gain in a sequential signal bandwidth spreading and despreading operation. The transmit part of the process may be accomplished with any one of the band-spreading modulation methods. Despreading is accomplished by correlating the received spread spectrum signal with a similar local reference signal. When the two signals are matched, the desired signal collapses to its original bandwidth (before spreading), whereas any unmatched input is spread by the local reference to the local reference bandwidth or more. A filter then rejects all but the desired, narrowband signal; that is, given a desired signal and its interference, a spread
spectrum receiver enhances the signal while suppressing the effects of all other inputs. It is not necessarily true that a processor with a given process gain can perform properly when faced with an interfering signal having a power level larger than the desired signal by the amount of the available process gain. Another term, jamming margin, which expresses the capability of a system to perform in such hostile environments, must be introduced.

Jamming margin is that quantity which is usually intended in the specification of spread spectrum systems, but it is less readily predicted from bandwidth and information-rate. One can be sure, however, that jamming margin in any given system is always less than the process gain available from that system. Jamming margin takes into account the requirement for a useful system output signal-to-noise ratio and allows for internal losses; that is (when expressed in dB):

\[
\text{JAMMING MARGIN} = G_p - \left[ L_{\text{sys}} + \left( \frac{S}{N} \right)_{\text{out}} \right] = M_j.
\] (3-8)

where \(G_p\) is again the process gain, \(L_{\text{sys}}\) are the system implementation losses, and \(\left( \frac{S}{N} \right)_{\text{out}}\) is the signal-to-noise ratio at the information output. For example, a system with 30-dB process gain, minimum \(\left( \frac{S}{N} \right)_{\text{out}}\) of 10 dB, and \(L_{\text{sys}}\) of 2 dB would have an 18-dB jamming margin \((M_j)\). It could not be expected to operate with interference more than 18 dB above the desired signal.

The output signal-to-noise ratio may be derived for the case in which the jammer power dominates the desired signal. It is given by [18]:

\[
\left( \frac{S}{N} \right)_{\text{out}} = G_p - \frac{J}{S} \quad \text{(in dB)}.
\] (3-9)
where $J$ is the power level of the jamming signal and $S$ is the power level of the desired signal. Expression (3-9) is valid for the region above the jamming threshold where the system can still operate in the presence of interference. Thus, it is clear that the available signal-to-noise ratio at the processor output is a function of not only the process gain but also of the ratio of jammer to signal powers. The output signal-to-noise ratio is essential in determining both bit error rate (BER) and message error rate (MER) for any given modulation format in a DS-SS system.

The jamming margin in any DS-SS system characterizes survival of some messages even when collisions do occur at the receiver. The collision study which will be discussed in great detail in subsequent sections was undertaken to quantify this behavior. It attempted to answer a question that has not been thoroughly investigated, namely, what happens to a spread spectrum signal that is being demodulated while another signal with an identical spreading code (the jamming signal) arrives at the receiver. It is important to note that defining the jamming signal as any signal within the receiver bandwidth with modulation and spreading code identical to those of the desired signal is slightly different from typical jammer definition found in [18]. There, the jammer or interferer is described more generally as any signal within the receiver bandwidth; and it is usually a continuous wave (CW) signal. However, when the proposed channel model is implemented using DS-SS, the biggest source of interference will come from system users operating with the same spreading code over a common frequency band.
3.2 EXPERIMENTAL METHODS AND SETUP

3.2.1 Overview of DS-SS systems employed in collision study

As was pointed out in Chapter 2, some message collisions will inevitably occur when the proposed traffic strategy is implemented, even though they will in fact be significantly reduced by employing the developed retransmission strategy (see Figure 2.1). It was also mentioned that the DS-SS system implementing this strategy will operate in a 900 MHz ISM band (902-928 MHz). This system will incorporate 4 independent channels, with each channel possessing a spreading code which is orthogonal (mutually transparent) to the spreading codes of the other 3 channels. Each user would then randomly select one of these channels for transmission. Therefore, the major goal of the collision study became to quantify the behavior of various DS-SS systems in the presence of interferers with the same spreading code as the message of interest. In other words, is there a relationship between the probability that a message of interest will survive a collision and the average powers of both that message and any interfering message(s)? This issue has implications that extend far beyond the implementation of the proposed system. For example, it is particularly important in understanding the behavior of spread ALOHA systems [20], which would incorporate the ALOHA transmission strategy over spread spectrum links.

The collision study was performed using two different DS-SS systems. Each system employs different modulation formats and receives spread spectrum signals using fundamentally different techniques. The first system, designed by Grayson Electronics, uses DPSK modulation and performs signal detection with a matched filter receiver. The
second system, manufactured by Loral, employs BPSK modulation and uses a sliding correlator receiver to detect incoming spread spectrum signals. In order to better understand the operation of each system, one needs to first grasp the major differences that exist in the operation of the two receiver types.

Before any signal can be properly received and demodulated in a spread spectrum system, synchronization on the part of the spreading code must be achieved between transmitters and receivers. Code synchronization is necessary in all spread spectrum systems because the code is the key to despreading the desired information and to spreading any undesired signals. In spread spectrum systems two general regions of uncertainty exist, with respect to synchronization. These are code-phase and carrier-frequency uncertainties, and both require resolution before a spread spectrum receiver can operate. The code phase must be resolved to better than one chip, and the center frequency, as seen at the receiver, must be resolved to the degree that the despread signal is within the aperture of the postcorrelation filter. Furthermore, the carrier frequency is often constrained to be accurate enough to work well with a demodulator.

The initial synchronization problem is the most difficult of all. When synchronization has already occurred, it is often possible to base a subsequent synchronization on the knowledge of timing gained. Many techniques for achieving synchronization have evolved, some with simple requirements, others with complex implementation implications. The two techniques that are most commonly used in DS-SS systems of today, the sliding correlator and the matched filter, are employed in the equipment under consideration.
3.2.2 Sliding correlator receiver

The simplest of all correlation techniques uses a sliding correlator, so called because the receiving system, in searching for synchronization, operates its code-sequence generator at a rate different from the transmitter’s code generator. The effect is that the two code sequences slip in phase with respect to each other, and if viewed simultaneously (say with an oscilloscope), as in Figure 3.3 they seem to slide past each other, stopping only when the point of coincidence is reached. This condition is referred to as a PN lock. Once lock is achieved, the receive clock signal is changed to match the transmitter’s clock signal. The sliding correlator synchronization is a coherent process since it requires the demodulator to acquire phase lock to the carrier as well as PN lock to the transmitter’s signal. Figure 3.4 illustrates the synchronization steps for this type of correlator.

The advantage of the sliding correlator is its simplicity in that nothing more is required than some way of shifting the code clock of the receiver to a different rate. The difficulty of using a simple sliding correlator for synchronization, however, is that when a large degree of uncertainty is encountered, examination of all possible code-phase positions is impractical because of the time involved. As a rule of thumb, one can expect to search at a rate approximately equal to the data rate for which the receiver has been designed. Thus, when a long maximal code is used (which is advantageous for better interference rejection capabilities), the code acquisition time with the sliding correlator could be much longer than the message duration. This leads to a significant degradation in system throughput. For example, the baseband data rate for the proposed system is 40 Kbits/sec and the message duration is 4.6 msec. The sliding correlator receiver employed in the collision study operates with a \((2^{16}-1)\) chip maximal code, which leads to a
Figure 3.3 Sliding correlator action: (a) sliding process in operation; (b) synchronized codes after acquisition [18].
Figure 3.4 Sliding correlator block diagram.
possibility of having the receiving code-sequence generator offset by more than 65,000
chips. Under these conditions, it could take more than 1.5 seconds (or 350 message
lengths) for the receiver to acquire synchronization, significantly decreasing the overall
throughput of the proposed system. Thus, when using a sliding correlator, careful
attention must be given to the tradeoff between the degree of interference rejection and
the code acquisition time.

One of the most effective techniques for making use of a sliding correlator employs
special code sequences, short enough to allow a search through all possible code positions
in some reasonable time but limited in how short they can be by correlation requirements.
A well-chosen code sequence (called a preamble when used for synchronization) is a good
solution to almost all synchronization problems. Typical synchronization preambles range
in length from several hundred chips to several thousand, depending on the specific
system’s requirements. Preamble synchronization methods have one significant weakness
that comes about as a result of the very code property that makes them work well; that is,
the relatively short sequence length which allows rapid synch acquisition tends to be more
vulnerable to false correlations and to possible reproduction by a would-be interferer.
With the exception of the possible vulnerability problem, however, preamble
synchronization is by far the least critical, easiest to implement, least complex, and best for
all around use.
3.2.3 Matched filter receiver

The second major type of DS-SS receiver is the matched filter synchronizer. This receiver generates a time-reversed replica of its desired input signal, when its input is an impulse. The transfer function of a matched filter is the complex conjugate of the signal to which it is matched [21]. Matched filter synchronizers are generally made up of delay elements called taps. They recognize a particular code sequence and that sequence only. Each delay element has a delay equal to the period of the expected code clock so that each element contains energy corresponding to only one code chip at any one time. The matched filter correlator sums the input sequence with a stored reference at each bit in the shift register. Sums from each tap are then summed and compared to a set threshold value at the correlator output. Correlation occurs when the sum exceeds a set threshold, representing the peak cross-correlation. From this threshold point, the message is despread. Figure 3.5 shows the block diagram of a baseband digital matched filter.

A significant point in the use of delay-line matched filter synchronizers is that they must accurately represent the clock period of the code sequence to be detected. When the delay-line elements exactly match an input signal, the signal summation is perfect. If, however, the code chip rate does not match the delay line, only partial correlation between delay line outputs occurs. Clock frequency drift, Doppler shift or any other cause for clock offset can result in the delay-element periods in the filter being mismatched with the incoming signal. For this reason it may be necessary to employ an array of delay-line matched filters with graduated delay periods whenever a large clock-rate uncertainty exists.
Figure 3.5 Baseband digital matched filter.
Typically, matched filter receivers have shorter code acquisition times than their sliding correlator counterparts (since they can theoretically achieve PN lock with the first bit of message information). However, they are also significantly more complex and costly than sliding correlator receivers. Thus, the decision of whether to employ a sliding correlator or a matched filter will depend largely on system requirements and cost considerations.

3.2.4 Basis for a single interferer collision study

As was pointed out previously, the goal of the collision study was to quantify the performance of the two DS-SS receivers in a closed loop environment in which a desired signal is initially received and then “collided” with a similar co-channel interference signal. In both cases, tests were performed using multiple interferer and desired signal powers, exploiting the full dynamic range of each receiver. Even though multiple message collisions are possible at the receiver, only a single interferer was employed in each case. This was done because collisions between more than two messages may be neglected when the proposed channel is implemented with the system parameters given in Section 2.2.1. The justification for using a single interferer in the collision study is as follows.

The average initial message arrival rate on the system is \( \lambda N \), where \( \lambda \) is the average rate of initial messages transmitted per user and \( N \) is the total number of users on the system. Assuming Poisson arrivals, the probability \( p(k) \) of \( k \) original arrivals in time interval \( t \) is given by expression (2-1), namely:
The collision window for the proposed channel model can be expressed as:

\[ t = 2\tau_{\text{eff}}. \] (3-11)

where \( \tau_{\text{eff}} \) is the effective collision parameter defined in terms of message length \( \tau \), number of retransmissions \( E \), and retransmission interval \( T \) by equation (2-18). For any practically realizable, small number of retransmissions, \( \tau_{\text{eff}} \) can be approximated by expression (2-19) as:

\[ \tau_{\text{eff}} \approx (E + 1)\tau. \] (3-12)

Consequently, the probability of having a double collision is the probability of 3 message arrivals during the collision window \( 2\tau_{\text{eff}} \) which is:

\[ p(3) = e^{-\lambda N 2\tau_{\text{eff}}} \left( \frac{\lambda N 2\tau_{\text{eff}}}{3} \right)^3. \] (3-13)

The probability of having a single collision is the probability of 2 message arrivals during the collision window \( 2\tau_{\text{eff}} \) which is:

\[ p(2) = e^{-\lambda N 2\tau_{\text{eff}}} \left( \frac{\lambda N 2\tau_{\text{eff}}}{2} \right)^2. \] (3-14)

Thus, the ratio of double to single collisions is given by:

\[ \frac{p(3)}{p(2)} = \frac{\lambda N 2\tau_{\text{eff}}}{3} = \frac{\lambda N 2(E + 1)\tau}{3}. \] (3-15)

Using the system parameter of \( \tau = 4.6 \text{ msec} \) together with the arrival rate of 1 msg/sec (a reasonably heavy arrival rate) and 10 retransmissions in (3-15) yields \( \frac{p(3)}{p(2)} = 0.034 \). It is
easy to see that double collisions are far less likely to occur than single collisions and can, therefore, be neglected without much loss of accuracy.

3.2.5 Grayson experimental setup

The first phase of the collision study involved the DS-SS system designed by Grayson Electronics. It was developed as a prototype for the proposed system which will utilize the transmission strategy described in Chapter 2. The goal of this phase of collision testing was to study the performance of Grayson prototype receiver in the presence of colliding messages. The experimental setup, diagrammed in Figure 3.6, consists of 2 prototype transmitters (set up in a master-slave configuration), a PC controlling them, 2 variable attenuators, a 3-dB coupler, a Grayson prototype matched filter receiver, and another PC for storing received messages.

The main components of each prototype transmitter include Analog Devices ADSP-2181 digital signal processing (DSP) chip, Motorola MC145190 phase-locked loop (PLL) frequency synthesis chip, and Maxim MAX2402 amplifier/transmitter. The DSP chip is used generate the differential coding (which is the basis for DPSK modulation, as was explained earlier) and spreading code for a given message. It applies the spreading code to every data bit in order to properly encode the given message. The DSP chip also provides the modulated data to the Maxim transmitter. Finally, it loads the correct divide ratios into the frequency synthesizer chip necessary to set the proper carrier frequency.

The prototype receiver consists of wireless measurement instrument (WMI) chassis and motherboard combined with a special decoder board supporting the spread
Figure 3.6 Experimental setup of Grayson receiver collision study.
spectrum demodulator chip and superheterodyne radio receiver. The superheterodyne radio receiver downconverts the incoming 900 MHz spread spectrum signal to 20 MHz. Subsequently, matched filter code synchronization is performed by the spread spectrum demodulator chip. After code synchronization is achieved and the message is despread, the demodulator chip performs DPSK demodulation. Finally, the despread and decoded data is processed using WMI software and serially sent to a PC control terminal.

The collision study was performed by having each transmitter initially generate a 184-bit message at a baseband bit rate of 40 Kbits/sec. Each message was then spread with a 50 chip code at a rate of 2 Mchips/sec. This produces a process gain of 20 dB since the transmission bandwidth of the signal (the main lobe of the sinc squared function) is approximately twice the chipping rate [18]. The resulting spread signal, DPSK modulating a 912 MHz carrier, was transmitted.

The ADSP-2181 chips of the two transmitters were connected so that their clocks would be set up in a master-slave configuration. This allowed the master transmitter to synchronize with the slave transmitter by enabling the slave transmitter prior to transmitting its own message. After it was enabled, the slave transmitter waited in a loop to receive an interrupt signal from the master transmitter. Subsequent to receiving the interrupt signal, the slave transmitter delayed a predetermined time before transmitting its own message. This time delay corresponded to approximately 48 information bits. Since each message contained 184 information bits, this ensured that there would be a significant overlap of messages at the receiver. In other words, message collisions would inevitably occur.
Each transmitter was programmed to generate 1,000 messages separated by approximately 300 msec. The transmitters generated different messages that were easily discernible at the receiver. The transmitter with the master clock, whose messages were tracked and later analyzed, was considered the transmitter of interest. The transmitter of interest sent a message ASCII-represented by “AAAABCDEFGHIJKLMNOPQRST”, while the interfering transmitter sent a message given by “AAAAAXXXXXXXXXXXXXXXXXXX”. The initial four A’s in each message constitute the preamble field necessary to achieve synchronization at the receiver. Each transmitter generated a +20 dBm signal that was attenuated to ensure received signal power within the receiver’s sensitivity range (-45 to -70 dBm). These reductions in signal power levels are comparable to loss in signal strength encountered in a typical microcellular environment [9].

The signal attenuation of the master transmitter was varied in 5 dB increments in order for the received signal power to be within the above-mentioned range. The interferer’s power levels were also varied in 2 dB steps from 0 to 14 dB relative to each power level for the message of interest. Five trials were performed for each combination of message and interferer powers, with approximately 1,000 messages in each trial. An output text file was created for each trial, and it stored ASCII representations of each message received during that particular trial. Since the message of interest was known, these files could later be analyzed to determine any and all errors that occurred.
3.2.6 Loral experimental setup

The second phase of the collision study involved the Loral EB-100 DS-SS evaluation system consisting of 2 transmitters and a receiver which utilizes sliding correlation. In addition to the spread spectrum evaluation system, the experimental setup shown in Figure 3.7 consisted of a PC control terminal, 2 Motorola 68HC11 microprocessors, 2 variable attenuators, a 3-dB coupler, and a switching circuit. Since the Loral system was not specifically designed for the proposed application, several modifications had to be made in order for this study to mirror the testing conditions of the previous phase as much as possible.

The architecture of the Loral DS-SS evaluation system is based on PA-100 Digital Demodulation Application Specific Integrated Circuit (ASIC) and XILINX Field Programmable Gate Array (FPGA). The PA-100 is very comprehensive and includes all major subsystems required for a spread spectrum receiver including carrier frequency and phase recovery, chip and data timing recovery, PN code acquisition and tracking, automatic gain control (AGC), and data recovery.

The Loral transmitter is controlled by an Intel 8751 microprocessor that directs the XILINX FPGA to convert the serial input data stream into the properly encoded format. Subsequently, the signal is upconverted from baseband to the modulated 70 MHz carrier frequency. One drawback to the transmitter (for the purposes of the collision study) is that the spreading code is always transmitted even if no serial inputs are present. Thus, it is difficult to determine on a spectrum analyzer when a message is being transmitted. A circuit, discussed later in this section, was designed to correct the problem of always transmitting.
Figure 3.7 Experimental setup of Loral receiver collision study.
The PA-100 also contains many of the functional blocks that make up the Loral receiver. It performs AGC on the incoming signal which equalizes the input signal power. Next, it performs sliding correlation to achieve code synchronization. After code synchronization is achieved, it despreads and demodulates the message in order to recover the original data.

Since the Loral transmitters were designed to transmit the PN sequence at all times, it was important to modify the circuit board to switch the interfering transmitter on and off. Therefore, the first HC11 was programmed to control the switching of both transmitter’s carriers. Upon starting the program, the microprocessor would enable the carrier in the transmitter of interest for a short period of approximately 6 msec. This would allow the receiver ample time to lock on to the PN and phase reference of the transmitter of interest. After this brief delay, the HC11 would simultaneously enable the carrier of the interfering transmitter and begin sending data to the transmitter of interest at a rate of 9.6 Kbits/sec. This is unlike the initial phase of the collision study, where messages were generated by the Grayson prototype transmitter itself. Subsequently, the interfering transmitter would receive data from the second HC11 which is programmed to continuously generate messages.

The Loral collision study was performed by having the transmitter of interest generate a 48-bit message at a baseband bit rate of 9.6 Kbits/sec in order to keep message duration for the two experiments approximately equal. The message was then spread with a \((2^{16}-1)\) chip maximal code at a rate of 8 Mchips/sec for a process gain of 32 dB. The resulting spread signal, BPSK (Binary Phase Shift Keying) modulating a 70 MHz carrier, was transmitted. A message ASCII-represented by “ahtst” was sent from the transmitter.
of interest, while the interfering transmitter sent an unmodulated spread carrier. As was
pointed out previously, the transmitter of interest was on (generating an unmodulated
spread carrier) for the entire duration of the test, while the interfering transmitter was
turned on and off to coincide with the initiation and termination of transmission from the
transmitter of interest. This ensured that the receiver would initially acquire the spreading
code for the message of interest since code acquisition time for a sliding correlator is
typically longer than it is for a matched filter [18]. The experiment was performed in
exactly the same manner as before, except that each transmitter generated a 0 dBm signal
and the interferer’s power levels were varied in 2 dB steps from 0 to 20 dB relative to
each power level for the message of interest. Five trials were again executed for each
combination of message and interferer powers, with approximately 1,000 messages in each
trial. An output text file was subsequently created for each trial storing ASCII
representations of each message received during that specific trial. These files were later
analyzed to determine if any message errors had occurred.

