4.0 Model Description

4.1 History

The models used in the current effort have been developed over the past seven years by various students in the Thermal Radiation Group at Virginia Tech as part of their master’s and doctoral research, as indicated in Chapter 2. The optical and radiative model was developed by Meekins [29] and Bongiovi [30], and provides the spectral and spatial distribution of energy within the radiometric channel. This model provides the boundary conditions for the transient thermal model of the radiometric channel structure, and the dynamic electrothermal model of the detector module assembly. The thermal diffusion model of the radiometric channels was completed by Savransky [32] in order to assess the possibility of the radiometric signal being contaminated by surface emission caused by the channel structure experiencing thermal transients. This model was used by Haeffelin [62] to verify that for the CERES geometry thermal contamination due to emission from these surfaces is not a significant factor for nominal on-orbit conditions. The dynamic electrothermal model of the detector module assembly was initially developed by Haeffelin [31] as part of his master’s research, in which he studied the issue of spatial equivalence of ERBE- and CERES-type thermistor bolometer detectors. The current author then took over responsibility for integrating Haeffelin’s model into a completed end-to-end model. As part of this effort, the code has been modified to take into account...
transient temperature changes in the aluminum substrates behind the two detectors, thus linking them thermally. In addition, the electronic model was updated to represent the full suite of signal-conditioning electronics of the CERES instrument. Finally, an optical front end was developed in order to be able to scan simulated Earth scenes as provided by Villeneuve [33]. The completed end-to-end model was validated by simulating the ground calibrations of the actual instrument, providing a virtual instrument with which to assess the performance of the actual flight instruments. A reduced-order version of the current model was supplied to Haeffelin as part of his doctoral research [62] in order to validate its performance and to assess generic concerns which may appear in operational scanning thermistor bolometer type radiometers.

4.2 Optical and Radiative Model

A Monte-Carlo ray-trace representation is used to compute the distribution of monochromatic radiation within the instrument. This distribution is quantified through the use of the monochromatic distribution factor, $D_{ijk}$. This factor is defined as the fraction of energy emitted from surface or volume element i in wavelength interval k which is absorbed by surface or volume element j. The distribution factor includes direct radiation from i to j as well as all possible diffuse and specular reflections and refraction through the bandpass filters. Basic assumptions allowing the use of this method are that thermal radiation transport between two elements may be assumed to occur in discrete energy bundles and the laws of chance may be used to determine the disposition of each energy bundle as it interacts with an element. The current model ignores diffraction effects since measurement of these effects on the flight instruments indicates that they are insignificant.

The enclosure specified by the instrument geometry is subdivided into surface elements whose absorptivities, degrees of specularity, and temperatures are specified. In addition, the bandpass filters may be subdivided into volume elements with known temperatures and transmissivities. A statistically significant number, typically in the millions, of optical energy bundles is then allowed to enter the enclosure through the baffle with directional and spatial distributions dependent upon the nature of the source field being simulated. A statistically
significant number of energy bundles is also diffusely emitted from the interior surfaces of the instrument. The path of each bundle is traced through the channel as it is reflected from, and transmitted through, the various surface and volume elements until it is eventually absorbed or exits the instrument through the telescope opening. The distribution factor $D_{ijk}$ is then estimated as the ratio of the number of energy bundles emitted from surface or volume element $i$ in wavelength interval $k$ and absorbed by surface or volume element $j$ to the number emitted from element $i$ in wavelength interval $k$.

Following the definition of the monochromatic distribution factor, the power absorbed by surface or volume element $j$, due to emission from surface element $i$, in the wavelength interval $k$, is given by

$$Q_{ijk} = \varepsilon_{ik} A_i e_{bk} (\Delta\lambda_k, T_i) D_{ijk}, \quad (4.1)$$

where $\varepsilon_{ik}$, $A_i$, and $T_i$ are the emissivity in wavelength interval $k$ (-), surface area ($m^2$) and temperature (K) of element $i$, and $e_{bk}$ is the emissive power of element $i$ in wavelength interval $k$ (W). In Eq. 4.1 the emissive power $e_{bk}$ of an element in a given wavelength interval is found by integrating Planck’s blackbody radiation distribution function over the wavelength band of interest,

$$e_{bk} = \int_{\lambda_i}^{\lambda_i+\Delta\lambda_k} \frac{2\pi C_1}{\lambda^5 \left(e^{C_2/\lambda T_i} - 1\right)} d\lambda, \quad (4.2)$$

where $C_1$ and $C_2$ are physical constants and $T_i$ is the temperature of the emitting element, $i$.

