CHAPTER TWO

2. Background and importance of crown shape/profile modeling

2.1 Introduction

The tree crown plays an essential role in tree productivity in that it is the location of the physiological activity that leads to growth and development. The location and distribution of branches and foliage which comprise tree crowns determine the shape of crowns. Many researchers have examined and/or modeled tree crowns in relation to their structures and shapes, while others have concentrated upon modeling the impact of shape on physiological activity of tree crowns.

2.2 Crown structure

Bertram (1989) identified the two main components to the structural design of branches in woody plants as: (i) the spatial organization of the branches, and (ii) the structural proportions of individual stems and branches. Both components have been used by researchers to describe or model the structure of tree crowns.

Ovington and Madgwick (1959) used branch length, branch weight, and foliage weight by whorl as crown structural descriptors in young Scots pine (Pinus sylvestris L.) trees. Stiell (1962) detailed the crown structure of 8 to 20 year old red pine (Pinus resinosa Ait.) noting (i) 8-14 live whorls per tree, (ii) more branches could be found on the southern aspect of the bole, (iii) branch diameters tended to increase and then decrease as one progressed from base live crown to the tree tip, and (iv) secondary
branches tended to occur in pairs. The intent of both of these early works was to use
crown structure to quantify the distribution of foliage within the tree crown.

Others recognized the link between crown size and tree growth. Horn (1971), for
example, noted the role that quantitative crown descriptions have in productivity research
while developing a model to predict crown radii. More recent efforts in describing or
modeling crown structure have had one of two purposes: (i) evaluate wood quality, (ii)
further quantification of the crown.

2.2.1 Wood quality

The location of branches and knots obviously affects wood quality. Inglis and
Cleland (1982) reported an equation predicting branch index (branch diameter for a given
5.5 m log length) based on site characteristics for thinned radiata pine stands. Tombleson
et al. (1990) studied the site and stocking effects on branch size in the same species.

Maguire et al. (1990) developed an equation to predict branch diameter near the
crown base in Douglas-fir (Pseudotsuga menziessii [Mirb.] Franco) while Pukkala et al.
(1992) focused on the diameter of the largest branch in Scots pine. The former modeled
the number of branches per annual shoot, maximum branch diameter per annual shoot,
and relative branch diameter using the Weibull distribution in 4- to 7-year-old Douglas-
fir. They noted that the branching patterns in young trees influenced the quality of the
wood recovered upon harvest. Doruska and Burkhart (1994) developed a system of
equations to generate the diameters and locations of branches within the crowns of
loblolly pine.
While studies of branch diameter and locations are important with respect to wood quality, the main thrust of this research effort is focused on the outer and inner profiles of the crown and the use of nonparametric regression to describe its shape.

### 2.2.2 Quantification of the crown

Recently, methods used to quantify the tree crown have changed dramatically from the descriptive efforts of Ovington and Madgwick (1959) and Stiell (1962). For example, Kellomäki (1986) chose to examine competition-density and the -3/2 power models with respect to tree crowns and branching by treating the individual tree crown as a population of branches much the same way a stand of trees can be considered a population of stems. Kellomäki found that a smaller branch density results in a greater branch volume than a greater branch density does on a tree level (note this is the opposite of the result obtained when considering a stand as a population of stems). Narrow crowns versus broader crowns were also discussed with respect to productivity (Kellomäki 1986).

Hashimoto (1990, 1991) used branching characteristics when reporting crown morphological changes in young sugi (*Cryptomeria japonica*) stands determining that (i) middle-story trees possessed higher branch inclinations than either over-story or under-story trees, with under-story trees possessing the lowest branch inclinations, and (ii) the ratio of crown length to crown diameter increased with age. The light environment was determined to be the primary determinant of crown morphology and structure.

More generally, Assmann (1970) described tree crowns, crown measures, and the importance of tree crowns with respect to productivity. Horn (1971) detailed crown shape and its effect on light interception. Along the lines of Horn (1971), Honda and Fisher
(1978) confirmed that branching angles observed in nature do result in the maximum leaf area in the tropical tree *Terminalia catappa* L.

As these studies suggest, research interest in the light environment and shape of tree crowns with respect to productivity was on the rise. The next section details the importance of modeling crown shape and describing foliage distribution.

### 2.3 Physiological impacts of crown shape

The volume of space occupied by tree crowns and the distribution and amount of foliage within tree crowns, in combination with the amount of solar radiant energy actually striking the crown, determine the amount of solar radiant energy absorbed by the crown (Grace 1990, Wang and Jarvis 1990b). Horn (1971) and Assmann (1970) were among the first to describe the effect of crown shape on productivity. Jahnke and Lawrence (1965) and Terjung and Louie (1972) were among the first to mathematically describe the relationships between crown shape, solar radiation, and productivity. Oker-Blom and Kellomäki (1982) continued such calculations noting that a conelike crown shape maximizes light absorption.