3.3 EXPERIMENTAL RESULTS AND DISCUSSION

3.3.1 Analysis of collision data

In order to analyze the experimental data files, two separate C modules were
written. One was developed to analyze Grayson collision data, while the other was used
to process Loral collision results. The source code for the Grayson program appears in
Appendix D. For each input test file, these modules generated an output file which
calculated the MER along with the distribution of message, byte, and bit errors. A listing of a typical output file can be found in Appendix E.

As was stated previously, five trials were performed for each combination of message and interferer powers with each receiver under consideration. Subsequently, the mean MER was calculated for the appropriate power levels by averaging the five corresponding MER values. The results are shown in Figures 3.8 and 3.9, with the former representing message collision dynamics of the Grayson matched filter receiver and the latter depicting collisional performance of the Loral sliding correlator receiver. In both figures, the experimental mean MER values are represented by data points while the solid lines show the corresponding best-fit curves.

Closer examination of Figures 3.8 and 3.9 leads to an important observation, namely that the best-fit curves are approximately parallel to each other. Also, the adjacent curves in both figures are displaced by about 5 dB with respect to each other. Since adjacent curves correspond to a 5 dB difference in message powers, these results clearly indicate that MER for both receivers under consideration is primarily a function of the ratio of signal powers, and not a function of the absolute power levels. Therefore, it is possible to develop a curve showing the probability of message survival \((1 - \text{MER})\) as a function of this ratio. This can be accomplished by taking sets of experimental data points which have a constant ratio of signal powers, even though their absolute message and interferer power levels differ, and interpolating a best-fit curve for the probability of message survival based on these sets of data. For each receiver, each set contains 6 data points with constant ratio of signal powers, corresponding to message powers of -45, -50, -55, -60, -65, and -70 dBm. Figures 3.10 and 3.11 show the best-fit curves for the
Figure 3.8  Experimental values (points) and best-fit curves for message error rate versus absolute power levels of interferer for a matched filter receiver.
Figure 3.9 Experimental values (points) and best-fit curves for message error rate versus absolute power levels of interferer for a sliding correlator receiver.
Figure 3.10  Experimental values (points) and best-fit curve for probability of message survival versus the ratio of message power to interferer power for a matched filter receiver.
Figure 3.11  Experimental values (points) and best-fit curve for probability of message survival versus the ratio of message power to interferer power for a sliding correlator receiver.
probability of message survival, along with the experimental points, for a matched filter and a sliding correlator receiver, respectively. These graphs exhibit analogous behavior and show the presence of three distinct regions for the probability of message survival with respect to the relative powers of colliding messages: the region where the probability of message survival is equal to 0, the region where it varies from 0 to 1 (the transition region), and the region where it is equal to 1. It is important to note that the transition region of a matched filter receiver is significantly narrower than that of a sliding correlator receiver (8 dB for the former versus 16 dB for the latter). This is an interesting result, and some of its possible explanations will be discussed further in Section 3.3.3.

3.3.2 Approximate mathematical expression for message collision dynamics

It was pointed out in Section 1.3 that the ultimate goal of this work is to incorporate the proposed traffic model and receiver collision dynamics in an actual microcellular wireless propagation environment in order to arrive at a more complete theory of the channel. As will be explained in subsequent chapters, the received signal power in wireless microcells is largely dependent on the transmitter-receiver (T-R) separation distance. Obviously, messages from users with different T-R separation distances will arrive at the receiver with different signal powers. It is essential to quantitatively determine, based on results of the collision study, how this difference in received signal powers will affect the survivability of a message undergoing collision. This necessitated the development of a mathematical formulation for collision dynamics of each receiver under test based on the experimental data presented in the previous section.
In order to develop the equation of the best-fit curve for the probability of message survival versus the ratio of message power to interferer power, a well known statistical technique of logistic regression was employed [22]. This technique is particularly applicable to cases where the collected data are binary in character, as is the case here since any given message either survives a collision with an interferer or is lost. The logistic regression model used to accommodate this binary response situation is given by:

\[ P(x_i) = y_i = \frac{1}{1 + e^{-x_i \cdot \beta}} \quad (i = 1, 2, \ldots, s). \] (3-16)

The above equation assumes \( s \) data points \((x_i, P(x_i))\). This model relates the probability of occurrence \( P(x_i) \) against the regressor variable \( x_i \). Here, the quantity \( x_i \cdot \beta = \beta_0 + \beta_1 x_i + \cdots + \beta_k x_i^k \) is the multiple linear regression contribution. The logistic function lies between zero and one and, of course, takes on an interesting S-shape as depicted in Figures 3.10 and 3.11. The coefficients \( \beta_0, \ldots, \beta_k \) are determined using the weighted least squares technique which minimizes the following quantity:

\[ \sum_{i=1}^{s} w_i (y_i - \hat{y}_i)^2 \] (3-17)

where \( \hat{y}_i \) is the fitted (predicted) response and \( w_i = 1/\sigma_i^2 \). That is, each least squares estimator is weighted by the factor \( w_i \) which is the reciprocal of the error variance \( \sigma_i^2 \) at that data point.

For Grayson collision data, it was found using SAS statistical software that the best-fit curve is satisfactorily approximated by a 3rd order logistic regression model. Table 3.1 contains the estimated values for each coefficient, their standard deviations and 95%
confidence intervals (defined as the estimate ± two standard deviations).

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>ESTIMATED VALUE</th>
<th>STANDARD DEVIATION</th>
<th>95% CONFIDENCE INTERVAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₀</td>
<td>8.812103932</td>
<td>0.1057519852</td>
<td>8.6032153308 9.0209925325</td>
</tr>
<tr>
<td>β₁</td>
<td>3.064703621</td>
<td>0.05651468237</td>
<td>2.9530753218 3.1763319199</td>
</tr>
<tr>
<td>β₂</td>
<td>0.407152832</td>
<td>0.00979013436</td>
<td>0.3878152727 0.4264903921</td>
</tr>
<tr>
<td>β₃</td>
<td>0.023494960</td>
<td>0.00055010646</td>
<td>0.0224083852 0.0245815354</td>
</tr>
</tbody>
</table>

Table 3.1 Summary of calculated results for Grayson logistic regression model.

Thus, the equation for the probability of message survival $P_{sur}$ as a function of the difference $ΔW$ (in dB) between message and interferer powers for the matched filter receiver under consideration is given by:

$$P_{sur}(ΔW) = \begin{cases} 
0 & \text{if } ΔW < -10 \\
\frac{1}{1+e^{-8.812103932-3.064703621(ΔW)-0.407152832(ΔW)^2-0.023494960(ΔW)^3}} & \text{if } -10 ≤ ΔW < -1.95 \\
1 & \text{otherwise}
\end{cases} \quad (3-18)$$

Equation (3-18) represents the mathematical expression for the behavior of the Grayson matched filter receiver. Figure 3.12 shows the plots of the logistic curve (3-18) together with the best-fit curve from Figure 3.10 that it approximates.

For Loral data, it was found, again using SAS, that the curve is also satisfactorily approximated by a 3rd order logistic regression model. Table 3.2 shows the estimated values for each coefficient, their standard deviations and 95% confidence intervals.

<table>
<thead>
<tr>
<th>COEFFICIENT</th>
<th>ESTIMATED VALUE</th>
<th>STANDARD DEVIATION</th>
<th>95% CONFIDENCE INTERVAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₀</td>
<td>13.89271517</td>
<td>0.15992712261</td>
<td>13.578091104 14.207339232</td>
</tr>
<tr>
<td>β₁</td>
<td>2.223902883</td>
<td>0.03471079025</td>
<td>2.154742459 2.291315201</td>
</tr>
<tr>
<td>β₂</td>
<td>0.12456799</td>
<td>0.00245558290</td>
<td>0.119737133 0.129398845</td>
</tr>
<tr>
<td>β₃</td>
<td>0.00270547</td>
<td>0.0005663426</td>
<td>0.002594057 0.002816890</td>
</tr>
</tbody>
</table>

Table 3.2 Summary of calculated results for Loral logistic regression model.
Figure 3.12  Probability of message survival versus the difference between message and interferer powers for Grayson receiver.
Therefore, the expression for the probability of message survival $P_{sur}$ as a function of the difference $\Delta W$ (in dB) between message and interferer powers for the sliding correlator receiver is:

$$P_{sur}(\Delta W) = \begin{cases} 
0 & \text{if } \Delta W < -23 \\
1 & \text{if } -23 \leq \Delta W < -66 \\
\frac{1}{1 + e^{-1.389271517 - 2.2230293(\Delta W) - 0.12456799(\Delta W)^2 - 0.00278547(\Delta W)^3}} & \text{otherwise}
\end{cases}$$

Equation (3-19) represents the mathematical expression for the behavior of the Loral matched filter receiver. Figure 3.13 shows the plots of the logistic curve (3-19) as well as the best-fit curve from Figure 3.11 that it approximates.

An important measure of how well an equation approximates any given data point is the value of the residual (also known as the error of fit) at that point which is expressed as:

$$r_i = y_i - \hat{y}_i$$

Ideally, the residuals should oscillate around zero. This is indeed verified in Figures 3.14 and 3.15 which diagram residuals versus predicted values for Grayson and Loral receivers, respectively.

### 3.3.3 Characterization of message survival through jamming margin

The collision study has enabled us to reach a much better understanding of the performance capabilities for the receivers under consideration. Significant insight into each receiver type can be gained by determining the jamming margin and the width of the transition region encountered in the two phases of the experiment. In addition, a much
Figure 3.13 Probability of message survival versus the difference between message and interferer powers for Loral receiver.
Figure 3.14 Residuals $r_i$ versus fitted (predicted) values $\hat{y}_i$ for Grayson receiver.

Figure 3.15 Residuals $r_i$ versus fitted (predicted) values $\hat{y}_i$ for Loral receiver.
clearer picture can now be drawn regarding the degree of interference rejection for both receivers.

With respect to the jamming margin, it can be seen from Figure 3.10 that the Grayson receiver operates with a jamming margin of approximately 2 dB. The total outage (the condition where the probability of message survival is 0) occurs at about 10 dB, corresponding to a transition region of 8 dB. By comparison, the Loral receiver operates with a jamming margin of about 7 dB (see Figure 3.11), and the outage condition is not reached until the jammer power exceeds the power in the wanted signal by 23 dB. This corresponds to a much wider transition region (approximately 16 dB) than in the first case. In order to arrive at some potential causes for this effect, the concept of the jamming margin and what happens as the jammer power increases needs to be understood. This concept is illustrated by Figure 3.16 below.

![Figure 3.16 Characterization of signal/jammer interaction through jamming margin.](image)
Figure 3.16 points out the fact that the jammer is seen as uncorrelated noise by the receiver. Even though the jammer itself uses the same spreading code as the wanted signal, it will be seen as this uncorrelated noise since the receiver locks on to the wanted signal first.

The wide transition region of the Loral receiver indicates that even in the case of the jammer power greatly exceeding the jamming margin, some messages still manage to arrive uncorrupted. This is an extremely desirable interference rejection characteristic specific to sliding correlator receivers. It is due to the fact that sliding correlator receivers employ coherent detection which provides additional interference rejection. The Loral EB-100 receiver, for example, uses a phase/frequency detector (PFD) and a phase loop filter (PLF) for carrier recovery. The PFD detects the phase and frequency of the incoming signal and then passes this information (in a binary format) on to the PLF. The PLF, in turn, converts the phase data to a control word for the digital phase shifter (DPS) which allows the DPS to track the phase of the incoming signal. The carrier recovery loop has the ability to track the incoming signal’s phase, thus allowing for message reception even in cases of excessive jammer power. It is important to note that the message duration of approximately 5 msec used in this experiment was on the order of magnitude of the PN lock time for the Loral receiver under test. The width of the transition region would have likely decreased had a much longer message been employed. This is due to the fact that since phase and PN lock are stochastic processes (with respect to time required to acquire each), shorter messages have a far greater chance to be demodulated (that is, survive a collision) before a receiver can lock on to a much stronger jamming signal.
The Grayson receiver, on the other hand, employs a matched filter to synchronize with the PN code of the incoming signal. This is a noncoherent or asynchronous type of detection. Since both the jammer and the desired signal possess the same spreading code, the receiver will despread the jammer at power levels which exceed the jamming margin as well as the minimum $S/N$. This is where the receiver experiences something similar to a capture effect. It has been shown that the probability of receiving the jamming signal over the desired signal is a function of the ratio of the two signal powers [7]. If the receiver loses synchronization with the desired signal, it will attempt to regain it. The receiver will begin to synchronize with the jamming signal when the jammer-to-signal ratio becomes greater than the capture ratio. In the Grayson collision study, the transmitter of interest sent 6 bytes of data before the jamming message was sent. During this time frame, the receiver synchronizes to the PN code of the transmitter of interest. However, when the jammer with powers above the jamming margin collides with the intended message, false detects begin to appear at the correlator threshold detector. At even greater jammer power levels, the jamming signal is received and the desired signal becomes interference.

To summarize, an important trade-off exists when one is deciding whether to employ a matched filter or a sliding correlator DS-SS receiver for a particular application. Matched filter receivers offer a faster PN lock acquisition, allowing a significant increase in the throughput of the system, than do sliding correlator receivers. However, the interference rejection capabilities of matched filter receivers are somewhat inferior to those of their sliding correlator counterparts. Therefore, a system designer must carefully consider the requirements of an application before deciding which receiver type will optimize system performance.
CHAPTER 4. ENHANCED THEORY OF THE CHANNEL WITH SURVIVAL OF COLLIDING MESSAGES

So far, we have considered two of the primary factors (transmission algorithm and receiver collision dynamics) that will affect the performance of the proposed communication system. Chapter 2 contained the developed probabilistic traffic model for the channel. With this model, we were able to determine the probability that any given message will be successfully transmitted based on such parameters as message length, retransmission interval, number of retransmissions, etc. It was assumed that any message suffering a collision with another message will automatically be lost. In Chapter 3, we saw, based on the collision study performed with two major types of DS-SS receivers, that survival of messages if often possible after collisions. Moreover, we were able to determine that message survival hinges on the relative received signal powers of colliding messages. The importance of this result lies in the fact that it can be directly included in the theoretical model developed in Chapter 2.

This task can be accomplished only if the positions of transmitters originating colliding messages in the wireless microcell intended for the proposed application could help us to determine the relative received signal powers of these messages. This is indeed possible, and can be achieved with the inclusion of radio wave propagation effects into the channel model. The goal of this undertaking is to ultimately be able to predict the probability of successful message transmission for a particular transmitter location within a given microcell.
4.1 INCLUSION OF COLLISION DYNAMICS INTO THEORETICAL MODEL

As was shown in Chapter 3, the collision study resulted in the development of closed-form expressions relating the probability of message survival to the difference between message and interferer powers for DS-SS systems under test (see expressions (3-18) and (3-19)). These empirical formulas are applicable in the linear range of receiver operation, which was determined to be sufficiently wide to handle the expected fluctuations of the received signal powers in various microcells. These results can be incorporated directly into the probabilistic traffic model. The following procedure outlines the necessary steps.

Recall that the probability of collision for any message transmission with the proposed channel is given by expression (2-15). By including survival of colliding messages, this expression can be written more precisely as:

\[
P_c = \left[ (1 - e^{-2\lambda N T_{	ext{off}}}) + (1 - e^{-\lambda N T}) (1 - (1 - \frac{2E\tau}{T - E\tau}) e^{-2\lambda N T_{	ext{off}}}) \right] (1 - P_s). \tag{4-1}
\]

where \( P_s \) is the probability of message survival and all other terms retain their previous meanings. \( P_s \) expresses the probability of message survival for a particular transmitter location. It depends on factors such as the type of microcell under consideration and the density of users in that microcell. It is important not to confuse \( P_s \) with the probability of message survival \( P_{\text{sur}} \) from Chapter 3. \( P_{\text{sur}} \) takes into account only the collisional dynamics of a given receiver, and it is just one of the factors used to determine \( P_s \). The approach to calculate \( P_s \) will be discussed in greater detail in Section 4.3. It should be
noted that if no collisional survival is assumed (as was the case with the probabilistic traffic model of the channel), $P_s = 0$ and (4-1) reduces to (2-15), as expected.

The introduction of expression (4-1) allows us to modify expression (2-16) for the message error rate of the proposed transmission strategy. It thus becomes:

$$\text{MER} = \left( P_s \right)^{E+1}$$

$$= \left[ \left( 1 - e^{-2\lambda N_{T,\text{eff}}} \right) + \left( 1 - e^{-\lambda N_{T}} \right) \left( 1 - \left( 1 - \frac{2E\tau}{T - E\tau} \right)^E \right) e^{-2\lambda N_{T,\text{eff}}} \left( 1 - P_s \right) \right]^{E+1} \quad (4-2)$$

Expression (4-2) represents the message error rate for a particular transmitter location in a given microcell. It incorporates the results of the developed traffic model (by way of expressions (2-15) and (2-16)), as well as the receiver collision dynamics and microcellular propagation effects (both of which are included in the $P_s$ term).

As was mentioned previously, in order to arrive at a more complete channel model (which is represented by expression (4-2)), a connection needs to be established between received signal powers and physical transmitter locations corresponding to those powers. This can only be accomplished by characterizing the propagation environment under consideration which no simple task given the extreme variability of the wireless transmission medium. The transmission path between the transmitter and the receiver can vary from simple line-of-sight to one that is severely obstructed by buildings, mountains, and foliage. Unlike wired channels that are highly predictable, radio channels are rather random and do not offer easy analysis. The following section addresses various approaches that are used to characterize the radio channel, as well as their applicability to the proposed system.
4.2 EFFECTS OF PROPAGATION IN WIRELESS MICROCELLS

4.2.1 Characteristics of the radio propagation channel

The radio channel is attractive due to its ability to provide wireless communication services, but it is also one of the most hostile mediums in which to operate. The radio channel places fundamental limitations on the performance of wireless communication systems. Terrestrial radio signals are not only subject to the same significant propagation-path losses that are encountered in other types of atmospheric propagation, but are also subject to the path-loss effects of terrestrial propagation. Electromagnetic waves are often reflected, scattered, diffracted, and attenuated by the surrounding environment. The scattered components interfere and build up an irregular field distribution, and the signal at the receiver is therefore attenuated and distorted. Terrestrial losses are greatly affected by the general topography of the terrain. In general, the texture and roughness of the terrain tend to dissipate propagated energy, reducing the received signal strength at the base station. Losses of this type, combined with free-space losses, collectively make up the propagation-path loss. Modeling the radio channel has historically been one of the most difficult parts of radio system design, and is typically done in a statistical fashion, which is verified by measurements made specifically for an intended communication system or spectrum allocation.

Propagation models have traditionally focused on predicting the average received signal strength at a given distance from the transmitter, as well as the variability of the signal strength in close spatial proximity to a particular location. Propagation models that
predict the mean signal strength for an arbitrary T-R separation distance are called large-scale propagation models, since they characterize signal strength over large T-R separation distances (several hundreds or thousands of meters). On the other hand, propagation models that characterize the rapid fluctuations of the received signal strength over very short travel distances (a few wavelengths) or short time durations (on the order of microseconds) are called small-scale or fading models.

When an instantaneously fading signal $s(t)$ is received at any time $t$ in a radio environment, this signal can be expressed as [23]:

$$s(t) = r(t)e^{j\psi(t)}.$$  \hspace{1cm} (4-3)

where $\psi(t)$ is the term for the phase of the signal $s(t)$ and $r(t)$ is the term for the envelope of the signal. Furthermore, $r(t)$ can be separated into two terms:

$$r(t) = m(t)r_0(t).$$ \hspace{1cm} (4-4)

where $m(t)$ represents large-scale path loss and $r_0(t)$ represents small-scale fading. Although random phase variation obviously does affect system performance, the phase information $\psi(t)$ is not used when calculating path loss. It is the envelope information $r(t)$ that is of primary importance for calculating path loss.

### 4.2.2 Large-scale path loss in context of the proposed system

Several models for explaining path loss have been developed, all based on extensive propagation measurements. One of the most popular methods for estimating the value of received signal power is the mean approach [24]. If $m(y)$ represents large-scale
path loss at any physical spot \( y \) corresponding to time \( t \) during the test runs, then equation (4-4) can be expressed as:

\[
r(y) = m(y)r_0(y).
\] (4-5)

The component \( m(y) \) is called local mean and its variation is due to the terrain contour between the transmitter and the receiver. The factor \( r_0(y) \) is called multipath fading and its variation is due to the waves reflected from the surrounding buildings and other structures. The local mean can be obtained from:

\[
m(x) = \frac{1}{2L} \int_{x-L}^{x+L} r(y)dy.
\] (4-6)

The problem of selecting a proper interval \( L \) for obtaining local means is extremely important, and it is discussed at length below.