Finally, the total power absorbed by surface or volume element $j$ due to emission from surface or volume element $i$, $Q_{ij}$, is given by

$$Q_{ij} = \sum_k Q_{ijk}, \quad (4.3)$$

where the summation is over the $k$ wavelength bands of interest. Note that the analysis implies the need for a separate monochromatic distribution factor matrix for each wavelength interval $k$.

Three properties of monochromatic distribution factors exist which may be used to reduce the time and effort required for their calculation. They are
\[
\sum_{j=1}^{n} D_{ijk} = 1.0, \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m \tag{4.4}
\]

\[
\varepsilon_{ik} A_i D_{ijk} = \varepsilon_{jk} A_j D_{jik}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m \tag{4.5}
\]

and,

\[
\sum_{i=1}^{n} \varepsilon_{ik} A_i D_{ijk} = \varepsilon_{jk} A_j, \quad j = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m. \tag{4.6}
\]

Equation 4.4 is a statement of conservation of energy for the special case of coherent scattering, while Eq. 4.5 expresses a reciprocity relationship. Equation 4.6 is a combination of Eqs. 4.4 and 4.5 and is obtained by summing Eq. 4.5 over \(i\) and applying Eq. 4.4 to the result. Equation 4.6 may be rearranged to represent the global error in a given column \(j\) of distribution factors corresponding to a given wavelength interval \(k\).

\[
\text{percent error} = \left[ 1 - \frac{\sum_{i=1}^{n} \varepsilon_{ik} A_i D_{ijk}}{\varepsilon_{jk} A_j} \right] \times 100, \quad j = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m. \tag{4.7}
\]

As the number of energy bundles emitted from each surface increases, the monochromatic distribution factors converge to their proper values, and the right hand side of Eq. 4.7 approaches zero.

### 4.3 Thermal Diffusion Model of the Detector Module Assembly

Thermal diffusion in the detector module assembly has been modeled with a fully implicit finite difference formulation. The diffusion problem is three-dimensional, unsteady, and contains nonlinear boundary conditions. Since the thermophysical properties of the detector materials vary from one layer to the next, each layer has been subdivided into control volumes such that the properties are constant within the domain of a given control volume. Nodes are located at the centers of the control volumes so that, in general, each node has six neighbors. The temperature of each node is directly related to the temperatures of its adjacent nodes. In addition, because the problem is unsteady, the temperature of the node at the current time step also depends on its temperature at the previous time step.
The thermistor bolometers depicted in Figure 3.8 are geometrically three-dimensional. Although we know that heat is conducted primarily in one direction in the detectors, the three-dimensionality of the problem is retained in order to accurately model diffusion in the aluminum substrate. In reality, the disks to which the detectors are attached have a circular cross-section. In the current effort they are modeled with a rectangular (27.62-by-27.62-mm) cross-section having the same surface area as the actual circular cross-section. This is justified since the modeling effort is concerned with determining the thermal environment in the vicinity of the detectors whose dimensions are small, 1.5-by-3.0-mm, relative to those of the disk, whose diameter is 30.76 mm. From the point of view of the energy flowing through the detectors, the disks are infinite plates and the diffusion path is essentially independent of whether or not the edge boundary is circular or rectangular.

The modeled boundary conditions may be stated as follows. The vertical edges of the detectors are considered to be insulated even though in reality a radiative boundary exists there. These edges are extremely thin and their temperature remains very close to that of the surroundings, so the net radiation to those edges may be considered negligible. The top surface of the active absorber layer receives radiation from the scene through the telescope, and also exchanges heat with its surroundings by radiation. The spatial distribution of energy absorbed by the top layer of the active absorber is known from Bongiovi’s optical and thermal radiative model. The top surface of the compensating detector exchanges energy with its surroundings, the isothermal enclosure defined by the endcap geometry. The vertical edges of the aluminum substrates are insulated as they are on the actual instrument. On the active disk, a known temperature condition is applied to the boundary between the disk and optics housing. This is justified since the temperature at this location is actively controlled to a constant 38.0°C on the actual instrument using electrical heaters. Similarly, the interface between the compensating disk and the endcap is modeled with a constant temperature boundary as well. This is justified on the basis that a calibrated Platinum Resistance Thermometer (PRT) is embedded in the endcap. This PRT is used in conjunction with a PRT on the optics housing in the control logic for the
maintenance of the detector module assembly temperature. A graphical description of the actual instrument and modeled boundary conditions may be seen in Figures 4.1 and 4.2.

### 4.4 Electrical Diffusion Model

The active and compensating thermistor bolometers are mounted in a detector bridge circuit, as shown in Figure 3.9. In each bolometer, electrical current passes through the thermistor layer which is connected to the circuit by platinum leads. The resistance provided by a square sheet of thermistor material is directly related to its temperature by Eq. 3.1.

