Appendix C

Derivation of the Shrinking Sphere Model
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(Rimstidt, personal communication)

The surface area of a sphere is

\[ A = 4\pi r^2 \]

The volume of a sphere is

\[ V = \frac{4}{3} \pi r^3 \]

The general relationship between the surface area and the volume is

\[ A = b V^\frac{2}{3} \]

\[ 4\pi r^2 = b \left(\frac{4}{3} \pi r^3\right)^\frac{2}{3} \]

\[ b = 4.84 \]

The volume, \( V \), of a material is defined as

\[ V = n V_m \]

where \( V_m \) is the molar volume of a substance and \( n \) is the number of moles.

\[ A = bn^\frac{2}{3} V_m^\frac{2}{3} \]

For a zeroth order rate law

\[ \frac{dn}{dt} = -Ak \]

\[ \frac{dn}{dt} = -b V_m^\frac{2}{3} kn^\frac{2}{3} \]
\[ \int_0^\infty \frac{dn}{n^2} = -bV_m^2 k \int_0^t dt \]

\[-3n^\frac{1}{3} = -bV_m^2 k \Delta t \]

\[ \Delta t = \frac{\frac{1}{3}n^\frac{1}{3}}{bV_m^\frac{2}{3}} \]

\[ \Delta t = \frac{3V_m^\frac{1}{3}}{bV_m bk} \]

\[ \Delta t = \frac{3r(\frac{4}{3})^\frac{1}{3} \pi^\frac{1}{3}}{V_m 4\pi^\frac{1}{3} (\frac{4}{3})^\frac{2}{3} k} \]

\[ \Delta t = \frac{r}{V_m k} \]

Substituting for \( d = 2r \).

\[ \Delta t = \frac{d}{2V_m k} \]