If no multipath fading is present, then the propagation-path loss is the only major factor that must be considered. However, if severe multipath fading is present in the radio environment, this means that \( r_0(y) \) in (4-5) cannot be treated as a constant and in order to obtain \( r_0(y) \) it is first necessary to obtain \( m(y) \) by estimation of \( \hat{m}(x = y) \). This is done as follows:

\[
\hat{m}(x) = m(x) \frac{1}{2L} \int_{x-L}^{x+L} r_0(y)dy.
\] (4-7)

Expression (4-7) assumes that \( m(x) \) is the true local mean and can therefore be brought outside the integral. The local means obtained from (4-7) are called “running means”; i.e., the data points within a length \( L \) on both the left and right sides of point \( x \) are used to obtain an average for that point.
The term \( \hat{m}(y) \) is derived from an averaging process applied to the envelope \( r(y) \) of an instantaneous fading signal \( s(y) \) at any spot \( y \). Thus, \( \hat{m}(y) \) will be factored out from \( r(y) \) in order to obtain \( r_0(y) \). The envelope of a fading signal contains both large-scale and small-scale fading components. The large-scale fading components, which contribute only to propagation-path loss, must be removed, and the small-scale fading components, which are the result of the multipath phenomenon, must be retained.

If the length \( L \) is not long enough, \( m(x) \) itself retains partial small-scale fading information and therefore \( \hat{m}(x) \) is different from \( m(x) \). When \( L \) is too long, the details of the local means are wiped out from the averaging process of expression (4-7); again, \( \hat{m}(x) \) is different from \( m(x) \). If the length of \( L \) is chosen properly, then \( \hat{m}(x) \to m(x) \) and the integral portion of (4-7) becomes:

\[
\frac{1}{2L} \int_{x-L}^{x+L} r_0(y) dy \to 1. \tag{4-8}
\]

The length of \( 2L \) has been determined to be about 40 wavelengths [25], since the spread in the value of local mean is less than 1 dB for any length over 40 wavelengths. Using up to 50 samples in an interval of 40 wavelengths is an adequate averaging process for obtaining the local means [26]. This implies that the interval of about 15 meters for 915 MHz (the center frequency of a 900 MHz ISM band that will carry signals in the proposed system) is a reasonable guideline for obtaining the local mean.

The proper selection of the separation distance used to obtain the local means results in the multipath fading term \( r_0(y) \) approaching a constant value over the averaging distance \( 2L \). This leads to a highly accurate estimate for the local mean. Figure 4.1
illustrates a typical envelope of a fading signal. Examination of Figure 4.1 shows that it is possible to draw a distinction between the small-scale multipath effects and the large-scale variations of the local mean. Indeed, it is convenient to go further and suggest that the radio signal consists of a local mean value, which is sensibly constant over a small area, superimposed upon which is the small-scale rapid fading. The distance-dependent reduction in the values of local means constitutes large-scale propagation-path loss. Rapid fading is usually observed over distances of about half a wavelength, and fades in excess of 30 dB are not uncommon. If we consider the nature of the multipath and its influence on the characteristics of radio wave propagation, it is apparent that it is pointless to pursue an exact or deterministic characterization and we must resort to the powerful tools of statistical communication theory.

Another measurement-based large-scale path loss model is the log-normal shadowing model. It assumes that the path loss \( PL(d) \) at a particular location is random and distributed log-normally (normal in dB) about the mean distance-dependent value \( \bar{PL}(d) \) which is expressed as:

\[
\bar{PL} \text{ (dB)} = \bar{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right),
\]  

where \( d_0 \) is the close-in reference distance and \( n \) is the path loss exponent. Thus, the path loss at a particular location is given by:

\[
PL(d) = \bar{PL}(d) + X_\sigma = \bar{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right) + X_\sigma.
\]  

where \( X_\sigma \) is a zero-mean Gaussian distributed random variable (in dB) with standard deviation \( \sigma \) (also in dB).
Figure 4.1 Typical envelope of a fading signal [24].
Log-normal shadowing implies that measured signal levels for a given T-R separation distance have a Gaussian distribution about the distance-dependent mean of equation (4-9), where the measured signal levels have values in dB units. Therefore, the random effects of shadowing are accounted for using the Gaussian distribution. In practice, the values of $n$ and $\sigma$ are determined from measured data, using linear regression analysis [27], in which a linear fit is made to the signal in dB versus T-R separation distance on a logarithmic scale. The value of $\overline{PL}(d_o)$ in equation (4-10) is typically based on a free space assumption from the transmitter to $d_o$.

Since $PL(d)$ is a random variable with a normal distribution in dB about the distance-dependent mean, so is the received signal power $W_r(d)$ which is expressed as:

$$W_r(d) = W_t - PL(d).$$  \hspace{1cm} (4-11)

with $W_t$ being the transmitted signal power. Thus, the Q-function or error function ($erf$) may be used to determine the probability that the received signal level will exceed (or fall below) a particular level. The Q-function is defined as:

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2} \left[ 1 - erf\left(\frac{z}{\sqrt{2}}\right) \right],$$  \hspace{1cm} (4-12)

with $Q(z) = 1 - Q(-z)$. The probability that the received signal level will exceed a certain value $\alpha$ can be determined from the cumulative density function as:

$$P\{W_r(d) > \alpha\} = Q\left[ \frac{\alpha - \overline{W_r}(d)}{\sigma} \right].$$  \hspace{1cm} (4-13)

Similarly, the probability that the received signal level will be below $\alpha$ is given by:
At this juncture, it is worth mentioning that local means take on added significance with respect to the collision dynamics of the proposed system. Experimental observations have shown that the statistics of the received signal exhibit temporal stationarity over spatial distances of only a few tens of wavelengths [24]. Thus, ensemble averages (expectations or means) do not depend on time. Therefore, the power levels of the wanted and interfering messages can be considered constant during the time that the collision occurs.

\[ P\{W_r(d) < \alpha\} = Q\left(\frac{W_r(d) - \alpha}{\sigma}\right) \]  
\hspace{1cm} (4-14)

4.2.3 Small-scale fading in context of the proposed system

The major physical factor in the radio propagation channel that influences small-scale fading is multipath propagation. The presence of reflecting objects and scatterers in the channel creates a constantly changing environment that distributes the signal energy in amplitude, phase, and time. These effects result in multiple versions of the transmitted signal that arrive at the receiving antenna, displaced with respect to one another in time and spatial orientation. The random phase and amplitudes of the different multipath components cause fluctuations in signal strength, thereby inducing small-scale fading, signal distortion, or both.

The complex representation of a signal propagating in the multipath radio channel is best represented by the impulse response of the radio channel [28]. The representation is a mathematical model illustrated as:
\[ h(t) = \sum_{m=1}^{N} A_m \delta(t - \tau_m) e^{j\varphi_m}. \]  

(4-15)

where the transmitted impulse \( \delta(t) \) is received as the sum of \( N \) components with amplitudes \( A_m \) and arrival times \( \tau_m \) with phases \( \varphi_m \). Thus, for an impulse transmitted by a proposed system transmitter, by the time this impulse is received at the base station it is no longer an impulse but rather a pulse with a spread width that we call the delay spread. The delay spread has the effect of stretching a signal in time such that the duration of a signal received is greater than that originally transmitted.

In order to understand the radio propagation transmission impairments, it is important to characterize the channels. Channels are often classified into narrowband and wideband channels, which are often differentiated by the maximum delay spread of the channel. The following relationship between the maximum delay spread and the bit period is often used to differentiate between narrowband and wideband channels:

\[
\begin{align*}
\tau_{\text{max}} & < T = \text{narrowband, normalized delay spread} < 1 \\
\tau_{\text{max}} & > T = \text{wideband, normalized delay spread} > 1 
\end{align*}
\]

(4-16)

where \( T \) is the bit period and \( \tau_{\text{max}} \) is the maximum delay spread of the channel.

Extensive measurements have been performed to determine maximum delay spreads around 900 MHz in different kinds of environments. They indicate that the maximum delay spread is less than 0.2 \( \mu \text{sec} \) for open areas, about 2 \( \mu \text{sec} \) for suburban areas, and approximately 3 \( \mu \text{sec} \) for urban areas [29].

A common way of representing delay spreads is by normalizing them to the bit period, \( \tau/T \), which can also be used to distinguish between narrowband and wideband systems. In narrowband systems, the path loss and fading statistics are often of interest.
These include the rate of decay of signal strength with distance and the statistics of the fading encountered. In wideband systems, the delay spread characteristics are usually of primary concern, as they may cause intersymbol interference.

A quantity called coherence bandwidth related to the delay spread has been defined to get a measure of the range of frequencies over which the channel can be considered “flat” (i.e., a channel which exhibits a high degree of similarity in either the amplitudes or the phases of two received signals) [23]. A coherence bandwidth for two fading amplitudes of two received signals is given by:

\[ B_c = \frac{1}{2\pi \tau_{\text{max}}}. \] (4-17)

where \( \tau_{\text{max}} \) is again the maximum delay spread. Similarly, a coherence bandwidth for two random phases of two received signals is expressed as:

\[ B_c = \frac{1}{4\pi \tau_{\text{max}}}. \] (4-18)

When the bandwidth of the transmitted signal is greater than the coherence bandwidth of the channel, frequency-selective fading is experienced, and when the transmitted bandwidth is less than the coherence bandwidth of the channel, flat fading is the fading mechanism. Since the coherence bandwidth of the channel is a function of the maximum delay spread of the channel, when the delay spread of the channel is high, there is a high probability that frequency-selective fading will occur.

Various statistical techniques have been used to describe the characteristics of the received envelope of a flat fading signal. In the absence of a dominant signal component, such as a line-of-sight propagation path, Rayleigh distribution is used to model the
behavior of the received signal envelope. Rayleigh fading is particularly characteristic of radio propagation in urban environments, where high density of buildings often makes line-of-sight propagation impossible. Thus, the envelope of the received E-field, $r_e$, has a probability density function (pdf) given by:

$$p(r_e) = \frac{r_e}{\sigma^2} \exp(-\frac{r_e^2}{2\sigma^2}).$$  \hspace{1cm} \text{(4-19)}$$

where $\sigma$ is the rms value of the received E-field prior to envelope detection. The probability that $r_e$ is less than level $A$ is the cumulative distribution function (cdf) of $r_e$ which is expressed as:

$$P(r_e \leq A) = \int_{0}^{A} \frac{r_e}{\sigma^2} \exp(-\frac{r_e^2}{2\sigma^2})dr_e = 1 - \exp(-\frac{A^2}{2\sigma^2}).$$  \hspace{1cm} \text{(4-20)}$$

The average power of the received signal is the mean of $r_e^2$ which is $2\sigma^2$. Figure 4.2 shows the cumulative distribution of an actual signal subjected to multipath scattering plotted versus the Rayleigh cdf. The model signal shows a generally good fit to the distribution. Measured signal distributions in a local area are typically this close to the Rayleigh curve.

When there is a dominant nonfading signal component present, the small-scale fading envelope distribution is Ricean. The Ricean distribution of the envelope for $r_e$, containing dominant signal peak amplitude $A$, is given by:

$$p(r_e) = \frac{r_e}{\sigma^2} \exp(-\frac{r_e^2 + A^2}{2\sigma^2})I_0\left(\frac{r_eA}{\sigma^2}\right).$$  \hspace{1cm} \text{(4-21)}$$

where $I_0(\cdot)$ is a zero-order modified Bessel function. The Ricean distribution is often described in terms a parameter $K$ which is defined as the ratio between the deterministic
Figure 4.2 Cumulative signal statistics compared with Rayleigh cdf [30].
signal power and the variance of the multipath. It is given by $K = A^2/(2\sigma^2)$ or, in terms of dB:

$$K (dB) = 10\log\left(\frac{A^2}{2\sigma^2}\right).$$  \hspace{1cm} (4-22)

The parameter $K$ is known as the Ricean factor and completely specifies the Ricean distribution. As $A \to 0$, $K \to -\infty dB$, and as the dominant signal decreases in amplitude, the Ricean distribution reduces to a Rayleigh distribution. Also, for $K >> 1$ (which is true for open area propagation where multipath is negligible compared to the dominant line-of-sight signal), the Ricean pdf is approximately Gaussian about the mean. This fact is illustrated by Figure 4.3 which shows the Ricean density function normalized to the local mean.

In the proposed system, the data rate of 40 Kbits/sec implies the information symbol period of 25 $\mu$sec which is significantly larger than the worst case delay spread of 3 $\mu$sec. Under these conditions, the received signal will undergo flat fading due to multipath time delay spread [17]. In flat fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However, the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath.

With respect to the collision dynamics of the receiver, it was pointed out in Chapter 3 that the average time that the colliding messages interact at the receiver is on the order of milliseconds. Therefore, even the worst case delay spread of 3 $\mu$sec will not significantly change the probability of survival in collision for a given message. Thus,
Figure 4.3 The Ricean density function normalized to the local mean [28].
small-scale fading effects can be safely neglected in a collision analysis to get a first-order answer.

4.3 GENERAL APPROACH TO CALCULATE MESSAGE SURVIVAL

In order to arrive at a complete expression for the probability of message survival $P_s$ in equation (4-1), it was pointed out in Section 4.1 that received signal powers must be reconciled with physical transmitter locations in a given microcell corresponding to those powers. This is done by incorporating actual propagation data of that microcell with the receiver collision dynamics. The geometry of the problem is outlined in Figure 4.4.

As can be seen in Figure 4.4, the transmitter of interest (that is, the transmitter whose message survival probability we are after) is located at point $T$ within the microcell. The polar coordinates of this point are $(R, \Phi)$. The selection of the polar coordinate system is logical for dealing with a microcell that is circular in shape. However, this is by no means the only coordinate system that can be employed. As will be discussed in Chapter 5, there is a great deal of variation in the shape of different wireless microcells. Some of the factors that are of primary importance in determining microcellular contours include terrain topology and surrounding environment, both natural and man-made. Therefore, the selection of a proper coordinate system will have to be done on a case-by-case basis.

The receiver in Figure 4.4 is located at the origin of the microcell (that is, its coordinates are $(r, \phi) = (0,0)$) and is designated by letter $R$. A sample interfering transmitter is located at point $I$ whose coordinates are $(r, \phi)$. The received signal power
Figure 4.4 Sample microcellular layout for determination of $P_s$. 
at \( R \) due to the transmitter at \( T \) will be denoted \( W_{TR} = W_{TR}(R, \Phi) \). The signal power received at \( R \) from an interfering transmitter at \( I \) will be represented by \( W_{IR} = W_{IR}(r, \phi) \).

Recall from expressions (3-18) and (3-19), as well as the discussion in Section 3.3.1, that the probability of message survival \( P_{sur} \) arising from the collisional dynamics of a given receiver is a function of the difference \( \Delta W \) (in dB) between message and interferer powers. For example, the difference between received signal powers from the transmitter of interest at \( T \) and an interferer at \( I \) is given by:

\[
\Delta W = W_{TR}(R, \Phi) - W_{IR}(r, \phi).
\]  

(4-23)

Also, recall that the general form of expressions (3-18) and (3-19) is as follows:

\[
P_{sur}(\Delta W) = \begin{cases} 0 & \text{if } \Delta W < \Delta W_0 \\ f(\Delta W) & \text{if } \Delta W_0 \leq \Delta W < \Delta W_i \\ 1 & \text{if } \Delta W \geq \Delta W_i \end{cases}
\]

(4-24)

where \( f(\Delta W) \) is the logistic function with receiver-dependent coefficients. The value \( \Delta W_0 \) in (4-24) represents the maximum difference between received message and interferer powers which results in certain annihilation of that message. Similarly, the value \( \Delta W_i \) in (4-24) is the minimum difference between received message and interferer powers which ensures definite survival of that message in collision. The innermost arc in Figure 4.4 represents possible interferer locations (with respect to the transmitter of interest at \( T \)) resulting in \( \Delta W \) taking on the value \( \Delta W_0 \). One such interferer location could be at point \((r_0, \phi_0)\) leading to:

\[
\Delta W_0 = W_{TR}(R, \Phi) - W_{IR}(r_0, \phi_0).
\]

(4-25)
Likewise, the middle arc of Figure 4.4 shows interferer locations (again with respect to the transmitter of interest at $T$) leading to $\Delta W$ being equal to $\Delta W_1$. Thus, if an interferer is located at $(r_1, \phi_1)$ on this contour, we have:

$$\Delta W_1 = W_{TR}(R, \Phi) - W_{IR}(r_1, \phi_1).$$

(4-26)

In other words, these arcs represent contours of constant signal attenuation. The actual shape of these contours exhibits tremendous variation for different microcells. The problem of predicting the shape of these contours for different propagation environments has been addressed by various researchers [31]-[32], and it will be discussed in greater detail in Chapter 5. At this point, the circular shape of these contours is selected for the purposes of outlining the general procedure to calculate $P_s$.

The outermost arc in Figure 4.4 shows the extent of the microcell. Transmitters located on that circle will have the maximum radial T-R separation distance. For example, a transmitter at point $(r_{lim}, \phi_{lim})$ on this contour will have its messages arrive at the receiver with the signal power of $W_{lim}$. Some of the more important factors in determining the extent of the microcell are transmitter power, antenna gains, surrounding environment and receiver sensitivity. However, microcellular environments typically imply T-R separation distances of no more than about 1 km.

The first step in calculating the probability of message survival $P_s$ for a transmitter located at point $T$ in Figure 4.4 is to determine the incremental survival probability $dP_s$ due to interferers located in the incremental area $dS = rdrd\phi$. This quantity is simply the ratio of interferers in this incremental area (scaled by their corresponding receiver-
dependent probability of survival \( P_{\text{sur}} \) to the total number of users in the microcell.

Mathematically, it is expressed as follows:

\[
dP_s = \frac{1}{2} \times g(r, \phi) \cdot r r d\phi \cdot P_{\text{sur}}(\Delta W) \int_0^{r_{\text{un}}} g(r, \phi) r r d\phi
\]

where \( g(r, \phi) \) represents the density of users in the microcell. The user density has a significant effect on the probability of survival and will be discussed at greater length in Chapter 5. The factor of \( 1/2 \) in expression (4-27) arises from the fact that the collision studies outlined in Chapter 3 forced the message of interest to arrive at the receiver and acquire carrier lock prior to the interfering message. In actuality, there is an equal likelihood that either one of these two messages will be the first to arrive and acquire lock, which necessitates the inclusion of \( 1/2 \) in (4-27).

