The parameters used in the characteristic equation determined in Chapter 4 are summarized as

\[ \eta_1 = \frac{U_1 Z'_{01}(U_1)}{Z_{01}(U_1)} , \quad U_1 = u_1 \sigma_1 \]  
(A.1)

\[ \eta_2 = \frac{U_2 Z'_{02}(U_2)}{Z_{02}(U_2)} , \quad U_2 = u_2 \sigma_2 \]  
(A.2)

\[ \eta_3 = \frac{U_2 Z'_{02}(U_2)}{\bar{Z}_{02}(U_2)} \]  
(A.3)

\[ \eta_4 = \frac{\bar{U}_2 Z'_{02}(\bar{U}_2)}{Z_{02}(\bar{U}_2)} , \quad \bar{U}_2 = u_2 \sigma_2 \]  
(A.4)

\[ \eta_5 = \frac{\bar{U}_2 Z'_{02}(\bar{U}_2)}{\bar{Z}_{02}(\bar{U}_2)} \]  
(A.5)

\[ \eta_6 = \frac{U_3 Z'_{03}(U_3)}{Z_{03}(U_3)} , \quad U_3 = u_3 \sigma_2 \]  
(A.6)

\[ \eta_7 = \frac{U_3 Z'_{03}(U_3)}{\bar{Z}_{03}(U_3)} \]  
(A.7)
\[ \eta_8 = \frac{U_3 Z'_{03}(U_3)}{Z_{03}(U_3)} , \quad \bar{U}_3 = u_3 \sigma_3 \] (A.8)

\[ \eta_9 = \frac{U_3 Z'_{03}(U_3)}{Z_{03}(U_3)} \] (A.9)

\[ \eta_{10} = \frac{U_4 Z'_{04}(U_4)}{Z_{04}(U_4)} , \quad U_4 = u_4 \sigma_3 \] (A.10)

\[ \xi_1 = \frac{Z_{02}(U_2)}{Z_{02}(U_2)} \frac{Z_{02}(U_2)}{Z_{02}(U_2)} , \quad \xi_2 = \frac{Z_{03}(U_3)}{Z_{03}(U_3)} \frac{Z_{03}(U_3)}{Z_{03}(U_3)} \] (A.11)

The field coefficients are \( A_i, B_{i+1}, C_i, \) and \( D_{i+1} \); where \( i = 1, 2, \) and \( 3 \).

\( A_i \) and \( B_{i+1} \) are determined in terms of \( A_1 \) and shown in Chapter 4. The rest of the amplitude coefficients, \( C_i \) and \( D_{i+1} \), are also determined in terms of \( A_1 \) and are summarized as

\[ C_1 = \frac{(T_2 T_{13} - T_3 T_{12}) D_2 - T_2 S_1}{T_1 T_{12} - T_2 T_{11}} \] (A.12a)

\[ C_2 = \frac{(T_3 T_{11} - T_1 T_{13}) D_2 + T_1 S_1}{T_1 T_{12} - T_2 T_{11}} \] (A.12b)

\[ C_3 = \frac{(T_9 T_{20} - T_{10} T_{19}) D_4 - T_9 S_3}{T_8 T_{19} - T_9 T_{18}} \] (A.12c)

121
\[ D_2 = \frac{(S_1 \overline{T}_4 - S_2 \overline{T}_2)}{\overline{T}_1 \overline{T}_4 - \overline{T}_2 \overline{T}_3} \quad \text{(A.13a)} \]

\[ D_3 = \frac{(T_{10}T_{18} - T_8T_{20}) D_4 + T_8S_3}{T_8T_{19} - T_9T_{18}} \quad \text{(A.13b)} \]

\[ D_4 = \frac{(S_2 \overline{T}_1 - S_1 \overline{T}_3)}{\overline{T}_1 \overline{T}_4 - \overline{T}_2 \overline{T}_3} \quad \text{(A.13c)} \]

where

\[ T_1 = Z_{21}(U_1), \quad T_{11} = U_1 Z'_{21}(U_1) \]
\[ T_2 = -Z_{22}(U_2), \quad T_{12} = -U_2 Z'_{22}(U_2) \]
\[ T_3 = -\overline{Z}_{22}(\overline{U}_2), \quad T_{13} = -U_2 \overline{Z}'_{22}(\overline{U}_2) \]
\[ T_4 = Z_{22}(\overline{U}_2), \quad T_{14} = \overline{U}_2 Z'_{22}(\overline{U}_2) \]
\[ T_5 = \overline{Z}_{22}(\overline{U}_2), \quad T_{15} = \overline{U}_2 \overline{Z}'_{22}(\overline{U}_2) \]
\[ T_6 = -Z_{23}(U_3), \quad T_{16} = -U_3 Z'_{23}(U_3) \]
\[ T_7 = -\overline{Z}_{23}(U_3), \quad T_{17} = -U_3 \overline{Z}'_{23}(U_3) \]
\[ T_8 = Z_{23}(\overline{U}_3), \quad T_{18} = \overline{U}_3 Z'_{23}(\overline{U}_3) \]
\[ T_9 = \overline{Z}_{23}(\overline{U}_3), \quad T_{19} = \overline{U}_3 \overline{Z}'_{23}(\overline{U}_3) \]
\[ T_{10} = -\overline{Z}_{24}(U_4), \quad T_{20} = -U_4 \overline{Z}'_{24}(U_4), \]