Grace *et al.* (1987a) reported that even small errors in crown shape representations and estimates of leaf area can have large impacts on predicted instantaneous measures of solar energy interception (the impacts were somewhat mitigated when interception was evaluated on a daily basis). Grace *et al.* (1987b) reported that estimates of net photosynthesis of a given tree crown can be in error by up to 40% for *radiata* pine when model assumptions disagree with measured rates of net photosynthesis. Net photosynthesis varies according to leaf area, which is dependent on estimates of crown
shape. Wickramasinghe (1988) chose to use evapotranspiration to model tree growth potential. Embedded in this model is the use of leaf area.

Clearly the ability to model crown profile and thus crown shape and/or leaf area and distribution is essential to physiological-based process models. The next sections examine how such shapes have been represented in the past.

### 2.4 Representations of crown shape

A variety of methods have been used to describe crown shape or profile. Previous research has often focused on quantifying crown volume. Crown volume can be used as a measure of competitive status (Wensel et al. 1987; Biging and Dobbertin 1992, 1995) as well as an assessment of wildlife habitat or abundance (Morrison et al. 1987) in addition to being used for process modeling.

#### 2.4.1 Geometric Shapes

Mawson et al. (1976), when assessing bird habitat, chose to calculate crown volume by assigning trees and shrubs one of five shapes (circle, ellipse, neiloid, triangle, or parabola) to their crown profiles and one of three shapes (circle, ellipse, triangle) to their crown bases. They noted that the shapes must be "carefully selected".

Smith (1990), similar to Mawson et al. (1976), also used cones and paraboloids to represent tree crowns, but intersected the geometric shapes when assessing inter-tree competition common in fully stocked stands. Biging and Wensel (1990) applied Smalian's formula when calculating crown volumes by sections, noting that a quadratic crown taper seemed appropriate.
Hatch et al. (1975) used cones to model crowns when studying light interception of tree crowns. Korzhukin and Ter-Mikaelian (1995) chose geometric shapes to represent crowns as planar screens also when assessing light interception.

2.4.2 Fractals

Fractal geometry can be used to describe many objects in nature (Mandlebrot 1983). As objects are viewed on finer and finer scales, the same repeating pattern defines the edges or borders of the natural objects. The fractal dimension of an object is invariant, and is used to describe the change in the frequency of the repeating pattern as the scale changes. Whereas Euclidean dimensions are common to all objects, i.e. 1 = a line, 2 = an area, and 3 = a volume, fractal dimensions are unique to objects and fall between the Euclidean categories, i.e. a decimal number between 2 and 3 for an area (Zeide and Gresham 1991).

Zeide (1990) introduced fractal geometry to forestry and examined the fractal dimensions of tree crowns with respect to shade tolerance. A fractal dimension of 2 suggests that a crown’s foliage mass is proportional to the crown’s surface area. A fractal dimension of 3 suggests the foliage mass is proportional to the volume of the crown. The former is indicative of a crown whose foliage is concentrated on its edge, while the latter is indicative of a tree whose foliage is evenly distributed throughout the crown.

Zeide (1990) showed that within a given crown class, tolerant species possess larger fractal dimensions than intolerant species because the former can maintain more shade foliage than the latter. Across crown classes, suppressed crowns are expected to have lower fractal dimensions than dominant crowns because suppressed crowns occupy
regions of very low light intensity, too low for foliage to be maintained in the interior crown.

Zeide and Gresham (1991) expanded the fractal analysis of tree crowns by relating fractal dimension to site quality. They found that crowns on better quality sites possessed larger fractal dimensions (fuller crowns). Osawa (1995) hypothesized that species with smaller crown fractal dimensions (gymnosperms and some shade-intolerant angiosperms) should have smaller thinning exponents when applying the self-thinning rule.

Zeide (1997) examined the fractal dimension of the data used herein and tried to relate the dimension to foliage density. His efforts, though, were unsuccessful. Therefore, the fractal dimension of the crown will not be examined in this project.

Both Zeide (1990) and Zeide and Pfeiffer (1991), however, report a method of calculating fractal dimensions of tree crowns. Calculating crown volumes is required when calculating a crown’s fractal dimension. Most work to date has used geometric representations of sections of the crown (similar to the work reported in Section 2.4.1). Therefore, the concept of fractal dimension is still linked to the use of geometric shapes to represent tree crowns. This research will examine a method to improve such representations.

2.4.3 Structural models

With time, researchers have begun to abandon the rigidity of using geometric shapes to represent tree crowns. Kranigk and Gravenhorst (1995) developed a three-dimensional model of Norway spruce (*Picea abies* [L.] Karst.) crowns via the parameterization of morphological characteristics. The model starts with a single shoot in
relation to its sprouting behavior within the crown. The shoot is then grown, with additional sprouting of buds. As the shoots continue to develop and/or die, a three dimensional model is formed. This model can be used to develop the spatial distribution of needle area.

Kurth and Lanwert (1995) likewise present a morphologically-based crown model for Norway spruce. Instead of building the outer shell of the crown as Kranigk and Gravenhorst (1995) did, they concentrated on the branching structure within the crown, forming a model somewhat reminiscent of pipe model theory (Shinozaki et al. 1964a, 1964b; Oohata and Shinozaki 1979; and Valentine 1985).