It is worth mentioning that the receiver-dependent probability of survival \( P_{\text{sur}}(\Delta W) \) in expression (4-27) is a function of the coordinates \((R, \Phi)\) for the transmitter of interest and the coordinates \((r, \phi)\) for the interfering transmitter. This is due to the fact that the difference \( \Delta W \) between message and interferer powers is a function of these coordinates, namely:

\[
\Delta W = W_{TR}(R, \Phi) - W_{IR}(r, \phi) = \Delta W(R, \Phi, r, \phi).
\]

Similarly, expressions (4-25) and (4-26) become:

\[
\Delta W_0 = W_{TR}(R, \Phi) - W_{IR}(r_0, \phi_0) = \Delta W_0(R, \Phi, r_0, \phi_0).
\]

and

\[
\Delta W_1 = W_{TR}(R, \Phi) - W_{IR}(r_1, \phi_1) = \Delta W_1(R, \Phi, r_1, \phi_1).
\]
Therefore, the incremental survival probability $dP_s$ in (4-27) can be more accurately expressed as:

$$dP_s = dP_s (R, \Phi, r, \phi) = \frac{1}{2} \times \frac{g(r, \phi) \cdot r dr \phi \cdot P_{sur} (R, \Phi, r, \phi)}{\int_{0}^{2\pi} \int_{r_0}^{r} g(r, \phi) r dr \phi}.$$

Integrating the numerator of (4-31) over all possible interferer locations will yield the expression for the probability of message survival $P_s$:

$$P_s = \frac{1}{2} \times \frac{1}{\int_{0}^{2\pi} \int_{r_0}^{r} g(r, \phi) r dr \phi} \times \int_{0}^{2\pi} \int_{r_0}^{r} g(r, \phi) \cdot P_{sur} (R, \Phi, r, \phi) \cdot r dr \phi.$$

Recall from expression (4-24) that $P_{sur}$ is defined in a piecewise manner. By employing (4-28) through (4-30), it becomes:

$$P_{sur} (\Delta W) = P_{sur} (R, \Phi, r, \phi) = \begin{cases} 0 & \text{if } \Delta W (R, \Phi, r, \phi) < \Delta W_0 (R, \Phi, r_0, \phi_0) \\ f(R, \Phi, r, \phi) & \text{if } \Delta W_0 (R, \Phi, r_0, \phi_0) \leq \Delta W (R, \Phi, r, \phi) < \Delta W_1 (R, \Phi, r_1, \phi_1) \\ 1 & \text{if } \Delta W (R, \Phi, r, \phi) \geq \Delta W_1 (R, \Phi, r_1, \phi_1) \end{cases}$$

Using (4-33) in (4-32) yields:

$$P_s = P_s (R, \Phi) = \frac{1}{2} \times \frac{1}{\int_{0}^{2\pi} \int_{r_0}^{r} g(r, \phi) r dr \phi} \times \left\{ \int_{0}^{2\pi} \int_{r_0}^{r_1} g(r, \phi) \cdot f(R, \Phi, r, \phi) \cdot r dr \phi + \int_{r_0}^{r_1} g(r, \phi) \cdot r dr \phi \right\}.$$

The first double integral within the braces of (4-34) represents the contribution of interferers located in the transition region with respect to the transmitter of interest at point $T$. In other words, the probability that a message emanating from the transmitter at $T$ will survive a collision with an interfering message originated by a transmitter located somewhere in this region will be in the range between 0 and 1. The second double integral
within the braces of (4-34) represents the contribution of interferers located in the region from which transmissions will have no effect on a message of interest originating at point $T$.

The radial limits of integration $r_0$ and $r_1$, as well as the cellular extent $r_{\text{lim}}$, in (4-34) will obviously depend on the propagation environment under consideration. This dependence will be addressed at length in Chapter 5. The exact values of $r_0$ and $r_1$ will also depend on the azimuthal position of interferer $\phi$. Therefore, they can be more accurately represented as $r_0 = r_0(\phi)$ and $r_1 = r_1(\phi)$.

Equation (4-34) represents the general form of the expression for the probability of message survival $P_s$ for a transmitter located at a given point in a microcell. In order to calculate the MER for a transmitter at this location, expression (4-34) would have to be substituted into equation (4-2). As was pointed out earlier, the exact expression for $P_s$ will depend on the propagation environment, and in Chapter 5 it will be developed for several different microcells.
CHAPTER 5. MESSAGE TRANSMISSION IN VARIOUS MICROCELLULAR PROPAGATION ENVIRONMENTS

In the process of laying out the general method to calculate message survival with the proposed application (see Section 4.3), it was mentioned that contours of constant signal attenuation would have to be determined for a given microcell. This is no easy task since the local means obtained from field-strength measurements taken at predetermined sampling intervals can vary over a large range of decibels. It is therefore advantageous to classify the characteristics of the various sampling environments from which the data for local means are collected. Rules can then be formulated based on predictions of the value of local means associated with classified types sampling environments. The two main factors that will determine the broad parameters for classification are the characteristics of the terrain surface and contour and the presence or absence of buildings, structures, and other man-made objects. The following general classifications exist in conjunction with the contour features of the surrounding terrain [23]:

1. Open land - Undeveloped or partially developed farmland with conventional small dwellings and barns, and sparsely populated.

2. Industrialized open land - Developed areas exhibiting large-scale farming activities and occasional industrial facilities.

3. Suburban areas - Mixed residential and clean industrial uses such as warehouses and shopping malls.

4. Small to medium-sized city - Densely populated residential and commercial areas with well-defined business districts containing a number of high buildings.
5. Large-sized city - Heavily commercial and industrial area with many high-rise residential structures and even skyscrapers. Typical examples would include cities like New York, Chicago, and Los Angeles.

Open land and industrialized open land are often grouped together under the general heading of rural propagation environments. Similarly, cities of various sizes are considered urban propagation environments. The above classes are listed in the order of increasingly harsh propagation environments. The following section describes a number of models that have been proposed to predict propagation-path loss. While all these models aim to predict signal strength at a particular receiving point or in a specific local area, the methods vary widely in their approach, complexity, and accuracy.

### 5.1 OVERVIEW OF PROPAGATION MODELS

#### 5.1.1 Free space propagation model

The free space propagation model is used to predict signal strength when the transmitter and receiver have a clear, unobstructed line-of-sight (LOS) path between them. As with most large-scale radio wave propagation models, the free space model predicts that received power decays as a function of the T-R separation distance raised to some power (i.e. a power law function). The free space power received by a receiver antenna which is separated from a radiating transmitter antenna by a distance $R$, is given by the Friis transmission equation [24]:

\[ P_R = \frac{P_T G_T G_R}{(4\pi R)^2} \]
where $W_t$ is the transmitted power, $W_r(R)$ is the received power which is a function of the T-R separation, $G_t$ is the transmitter antenna gain, $G_r$ is the receiver antenna gain, $R$ is the T-R separation distance in meters, and $\lambda$ is the wavelength in meters. The gain of an antenna is related to its effective aperture, $A_e$, by:

$$G = \frac{4\pi A_e}{\lambda^2}. \quad (5-2)$$

The effective aperture $A_e$ is related to the physical size of the antenna, and $\lambda$ is related to the carrier frequency by:

$$\lambda = \frac{c}{f}. \quad (5-3)$$

where $f$ is the carrier frequency in Hertz and $c$ is the speed of light given in meters/sec. The values for $W_t$ and $W_r$ are expressed in the same units, and $G_t$ and $G_r$ are dimensionless quantities.

The Friis transmission equation shows that the received power falls off as the square of the T-R separation distance. This implies that the received power decays with distance at a rate of 20 dB/decade. The path loss, which represents signal attenuation as a positive quantity measured in dB, is defined as the difference (in dB) between the effective transmitted power and the received power, and may or may not include the effect of the antenna gains. The path loss for the free space model when antenna gains are included is given by:
When antenna gains are excluded, the antennas are assumed to have unity gain and path loss is given by:

\[
PL(dB) = 10\log \frac{W}{W_r} = -10\log \left\{ G_r G_t \left[ \frac{\lambda}{4\pi R} \right]^2 \right\}.
\]

(5-4)

The Friis transmission model is only a valid predictor for \( W_r \) for values of \( R \) which are in the far-field of the transmitting antenna. The far-field, called Fraunhofer region, of a transmitting antenna is defined as the region beyond the far-field distance \( R_f \), which is related to the largest linear dimension of the transmitter antenna aperture and the carrier wavelength. The Fraunhofer distance is given by:

\[
R_f = \frac{2D^2}{\lambda}.
\]

(5-6)

where \( D \) is the largest physical linear dimension of the antenna. Additionally, in order to be in the far-field region, \( R_f \) must satisfy the following two conditions:

\[
R_f \gg D.
\]

(5-7)

and

\[
R_f \gg \lambda.
\]

(5-8)

Since expression (5-1) is obviously not valid for \( R = 0 \), large-scale propagation models employ a close-in reference distance \( R_0 \). The received power \( W_r(R) \) at any distance \( R > R_0 \) is then related to \( W_r \) at \( R_0 \), which in turn is calculated using equation (5-1).

In a wireless radio channel, a single direct path between the transmitter and the receiver is seldom the only physical means for propagation, and therefore the free space
transmission model of expression (5-1) is in most cases inaccurate when used alone. Another model, called the two-ray ground reflection model, is often used to more accurately predict propagation-path loss.

5.1.2 Two-ray ground reflection model

The two-ray ground reflection model (or simply two-ray model) is a useful propagation model that is based on geometric optics, and considers both the direct path and a ground reflected propagation path between the transmitter and the receiver. The two-ray model is depicted in Figure 5.1 for transmitting antenna of height $h_1$ and receiving antenna of height $h_2$. By summing the contribution from each ray, the received signal $W_r$ for isotropic antennas can be expressed as [10]:

$$W_r = W_t \left( \frac{\lambda}{4\pi} \right)^2 \left| \frac{1}{r_1} \exp(-jkr_1) + \Gamma(\alpha) \frac{1}{r_2} \exp(-jkr_2) \right|^2.$$  \hspace{1cm} (5-9)

where $W_t$ is again the transmitter power, $r_1$ is the direct distance from the transmitter to the receiver, $r_2$ is the distance through reflection on the ground, $\Gamma(\alpha)$ is the reflection coefficient, and $k = 2\pi/\lambda$. The reflection coefficient, which depends on the angle of incidence $\alpha$, and the polarization, is given by:

$$\Gamma(\theta) = \frac{\cos \theta - a \sqrt{\varepsilon_r - \sin^2 \theta}}{\cos \theta + a \sqrt{\varepsilon_r - \sin^2 \theta}}.$$  \hspace{1cm} (5-10)

where $\theta = 90^\circ - \alpha$ and $a = 1/\varepsilon_r$ or 1 for vertical or horizontal polarization, respectively.

For average ground, the relative dielectric constant is $\varepsilon_r = 15 - j60\sigma\lambda$, and the
Figure 5.1 Two-ray model showing the ray paths [10].
conductivity $\sigma$ of the surface is usually taken to be 0.005 mho/m [33].

For large distances, $\alpha$ is small ($\theta \sim 90^\circ$), and $\Gamma(\theta)$ is approximately equal to -1. However, when $\alpha$ increases (that is, the T-R separation distance decreases), the value of $\Gamma(\theta)$ decreases and it can even be zero for vertical polarization at the Brewster angle (the angle at which no reflection occurs in the medium of origin). Figure 5.2 shows the received power given by equation (5-9) plotted as a function of distance for the cases of vertical and horizontal polarization, as well as the case assuming $\Gamma(\theta) = -1$, where $W_t = 1$ W (0 dBW), $f = 900$ MHz, $h_1 = 8.7$ m, and $h_2 = 1.6$ m. Upon closer examination of the plots in Figure 5.2, it is obvious that the approximation of $\Gamma(\theta) = -1$ overestimates the peaks of the signal as well as the depth of the fades. Because $|\Gamma(\theta)|$ is larger for horizontal polarization than for vertical polarization, the signal variation for vertical polarization is much less severe than for horizontal polarization, even up to a few hundred meters.

The two-ray model has been found reasonably accurate for predicting the large-scale signal strength in LOS microcellular channels in rural environments [34]. However, in urban and suburban areas where lateral obstacles such as buildings are present, additional reflections of the radio signal generated by the rows of buildings on both sides of the street should be taken into account. This necessitates using multi-ray models to predict propagation-path loss.
Figure 5.2 Two-ray model showing the receiving power for vertical and horizontal polarization and assuming $\Gamma = -1$ ($W_t = 1$ W, $f = 900$ MHz, $h_1 = 8.7$ m, and $h_2 = 1.6$ m) [10].
5.1.3 Multi-ray models

Several different multi-ray models have been proposed to predict large-scale signal strength in urban environments. The received signal given by these models is calculated using the following general expression:

\[ W_r = W_i \left( \frac{\lambda}{4\pi} \right)^2 \left| \sum_{i=1}^{n} \frac{\Gamma_i(\theta_i)}{r_i} \exp(-jkr_i) \right|^2. \] (5-11)

where \( n \) is the number of rays considered, and all other terms retain the same meanings as in equation (5-9). It is easy to see that equation (5-9) is a particular case of (5-11) for \( n = 2 \). The multi-ray model with the fewest number of rays that has been considered is the four-ray model [27]. Figure 5.3 illustrates the ray paths for this model. It shows some possible building-reflected ray paths (r3 and r4), as well as the direct ray (r1) and the ray reflected from the ground (r2).

The reflection coefficient \( \Gamma(\theta) \) in (5-11) is an important parameter to be considered. For the direct ray, \( \Gamma_1(\theta_1) \) is taken as 1, while \( \Gamma_2(\theta_2) \) for the second ray is given by expression (5-10) for vertical polarization case. The differences between the reflection on the ground and that on the lateral buildings are the polarization and the dielectric constant of the reflecting surface. For example, if the transmission is made with a vertical polarization, the reflection by the ground is considered as vertically polarized with respect to the ground, and the reflection by the buildings is viewed as horizontally polarized with respect to the building surface.

A comparison of the two-ray and four-ray models shows that the two-ray model gives only the trend of the spatial average for the signal variation predicted by the four-ray
Figure 5.3 Multiple ray configuration [27].
model, but it does not predict the rapid variations of the received signal resulting from multipath interference that is so prevalent in urban environments. The four-ray model is also able to predict the oscillations that exist for large T-R separation distances, where the two-ray model shows a smooth decrease (see Figure 5.2). Therefore, the four-ray model is much more useful than the two-ray model to study specular reflection from buildings in urban and suburban areas.

Several other multi-ray models have been considered in literature. In addition to single reflections of the four-ray model, these models also consider double reflections off the building. One such model is the six-ray model which considers all the rays present in the four-ray model plus a pair of doubly building-reflected rays [9]. This model was found to offer some additional improvement, compared with the four-ray model, in predicting mean received power in urban environments. However, the six-ray model is not sufficient to determine the detailed fluctuations of the received power. For this purpose, a model containing 10 rays was employed [31]. These are the direct and ground reflected rays. Also, two of each singly, doubly, and triple building-reflected rays, as well as building-ground and ground-building reflected rays for a total of 10 rays. This model was found well suited to predict the detailed fluctuations of the received power in lineal urban environments (that is, areas in which there is nominally a LOS path between the transmitter and the receiver).
5.1.4 Non-ray tracing models

In addition to the ray tracing models described in Sections 5.1.1 through 5.1.3, a number of other measurement based models have been proposed. One such model was proposed by Okumura [35], and later given an empirical formulation by Hata [36]. This model is applicable for frequencies in the range 150 MHz to 3000 MHz. The Okumura model is a set of curves based on an extensive series of propagation measurements in and around Tokyo. It is basically an empirical method for signal strength prediction based on determining the free space path loss between the transmitter and the receiver, adding an urban loss, and then adding or subtracting numerous correction factors to account for the nature of the terrain, the extent of urbanization, the heights of the antennas and street orientation. The basic formulation of this technique can be expressed as [24]:

\[
\text{path loss} = L_f + A_{mn} - H_m - H_r \quad \text{dB.}
\] (5-12)

In this expression, \(L_f\) is the free space path loss and \(A_{mn}\) is the median attenuation relative to \(L_f\) in urban areas over what is defined as ‘quasi-smooth’ terrain with a transmitter antenna height \(h_t\) of 200 m and a receiver antenna height \(h_r\) of 3 m. \(A_{mn}\) is a function of frequency and range and is expressed in graphical form by the series of curves shown in Figure 5.4. \(H_m\) and \(H_r\) are correction factors to account for antennas not at the reference heights of 200 m and 3 m; they are termed the height-gain factors and are also functions of frequency and range. Okumura’s paper contained graphs from which the appropriate values for any specific situation could be extracted.

Another model for predicting propagation-path loss was proposed by Ibrahim and Parsons [24],[37]. They took the approach that propagation in the urban environment
Figure 5.4 Median attenuation relative to free space in urban areas over quasi-smooth terrain (after Okumura) [24].
depends on such things as the density of buildings, the heights of buildings, and land use in
general. Furthermore, urban models suffer from an inherent vagueness associated with the
qualitative description of the urban environment. The empirical behavior was extracted
from measured data of propagation with regard to such factors as land usage factor,
degree of urbanization, and a varying terrain height for the mobile. The data was collected
in 500 m squares in London, England. The basic parameters for the London model are
summarized in Table 5.1. The “best fit” model based on measurements in London is:

\[
L_p = \left[ -20 \log(0.7 H_b) - 8 \log(H_m) + \frac{f}{40} + 26 \log\left(\frac{f}{40}\right) \right] \\
-861 \log\left(\frac{f + 100}{156}\right) + \left[ 40 + 14.15 \log\left(\frac{f + 100}{156}\right) \right] \log(1000d) \\
+0.265L - 0.37H + 0.087U - 5.5
\]  

(5-13)

The definitions of each term in equation (5-13) can be found in Table 5.1. Compared with
measurement, the rms errors produced by this model are 2.1 dB at 168 MHz, 3.2 dB at
455 MHz, and 4.2 dB at 900 MHz. Figure 5.5 shows an example for the case where
\(H_b = 100\) m, \(H_m = 1.5\) m, \(L = 50\%\), \(U = 16\%\), and \(H = 0\).

The two models described in this section have been used to predict the path loss or
signal strength degradation for land mobile radio services in urban environments. These
models, although widely used for conventional mobile radio, have not been found to be
suitable for the field-strength prediction in microcells in high-density urban environments.
This is due to the complex nature of the environment, and advanced techniques such as the
ray tracing models outlined in Sections 5.1.1 through 5.1.3 should be used instead [28].
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Range of Validity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_o$</td>
<td>Ibrahim and Parsons propagation, median, dB</td>
<td>-</td>
</tr>
<tr>
<td>$H_b$</td>
<td>Base antenna height, m</td>
<td>30–300</td>
</tr>
<tr>
<td>$H_m$</td>
<td>Mobile antenna height, m</td>
<td>&lt; 3</td>
</tr>
<tr>
<td>$L$</td>
<td>Land use factor, percentage of grid covered by buildings</td>
<td>3–50</td>
</tr>
<tr>
<td>$H$</td>
<td>Height difference between grid containing the fixed site and grid containing the mobile, m</td>
<td>-</td>
</tr>
<tr>
<td>$U$</td>
<td>Urbanization factor, percentage of buildings in grid taller than 3 levels; outside city center $U = 63.2$</td>
<td>0–100</td>
</tr>
<tr>
<td>$d$</td>
<td>Range, km (not beyond radio horizon)</td>
<td>&lt; 10</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency, MHz</td>
<td>150–1,000</td>
</tr>
</tbody>
</table>

Table 5.1 Parameters for Ibrahim and Parsons London model [30].
Figure 5.5 Ibrahim and Parsons London propagation model [30].
An alternative model for predicting propagation-path loss in urban microcells has recently been proposed by Tan and Tan [38]. This model is based on the uniform theory of diffraction (UTD), and it uses the multiple image concept and the generalized Fermat’s principle to describe the multiple reflections and diffractions. It includes all possible building and ground reflections and corner diffractions in the main street, side streets, and parallel streets of a microcell. This model has been found to be in good agreement with measurements which have been reported for city streets in Tokyo at 1.5 GHz [39] and New York City at 900 MHz [9] for various values of the propagation parameters. However, additional studies need to be done in order to determine the general applicability of this model.

5.2 CALCULATIONS FOR MESSAGE ERROR RATE IN A RURAL MICROCELL

5.2.1 Measured propagation characteristics

As was pointed out earlier in this chapter, a rural microcell represents the least severe propagation environment in which the proposed channel would be utilized. In other words, propagation-path loss for a given T-R separation distance would typically be smaller in a rural area than in any other type of propagation environment (namely, urban and suburban areas). Therefore, a rural microcell represents a good starting point for quantitative investigation into the performance of the proposed system.

The propagation data used in the subsequent analysis were gathered in open rural areas of Marlboro and Sandy Hook, New Jersey [9]. The measurements were made at
900 MHz by transmitting a CW signal from a mobile source to a fixed base, and recording the signal envelope variations as a function of mobile location. The microcell base antennas were placed at the side of the street or road at heights of up to 30 ft, simulating lamp post or utility pole mountings, and the mobile antennas were mounted on the roof of a minivan at a height of about 6 ft. Data was taken with omnidirectional antennas at both the base and mobile. The 900-MHz transmitter used in the study is a cellular radio transceiver driving an amplifier with a power output of up to 10 W. The receiver is a simple balanced mixer followed by a narrowband logarithmic IF amplifier/envelope detector.

The measured mean power at 900 MHz in rural environments is illustrated in Figure 5.6 by a solid curve. The calculated curves in this figure (dash-dot) represent the power received in a two-ray environment (see Section 5.1.2), including the antenna radiation patterns. No measurements were made to determine the actual antenna gains or the losses in the various transmission components of the measuring system. Thus, the measured and calculated curves are not compared on an absolute basis. The similarity of the basic propagation mechanism is demonstrated on a relative basis where the vertical offsets between the appropriate curves are chosen to facilitate clear comparison. The measured mean power plotted in Figure 5.6 is averaged over a 10-ft window.