\[ \overline{T}_1 = T_5 + T_4 \hat{T}_1 \]
\[ \overline{T}_2 = T_6 \hat{T}_3 + T_7 \hat{T}_5 \]
\[ \overline{T}_3 = T_{15} + T_{14} \hat{T}_1 \]
\[ \overline{T}_4 = T_{16} \hat{T}_3 + T_{17} \hat{T}_5, \]
\[
\hat{T}_1 = (T_3 T_{11} - T_1 T_{13}) / (T_1 T_{12} - T_2 T_{11})
\]
\[
\hat{T}_2 = T_1 / (T_1 T_{12} - T_2 T_{11})
\]
\[
\hat{T}_3 = (T_9 T_{20} - T_{10} T_{19}) / (T_8 T_{19} - T_9 T_{18})
\]
\[
\hat{T}_4 = -T_9 / (T_8 T_{19} - T_9 T_{18})
\]
\[
\hat{T}_5 = (T_{10} T_{18} - T_8 T_{20}) / (T_8 T_{19} - T_9 T_{18})
\]
\[
\hat{T}_6 = T_8 / (T_8 T_{19} - T_9 T_{18}),
\]
\[
S_1 = -A_1 U_1^2 Z^{*}_{01}(U_1) + A_2 U_2^2 Z^{*}_{02}(U_2) + B_2 U_2^2 Z^{*}_{02}(U_2)
\]
\[
S_2 = -A_2 \bar{U}_2^2 Z^{*}_{02}(\bar{U}_2) - B_2 \bar{U}_2^2 Z^{*}_{02}(\bar{U}_2) + A_3 U_3^2 Z^{*}_{03}(U_3) + B_3 U_3^2 Z^{*}_{03}(U_3)
\]
\[
S_3 = -A_3 \bar{U}_3^2 Z^{*}_{03}(\bar{U}_3) - B_3 \bar{U}_3^2 Z^{*}_{02}(\bar{U}_3) + B_4 U_4^2 Z^{*}_{04}(U_4),
\]

and
\[
\bar{S}_1 = -[T_4 \hat{T}_2 S_1 + (T_6 \hat{T}_4 + T_7 \hat{T}_6) S_3]
\]
\[
\bar{S}_2 = -[T_{14} \hat{T}_2 S_1 + (T_{16} \hat{T}_4 + T_{17} \hat{T}_6) S_3] + S_2.
\]
Some parameters used in the field expressions are summarized as

\[ f_1(r) = (-jk_o/u^2)[A_1(\tilde{\beta}u)J'_\nu(ur) + B_1(vZ_o/r)J_\nu(ur)], \quad (B.1) \]
\[ f_2(r) = (jk_o/w^2)[A_2(\tilde{\beta}w)K'_\nu(wr) + B_2(vZ_o/r)K_\nu(wr)], \quad (B.2) \]
\[ f_3(r) = (-jk_o/u^2)[A_1(\tilde{\beta}u)J'_\nu(ur) + B_1(uZ_o)J_\nu(ur)], \quad (B.3) \]
\[ f_4(r) = (jk_o/w^2)[A_2(\tilde{\beta}w)K'_\nu(wr) + B_2(wZ_o)K_\nu(wr)], \quad (B.4) \]
\[ g_1(r) = (jk_o/u^2)[A_1(n_1^2v/Z_o r)J'_\nu(ur) + B_1(\tilde{\beta}u)J_\nu(ur)], \quad (B.5) \]
\[ g_2(r) = (-jk_o/w^2)[A_2(n_2^2v/Z_o r)K'_\nu(wr) + B_2(\tilde{\beta}w)K_\nu(wr)], \quad (B.6) \]
\[ g_3(r) = (-jk_o/u^2)[A_1(n_1^2u/Z_o)J'_\nu(ur) + B_1(\tilde{\beta}v/r)J_\nu(ur)], \quad (B.7) \]
\[ g_4(r) = (jk_o/w^2)[A_2(n_2^2w/Z_o)K'_\nu(wr) + B_2(\tilde{\beta}v/r)K_\nu(wr)], \quad (B.8) \]

where \( Z_o = (\mu_o/\varepsilon_o)^{1/2} \).