When the active bolometer receives a radiative heat input, its temperature distribution changes rapidly which, in turn, changes the overall resistance of the thermistor. Equation 3.1 cannot be used to relate the average temperature of the thermistor to its overall resistance. Instead, given an assumed electrical potential difference across the detector, a discrete electric field is computed using the two-dimensional electric diffusion equations. Local electrical conductivity is computed from local temperatures given by the thermal analysis. The current density passing through the detector is then computed by applying Ohm’s law locally. Finally, the overall resistance of the thermistor is computed as an equivalent resistance; that is, as the ratio of the applied potential difference to the computed total current. Complete details of this analysis are given in Haeffelin [31].

### 4.5 Detector Circuit Model

The detector signal-conditioning electronics, shown in Figures 3.9 and 3.10, transform variations of the thermistor electrical resistance into fluctuations of the electrical output voltage. The active and compensating thermistor bolometers are mounted in adjacent arms of a bridge circuit and fixed resistors are mounted in the other two arms. When the radiative power incident to the active bolometer imparts a condition such that the bridge is balanced, no bridge output voltage is produced. The idea behind the bridge is that if the heatsink temperature changes it will affect both thermistors in the same way. Therefore, the balance (or unbalance) of the bridge will remain unchanged, as will the output signal.
The compensating bolometer sees only minimal changes in radiative heat input and so its temperature, and thus its electrical resistance, might be expected to remain constant. However, when the radiative heat input to the active bolometer changes, the resistance of the active thermistor changes, creating a time-varying current flow through both the active and compensating detectors. This means that the levels of self-heating in both thermistors change with any change in radiative input to the active bolometer. The current model correctly describes this feedback loop.

Once the electrical resistances of the active and compensating bolometers are established, the signal response of the detector bridge is computed in two steps. The pre-amplifier portion of the signal conditioning electronics consists of a standard instrumentation amplifier and low-pass filter. For the current effort the electronic components are treated as ideal. This is justified by the fact that very stable high-performance flight components are used in their fabrication. With this assumption the signal leaving the instrumentation amplifier $V_{10}$ in Figure 3.9 may be computed by the relation

$$V_{10} = V_{\text{bridge}} \left[ \frac{R_{11}}{R_9} \left( \frac{R_5 + 2R_6}{R_5} \right) \right],$$

(4.1)

where the bridge potential difference $V_{\text{bridge}}$ is defined by the relation

$$V_{\text{bridge}} = V_2 - V_1.$$  

(4.2)

Similarly, the low-pass filter transfer function may be represented by the relation

$$V_{\text{out}} = -\frac{1}{R_{12}C_1} \int V_{10} \, dt.$$  

(4.3)

However the resistance $R_{14}$ limits the low-frequency gain of the circuit; thus Eq. 4.3 is only accurate for input frequencies somewhat greater than

$$f (\text{Hz}) = \frac{1}{2\pi R_{14}C_1}.$$  

(4.4)

Sufficiently far below this frequency, the low-pass filter acts only as an amplifier with gain determined by
\[ V_{\text{out}} = V_{10} \left( \frac{R_{14}}{R_{12}} \right). \] (4.5)

Upon leaving the low-pass filter, the signal is split and effectively doubled by the use of an inverting amplifier \( A_5 \), as seen in Figure 3.9. The signal is immediately reduced by a factor of two with the amplifier \( A_1 \), as seen in Figure 3.10. The reason for this quick succession of amplification and reduction is that the signal passes through a connector at this point and by doubling the signal before passing through the connector, any electronic noise introduced by the connector is effectively cut in half. After leaving the pre-amplification portion of the signal-conditioning electronics, the signal passes through the four-pole Bessel filter discussed in Chapter 3 and shown in Figure 3.10. The Bessel filter has an electronic gain of unity for frequencies below the corner frequency of 22 Hz, as seen in Figure 3.11.

The Bessel filter is integrated numerically by treating the signal entering it as a series of step inputs whose magnitudes are equivalent to the change in input signal between the present and previous time step. These step inputs are individually filtered for 80 ms by the filtering function shown in Figure 3.13. A running sum for the filtered inputs currently present in the moving 80-ms window is then calculated.

After passing through the Bessel filter, the electronic signal is amplified by 25 percent and immediately mapped from the analog domain into a 12-bit digital domain in an effort to eliminate the possibility of picking up any additional electronic noise. By converting the output to a 12-bit digital domain, a total of 4096 counts are obtained to cover the full-scale voltage of 0-to-10-V. This provides a conversion factor, \( C \), of 409.5 counts/V.

The result of the signal-conditioning electronics design is an attenuation of any high-frequency electronic noise above 22 Hz to prevent aliasing, an electronic gain of approximately 2200 and a time shift in the output of the digital signal of approximately 40 ms due to the Bessel filter.