It is easy to see that there is good agreement between the measured and calculated (using the two-ray model) results in Figure 5.6. The differences between the shape of the measured and theoretical curves are small. They could be attributed to scattering by small objects within the environment and are not expected to influence the potential quality of
Figure 5.6 Measured and calculated (two-rays) mean power in rural environments [9].
It is interesting to note that the mean power decays with distance (referenced to 1000 ft) faster than $1/r^2$ for distances exceeding the reference. Referenced to 1500 ft, the power decay reaches the asymptotic $1/r^4$ at distances exceeding 1500 ft. This is due to the first-order mutual cancellation of the LOS and ground reflected rays [33].

In order to apply the measured results of Figure 5.6 to the calculations for MER of the proposed channel in a rural environment, several assumptions need to be made. First, we must postulate that interferers located in adjacent microcells have a negligible effect on the transmitter of interest in comparison with interferers in the same cell. The validity of this assumption has been verified by a number of experiments performed at 900 MHz [40]. These experiments showed that only marginal co-channel interference occurred for received signal strengths above -100 dBm when the two transmitters were separated by a distance of 3.4 km. For a maximum microcell radius of 1 km to be used for the proposed application, this practically eliminates any effect that adjacent cell interferers may have on the transmitter of interest. Secondly, we must assume that the measured mean power shown in Figure 5.6 is radially symmetric (that is, the same in all directions). This is a rather good assumption for open rural areas with a flat terrain, such as the ones used to obtain the results of Figure 5.6.
5.2.2 Calculation procedure

The first step to calculate the MER in the rural microcell is to determine the best-fit curve for the measured path loss data in Figure 5.6. Using MATHCAD, the equation of the best-fit curve was found to be given by the following polynomial:

\[
PL(r) = \frac{3}{r^3} + \frac{3}{r^2} + \frac{3}{r} - 19.77 - 0.064r + (4.257 \cdot 10^{-5})r^2 - (1.188 \cdot 10^{-8})r^3 \quad dB. \quad (5-14)
\]

where the radial T-R separation distance \( r \) is now expressed in meters. Figure 5.7 shows this curve along with the experimental points, while Figure 5.8 depicts the absolute values of the differences between the path loss predicted by expression (5-14) and the corresponding experimental points from Figure 5.6. It can be seen from Figure 5.8 that there is generally a good agreement between the experimental values and those given by the best-fit curve.

Having determined the closed-form expression for the path loss, it becomes possible to give the exact form of equation (4-23) for the difference \( \Delta W \) between received signal powers from the transmitter of interest located at point \( T \) in Figure 4.4 and an interferer at point \( I \) in the same figure. For a radially symmetric case, it is given by:

\[
\Delta W(r, R) = W_{TR}(R) - W_{RI}(r) = PL(R) - PL(r) = \left[ \frac{3}{R^3} + \frac{3}{R^2} + \frac{3}{R} - 19.77 - 0.064R + (4.257 \cdot 10^{-5})R^2 - (1.188 \cdot 10^{-8})R^3 \right] - \left[ \frac{3}{r^3} + \frac{3}{r^2} + \frac{3}{r} - 19.77 - 0.064r + (4.257 \cdot 10^{-5})r^2 - (1.188 \cdot 10^{-8})r^3 \right]. \quad (5-15)
\]

with \( R \) being the T-R separation distance for the transmitter of interest and \( r \) being the T-R separation distance for the interfering transmitter. Expression (5-15) was derived by
Figure 5.7  Experimental values (points) and best-fit curve for path loss (PL) versus T-R separation distance (r) in a rural microcell.
Figure 5.8 Errors of fit for expression (5-14).
using (5-14) in (4-23) and assuming identical transmit powers for both the transmitter of interest and the interfering transmitter (which of course will be the case in the proposed system). By using (5-15) and (4-24) in (3-18), we can arrive at the expression for the probability of message survival \( P_{\text{sur}} \) with the Grayson receiver (which is intended for the proposed application) as a function of T-R separation distances \( r \) and \( R \). It is expressed as:

\[
P_{\text{sur}}(\Delta W) = P_{\text{sur}}(r, R)
\]

\[
= \begin{cases} 
0 & \text{if } \Delta W(r, R) < -10 \\
1 + \exp \left\{ -8.812103932 - 3.064703621 \Delta W(r, R) - 0.407152832 \Delta W(r, R)^2 - 0.023494960 \Delta W(r, R)^3 \right\} & \text{if } -10 \leq \Delta W(r, R) < -1.95 \\
1 & \text{otherwise}
\end{cases}
\]

where \( \Delta W(r, R) \) is given by (5-15).

Next, the incremental survival probability \( dP_s \) due to interferers located in the ring \( 2\pi r dr \) is determined by using (5-16) in (4-27). With this substitution, it becomes:

\[
dP_s = \frac{1}{2} x \frac{g(r) \cdot 2\pi r dr \cdot P_{\text{sur}}(r, R)}{\int_0^{r_{\text{min}}} g(r) 2\pi r dr}.
\]
with $r_{\text{lim}} = 1$ km (1,000 m) in this case. If we assume that 250 users (which is the maximum expected number of users per channel in a microcell of the proposed system) are uniformly distributed throughout the microcell, and no user is within 10 meters of the base station receiver, the expression for the density of users in the microcell $g(r)$ is independent of $r$ and is given by:

$$g(r) = g = \frac{250}{\pi (r_{\text{lim}}^2 - r_{\text{min}}^2)}.$$  \hfill (5-18)

where $r_{\text{min}} = 10$ m. Note that since the user density in the above equation is independent of $r$, it can be brought out of the integral in (5-17). The uniform user density is by no means the only distribution that can be employed. The effect of selecting other user densities on the probability of message survival $P_s$ in this microcell will be addressed in Section 5.2.3.

Before the probability of message survival $P_s$ can be arrived at, the radial limits of the transition region $r_0(R)$ and $r_1(R)$ for the transmitter of interest must be determined. They can be calculated by using (5-14) in the following equations:

$$PL(r_0) - PL(R) = 10 \quad (5-19)$$

and

$$PL(r_1) - PL(R) = 1.95 \quad (5-20)$$

If either $r_0$ in (5-19) or $r_1$ in (5-20) is less than $r_{\text{min}}$, it will be set to $r_{\text{min}}$ in order to prevent integrating over a region that does not have any users.

Recall that the general form of the expression for the probability of message survival $P_s$ for a transmitter located at a given point within a microcell is determined by
integrating over all possible interferer locations and is given by (4-34). For the case of uniform user density and radial symmetry, it reduces to:

\[
P_s = P_s(R) = \frac{1}{2} \times \frac{1}{\pi(r_{\text{lim}}^2 - r_{\text{min}}^2)} \times \left\{ \int_{r_{1}(R)}^{r_{1}(R)} f(r, R) \cdot 2\pi r dr + \pi \left( r_{\text{lim}}^2 - (r_{1}(R))^2 \right) \right\}
\]

(5-21)

where \( f(r, R) \) is given in equation (5-16), \( r_{\text{lim}} \) is again 1 km, and \( r_{\text{min}} \) is still 10 m.

Figure 5.9 shows the probability of message survival \( P_s \) from (5-21) as a function of the radial T-R separation distance \( R \) under the condition of uniform user density. It illustrates the fact that message survival is possible even for transmitters located at the outer edges of the microcell. For example, in the case of a transmitter with radial T-R separation distance of 1 km the probability of message survival is approximately 27 percent. The probability of message survival increases with the decreasing distance from the transmitter to the receiver, as expected. For transmitters located within 150 meters of the base station receiver, the survival probability is about 50 percent. In other words, the probability of message survival for close-in transmitters is simply the probability that their message arrives at the receiver and acquires lock prior to the interfering message. Since the probability of message survival is not negligible for any T-R separation distance in this microcell, we can expect a significant improvement in the values of the MER compared to the one predicted by the probabilistic traffic model of the channel (see equation (2-16)) which assumes no collisional survival (that is, \( P_s = 0 \)).

Expression (5-21) can now be substituted into equation (4-2) in order to calculate the MER for a transmitter at any given T-R separation distance in a rural microcell under...
Figure 5.9  Probability of message survival ($P_s$) versus radial T-R separation distance (R) for a rural microcell with uniform user density.
the assumption of uniform user density. The MER curves for several different T-R separation distances, together with the MER curve calculated using equation (2-16) which assumes no survival of colliding messages, are shown in Figure 5.10 for initial message arrival rate of 4 messages/sec. All other channel parameters used to generate these curves were identical to those used in Figure 2.1. It is easy to see that any transmitter in this microcell will require significantly fewer retransmissions to achieve the MER value of $1 \cdot 10^{-5}$ than is predicted by the probabilistic traffic model through expression (2-16) (which of course is independent of the T-R separation distance since it neglects survival of colliding messages). For example, transmitters situated up to 500 meters from the base station receiver will require no more than 4 retransmissions in 30 seconds to achieve MER of $1 \cdot 10^{-5}$, compared to 10 retransmissions in 30 seconds predicted by equation (2-16). Even for a transmitter 1 km from the receiver, only 6 retransmissions in 30 seconds will be needed to achieve the desired MER value. The resulting elimination of unnecessary retransmissions will allow the proposed channel to support even greater initial message arrival rates than those predicted in Chapter 2.

5.2.3 Effect of user density on probability of message survival

As was pointed out in the previous section, the probability of message survival $P_s$ in a microcell is significantly affected by the user density in that microcell. The calculations in Section 5.2.2 assumed uniform user density. However, certain microcells may have a greater concentration of users near the base station receiver than towards the edges of the microcell. In other words, they have user densities that vary as $1/r$ and
Figure 5.10  MER versus number of retransmissions in a rural microcell for initial message arrival rate equal to 4 messages/sec.
sometimes even as \(1/r^2\). We shall now proceed with a general formulation for calculating
the probability of message survival for any radially symmetric user density function.

The first step in this process involves the determination of the exact form of the
function. For example, if user density \(g(r)\) varies as \(1/r\), then it has the general form
\(g(r) = C/r\) where \(C\) is some unknown constant. This constant can be determined by
solving the following equation:

\[
\int_{r_{\text{min}}}^{r_{\text{lim}}} g(r) \cdot 2\pi rdr = \int_{r_{\text{min}}}^{r_{\text{lim}}} \frac{C}{r} \cdot 2\pi rdr = N. \quad (5-22)
\]

where \(N\) is the total number of users in a microcell and all other terms retain their
previous meanings. If user density is a function of \(1/r^2\), then it is given by \(g(r) = C/r^2\),
and the constant is obtained from:

\[
\int_{r_{\text{min}}}^{r_{\text{lim}}} g(r) \cdot 2\pi rdr = \int_{r_{\text{min}}}^{r_{\text{lim}}} \frac{C}{r^2} \cdot 2\pi rdr = N. \quad (5-23)
\]

Having determined the value of \(C\) (and therefore the exact form of \(g(r)\)), the
probability of message survival \(P_s\) can be calculated as follows:

\[
P_s(R) = \frac{1}{2} \times \frac{1}{\int_{r_{\text{min}}}^{r_{\text{lim}}} g(r) \cdot 2\pi rdr} \times
\left\{ \int_{r_0(R)}^{r_1(R)} g(r) \cdot f(r,R) \cdot 2\pi rdr + \int_{r_1(R)}^{r_{\text{lim}}} g(r) \cdot 2\pi rdr \right\}. \quad (5-24)
\]

It is clear that the above expression reduces to equation (5-21) when \(N = 250\) and the
user density is given by (5-18). Figure 5.11 shows the probability of message survival
from (5-24) versus the radial T-R separation distance for different user densities and with
\(N = 250\). These plots point out a result that is entirely expected; namely, that distant
transmitters are at a distinct disadvantage for greater user densities near the receiver.
Figure 5.11 Probability of message survival ($P_s$) versus radial T-R separation distance (R) for a rural microcell with various user densities.
5.3 CALCULATIONS FOR MESSAGE ERROR RATE IN AN URBAN MICROCELL

5.3.1 Determining microcellular contours

Characterizing the urban propagation environment is one of the more difficult tasks that any wireless system designer has to undertake. The major difficulty in characterizing these environments stems from the fact that urban areas exhibit a large degree of variability. While some cities (such as New York City) feature rectilinear street plans, others (such as Boston) have decidedly asymmetric layouts. In addition, there is a great variation in terrain profiles from one city to another. Cities like New York City and Chicago are relatively flat, while cities like San Francisco are rather hilly. Other features that vary for different urban areas include street width, distance between streets, building height, and building density. However, it is generally safe to say that a signal will encounter most severe attenuation in urban environments. Therefore, it is extremely important to examine the viability of operating the proposed system in such an environment.

Urban propagation is generally broken down into two categories: LOS propagation, pertaining to paths between transmitters and receivers within view of each other (that is, where users are on the same street as the base station receiver); and non-LOS propagation, pertaining to paths that are blocked by buildings (that is, where users are on streets perpendicular or parallel to the one containing the base station receiver) [41]. Figure 5.12 shows how the reflected waves enter the non-LOS streets at near and far distances. Extensive studies have been performed to quantify both LOS and non-LOS
Figure 5.12 Rays entering non-LOS streets [42].
propagation [9],[40],[43]-[47]. In addition, a multi-ray model has been developed for LOS propagation [31] and has been used to study the performance of several microcellular systems [48]-[49]. However, characterizing non-LOS microcell propagation is a much more difficult task. The subsequent analysis will utilize a model that has been developed by Goldsmith and Greenstein to predict non-LOS coverage areas for cities with rectilinear street plans like that in Figure 5.12 [32]. This model is based on an extensive 900 MHz propagation study conducted along numerous streets in Manhattan (New York City) [9]. They discovered that the contours of constant signal power attenuation (called the local mean attenuation, or LMA) are shaped like concave diamonds, symmetrical about the location of the base station receiver (see Figure 5.13). The model considers the propagation mechanism of radio signals in urban microcells which involves coupling of some of the energy transmitted along the main street (the street with the base station receiver) into the cross streets. Subsequently, some of that energy is coupled into the parallel streets. Based on this mechanism, the falloff with distance of received signal power on the LOS street decreases as (distance)$^{-\gamma}$, where $\gamma$ is typically between 2 and 4 [46]. For users on perpendicular or parallel streets (non-LOS propagation), the received power at a given distance is generally much lower than for the same distance along the main street. This is due to both the extra distance traveled by the signal and the added losses around corners.

The idea behind this model is to be able to generate contours of constant LMA. This is accomplished by mathematically approximating the variation of LMA along any given street by a simple function of distance, and to statistically characterize the
Figure 5.13  Contours connecting data-derived points of constant LMA in a Manhattan neighborhood [32].
fluctuations of the true LMA about this fit. Thereafter, these results are employed to predict constant LMA contours.

5.3.2 Calculation procedure

As was pointed out in the previous section, the first step in generating constant-LMA contours is to determine the fitted attenuation $L'_m(x)$ (in dB) as a function of the distance variable $x$. At this point, it is worth mentioning that for streets containing the base station receiver (main streets) and for parallel streets, the distance variable will be denoted by $x$. For streets perpendicular to a main street (cross streets), the distance variable will be denoted by $y$. Thus, the subscript $m$ denotes the fact that $L'_m(x)$ is the fitted attenuation along the main street. Figure 5.14 shows the plots of both measurement-based and model-based fitted attenuation $L'_m(x)$ for Lexington Avenue in Manhattan used by Goldsmith and Greenstein in [32]. The measurement-based plot is taken from the 900 MHz propagation study outlined in [9], while the model-based plot is from the multi-ray LOS model in [31] applied to the main street. It is interesting to note that there is generally a very good agreement between the measurement-based and model-based results of Figure 5.14. The agreement is within 2 dB over the major region of $x$. The major implication of this finding is that it may be possible to accurately predict the coverage area of an urban microcell. No measurements are needed, and all that must be known are basic properties of the street geometry and antenna positions and patterns.

With the aid of MATHCAD, the measurement-based curve of $L'_m(x)$ in Figure
Figure 5.14 Plots of $L_m(x)$, comparing measurement-based results with results based on the multi-ray LOS model for Lexington Avenue [32].
5.14 was determined to be well described by the following equation:

\[ L^*_m(x) = \frac{3}{x^3} + \frac{3}{x^2} + \frac{3}{x} - 64.468 - 0.07x + (7.563 \times 10^{-5})x^2 - (3.497 \times 10^{-8})x^3. \]  

(5-25)

where the T-R separation distance \( x \) has been converted from feet to meters. Expression (5-25), together with the experimental points that were used to generate it, is plotted in Figure 5.15. Figure 5.16 shows the absolute value of the differences between the fitted attenuation predicted by equation (5-25) and the corresponding points from Figure 5.15. Upon closer examination of Figure 5.16, it is clear that there is an excellent agreement (less than 1 dB difference) between the experimental values and those given by the best-fit curve of (5-25).

The next step in quantifying the performance of the proposed system in this environment is to determine microcellular extent. This is done by finding the functional dependence of the distance variable \( y \) along the cross streets on the distance variable \( x \) along the parallel and main streets. This is done with the aid of the following formula developed in [32]:

\[ y(x) = \pm 10^{\frac{1}{100}(L_{w(x)} - L_0 + D_0)}. \]  

(5-26)

The above expression represents the contour for a particular LMA value \( L_0 \). \( D_0 \) in (5-26) is defined as:

\[ D_0 = D_m + D_c - D_{c,0} + L_p + 10\log\left(\frac{A_c}{C_c}\right). \]  

(5-27)

\( D_0 \) is the dB fitting error between the actual value of LMA \( L(z) \) and the fitted LMA \( L^*(z) \), with \( z \) being either the x- or the y-coordinate. Since there is typically a statistical distribution in the values of \( L(z) \), \( D_0 \) is also statistically distributed. The 90th percentile
Figure 5.15  Experimental values (points) and best-fit curve for fitted attenuation \((L_m^*)\) versus T-R separation \((x)\) distance in an urban microcell.
Figure 5.16 Errors of fit for expression (5-25).
of $D_0$ will be utilized in the subsequent analysis. $D_m$ in (5-27) represents the dB fitting error at the intersection of the main street and a particular cross street, $D_c$ is the dB fitting error along the cross street at distance $y$ from the main street, and $D_{c,0}$ is the dB fitting error along the cross street for $y = 0$. The attenuation component $L_p$ in (5-27) is a statistical quantity introduced to account for the possibility that at some lower value of $y$, there is an intersection with a parallel street and a coupling loss into that street that will cause the LMA to drop to $L_0$ or below. The coefficients $A_c$ and $C_c$ in (5-27) result from modeling the fitted LMA along the cross street by the following function, as in [32]:

$$L_c^*(y) = -10 \log \left( A_c + B_c y^2 + C_c y^4 \right).$$

(5-28)

Table 5.2 contains the mean values for the components of $D_0$ calculated in [32]. These values were derived from the propagation data of [9] collected on Lexington Avenue.

<table>
<thead>
<tr>
<th></th>
<th>$D_m$</th>
<th>$D_c$</th>
<th>$D_{c,0}$</th>
<th>$L_p$</th>
<th>$10\log \left( \frac{A_c}{C_c} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN</td>
<td>0 dB</td>
<td>0 dB</td>
<td>-3.7 dB</td>
<td>-4.0 dB</td>
<td>-35.2 dB</td>
</tr>
</tbody>
</table>

**Table 5.2** Calculated components of dB fitting error $D_0$.

For the proposed application, the extent of the microcell is governed by the sensitivity of the Grayson receiver whose linear region of operation extends down to -100 dBm. Coupled with the fact that all system transmitters will have identical signal powers of +20 dBm, the microcellular extent $y_{\lim}(x)$ will be determined by the maximum...
allowable LMA value of $L_0 = -120$ dB. This contour is calculated by solving equation (5-26) with $L_0 = -120$, together with expression (5-27) and the values from Table 5.2. The resulting contour is depicted in Figure 5.17 with $\times$ representing the location of the base station receiver and each coordinate limited to no more than 1 km. The microcell of Figure 5.17 is essentially a concave diamond centered on the base station, as expected. It is fairly symmetrical about the base station location, and clearly elongated along the main street. This elongation is related to the fact that LOS propagation is stronger than non-LOS propagation.

Having determined both microcellular shape and extent, the next step is to determine the LMA value $L_{0X}(X,Y)$ for an arbitrary transmitter of interest located at point $(X,Y)$ within the microcell. This is accomplished by rewriting equation (5-26) as follows:

$$L_0(X,Y) = L^*_m(X) + D_0 - 40 \log(Y).$$

where $L^*_m(X)$ is calculated from expression (5-25).

