Partitioning Methods and Algorithms for Configurable Computing Machines

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(ABSTRACT)

This thesis addresses the partitioning problem for configurable computing machines. Specifically, this thesis presents algorithms to partition chain-structured task graphs across configurable computing machines. The algorithms give optimal solutions for throughput and total execution time for these problems under constraints on area, pin count, and power consumption. The algorithms provide flexibility for applying these constraints while remaining polynomial in complexity. Proofs of correctness as well as an analysis of runtime complexity are given. Experiments are performed to illustrate the runtime of these algorithms.
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# TABLE OF CONTENTS

1 Introduction  
1.1 Motivation 1  
1.2 The problem 3  
1.3 The approach 4  
1.4 Thesis organization 4

2 Background 6  
2.1 Definitions, terms, and model used in the thesis 6  
2.2 Past and present partitioning methods used for FPGAs 14  
2.3 Comparison of distributed and configurable computing systems 16  
2.4 Related work in distributed computing systems 18  
2.5 Bokhari’s method for solving the linear module and processor case 19  
\hspace{.5cm} 2.5.1 Illustration of Bokhari’s algorithm 20  
\hspace{.5cm} 2.5.2 Basis for using Bokhari’s method for CCMs 25

3. Generalization of Bokhari’s Algorithm for the case of CCMs 28  
3.1 Partitioning chain structured modules onto a chain of PEs 28  
\hspace{.5cm} 3.1.1 The problem 28  
\hspace{.5cm} 3.1.2 Assumptions 29  
3.2 The solution 29  
\hspace{.5cm} 3.2.1 Layered assignment graph $G_a$ 29  
\hspace{.5cm} 3.2.2 Minimum bottleneck assignment 30  
\hspace{.5cm} 3.2.3 Assignment of weights to the nodes of graph $G$ 31  
3.3 Proof of correctness for Algorithm 3.1 33  
3.4 Improvement in the speed of execution 34
4. Solution to Partitioning Series Module Graphs onto Arbitrary PE Graphs 37

4.1 Mapping chain structured modules onto a general PE graph 37
   4.1.1 The problem 38
   4.1.2 Assumptions 38
4.2 The solution 38
   4.2.1 Construction of layered graph for each of the $G_{px}$ 41
   4.2.2 Assignments of weights to the nodes of the graph $G_a$ 42
4.3 Proof of correctness of the algorithm 45
4.4 Complexity 47

5. Module Graph having a Fork onto a Chain of PEs 48

5.1 Partitioning a module graph having a fork onto a chain of PEs 48
   5.1.1 The problem 49
   5.1.2 Assumptions 49
5.2 The solution 50
   5.2.1 Construction of the assignment graph 51
   5.2.2 Construction of nodes in the assignment graph $G_{al}$ 52
   5.2.3 Assignment of edges for the assignment graph $G_{al}$ 53
   5.2.4 Application of constraints 53
   5.2.5 Assignment of weights 54
5.3 Proof of correctness of Algorithm 5.1 55

6. Results 57

6.1 Implementation of Algorithms 3.1, 4.1, 4.2, and 5.1 57
   6.1.1 Results for Algorithm 3.1 implementation 57
   6.1.2 Results for Algorithm 4.1 and 4.2 implementation 62
   6.1.3 Results for the implementation of Algorithm 5.1 67
7. Conclusions and Suggestions

7.1 Conclusions on the thesis
7.2 Suggestions for future work

References

Vita
LIST OF FIGURES

1.1 General structure of a contemporary CCM. 2

2.1 A module graph. 7

2.2 Four PEs connected by links. 8

2.3 A module graph with five modules. 21

2.4 Nodes of the assignment graph. 21

2.5 Edge creation for the assignment graph. 22

2.6 Weights on the nodes of assignment graph. 23

2.7 Highlighted critical path representing minimum bottleneck assignment. 24

2.8 A nine-module chain mapped to a four-processor chain. 25

3.1 Node creation for graph $G_a$. 31

3.2 Edge creation for graph $G_a$. 32

3.3 Algorithm 3.1 for finding minimum bottleneck path or minimum total execution time from the assignment graph $G_a$. 32

3.4 A layered graph for a linear array problem with nine modules and four processors. 34
3.5 An improved layered graph for a linear array problem with nine modules and four processors. 35

4.1 A chain module graph. 37

4.2 A PE graph shown having nine PEs and their links. 38

4.3 Example of a trail. 39

4.4 Partitioning problem A. 40

4.5 Partitioning problem B. 40

4.6 Algorithm 4.1. 44

4.7 Algorithm 4.2. 44

5.1 A PE graph with five PEs connected as a chain. 48

5.2 A module graph having a single fork. 49

5.3 Module graph showing the set of modules $V_{a1}$, $V_{a2}$, and $V_M$. 51

5.4 Module graph $G_{ml}$ showing the set of modules $V_{Cl}$ and $V_{a2}$. 51

5.5 Algorithm 5.1 for $G_m$. 54

6.1 Plot of bottleneck values versus the limitation of modules on a PE. 60

6.2 Comparison of two plots representing the runtimes of two different implementations of Algorithm 3.1. 61
6.3 Directed arrows showing a trail in a PE graph. 62

6.4 A PE graph with four PEs and four links. 65
LIST OF TABLES

6.1 Table containing the results for Algorithm 3.1 implementation. 58

6.2 Results for Algorithm 3.1 under increasing constraints. 59

6.3 Results of Algorithm 4.1 for the case of PE inter-connection being a ring of 10 PEs. 63

6.4 Results for the two dummy PE case for the trail shown in Figure 6.3. 63

6.5 Results showing reduction in graph size and execution time of Algorithm 4.1 under increasing constraints. 64

6.6 Results for Algorithm 4.2. 65

6.7 Weights of the modules. 66

6.8 Runtimes for implementation of Algorithm 4.1 for CCM of Figure 1.1. 67

6.9 Results for the implementation of Algorithm 5.1. 67