Others have held somewhat to geometric shapes by using mathematical equations. However, the shapes represented by such mathematical equations are flexible to accommodate a wide variety of tree crown profiles. Parameterization of such equations are typically functions of tree attributes such as diameter at breast height (DBH) and tree height (HT). These recent efforts are outlined in the next section.

### 2.4.4 Mathematical Equations

Mohren (1987), as reported by Nepal et al. (1996), used the following equation form to describe the crown shapes of Douglas-fir:

\[
R_i = R_B(1 - R_{CH_i})^\phi + \omega_i
\]  

(2.1)

where \( R \) is the crown radius calculated at a relative crown height \( (R_{CH}) \) within the crown given the radius at the crown base \( (R_B) \) and a shape parameter \( \phi \). The term \( \omega \) is an error term with the standard assumptions regarding error terms. This model form will be hereafter referred to as equation (2.1) or the Mohren model. The value of the shape
parameter was related to differing ages of stand development, as crowns progress from a cone shape \((\phi = 1)\) to a cylinder \((\phi = 0)\). A parabolic shape results when \(0 < \phi < 1\). This model form could be and was fit to the data in this study.

Baldwin and Peterson (1997) presented a series of equations to predict crown shape of loblolly pine. The parameterization of the outer and inner crown profile equations were based upon individual tree attributes. The reported equation for outer crown radius \((OCR)\) was:

\[
OCR = (-4.5121 + 0.5176DBH + 4.3529R \left(\frac{RCH - 1}{RCH + 1}\right) + (4.4749 - 0.0175AGE - 0.4985DBH - 6.0414R)(RCH - 1) 
\]  
(2.2)

and the equation for inner crown radius \((ICR)\) was reported to be:

\[
ICR = (0.0168DBH + 0.0155FL) + (-0.0233DBH)(RCH) 
\]  
(2.3)

\(R\) is equal to crown ratio and can be calculated via:

\[
R = (HT - HBLF) / HT 
\]  
(2.4)

Baldwin and Peterson provided an equation to calculate height to base live foliage \((HBLF)\) which is used in the calculation of \(R\).

\[
HBLF = 09326HLC - 0.0267 + 0.1006HT 
\]  
(2.5)

Note that equation (2.2) assumes the approximate shape of an ellipsoid (it is flexible based upon tree attributes) for the outer crown radius, while equation (2.3) is linear with respect to modeling the inner crown radius. Baldwin and Peterson (1997) were among the first to address the inner crown profile in addition to the outer profile. Both obviously affect the distribution and location of foliage within the crown. Equations (2.2) and (2.3)
could be and were applied to the data in this research effort and will be hereafter referred to as the Baldwin model or equations (2.2) and (2.3).

Nepal et al. (1996) noted that most crown radius models calculate an average crown radius (the Baldwin model outlined above excepted of course) while the maximum crown radii determine the crown shape and thus solar energy interception. They proposed the use of stochastic frontier modeling approaches to model maximum crown radii.

Stochastic frontier modeling (see Aigner et al. 1977; Meeusen and van den Broeck 1977; Greene 1980; Huang 1984) has its foundation in forest econometrics, but can be applied wherever a maximum (or minimum) relationship between two variables is of interest. The error term in the stochastic frontier model is divided into two components. The first component is the standard error component or random noise present in the data. The second error component extends the average relationship between the two variables to the frontier (either the maximum or minimum relationship) being examined.

In their modeling efforts, Nepal et al. (1996) found that the stochastic frontier model did strictly exceed the radii calculated by a least squares model fit representing the average crown radius along the entire bole length. Radii of all branches within a crown are required to fit the stochastic frontier models; thus, it could not be applied within this research effort. However, based upon the graphic representations in Nepal et al. (1996), the stochastic frontier models mimic the shape of the average crown radius models (in much the same manner that anamorphic site index curves mimic one another.) The crown profiles at the actual extent of the foliage do not always follow this average shape. The
radii at the extent of the foliage, and not the average radius of the crown along the bole, is the target of this particular project.

2.4.5 Inclusion of stochasticity

Crown shapes can be and are quite often distinct from tree to tree within a given stand. Thus, some may argue that assigning any definable shapes to such objects may not be plausible. Rather, obtaining measurements from each tree or adding a stochastic component to crown representations might be used to model such variations in shape. Biging and Gill (1997) inject just such a stochastic notion into representations of tree crowns by employing autoregressive moving average (ARMA) models in conjunction with quadratic or cubic trends. The trend was used to determine the average profile of tree crowns, with stochasticity about this average profile entering via the inclusion of random errors.

There is no doubt the crowns (of conifer species of the Sierra Nevada) simulated by Biging and Gill (1997) are indeed quite realistic. However, their method involves a somewhat heavy data requirement per tree (typically 35 to 50 measures of crown radii were used in each tree crown simulated.) As such, this model type could not be employed to the data used herein.

The notion of crowns having shapes that cannot be mathematically defined, though, is key to this modeling effort. In fact, the entire thrust behind nonparametric regression is to use a data-driven technique to determine the shape of a curve as opposed to having the modeler impose one onto the data.