APPENDIX A

CALCULATION OF MOISTURE DIFFUSION CONDITIONS

Chemical Potential

For the chemical potential model, Fick’s law takes the form of

\[ J = -K_{\mu} \frac{d\mu}{dx} \]  \hspace{1cm} (A.1)

In order to validate this model applying the null flow condition, the chemical potential must be the same for both sides of the specimen as indicated below,

\[ \mu_1 = \mu_2 \]  \hspace{1cm} (A.2)

Given: \( T_1, R.H._1 \) and \( T_2 \)
Find: \( R.H._2 \)
Assumption: Ideal gas for the water vapor

Gibbs free energy can be written in terms of the enthalpy \( (h) \), entropy \( (s) \) and temperature \( (T) \) as,

\[ \mu(P_v, T) = h_v(P_v, T) - T s(P_v, T) \]  \hspace{1cm} (A.3)

Entropy is derive from the second law of thermodynamics as

\[ s_v(P_v, T) = s^o(P_v, T_o) + \int_{T_o}^{T} \frac{C_{PM}(T)}{T} dT - R_v \ln \left( \frac{P_v}{P_{o_v}} \right) \]  \hspace{1cm} (A.4)
where the vapor pressure \( (P_v) \) is a function of the relative humidity (\( \phi \)) and temperature (\( T \)), the absolute entropy (\( s_o \)) at the reference point \( P_o \) and \( T_o \). Equation A.4 can be simplified assuming the specific heat (\( C_{pv} \)) is constant on the interval of interest

\[
s_v(P_v, T) = s_o(P_o, T_o) + C_{pv} \ln \left( \frac{T}{T_o} \right) - R_v \ln \left( \frac{P_v}{P_o} \right)
\]  \hspace{1cm} (A.5)

where the vapor pressure (\( P_v \)) is a function of the relative humidity and temperature and given by,

\[
P_v = \phi \cdot P_{sat}(T)
\]  \hspace{1cm} (A.6)

The gas constant is given in terms of the universal gas constant (\( R \)) and the molecular weight of water (\( M_v \)).

\[
R_v = \frac{R}{M_v} = 0.4615 \text{ kJ/kg K}
\]  \hspace{1cm} (A.7)

Now recall chemical potential is given by,

\[
\mu = h_v - T s_v
\]  \hspace{1cm} (A.8)

For the validation of this model the chemical potential must be the same for both chambers, hence,

\[
h_{v,1} - T_1 s_{v,1} = h_{v,2} - T_2 s_{v,2}
\]  \hspace{1cm} (A.9)

The above expression can be written as,

\[
s_{v,2} = \frac{h_{v,2} - h_{v,1} + T_1 s_{v,1}}{T_2}
\]  \hspace{1cm} (A.10)
Since by definition the specific heat is,

$$C_{pv} = \frac{\partial h}{\partial T} \Bigg|_p$$  \hspace{1cm} (A.11)

The difference in the enthalpies can be replaced by,

$$h_{v,2} - h_{v,1} = C_{pv}(T_2 - T_1)$$  \hspace{1cm} (A.12)

The entropy for state 2 can be solved by substituting Eq. A.12 into Eq. A.10,

$$s_{v,2} = \frac{C_{pv}(T_2 - T_1) + T_1s_{v,1}}{T_2}$$  \hspace{1cm} (A.13)

Now the corresponding vapor pressure for 2 can be found by using,

$$s_{v,2} = s^o + C_{pv} \ln \left( \frac{T_2}{T_o} \right) - R_v \ln \left( \frac{P_{v,2}}{P_o} \right)$$  \hspace{1cm} (A.14)

Solving for the vapor pressure from Eq. A.14,

$$P_{v,2} = P_o \exp \left[ \frac{s_o - s_{v,2} + C_{pv} \ln \left( \frac{T_2}{T_o} \right)}{R_v} \right]$$  \hspace{1cm} (A.15)

The relative humidity for 2 ($\phi_2$) can be solved by manipulating Eq. A.4,

$$\phi_2 = \frac{P_{v,2}}{P_{sat}(T_2)}$$  \hspace{1cm} (A.16)

In this fashion the conditions needed to satisfy the chemical potential model would be, $T_1, \phi_1$ for one side of the specimen and $T_2, \phi_2$ for the other.

The conditions used for the validation of the chemical potential were:

Top Chamber: 25°C, 45% R.H.  \hspace{1cm} Bottom Chamber: 30°C, 52% R.H.
Activated Moisture Molecules

Skaar and Siau (1981) proposed a nonisothermal moisture diffusion model based on the gradient of activated moisture molecules. Once again Fick's law was modified to the form

\[ J = -K_{M^*} \frac{dM^*}{dx} \]  

(A.17)

where \( K_{M^*} \) is the coefficient of diffusion based on a gradient of activated moisture, and \( M^* \) is the content of activated moisture in wood. The content of activated moisture can be expressed as,

\[ M^* = \gamma \exp\left(\frac{-E_b}{RT}\right) \]  

(A.18)

where \( E_b \) is the activated energy and \( \gamma \) is the moisture content. The activated energy can be calculated with use of,

\[ E_b = 38,500 - 290\gamma \left(\frac{J}{\text{mol}}\right) \]  

(A.19)

The moisture content (\( \gamma \)) is a function of the relative humidity and given in the form of,

\[ \gamma = \frac{A1\phi}{(1 + A2\phi)(1 - A3\phi)} \]  

(A.20)

where the \( A \) coefficients are the moisture storage coefficients for the material, and both \( \gamma \) and \( \phi \) are expressed as a fraction. The above equation is known as the sorption isotherm function. The moisture storage coefficients are based upon measurements performed by Burch and Thomas (1992), (Burch and Thomas, 1993).

\[ A1 = 0.2121 \]
\[ A2 = 3.427 \]
\[ A3 = 0.8106 \]  

(A.21)
The activated moisture content can then be calculated as a function of relative humidity. In order to properly use the null flow condition to validate this model the activated moisture content gradient must be zero.

For a given set of conditions for one of the chambers and the temperature for the other, it is necessary to calculate the relative humidity for which the activated moisture content of both chambers is the same. This procedure was accomplished by using a graphical method,

![Activated moisture content plot.](image)

**Figure A.1** Activated moisture content plot.

The experimental conditions for this model were:

**Top Chamber:** 25°C, 45% R.H.  
**Bottom Chamber:** 35°C, 31.9% R.H.
APPENDIX B

ASHRAE APPARATUS DATA AND RESULTS