In order to calculate the probability of message survival $P_s$ for a transmitter located at point $(X,Y)$, it is necessary to first determine the contours $y_0(x,X,Y)$ and $y_1(x,X,Y)$ corresponding to the edges of the transition region with respect to the receiver-dependent probability of message survival $P_{sur}$ (that is, where $P_{sur}$ is between 0 and 1) for that transmitter location. Recalling equation (3-18), we can see that, with the Grayson receiver, these contours correspond to the LMA values of $L_{00}(X,Y)$ and $L_{01}(X,Y)$, respectively, which are expressed as:

$$L_{00}(X,Y) = L_0(X,Y) + 10$$

(5-30)
Figure 5.17 Contour of urban microcell for proposed application (LMA = -120 dB).
and

\[ L_{01}(X, Y) = L_0(X, Y) + 1.95 \quad (5-31) \]

where \( L_0(X, Y) \) is given by expression (5-29). Using (5-30) and (5-31) in (5-26) leads to the following expressions for \( y_0(x, X, Y) \) and \( y_1(x, X, Y) \):

\[ y_0(x, X, Y) = \pm 10^{10^{40\{L_0(X) - L_0(Y) + D_b\}}} \quad (5-32) \]

and

\[ y_1(x, X, Y) = \pm 10^{10^{40\{L_0(X) - L_0(Y) + D_b\}}} \quad (5-33) \]

Figures 5.18 through 5.20 show the regions formed by the contours \( y_0(x, X, Y) \), \( y_1(x, X, Y) \), and \( y_{lim}(x) \), for transmitters of interest located at \((X, Y) = (310, 390)\), \((200, 200)\), and \((600, 200)\), respectively. Note that the location of the base station receiver in each of these figures is represented by a solid box, and only the first quadrant (that is, due to symmetry, 1/4 of each region) is shown. Point \((X, Y) = (310, 390)\) was selected because it represents the coordinates of the point at the microcell edge which is closest to the base station receiver (498 m).

Since the LMA value \( L_0(X, Y) \) for an arbitrary transmitter of interest located at point \((X, Y)\) within the microcell can now be determined using equation (5-29), it becomes possible to extract the exact form of expression (4-23) for the difference \( \Delta W \) between received signal powers from the transmitter of interest located at point \((X, Y)\) and an interferer at point \((x, y)\). Here, it is given by:

\[ \Delta W(x, y, X, Y) = L_0(X, Y) - L_0(x, y). \quad (5-34) \]
Figure 5.18 Regions of interest calculated using (5-32) and (5-33) for transmitter at $(X,Y) = (310,390)$. 
Figure 5.19 Regions of interest calculated using (5-32) and (5-33) for transmitter at $(X,Y) = (200,200)$. 
Figure 5.20 Regions of interest calculated using (5-32) and (5-33) for transmitter at $(X, Y) = (600, 200)$.
where we have again made use of the fact that the transmit powers for the transmitter of interest and the interfering transmitter are identical. By using (5-34) and (4-24) in (3-18), the expression for the probability of message survival $P_{\text{sur}}$ with the Grayson receiver as a function of transmitter positions $(X,Y)$ and $(x,y)$ becomes:

$$P_{\text{sur}}(\Delta W) = P_{\text{sur}}(x,y,X,Y)$$

$$= \begin{cases} 
0 & \text{if } \Delta W(x,y,X,Y) < -10 \\
1 & \text{if } -10 \leq \Delta W(x,y,X,Y) < -1.95 \\
1 + \exp \left\{ -8.812103932 - 3.06470362 \Delta W(x,y,X,Y) - 0.407152832 \Delta W(x,y,X,Y)^2 - 0.023494960 \Delta W(x,y,X,Y)^3 \right\} & \text{otherwise}
\end{cases}$$

$$= \begin{cases} 
0 & \text{if } \Delta W(x,y,X,Y) < -10 \\
f(x,y,X,Y) & \text{if } -10 \leq \Delta W(x,y,X,Y) < -1.95 \\
1 & \text{otherwise}
\end{cases}$$

(5-35)

where $\Delta W(x,y,X,Y)$ is given by (5-34).

Next, the incremental survival probability $dP_s$ due to interferers located in the incremental area $dxdy$ is determined by using (5-35) in (4-27) and noting the difference in coordinate systems. With this substitution, it becomes:

$$dP_s = \frac{1}{2} \times g(x,y) \cdot dxdy \cdot P_{\text{sur}}(x,y,X,Y) \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{y_{\text{min}}(x)}^{y_{\text{max}}(x)} g(x,y)dy \, dx.$$  

(5-36)
with \( x_{\text{min}} = 10 \text{ m} \) and \( x_{\text{lim}} = 1,000 \text{ m} \) in this particular case. If we again assume that 250 users are uniformly distributed throughout the microcell, the expression for the density of users in the microcell \( g(x, y) \) is independent of both \( x \) and \( y \), and is given by:

\[
g(x, y) = g = \frac{250}{4\int_{x_{\text{min}}}^{x_{\text{lim}}} y_{\text{lim}}(x) dx}.
\] (5-37)

where the factor 4 arises from the fact that all four quadrants need to be considered. By integrating equation (5-36) over all possible interferer locations, we can arrive at the expression for the probability of message survival \( P_s \) for a transmitter located at a given point \( (X, Y) \) within a microcell. For the case of uniform user density, it reduces to:

\[
P_s = P_s(X, Y)
= \frac{1}{2} \times \left( \frac{1}{\int_{x_{\text{min}}}^{x_{\text{lim}}} y_{\text{lim}}(x) dx} \times \right.
\left. \left\{ \int_{x_{\text{min}}}^{x_{lim}} y_0(x, X, Y) dy (x, y, X, Y) dx dy + \int_{x_{\text{min}}}^{x_{\text{lim}}} (y_{\text{lim}}(x) - y_1(x, X, Y)) dx \right\} \right)
\] (5-38)

where \( y_0(x, X, Y) \), \( y_1(x, X, Y) \), and \( f(x, y, X, Y) \) are obtained from (5-32), (5-33), and (5-35), respectively. The first term in the sum within the braces of (5-38) again represents the contribution of interferers located in the transition region with respect to the transmitter of interest at point \( (X, Y) \), while the second term in this sum represents the contribution of interferers located in the region from which transmissions will have no effect on a message of interest originating at point \( (X, Y) \).

Figure 5.21 depicts the probability of message survival \( P_s \) from (5-38) as a function of the \( y \)-coordinate \( Y \) for the transmitter of interest given the following \( x \)-coordinates: \( X = 10, 200, 310, 600, \) and \( 1,000 \). Each curve in Figure 5.21 is shown up to
Figure 5.21 Probability of message survival for various transmitter locations.
the maximum allowed $Y$ value for its respective $X$.

Figure 5.21 illustrates the fact that message survival is again possible even for transmitters located at the outer edges of the urban microcell. For example, in the case of a transmitter located at $(X,Y) = (1000,170)$, which is the furthest point from the base station receiver in this microcell (the T-R separation distance for this transmitter is about 1,014 m), the probability of message survival is approximately 16 percent. For transmitter locations close to the base station receiver, the survival probability approaches 50 percent, as expected. It is also interesting to note that all curves in Figure 5.21 terminate at about the same $P_s$ value, signifying that the probability of message survival is approximately the same for all transmitters located at the outer edges of the microcell.

Since the probability of message survival is not insignificant for any transmitter location in this microcell, we can once again expect a notable improvement in the values of the MER compared to the ones predicted in expression (2-16) by the probabilistic traffic model. This is verified by substituting equation (5-38) into equation (4-2), which can be done for any given transmitter location $(X,Y)$. Figure 5.22 illustrates the MER curves for the same transmitter locations as in Figures 5.18 through 5.20, together with the MER curve calculated using expression (2-16) which assumes total annihilation of all colliding messages, for initial message arrival rate of 4 messages/sec. All other channel parameters used to generate these curves are identical to those used in Figure 2.1. We can again ascertain that any transmitter in this microcell will require significantly fewer retransmissions to achieve the MER value of $1 \cdot 10^{-5}$ than is predicted by the probabilistic traffic model through equation (2-16). Even for a transmitter located at the outer edge of the microcell (at point $(X,Y) = (310,390)$), no more than 7 retransmissions in 30 seconds
Figure 5.22  MER versus number of retransmissions in an urban microcell for initial message arrival rate equal to 4 messages/sec.
will be required to achieve MER of $1 \cdot 10^{-5}$, compared to 10 retransmissions in 30 seconds predicted by expression (2-16). This will again eliminate unnecessary retransmissions and allow the proposed DS-SS channel operating in an urban microcell to support even greater initial message arrival rates than those predicted in Chapter 2.

Some additional information about channel performance in an urban microcell can be deduced from examining the MER as a function of the number of retransmissions and the y-coordinate for the transmitter of interest, with its x-coordinate fixed at a given value. Figure 5.23 illustrates this relationship for $X = 500$ m. It can be seen that for a transmitter located at $(X,Y) = (500,10)$ only 4 retransmissions in 30 seconds are necessary to achieve MER of $1 \cdot 10^{-5}$, while for a transmitter at the edge of the microcell (at point $(X,Y) = (500,290)$) 7 retransmissions in 30 seconds will be required to achieve the same MER. In addition, increasing the number of retransmissions for near transmitters seems to result in a greater decrease of their MER than it does for transmitters located further away from the base station receiver. This is due to the fact that near transmitters have a greater probability of message survival $P_s$, thus magnifying the effect of each additional retransmission.

### 5.4 MICROCELLULAR REGIONS WITH CONSTANT NUMBER OF RETRANSMISSIONS

In Section 5.2, we have developed the technique to calculate the MER as the function of the number of retransmissions for a transmitter at a particular T-R separation distance in a rural microcell, using the maximum expected initial message arrival rate as
Figure 5.23  MER versus number of retransmissions and y-coordinate $Y$ in an urban microcell for initial message arrival rate equal to 4 messages/sec and with x-coordinate $X = 500$ m.
a parameter. In Section 5.3, this calculation was performed for a transmitter at a given point \((X,Y)\) in an urban microcell. However, a question that a system designer would often face in developing the proposed system is how many retransmissions would be required for a transmitter at a particular location in each microcellular environment in order to achieve the desired MER. In other words, the regions with constant number of retransmissions would have to be determined for each type of microcell. This task can be accomplished by performing the procedure outlined below.

The first step in the calculations for each type of microcell involves rewriting equation (4-2) to solve for the probability of survival \(P_s\) in terms of all other channel parameters. This leads to the following expression:

\[
P_s(E) = 1 - \frac{1}{\text{MER}^{E+1}} \left(1 - e^{-2\lambda N_{\text{ef}}^2} + (1 - e^{-\lambda NT})(1 - \left(1 - \frac{2ET}{T - ET}\right)^E) e^{-2\lambda N_{\text{ef}}^2}\right).
\]

(5-39)

By specifying the required MER in the above equation, the probability of survival can be determined for any given number of retransmissions \(E\).

Recalling from Figure 5.9 that the probability of survival under the assumption of uniform user density in the rural microcell considered ranged from 27 to 50 percent, only numbers of retransmissions resulting in survival probability of (5-39) within this range will be used. For a minimum required MER of \(1 \cdot 10^{-5}\) and all other channel parameters identical to those used in Figure 2.1, only values \(P_s(4)\) and \(P_s(5)\) fall in the above-mentioned range. Thus, transmitters in this microcell would need to use between 4 and 6 retransmissions (depending on their radial T-R separation distance) in order to achieve the
required MER, where the region requiring 6 retransmissions is bounded by the limit of the microcell.

Next step involves determining the maximum radial T-R separation distance $R$ for each number of retransmissions in the above-mentioned range. The calculations will be shown for the case of 4 retransmissions since all other cases are handled in an analogous manner. Using (5-39), the calculated value of $P_s(4)$ is 0.41563. For uniform user density, we now need to equate expression (5-21) to the value of $P_s(4)$ from above and solve the following resulting expression for $R$:

$$\frac{1}{2} \times \frac{1}{\pi \left( r_{\text{lim}}^2 - r_{\text{min}}^2 \right)} \left\{ \int_{r_i(R)}^{r_i(R)} f(r, R) \cdot 2\pi r dr + \pi \left( r_{\text{lim}}^2 - (r_i(R))^2 \right) \right\} = P_s(4). \quad (5-40)$$

where the function $f(r, R)$ is given in equation (5-16). The value of $R$ determined in (5-40) corresponds to the maximum T-R separation distance where 4 retransmissions are sufficient to achieve MER of $1 \times 10^{-5}$.

Subsequently, expression (5-40) is solved for $P_s(5)$ and $P_s(6)$ to determine the radial bounds of the regions where 5 and 6 retransmissions are necessary for MER of $1 \times 10^{-5}$.

The results of calculations are shown in Figure 5.24.

It is interesting to note that transmitters located within 10-600 meters (the actual calculated value being 601.7 m) of the base station receiver will require just 4 retransmissions to achieve the required MER, compared with 10 retransmissions predicted by the probabilistic traffic model of equation (2-16) alone. In addition, only the transmitters located at the outer limits of this rural microcell (those with radial T-R separation distance within 979.3-1,000 m) will require as many as 6 retransmissions to
Figure 5.24 Number of retransmissions in 30 seconds required to achieve MER of no greater than $1 \times 10^{-5}$ versus radial T-R separation distance in a rural microcell.
achieve MER of $1 \cdot 10^{-5}$. This is still significantly less than the number of retransmissions predicted by the traffic model, and it again points out that DS-SS operation will ensure a substantial improvement in the performance of the proposed channel.

For the urban microcell, the determination of regions with constant number of retransmissions again requires us to first solve expression (5-39) to find the probability of survival $P_s(E)$ for each number of retransmissions $E$. Recalling from Figure 5.21 that the probability of survival under the assumption of uniform user density in the urban microcell considered ranged from 14 to 50 percent, only numbers of retransmissions resulting in survival probability of (5-39) within this range can be employed. For a minimum required MER of $1 \cdot 10^{-5}$ and all other channel parameters identical to those used in Figure 2.1, only values $P_s(4)$ through $P_s(6)$ fall in the above-mentioned range. Therefore, transmitters in this microcell would have to employ between 4 and 7 retransmissions in order to achieve the required MER. The calculations will again be shown only for the case of 4 retransmissions since all other cases are treated in a similar manner.

After the value $P_s(4)$ is determined from (5-39), it can be substituted into equation (5-38) yielding the following expression:

$$\frac{1}{2} \times \frac{1}{\int_{y_{\min}}^{y_{\max}} y_{\lim}(x)dx} \times \left\{ \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} f(x, y, X, Y) dy dx \right\} = P_s(4). \quad (5-41)$$

It is again worth mentioning that lower-case coordinate representations $x$ and $y$ are general in nature, while the upper-case coordinates $X$ and $Y$ represent a particular transmitter location. Equation (5-41) is solved for the coordinate $Y$ of the transmitter of
interest by setting its coordinate \( X \) to any value within the allowed range between \( x_{\text{min}} \) and \( x_{\text{lim}} \) (see Section 5.3.2). This is done in order to determine one possible transmitter position \((X,Y)\) at the outer edge of the region where 4 retransmissions would be needed to achieve the required MER. Setting \( X = 10 \) in expression (5-41) results in the calculated value of \( Y = 224 \). Thus, point \((X,Y) = (10, 224)\) represents a possible transmitter location on the edge of the region requiring 4 retransmissions.

Once the task of determining one possible location is accomplished, the LMA value \( L_{0}(X,Y) \) needs to be calculated for this transmitter position. This step is necessary since all other transmitters located at the outer edge of this region will have the same value of LMA. In other words, this region will be bounded by the contour of constant LMA. This value can be determined by solving expression (5-29), namely:

\[
L_{0}(X,Y) = L'_{m}(X) + D_{0} - 40 \log(Y). \tag{5-42}
\]

where \( L'_{m}(X) \) is calculated from expression (5-25). This results in the calculated value of \( L_{0}(10,224) = -94.98 \) dB. Thereafter, the contour of the region requiring 4 retransmissions, \( y_{4}(x) \), can be determined by solving equation (5-26) as follows:

\[
y_{4}(x) = \pm 10^{\frac{1}{40}(L'_{m}(x)-L_{0}+D_{0})} \tag{5-43}
\]

with \( L_{0} = -94.98 \) dB, as calculated in expression (5-42). The contour \( y_{4}(x) \) describes all points \((x,y)\) lying at the outer edge of the region where 4 retransmissions would be needed.

The above procedure is then repeated to determine the contours of the regions requiring 5 and 6 retransmissions, \( y_{5}(x) \) and \( y_{6}(x) \). The region requiring 7
retransmissions will be bounded by the microcellular extent $y_{lim}(x)$, which is again solved by using equation (5-43) with $L_0 = -120$ dB.

The resulting regions are illustrated in Figure 5.25, with again only the first quadrant shown.

It is easy to see that most of the transmitters in this microcell will require between 4 and 6 retransmissions. Thus, significant improvement over the number of retransmissions predicted by the probabilistic traffic model alone is again achieved.

It is obvious that the determination of the regions with constant number of retransmissions will have to be done on a case by case basis for each particular microcellular environment. In addition, these calculations lead us to conclude that all transmitters do not need to send identical number of retransmissions in order to achieve the required MER. The actual number of retransmissions required for each transmitter will depend on the transmitter’s position in a given microcellular environment.

5.5 DS-SS CHANNEL PERFORMANCE IMPROVEMENT: INITIAL MESSAGE RATE AND MESSAGE SURVIVAL

The method of calculating all parameters pertinent to the channel design was introduced in Section 2.2.2. This method was based on the probabilistic traffic model which assumed annihilation of colliding messages. For MER of less than or equal to $1 \cdot 10^{-5}$, the results were tabulated (see Table 2.1) and illustrated in Figure 2.2. We are particularly interested in the case of $E = 10$, $T = 30$ sec, and $\lambda N = 4.01$ messages/sec, since this case represents the heaviest expected load on the channel under development.
Figure 5.25 Regions with constant number of retransmissions in an urban microcell.
It is clear that the ability of transmissions to survive collisions in a DS-SS environment will only serve to increase the maximum supportable initial message arrival rates. The mean number of retransmissions throughout the microcell should be calculated for each type of microcellular environment, and later used to determine the mean supportable initial message arrival rate for a given retransmission interval and number of retransmissions.

For the rural microcell, the mean number of retransmissions can be determined from the results of calculations (graphically represented in Figure 5.24), and it is given by:

\[
E = 4 \int_{r_{\min}}^{R_4} 2\pi r dr + 5 \int_{R_4}^{R_5} 2\pi r dr + 6 \int_{R_5}^{r_{\lim}} 2\pi r dr = 4.679
\]  

(5-44)

The terms \( R_4 \) and \( R_5 \) in expression (5-44) are maximum T-R separation distances from equation (5-40) for 4 and 5 retransmissions, respectively, while the terms \( r_{\min} \) and \( r_{\lim} \) in (5-44) retain their previous meanings and values.

Having determined the mean number of retransmissions, we can then estimate the initial message arrival rate that the proposed channel operating in a DS-SS environment with the same set of parameters would actually be able to support while maintaining MER of no greater than \( 1 \cdot 10^{-5} \). This can be accomplished by recalling from Section 2.2.4 that the aggregate rate of messages attempting transmission over the channel (newly generated plus retransmitted ones) is simply \( \lambda N (E + 1) \) messages/sec, where \( \lambda N \) is still the initial message arrival rate and \( E \) is the number of retransmissions. It can also be seen from Table 2.1 that the probabilistic traffic model alone stipulates that 10 retransmissions in 30 seconds would be required to support an initial message arrival rate of 4.01 messages/sec with MER of less than \( 1 \cdot 10^{-5} \). Under these conditions, the aggregate message rate would
equal 44.11 messages/sec. However, we saw in (5-44) that DS-SS operation of the channel allows us to have a mean number of only 4.679 retransmissions in this microcell for MER of $1 \cdot 10^{-5}$. Substituting this value into the expression for the aggregate message rate and equating it to 44.11 message/sec yields the initial message arrival rate of $\lambda N = 7.77$ messages/sec that the channel would actually be able to support. This corresponds to a 94 percent improvement over the value predicted by the traffic model alone, and it clearly demonstrates the significant advantage that is derived from operating this channel in a DS-SS environment.