The transverse field components are summarized as

\[ E_r = f_1(r)\cos(\nu\phi + \bar{\phi}_o), \quad r < \sigma \quad (B.9a) \]
\[ E_r = f_2(r)\cos(\nu\phi + \bar{\phi}_o), \quad r > \sigma \quad (B.9b) \]
\[ E_\phi = -f_3(r)\sin(\nu\phi + \bar{\phi}_o), \quad r < \sigma \quad (B.10a) \]
\[ E_\phi = -f_4(r)\sin(\nu\phi + \bar{\phi}_o), \quad r > \sigma \quad (B.10b) \]
\[ H_r = -g_1(r)\sin(\nu\phi + \bar{\phi}_o), \quad r < \sigma \quad (B.11a) \]
\[ H_r = -g_2(r)\sin(\nu\phi + \bar{\phi}_o), \quad r > \sigma \quad (B.11b) \]
\[ H_\phi = g_3(r)\cos(\nu\phi + \bar{\phi}_o), \quad r < \sigma \quad (B.12a) \]
\[ H_\phi = g_4(r)\cos(\nu\phi + \bar{\phi}_o), \quad r > \sigma \quad (B.12b) \]
The following coefficients are defined as

\[ Q_1 = \beta k_0^2 \left( \frac{1}{u^2} \right) \left( Z_o B_1^2 + n_1^2 A_1^2 / Z_o \right) \] \hspace{1cm} (B.13)  
\[ Q_2 = \left( \frac{1}{u^4} \right) \left( \frac{2k_0^2}{u^2} \right) A_1 B_1 \left( \beta^2 + n_1^2 \right) - 2Q_1 \] \hspace{1cm} (B.14)  
\[ Q_3 = \beta k_0^2 \left( \frac{1}{w^2} \right) \left( Z_o B_2^2 + n_2^2 A_2^2 / Z_o \right) \] \hspace{1cm} (B.15)  
\[ Q_4 = \left( \frac{1}{w^2} \right) \left( \frac{2k_o^2}{w^2} \right) A_2 B_2 \left( \beta^2 + n_2^2 \right) - 2Q_3. \] \hspace{1cm} (B.16)

The field coefficients are \( A_1, B_1, A_2 \) and \( B_2 \). Here, we choose \( B_1 \) as the independent coefficient and use the boundary conditions at \( r = \sigma \) to express \( B_2, A_1 \) and \( A_2 \) in terms of \( B_1 \). From (5.2),

\[ B_1 J_\nu(U) = B_2 K_\nu(W) \]

then \[ B_2 = B_1 J_\nu(U) / K_\nu(W) \] \hspace{1cm} (B.17)  

From (5.1),

\[ A_1 J_\nu(U) = A_2 K_\nu(W) \]

then \[ A_2 = A_1 J_\nu(U) / K_\nu(W) \] \hspace{1cm} (B.18)

using boundary conditions for \( E_\phi \) and \( H_\phi \), and substituting (B.17) and (B.18) in these expressions, we can obtain the following,

\[ A_1 = -\left( Z_o / \nu \right) \left( 1 / \beta \right) \left( U W / V \right)^2 (\eta_1 + \eta_2) B_1 \] \hspace{1cm} (B.19)  

and \[ A_2 = -\left( Z_o / \nu \right) \left( 1 / \beta \right) \left[ J_\nu(U) / K_\nu(W) \right] (\eta_1 + \eta_2) B_1. \] \hspace{1cm} (B.20)

**Calculation of Power Flow, P:**

To calculate the power flow, \( P \), we write
\[(E \times H^*).a_z = E_r H_{\phi}^* - E_\phi H_r^* \]

The expression for power flow is obtained by substituting the field components in the expression above, which yields

\[
P = (\varphi_o/8) \left\{ Q_1 \sigma^2 [J_{v-1}(U) - J_v(U) J_{v-2}(U)] + vQ_2 J_v^2(U) + Q_3 \sigma^2 [-K_{v-1}(W) + K_v(W) K_{v-2}(W)] - vQ_4 K_v^2(W) \right\}. \quad (B.21)
\]

**Calculation of Power Loss, \( P_L \):**

To calculate the power loss, \( P_L \), we start with

\[
J_s = a_n \times H
\]

\[
J_s = J_s^1 = a_\phi \times H = -H_\phi a_z + H_z a_r, \quad \varphi = 0
\]

\[
J_s = J_s^2 = -a_\phi \times H = H_\phi a_z - H_z a_r, \quad \varphi = \varphi_0
\]

\[
|J_s|^2 = |H_\phi|^2 + |H_z|^2.
\]

After simplification, the expression for conductor power loss is obtained as

\[
P_{lc} = R_s \left\{ \int_0^a (B_1 J_v^2(Ur) + |g_1(r)|^2)dr + \int_{\varphi}^\varphi (B_2 K_v^2(Wr) + |g_2(r)|^2)dr \right\}. \quad (B.22)
\]

The expression for dielectric power loss, \( P_{ld} \), is obtained as

\[
P_{ld} = (1/2) \sigma_d \int_0^\varphi \cos^2(\nu \varphi) d\varphi \int_0^a [A_1^2 J_v^2(ur) + |f_1(r)|^2 + |f_2(r)|^2] rdr. \quad (B.23)
\]