The calculation procedure for the urban microcell is analogous to that of the rural microcell, except that the mean number of retransmissions is determined with the aid of the following expression:

$$
E = \frac{4 \int_{x_{\text{min}}}^{x_{\text{max}}} y_4(x) \, dx + 5 \int_{x_{\text{min}}}^{x_{\text{lim}}} [y_5(x) - y_4(x)] \, dx}{\int_{x_{\text{min}}}^{x_{\text{lim}}} y_{\text{lim}}(x) \, dx} + \frac{6 \int_{x_{\text{lim}}}^{x_{\text{lim}}} [y_6(x) - y_5(x)] \, dx + 7 \int_{x_{\text{lim}}}^{x_{\text{lim}}} [y_{\text{lim}}(x) - y_6(x)] \, dx}{\int_{x_{\text{lim}}}^{x_{\text{lim}}} y_{\text{lim}}(x) \, dx}
$$

(5-45)

where the contours $y_4(x)$, $y_5(x)$, $y_6(x)$, and $y_{\text{lim}}(x)$ are again arrived at by solving expression (5-43) with appropriate LMA values $L_0$, and the terms $x_{\text{min}}$ and $x_{\text{lim}}$ retain their previous meanings and values.

The calculation procedure leads to the supportable initial message arrival rate in the urban microcell of 7.08 messages/sec. This corresponds to a 77 percent improvement over the value of 4.01 messages/sec predicted by the traffic model. It is still a rather significant betterment for the purposes of increasing the throughput of the proposed channel, and it highlights the fact that system performance will greatly surpass predictions.
of the traffic model even in exceptionally adverse propagation environments such as an urban microcell in New York City.

As we have seen, the actual supportable initial message arrival rate depends on the propagation characteristics of the microcell. Therefore, the analysis outlined above would have to be performed for each microcellular environment based on the actual propagation data for that environment. The estimated results for the initial message arrival rates given above are obviously only approximations. However, they allow us to conclude that the actual supportable initial message arrival rates will greatly exceed the predictions of the probabilistic traffic model.
CHAPTER 6. SUMMARY AND CONCLUSIONS

The work presented in this dissertation was motivated by the need for a unidirectional communication channel for a home-shopping network application. It was somewhat surprising to learn that previously developed and analyzed simplex channel models were not well suited for this particular application. They made no attempt to develop a retransmission strategy which would certainly be necessary during periods of heavy channel loading to improve the chances of each user having their message received correctly. In addition, these models made no provision for each user to be able to initiate a transmission at any given time. These features are of particular importance in a home-shopping network where product advertisements which prompt user response occur continuously at random points throughout any given day. Therefore, a need arose for the development of a new random access protocol with retransmissions which could improve the probability of successful message transmission in a one-way communication environment.

The theoretical formulation of the channel model is contained in Chapter 2. Several interesting points should be noted in connection with the developed channel model. First of all, the initial traffic from each user is modeled using Poisson statistics. Poisson statistics have been shown to serve as a good approximation in modeling the arrival of a large number of messages from totally uncorrelated sources, as is the case with the application under consideration. Secondly, each user is provided with a retransmission period immediately following their initial transmission of a message. This period is subsequently broken down into a number of subintervals of equal duration. All message
retransmissions for that user are randomly distributed (using uniform distribution) and take place during the retransmission period, with only one retransmission occurring in each subinterval.

The complex nature of this channel model necessitated the development of a mathematical approach in order to arrive at a closed-form expression for the probability of successful message transmission. Towards that end, the concept of an effective collision parameter was introduced and mathematically formulated. This allowed independent treatment, from a collisional dynamics point of view, of the initial message generation and retransmission processes, thus greatly simplifying the analysis. The resulting closed-form solution was verified with a computer program written to simulate channel behavior. Good agreement between theoretically predicted and simulated results was observed for a wide range of channel parameters. In addition, the proposed channel model was found to significantly improve channel reliability compared to using a single transmission of each message.

The model developed in Chapter 2 served as a worst-case estimate of channel performance since it assumed that all colliding messages would be lost. The fact that this channel was envisioned to operate in a DS-SS environment served as a major motivation for quantifying any possible improvement that such operation would provide. Specifically, the collisional performance of messages with identical spreading codes needed to be studied, since this issue was neither widely addressed nor well understood up to this point. Thus, a collision study outlined in Chapter 3 was performed in order to quantify the effect that an interferer with the same spreading code would have on a message of interest.
The collision study was designed to analyze the performance of two major DS-SS systems currently in use. These two systems differ in the way that the pseudonoise spreading code is acquired at the receiver. One system performs signal detection with a matched filter receiver, while the other uses a sliding correlator to detect incoming spread spectrum signals. The major goal of the collision study was to determine how these different receiver types performed in the presence of system interferers. In other words, the study set out to quantify interference rejection capability of each system.

Several interesting findings resulted from this collision study. First and foremost, the possibility of messages surviving a collision with an interferer of the same spreading code was verified for both DS-SS systems under test. These results lead us to believe that the actual system performance would exceed the predictions of the probabilistic traffic model. Secondly, it was determined that the probability of message surviving a collision was primarily a function of the ratio (or difference in dB) of signal powers, and not a function of the absolute power levels. This property holds for both receivers in the linear region of their operation, and it is of extreme importance in the context of system performance. It allows us to incorporate the results of the collision study directly with the propagation data for any given microcellular environment in which this system would operate. In other words, we would be able to make a direct transition from the difference in received signal powers between the message of interest and the interfering message to spacial positions of the transmitters emitting these signals in a microcell. The mathematical approach to incorporate the collision results in the enhanced theory of the channel is outlined in Chapter 4.
Important additional observations could be made from the results of the collision study. The collision dynamics for each receiver showed the presence of three distinct regions for the probability of message survival with respect to the relative powers of colliding messages: the region where the probability of message survival was equal to 0, the region where it varied from 0 to 1 (called the transition region), and the region where it was equal to 1. The width of the transition region was observed to be significantly greater for the sliding correlator receiver than for the matched filter receiver. This result pointed out the fundamental trade-off that exists between the two receiver types. The improvement in the interference rejection capability of a sliding correlator receiver comes at a price of significantly longer pseudonoise code acquisition time. Therefore, systems for which the throughput is a primary concern would be probably better served by employing matched filters for signal detection, while systems for which interference rejection is a major issue should use sliding correlator signal detection.

As was stated previously, the last major step in understanding the total system performance was to incorporate the results of the collision study with signal attenuation data for two widely different microcellular environments. Chapter 5 contains this analysis for a rural and an urban microcell. The propagation data in a rural microcell was based on measurements performed in several locales in New Jersey, while the urban microcell analysis was based on signal attenuation measurements performed in New York City. Only large-scale variations in the received signal power were considered since the worst case delay spread was significantly shorter than the bit period. Therefore, small-scale fading effects did not substantially change the probability of survival in collision for a
given message, and thus they were neglected in the collision analysis to get a first-order answer.

The results for both microcellular environments lead to several important conclusions. First of all, significant increase in the probability of successful message transmission compared to the values predicted by the probabilistic traffic model is achieved. This holds true even for users located at the edges of each microcell (that is, furthest away from the base station receiver). As a result, the number of retransmissions that is required to achieve a given MER value is markedly less than the number predicted by the channel model, often by more than 50 percent. The subsequent decrease in the total channel traffic will allow far greater arrival rates of initial messages to be supported.

In addition, the ability to predict the number of retransmissions that would be required to achieve a given MER at a particular user location could allow an implementation of an adaptive retransmission scheme. In other words, if we are able to determine the location of a transmitter with respect to the base station receiver, the appropriate number of retransmissions for that transmitter can be determined and subsequently sent. This would be similar in purpose to various power control schemes that are currently implemented in many cellular systems, and it would lead to a far more efficient utilization of the available channel resources.

An interesting feature of the analysis technique developed in this dissertation is its modularity. The analysis is initially broken down into three separate components: the probabilistic traffic model, the collisional receiver dynamics, and the microcellular propagation phenomena. Subsequently, a mathematical technique is outlined for incorporating each of these components into a complete channel model. This modularity
allows us to seamlessly substitute for any of these components, whether it is a different transmission strategy, a different receiver, or a different microcellular propagation environment. In each case, the general approach to analyzing the system performance remains unchanged, and it is this universality that significantly adds to the value of the developed technique from a system designer’s point of view.

The work presented in this dissertation by no means signals that a complete understanding of system performance has been reached. To the contrary, a number of important future research directions have been identified as the result of this work. With respect to the actual system under consideration, the statistical distribution of local-mean values could be incorporated into the analysis of system performance. In addition, the indoor-to-outdoor attenuation effects could be included in the complete channel model. The analysis in this dissertation was restricted to errors on the message level. In the future, this work could be extended to include error analysis on the bit level. Finally, the overlap time of colliding messages at the receiver could be varied, and its effect on the collisional receiver dynamics subsequently quantified.

With respect to the general performance of DS-SS receivers in the presence of co-channel interferers with the same spreading code, the work presented in this dissertation has helped to identify an additional important research direction. Namely, it may be possible to develop a method of predicting both the location and the width (with respect to the power differences between the colliding messages) of the transition region for the probability of message survival with a particular receiver. This would be done in terms of the major parameters of a DS-SS receiver, such as, for example, its process gain and the length of its spreading code. Developing this method would enable system designers to
more efficiently utilize the interference rejection capabilities of spread spectrum communication systems, thus paving the way for improved communication systems of the future.
APPENDIX A. CALCULATION OF MAXIMUM SUPPORTABLE INITIAL MESSAGE ARRIVAL RATE

*This MATHCAD 6.0 module contains calculations of the maximum message rate that a channel can support and still ensure a message error rate of less than 0.00001 for retransmission intervals T=5, 10, 15, 20, 25, and 30 sec. The channel bit rate is 40 Kbps and the message is 184 bits long.*

\[ \tau := 0.0046 \]
\[ E := 1, 2, \ldots, 15 \]
\[ j := 1, 2, 491 \]
\[ \text{rate}_j := 0.01j + 0.09 \]
\[ T := 5 \]
\[ \tau_{\text{eff}, E, j} := (E + 1) \cdot \tau + \frac{1}{2 \cdot \text{rate}_j} \ln \left[ e^{-\text{rate}_j \cdot T} + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) \right] \]
\[ P_{5, E, j} := \left( 1 - e^{-2 \cdot \text{rate}_j \cdot \tau_{\text{eff}, E, j}} \right) + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left[ 1 - \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) e^{-2 \cdot \text{rate}_j \cdot \tau_{\text{eff}, E, j}} \right]^{E + 1} \]
\[ \text{MER}_{5, E} := \left( P_{5} \right)^{E} \]
\[ T := 10 \]
\[ \tau_{\text{eff}, E, j} := (E + 1) \cdot \tau + \frac{1}{2 \cdot \text{rate}_j} \ln \left[ e^{-\text{rate}_j \cdot T} + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) \right] \]
\[ P_{10, E, j} := \left( 1 - e^{-2 \cdot \text{rate}_j \cdot \tau_{\text{eff}, E, j}} \right) + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left[ 1 - \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) e^{-2 \cdot \text{rate}_j \cdot \tau_{\text{eff}, E, j}} \right]^{E + 1} \]
\[ \text{MER}_{10, E} := \left( P_{10} \right)^{E} \]
\[ T := 15 \]
\[ \tau_{\text{eff}, E, j} := (E + 1) \cdot \tau + \frac{1}{2 \cdot \text{rate}_j} \ln \left[ e^{-\text{rate}_j \cdot T} + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) \right] \]
\[ P_{15, E, j} := \left( 1 - e^{-2 \cdot \text{rate}_j \cdot \tau_{\text{eff}, E, j}} \right) + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left[ 1 - \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) e^{-2 \cdot \text{rate}_j \cdot \tau_{\text{eff}, E, j}} \right]^{E + 1} \]
\[ \text{MER}_{15, E} := \left( P_{15} \right)^{E} \]
\[ T := 20 \]
\[ \tau_{\text{eff}, E, j} := (E + 1) \cdot \tau + \frac{1}{2 \cdot \text{rate}_j} \ln \left[ e^{-\text{rate}_j \cdot T} + \left( 1 - e^{-\text{rate}_j \cdot T} \right) \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) \right] \]
\[
\begin{align*}
P_{20,E,j} & := \left[ 1 - e^{-2 \cdot rate_j \cdot \tau_{eff,E,j}} \right] + \left[ 1 - e^{-rate_j \cdot T} \right] \left[ 1 - \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right)^E \right] e^{-2 \cdot rate_j \cdot \tau_{eff,E,j}} \right]^{E+1} \\
MER_{20,E} & := \left( P_{20,E} \right)^T \\
T & := 25 \\
\tau_{eff,E,j} & := (E+1) \cdot \tau + \frac{1}{2 \cdot rate_j} \ln \left[ e^{-rate_j \cdot T} + \left( 1 - e^{-rate_j \cdot T} \right) \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) \right] \\
P_{25,E,j} & := \left[ 1 - e^{-2 \cdot rate_j \cdot \tau_{eff,E,j}} \right] + \left[ 1 - e^{-rate_j \cdot T} \right] \left[ 1 - \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right)^E \right] e^{-2 \cdot rate_j \cdot \tau_{eff,E,j}} \right]^{E+1} \\
MER_{25,E} & := \left( P_{25,E} \right)^T \\
T & := 30 \\
\tau_{eff,E,j} & := (E+1) \cdot \tau + \frac{1}{2 \cdot rate_j} \ln \left[ e^{-rate_j \cdot T} + \left( 1 - e^{-rate_j \cdot T} \right) \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right) \right] \\
P_{30,E,j} & := \left[ 1 - e^{-2 \cdot rate_j \cdot \tau_{eff,E,j}} \right] + \left[ 1 - e^{-rate_j \cdot T} \right] \left[ 1 - \left( 1 - \frac{2 \cdot E \cdot \tau}{T - E \cdot \tau} \right)^E \right] e^{-2 \cdot rate_j \cdot \tau_{eff,E,j}} \right]^{E+1} \\
MER_{30,E} & := \left( P_{30,E} \right)^T \\
s(MER, \text{thresh}) & := \begin{cases} j - 1 & \text{break if } \min(MER) > \text{thresh} \\ j & \text{while } MER \leq \text{thresh} \\ j + 1 & \text{rate} \\ \end{cases} \\
max_rate & := s(MER, 1 \cdot 10^{-5}) \\
t(MER, \text{thresh}) & := \begin{cases} j - 1 & \text{break if } \min(MER) > \text{thresh} \\ j & \text{while } MER \leq \text{thresh} \\ j + 1 & \text{rate} \\ \end{cases} \\
max_rate & := t(MER, 1 \cdot 10^{-5}) \\
u(MER, \text{thresh}) & := \begin{cases} j - 1 & \text{break if } \min(MER) > \text{thresh} \\ j & \text{while } MER \leq \text{thresh} \\ j + 1 & \text{rate} \\ \end{cases} \\
max_rate & := u(MER, 1 \cdot 10^{-5})
\end{align*}
\]
\begin{align*}
v^{\langle \text{MER}_{20}, \text{thresh} \rangle} & := \\
& \begin{cases}
  j \leftarrow 1 \\
  \text{break if } \min^{\langle \text{MER}_{20} \rangle} > \text{thresh} \\
  \text{while } \text{MER}_{20} \leq \text{thresh} \\
  \text{rate}_j \leftarrow j + 1 \\
  j \leftarrow j + 1
\end{cases} \\
\text{max}_{\text{rate}}_{20}^{E} & := v^{\langle \text{MER}_{20}^{E}, 1 \cdot 10^{-5} \rangle} \\
\text{max}_{\text{rate}}_{20}^{E} & := w^{\langle \text{MER}_{25}^{E}, 1 \cdot 10^{-5} \rangle} \\
\text{max}_{\text{rate}}_{20}^{E} & := z^{\langle \text{MER}_{30}^{E}, 1 \cdot 10^{-5} \rangle} \\
\text{max}_{\text{rate}}_{30}^{E} & := z^{\langle \text{MER}_{30}^{E}, 1 \cdot 10^{-5} \rangle} \\
\text{max}_{\text{rate}}_{30}^{E} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{5}, \text{max}_{\text{rate}}_{10} \rangle} \\
\text{max}_{\text{rate}}_{5} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{5}, \text{max}_{\text{rate}}_{10} \rangle} \\
\text{max}_{\text{rate}}_{10} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{10}, \text{max}_{\text{rate}}_{15} \rangle} \\
\text{max}_{\text{rate}}_{15} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{15}, \text{max}_{\text{rate}}_{20} \rangle} \\
\text{max}_{\text{rate}}_{20} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{20}, \text{max}_{\text{rate}}_{25} \rangle} \\
\text{max}_{\text{rate}}_{25} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{25}, \text{max}_{\text{rate}}_{30} \rangle} \\
\text{max}_{\text{rate}}_{30} & := \text{augment}^{\langle \text{max}_{\text{rate}}_{30}, \text{max}_{\text{rate}}_{30} \rangle} \\
\text{max}_{\text{rate}} & := \text{max}_{\text{rate}}_{\text{temp}}
\end{align*}
APPENDIX B. SAMPLE SIMULATION FILE

Simulation run with varying parameters:
Constants: 15000 0.004600 30.000000 (Duration, Transmission time, and Maximum transmission time)
Average message density varies from 4.000000 to 4.000000 by 1.000000.
Number of retransmissions varies from 1 to 15 by 1.
Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 1 retransmissions and an average message density of 4.000000 msg/sec.
119768 8346 0.069685 (transmissions, w/ collisions, prob)
59884 287 0.004793 (messages, errors, probability)
Elapsed time in seconds: 13 (3+3+7)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 2 retransmissions and an average message density of 4.000000 msg/sec.
178590 18917 0.105924 (transmissions, w/ collisions, prob)
59530 70 0.001176 (messages, errors, probability)
Elapsed time in seconds: 19 (4+4+11)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 3 retransmissions and an average message density of 4.000000 msg/sec.
238192 32678 0.137192 (transmissions, w/ collisions, prob)
59548 24 0.000403 (messages, errors, probability)
Elapsed time in seconds: 26 (5+6+15)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 4 retransmissions and an average message density of 4.000000 msg/sec.
299375 50942 0.170161 (transmissions, w/ collisions, prob)
59875 13 0.000217 (messages, errors, probability)
Elapsed time in seconds: 35 (7+8+20)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.
Run for 5 retransmissions and an average message density of 4.000000 msg/sec.
359160  70925  0.197475  (transmissions, w/ collisions, prob)
59860   5   0.000084  (messages, errors, probability)
Elapsed time in seconds: 44 (8+12+24)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 6 retransmissions and an average message density of 4.000000 msg/sec.
418453  95159  0.227407  (transmissions, w/ collisions, prob)
59779   1   0.000017  (messages, errors, probability)
Elapsed time in seconds: 56 (9+15+32)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 7 retransmissions and an average message density of 4.000000 msg/sec.
478960  122871 0.256537  (transmissions, w/ collisions, prob)
59870   2   0.000033  (messages, errors, probability)
Elapsed time in seconds: 63 (10+19+34)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 8 retransmissions and an average message density of 4.000000 msg/sec.
538614  152019 0.282241  (transmissions, w/ collisions, prob)
59846   0   0.000000  (messages, errors, probability)
Elapsed time in seconds: 76 (11+24+41)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 9 retransmissions and an average message density of 4.000000 msg/sec.
598630  185420 0.309741  (transmissions, w/ collisions, prob)
59863   1   0.000017  (messages, errors, probability)
Elapsed time in seconds: 87 (13+29+45)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 10 retransmissions and an average message density of 4.000000 msg/sec.
658757  219867 0.333760  (transmissions, w/ collisions, prob)
59887   1   0.000017  (messages, errors, probability)
Elapsed time in seconds: 100 (14+36+50)
Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 11 retransmissions and an average message density of 4.000000 msg/sec.
716532 255965 0.357228 (transmissions, w/ collisions, prob)
59711 0 0.000000 (messages, errors, probability)
Elapsed time in seconds: 114 (16+42+56)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 12 retransmissions and an average message density of 4.000000 msg/sec.
778154 297206 0.381937 (transmissions, w/ collisions, prob)
59858 1 0.000017 (messages, errors, probability)
Elapsed time in seconds: 129 (16+50+63)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 13 retransmissions and an average message density of 4.000000 msg/sec.
837578 338085 0.403646 (transmissions, w/ collisions, prob)
59827 1 0.000017 (messages, errors, probability)
Elapsed time in seconds: 142 (18+58+66)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 14 retransmissions and an average message density of 4.000000 msg/sec.
898035 382542 0.425977 (transmissions, w/ collisions, prob)
59869 0 0.000000 (messages, errors, probability)
Elapsed time in seconds: 158 (20+67+71)

Limiting the maximum active messages to 1200 by removing 0 of 60000 messages.

Run for 15 retransmissions and an average message density of 4.000000 msg/sec.
958064 427438 0.446148 (transmissions, w/ collisions, prob)
59879 0 0.000000 (messages, errors, probability)
Elapsed time in seconds: 175 (21+77+77)
data sim01;
input defect msgsent;
MER = defect/msgsent;
cards;
289 59564
298 59611
278 59742
291 59872
286 59611
302 59611
289 59892
297 59823
295 59793
298 59704
279 59246
301 59889
283 59886
263 59178
286 59596
255 59741
287 59884;
;
proc iml;
use sim01;
read all into dataset;
options pageno = 1 ls = 64;
MER = dataset[.,3];
n = nrow(MER);
alpha = 0.05;
nboot = 1000;
xbar = j(nboot,1,0);
xnew = j(1,n,0);
do boot = 1 to nboot;
do i = 1 to n;
   u = int(ranuni(0)*n)+1;
xnew[i] = MER[u];
end;
xbar[boot] = xnew[:];
end;
rkxbar = rank(xbar);
copt = xbar[<>]-xbar[<>];
nci = int(nboot*(1-alpha));
do pointer = 1 to nboot-nci+1;
cwidth = xbar[loc(rkxbar = nci+pointer-1)]-
    xbar[loc(rkxbar = pointer)];
if cwidth<copt then do;
cilower = xbar[loc(rkxbar = pointer)];
ciupper = xbar[loc(rkxbar = nci+pointer-1)];
pointopt = pointer;
copt = cwidth;
end;
end;

print nci pointopt cilower ciupper;
quit;
APPENDIX D. C MODULE FOR ANALYSIS OF GRAYSON COLLISION DATA

```c
#include <stdlib.h>
#include <stdio.h>
#include <dos.h>
#include <string.h>
#define TRUE 1
#define FALSE 0

main(void)
{
    int i,j,ans,biterr,biterr1,biterr2,biterr3,biterr4,biterr5,biterr6,
        biterr7,biterr8,tempans,leftover, bytmerr, bytmerr 1, bytmerr 2,
        bytmerr 3, bytmerr 4, bytmerr 5, bytmerr 6, bytmerr 7, bytmerr 8, bytmerr 9,
        bytmerr 10, bytmerr 11, bytmerr 12, bytmerr 13, bytmerr 14, bytmerr 15,
        bytmerr 16, bytmerr 17, bytmerr 18, bytmerr 19, bytmerr 20, bytmerr 21,
        bytmerr 22, bytmerr 23, msgerr, newmsg, truefile, temp, endofmsg, l,
        delock, newtemp, trans2mes, truemsg;
    float mer, k, totmsgerr, totbiter, totbyter, ber, trans2rate;
    char mask[23] = "AAAAABCDEFHIJKLMNOPQRST", message[100],
                    inpfile[20], outpfile[20];
    unsigned char arr[23];
    FILE *in, *out;

    j=0;
    k=0;
    l=0;

    bytmerr=0;
    biterr1=0;
    biterr2=0;
    biterr3=0;
    biterr4=0;
    biterr5=0;
    biterr6=0;
    biterr7=0;
    biterr8=0;
    totbiter=0;
    totbyter=0;
```

totmsgerr=0;
bytmerr1=0;
bytmerr2=0;
bytmerr3=0;
bytmerr4=0;
bytmerr5=0;
bytmerr6=0;
bytmerr7=0;
bytmerr8=0;
bytmerr9=0;
bytmerr10=0;
bytmerr11=0;
bytmerr12=0;
bytmerr13=0;
bytmerr14=0;
bytmerr15=0;
bytmerr16=0;
bytmerr17=0;
bytmerr18=0;
bytmerr19=0;
bytmerr20=0;
bytmerr21=0;
bytmerr22=0;
bytmerr23=0;
truefile=TRUE;
trans2mes=0;

printf("Which file would you like to process? \n");
gets(inpfile);
printf("\n");
if (( in=fopen(inpfile,"rb"))==NULL) //the name of the file
{
    //is incorrect
    printf("File %s cannot be opened \n",inpfile);
    truefile=FALSE;
}

while ((k<1000) && (truefile==TRUE))
{
    //while less than
    newmsg=0; //certain # of messages
    i=0;     //encountered, proceed
    j=0;
    endofmsg=FALSE;
    truemsg=FALSE;
    delock=FALSE;
    bytmerr=0;
msgerr=0;
k=k+1;

//The program block below reads in all the characters from the
//current line of the input data file

do {
    temp=fgetc(in);
    if (temp!=EOF)
    {
        if (temp=='\n') //newline is encountered
        {
            newtemp=fgetc(in);
            if (newtemp=='\r') //the end of a message
                endofmsg=TRUE;
            else
            {
                if (newtemp=='\n') //the end of a message
                    //could be next
                    message[j]=temp;
                    j=j+1;
            }
            else //not yet the end of a
            {
                //message
                message[j]=temp;
                j=j+1;
                message[j]=newtemp;
                j=j+1;
                endofmsg=FALSE;
            }
        }
        else //character other than
        {
            if (temp=='\r' && newtemp=='\n') //newline is encountered
                endofmsg=TRUE; //the end of a message
            else
            {
                message[j]=temp;
                j=j+1;
                endofmsg=FALSE;
            }
        }
    } //EOF is encountered
endofmsg=TRUE;
} while (endofmsg==FALSE && temp!=EOF);

//The program block below is used to determine if the data on the
//current line is valid for message analysis

newmsg=j-1;
j=0;
l=0;
while (j<=newmsg)
{
    if (message[j]=='A')
    {
        if (((j+5)<= newmsg && message[j+1]=='A' &&
        {
            //message on the current line
            truemsg=TRUE;  //is valid
            while (j<=newmsg && l<=22)
            {
                arr[l]=message[j];
                if (l>0 && arr[l-1]=='X' && arr[l]=='X')
                    delock=TRUE;
                j=j+1;
                l=l+1;
            }
            j=newmsg+1;  //forcing loop termination
        }
        else  //so far, the valid message
        {
            //header is not seen
            j=j+1;
            truemsg=FALSE;
        }
    }
    else
        //letter A is yet to be seen
        j=j+1;
}

//The program block below performs the message analysis
//for a valid message

if (truemsg==FALSE)
    k=k-1;
else  //current message is valid
if (delock==TRUE)
    trans2mes=trans2mes+1;
newmsg=l-1;
if (newmsg<22)                                        //less than the expected
    //number of characters;
    j=newmsg;                                        //fill in the rest with
while (j<22)                                           //exact complements of
    //the expected character
    j=j+1;
    arr[j]=~(mask[j]);
}
newmsg=j;
}

j=0;
while (j<=newmsg)
{
    i=j;
    if (arr[i]=='X' && delock==TRUE)             //treat switchover characters
        arr[i]=~(mask[i]);                      //as 8-bit errors
    ans=mask[i]^arr[i];
    if (ans!=0)                                   //byte error in the current
    {
        //message is encountered
        tempans=ans;
        biterr=0;
        while (tempans!=0)
        {
            leftover=tempans%2;
            if (leftover==1)
            {
                biterr=biterr+1;
                tempans=tempans/2;
            }
            else
            tempans=tempans/2;
        }
        if (biterr>0)
            bytmerr=bytmerr+1;
        if (bytmerr>0 && msgerr==0)
        {
            msgerr=msgerr+1;
            totmsgerr=totmsgerr+msgerr;
        }
    }
switch (biterr)
{
  case 1: biterr1=biterr1+1; break;
  case 2: biterr2=biterr2+1; break;
  case 3: biterr3=biterr3+1; break;
  case 4: biterr4=biterr4+1; break;
  case 5: biterr5=biterr5+1; break;
  case 6: biterr6=biterr6+1; break;
  case 7: biterr7=biterr7+1; break;
  case 8: biterr8=biterr8+1; break;
}
} //end of IF (ANS!=0) statement
else
  biterr=0;

j=j+1;
} //end of WHILE (J<=NEWMSG) loop

switch (bytmerr)
{
  case 1: bytmerr1=bytmerr1+1; break;
  case 2: bytmerr2=bytmerr2+1; break;
  case 3: bytmerr3=bytmerr3+1; break;
  case 4: bytmerr4=bytmerr4+1; break;
  case 5: bytmerr5=bytmerr5+1; break;
  case 6: bytmerr6=bytmerr6+1; break;
  case 7: bytmerr7=bytmerr7+1; break;
  case 8: bytmerr8=bytmerr8+1; break;
  case 9: bytmerr9=bytmerr9+1; break;
  case 10: bytmerr10=bytmerr10+1; break;
  case 11: bytmerr11=bytmerr11+1; break;
  case 12: bytmerr12=bytmerr12+1; break;
  case 13: bytmerr13=bytmerr13+1; break;
  case 14: bytmerr14=bytmerr14+1; break;
  case 15: bytmerr15=bytmerr15+1; break;
  case 16: bytmerr16=bytmerr16+1; break;
  case 17: bytmerr17=bytmerr17+1; break;
  case 18: bytmerr18=bytmerr18+1; break;
  case 19: bytmerr19=bytmerr19+1; break;
  case 20: bytmerr20=bytmerr20+1; break;
  case 21: bytmerr21=bytmerr21+1; break;
  case 22: bytmerr22=bytmerr22+1; break;
  case 23: bytmerr23=bytmerr23+1; break;
}
} //end of IF (TRUEMSG!=FALSE) loop
if (temp==EOF)
//EOF encountered; terminate
    break;                             //outermost message loop
} //end of WHILE (K<1000) && (TRUEFILE==TRUE) loop

if (truefile==TRUE)
{
    printf("\n");
    printf("Which file would you like to save the output to? \n");
    gets(outpfile);
    printf("\n");
    out=fopen(outpfile,"wt");

    //total calculations
    totbiter=(long)biterr1+(2*biterr2)+(3*biterr3)+(4*biterr4)+
        (5*biterr5)+(6*biterr6)+(7*biterr7)+(long)(8*biterr8);
    totbyter=(long)bytmerr1+(2*bytmerr2)+(3*bytmerr3)+(4*bytmerr4)+
        (5*bytmerr5)+(6*bytmerr6)+(7*bytmerr7)+(8*bytmerr8)+
        (9*bytmerr9)+(10*bytmerr10)+(11*bytmerr11)+(12*bytmerr12)+
        (13*bytmerr13)+(14*bytmerr14)+(15*bytmerr15)+(16*bytmerr16)+
        (17*bytmerr17)+(18*bytmerr18)+(19*bytmerr19)+(20*bytmerr20)+
        (21*bytmerr21)+(22*bytmerr22)+(long)(23*bytmerr23);

    mer=totmsgerr/k;
    trans2rate=trans2mes/k;
    ber=totbiter/(184*k);

    //printing the results
    //to screen
    printf(" Message error distribution \n");
    printf("total # of messages with 1 byte error : %d\n",bytmerr1);
    printf("total # of messages with 2 byte errors : %d\n",bytmerr2);
    printf("total # of messages with 3 byte errors : %d\n",bytmerr3);
    printf("total # of messages with 4 byte errors : %d\n",bytmerr4);
    printf("total # of messages with 5 byte errors : %d\n",bytmerr5);
    printf("total # of messages with 6 byte errors : %d\n",bytmerr6);
    printf("total # of messages with 7 byte errors : %d\n",bytmerr7);
    printf("total # of messages with 8 byte errors : %d\n",bytmerr8);
    printf("total # of messages with 9 byte errors : %d\n",bytmerr9);
    printf("total # of messages with 10 byte errors: %d\n",bytmerr10);
    printf("total # of messages with 11 byte errors: %d\n",bytmerr11);
printf("total # of messages with 12 byte errors:  \%d\n", bytmerr12);
printf("total # of messages with 13 byte errors:  \%d\n", bytmerr13);
printf("total # of messages with 14 byte errors:  \%d\n", bytmerr14);
printf("total # of messages with 15 byte errors:  \%d\n", bytmerr15);
printf("total # of messages with 16 byte errors:  \%d\n", bytmerr16);
printf("total # of messages with 17 byte errors:  \%d\n", bytmerr17);
printf("total # of messages with 18 byte errors:  \%d\n", bytmerr18);
printf("total # of messages with 19 byte errors:  \%d\n", bytmerr19);
printf("total # of messages with 20 byte errors:  \%d\n", bytmerr20);
printf("total # of messages with 21 byte errors:  \%d\n", bytmerr21);
printf("total # of messages with 22 byte errors:  \%d\n", bytmerr22);
printf("total # of messages with 23 byte errors:  \%d\n", bytmerr23);
printf(" Byte error distribution \n");
printf("total # of bytes with 1 bit error     :  \%d\n", biterr1);
printf("total # of bytes with 2 bit errors    :  \%d\n", biterr2);
printf("total # of bytes with 3 bit errors    :  \%d\n", biterr3);
printf("total # of bytes with 4 bit errors    :  \%d\n", biterr4);
printf("total # of bytes with 5 bit errors    :  \%d\n", biterr5);
printf("total # of bytes with 6 bit errors    :  \%d\n", biterr6);
printf("total # of bytes with 7 bit errors    :  \%d\n", biterr7);
printf("total # of bytes with 8 bit errors    :  \%d\n", biterr8);
printf("\n");
printf("***********   T O T A L S   *********** \n");
printf("\n");
printf("total # of messages                   :  %1.0f\n", k);
printf("total # of wrong messages             :  %1.0f\n", totmsgerr);
printf("total # of switchovers                :  %1d\n", trans2mes);
printf("message error rate                    :  %.5f\n", mer);
printf(" switchover rate                       :  %.5f\n", trans2rate);
printf("total # of wrong bytes                :  %1.0f\n", totbyter);
printf("total # of wrong bits                 :  %1.0f\n", totbiter);
printf("bit error rate                        :  %.5f\n", ber);

//printing the results

//to file
fprintf(out," Message error distribution \n");
fprintf(out,"total # of messages with 1 byte error  \%d\n", bytmerr1);
fprintf(out,"total # of messages with 2 byte errors  \%d\n", bytmerr2);
fprintf(out,"total # of messages with 3 byte errors  \%d\n", bytmerr3);
fprintf(out,"total # of messages with 4 byte errors  \%d\n", bytmerr4);
fprintf(out,"total # of messages with 5 byte errors  \%d\n", bytmerr5);
fprintf(out,"total # of messages with 6 byte errors  \%d\n", bytmerr6);
fprintf(out,"total # of messages with 7 byte errors  \%d\n", bytmerr7);
fprintf(out,"total # of messages with 8 byte errors  \%d\n", bytmerr8);
fprintf(out,"total # of messages with 9 byte errors : \\
",bytmerr9);\nfprintf(out,"total # of messages with 10 byte errors: \\
",bytmerr10);\nfprintf(out,"total # of messages with 11 byte errors: \\
",bytmerr11);\nfprintf(out,"total # of messages with 12 byte errors: \\
",bytmerr12);\nfprintf(out,"total # of messages with 13 byte errors: \\
",bytmerr13);\nfprintf(out,"total # of messages with 14 byte errors: \\
",bytmerr14);\nfprintf(out,"total # of messages with 15 byte errors: \\
",bytmerr15);\nfprintf(out,"total # of messages with 16 byte errors: \\
",bytmerr16);\nfprintf(out,"total # of messages with 17 byte errors: \\
",bytmerr17);\nfprintf(out,"total # of messages with 18 byte errors: \\
",bytmerr18);\nfprintf(out,"total # of messages with 19 byte errors: \\
",bytmerr19);\nfprintf(out,"total # of messages with 20 byte errors: \\
",bytmerr20);\nfprintf(out,"total # of messages with 21 byte errors: \\
",bytmerr21);\nfprintf(out,"total # of messages with 22 byte errors: \\
",bytmerr22);\nfprintf(out,"total # of messages with 23 byte errors: \\
",bytmerr23);\nfprintf(out," Byte error distribution \\
");\nfprintf(out,"total # of bytes with 1 bit error     : \\
",biterr1);\nfprintf(out,"total # of bytes with 2 bit errors    : \\
",biterr2);\nfprintf(out,"total # of bytes with 3 bit errors    : \\
",biterr3);\nfprintf(out,"total # of bytes with 4 bit errors    : \\
",biterr4);\nfprintf(out,"total # of bytes with 5 bit errors    : \\
",biterr5);\nfprintf(out,"total # of bytes with 6 bit errors    : \\
",biterr6);\nfprintf(out,"total # of bytes with 7 bit errors    : \\
",biterr7);\nfprintf(out,"total # of bytes with 8 bit errors    : \\
",biterr8);\nfprintf(out,"***********   T O T A L S   *********** \\
");\nfprintf(out,"total # of messages                   : \\
",k);\nfprintf(out,"total # of wrong messages             : \\
",totmsgerr);\nfprintf(out,"total # of switchovers                : \\
",trans2mes);\nfprintf(out,"message error rate                    : \\
",mer);\nfprintf(out,"switchover rate                       : \\
",trans2rate);\nfprintf(out,"total # of wrong bytes              : \\
",totbyter);\nfprintf(out,"total # of wrong bits                : \\
",totbiter);\nfprintf(out,"bit error rate                        : \\
",ber);\n
fclose(in);\nfclose(out);\n}
return 0;
APPENDIX E. SAMPLE OUTPUT FILE FOR GRAYSON COLLISION STUDY

Message error distribution
total # of messages with 1 byte error : 13
total # of messages with 2 byte errors : 21
total # of messages with 3 byte errors : 23
total # of messages with 4 byte errors : 22
total # of messages with 5 byte errors : 29
total # of messages with 6 byte errors : 41
total # of messages with 7 byte errors : 59
total # of messages with 8 byte errors : 29
total # of messages with 9 byte errors : 31
total # of messages with 10 byte errors: 11
total # of messages with 11 byte errors: 2
total # of messages with 12 byte errors: 0
total # of messages with 13 byte errors: 0
total # of messages with 14 byte errors: 0
total # of messages with 15 byte errors: 0
total # of messages with 16 byte errors: 0
total # of messages with 17 byte errors: 0
total # of messages with 18 byte errors: 0
total # of messages with 19 byte errors: 0
total # of messages with 20 byte errors: 0
total # of messages with 21 byte errors: 0
total # of messages with 22 byte errors: 0
total # of messages with 23 byte errors: 0

Byte error distribution
total # of bytes with 1 bit error     : 420
total # of bytes with 2 bit errors    : 446
total # of bytes with 3 bit errors    : 295
total # of bytes with 4 bit errors    : 343
total # of bytes with 5 bit errors    : 108
total # of bytes with 6 bit errors    : 30
total # of bytes with 7 bit errors    : 5
total # of bytes with 8 bit errors    : 12

***********   T O T A L S   ***********

total # of messages                        : 784
total # of wrong messages             : 281
<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>total # of switchovers</td>
<td>0</td>
</tr>
<tr>
<td>message error rate</td>
<td>0.35842</td>
</tr>
<tr>
<td>switchover rate</td>
<td>0.00000</td>
</tr>
<tr>
<td>total # of wrong bytes</td>
<td>1659</td>
</tr>
<tr>
<td>total # of wrong bits</td>
<td>4420</td>
</tr>
<tr>
<td>bit error rate</td>
<td>0.03064</td>
</tr>
</tbody>
</table>
REFERENCES


VITA

Boris Davidson was born in Leningrad (now St. Petersburg), Russia in 1967. He came to the United States with his family in 1979 and settled in New York City. He received the B.S. and M.S. degrees in electrical engineering from Columbia University, New York, NY, in 1990 and 1992, respectively. His master’s topic concerned pump feedback techniques in erbium-doped fiber amplifiers. From 1992 to 1994, he was a Systems Engineer for Satellite Transmission Systems, Inc., Hauppauge, NY, where he worked on the design of FDMA and TDMA Standard A Satellite Earth Stations. In August 1994, he came to Virginia Tech to pursue the Ph.D. degree in electrical engineering.

Boris Davidson is a member of IEEE and ETA KAPPA NU. His current research interests lie in the areas of multiple access techniques for wireless communication systems, traffic modeling, and spread spectrum communications.